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Abstract

For equilibria stable with respect to ideal ballooning modes it is shown by means of a simple argument that, owing to plasma compressibility, resistive ballooning modes with small growth rates can only exist if they are extended along a field line. In particular, a small region of bad curvature along a field line is not sufficient for the existence of unstable modes.

With due allowance for the effect of compressibility and related sound wave propagation, the equations describing resistive ballooning modes in general three-dimensional equilibria were introduced in [1, 2]. Written in dimensionless form, these equations are

$$\frac{d}{dy} \left[\frac{c^2}{1 + \frac{B^2}{\langle B^2 \rangle} \frac{1}{y_0^2 Q} c^2} \frac{dF}{dy} \right] + \mathcal{D}_R F - \frac{1}{M} \frac{Q^2}{y_0^2} c^2 F = -\mathcal{D}_R D, \quad (1)$$

$$\begin{aligned} \frac{d}{dy} \left[\frac{\langle B^2 \rangle}{B^2} \frac{dD}{dy} \right] - \frac{1}{M} \frac{\langle B^2 \rangle}{B^2} \left[1 + M \frac{B^2}{\langle B^2 \rangle} G \right] \frac{Q^2}{y_0^2} D \\ - \frac{1}{M} c^2 \frac{Q^2}{y_0^2} \frac{1}{y_0^2 Q} D - \mathcal{D}_R \frac{1}{y_0^2 Q} D = (1 + KQ^3) \mathcal{D}_R \frac{1}{y_0^2 Q} F, \end{aligned} \quad (2)$$

where

$$c^2 = \frac{\langle B^2 / |\nabla v|^2 \rangle}{\dot{q}^2 B^2} C^2, \quad (3)$$

$$\mathcal{D}_R = \frac{\langle B^2 / |\nabla v|^2 \rangle}{\dot{q}^2} \frac{2\dot{p}}{\dot{\chi}^4} (\kappa_v + \dot{q}y\kappa_\phi) . \quad (4)$$

The quantities appearing in these equations are in standard notation and were extensively explained in [1].

If the effect of sound wave propagation is neglected by setting $G \rightarrow \infty$ (this means vanishing sound velocity), it then follows that $D = 0$, leaving an equation for F alone. If $Q^2/y_0^2 \ll 1/Qy_0^2$ (small growth rate) and if the mode is *not extended* along y ($c^2 \sim o(1)$), the third term in equation (1) can also be neglected. If the curvature term \mathcal{D}_R is positive at least in a small y -interval, then purely growing modes always exist [3].

The situation is essentially different if the effect of sound wave propagation (finite G) is taken into account. Then, D does not vanish everywhere and, for $Q^2/y_0^2 \ll 1/Qy_0^2$, i. e. $Q^3 \ll 1$ ($1/Qy_0^2$ can be either finite or large) and $c^2 \sim o(1)$, equations (1) and (2) reduce to

$$\frac{d}{dy} \left[\frac{c^2}{1 + \frac{B^2}{\langle B^2 \rangle} \frac{1}{y_0^2 Q} c^2} \frac{dF}{dy} \right] = -\mathcal{D}_R b , \quad (5)$$

$$\frac{d}{dy} \left[\frac{\langle B^2 \rangle}{B^2} \frac{dD}{dy} \right] = \frac{1}{y_0^2 Q} \mathcal{D}_R b , \quad (6)$$

where $b = F + D$.

Substituting $\mathcal{D}_R b$ from equation (5) into equation (6) one obtains, after integration,

$$\frac{db}{dy} = \frac{1}{1 + \frac{B^2}{\langle B^2 \rangle} \frac{1}{y_0^2 Q} c^2} \frac{dF}{dy} , \quad (7)$$

and inserting this expression for $\frac{dF}{dy}$ into equation (5) yields

$$\frac{d}{dy} \left(c^2 \frac{db}{dy} \right) + \mathcal{D}_R b = 0 . \quad (8)$$

This equation is identical to the ideal ballooning mode equation, with b as eigenfunction. It has no square integrable solutions b in $-\infty < y < \infty$, since it is assumed that the

equilibrium is stable with respect to ideal ballooning modes. The argument is not valid, of course, for extended modes. For these, the terms containing $Q^2 c^2 / y_0^2$ can become large even for small Q and cannot be neglected in the equations. Thus, if resistive ballooning modes with small growth rates $Q^3 \ll 1$ exist at all, they *have to be extended* along y . The existence of a finite interval of bad curvature along the field line does not guarantee the existence of unstable modes, contrary to the case in which the stabilizing effect of the sound wave propagation is neglected.

References

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