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Rate for a Class of 3-D MHD Equilibria

DARÍO CORREA-RESTREPO

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Rate for a Class of 3-D MHD Equilibria**

Darío Correa-Restrepo

Max-Planck-Institut für Plasmaphysik

EURATOM Association

D-8046 Garching bei München, Federal Republic of Germany

Abstract

A previous derivation of formulae for calculating the growth rate of resistive ballooning modes which are described by a simple model equation is extended here to a large class of general tokamak and stellarator equilibria.

Resistive ballooning modes in general two- or three-dimensional MHD equilibria are described by two coupled, second-order, ordinary differential equations for the eigenfunctions F and D . Using the standard notation introduced and explained in detail in [1, 2], one can write these equations in the dimensionless forms,

$$\frac{d}{dy} \left[\frac{c^2}{1 + \frac{B^2}{\langle B^2 \rangle} \frac{1}{y_0^2 Q} c^2} \frac{dF}{dy} \right] + \mathcal{D}_R F - \frac{1}{M} \frac{Q^2}{y_0^2} c^2 F = -\mathcal{D}_R D, \quad (1)$$

$$\begin{aligned} & \frac{d}{dy} \left[\frac{\langle B^2 \rangle}{B^2} \frac{dD}{dy} \right] - \frac{1}{M} \frac{\langle B^2 \rangle}{B^2} \left[1 + M \frac{B^2}{\langle B^2 \rangle} G \right] \frac{Q^2}{y_0^2} D \\ & - \frac{1}{M} c^2 \frac{Q^2}{y_0^2} \frac{1}{y_0^2 Q} D - \mathcal{D}_R \frac{1}{y_0^2 Q} D = (1 + KQ^3) \mathcal{D}_R \frac{1}{y_0^2 Q} F, \end{aligned} \quad (2)$$

by defining

$$c^2 = \frac{\langle B^2 / |\nabla v|^2 \rangle}{\dot{q}^2 B^2} C^2, \quad (3)$$

$$\mathcal{D}_R = \frac{\langle B^2/|\nabla v|^2 \rangle}{\dot{q}^2} \frac{2\dot{p}}{\dot{\chi}^4} (\kappa_v + \dot{q}y\kappa_\phi) . \quad (4)$$

A powerful method of solving these equations in many interesting situations is afforded by the theory of matched expansions. The application of this method in [1,2] yielded an explicit expression for the normalized growth rate Q of resistive MHD equilibria which are at, or near, marginal ideal stability, provided that the Mercier exponent s_M does not exceed the value $1/2$, as is the case in numerous configurations, including many tokamak equilibria. There are, however, other cases (such as some tokamak configurations near the second ideal stability limit and many stellarator equilibria) for which the Mercier exponent exceeds the value $1/2$. In this case, the correct application of the method of asymptotic expansions requires the inclusion of higher-order terms (in an expansion with respect to the small parameter which appears in the problem). This situation was treated in detail in Appendix B of [2] for a *model equation* of a class of tokamak equilibria, and it is those results which are now extended to general tokamak and stellarator configurations.

As in previous work, the quantity G (roughly the inverse plasma β) is considered to be a large number. Since the normalized growth rate Q is not small for equilibria which are at, or near, marginal ideal stability, it is possible in this case to simplify the treatment of equations (1), (2), as explained in [1,2]. Following the same approach as in Appendix B of [2], one obtains an *explicit expression* for the normalized growth rate of general three-dimensional configurations which have $s_M > 1/2$.

For equilibria with marginal ideal stability the growth rate Q is given by

$$Q^3 = \frac{\int_0^\infty \frac{B^2 c^4}{\langle B^2 \rangle} \left| \frac{dF_0}{dy} \right|^2 dy}{\int_0^\infty \frac{1}{M} c^2 |F_0|^2 dy} , \quad (5)$$

where F_0 is the solution of the marginal ideal ballooning mode equation. This expression corresponds to equation (B.40) of [2].

Near ideal marginal stability, the growth rate is given by

$$\frac{1}{\Delta'} + \frac{a_{0-}}{a_{1+}} \frac{1}{y_0^2 Q} \int_0^\infty \left[\frac{c^2}{M} Q^3 |F_0|^2 - \frac{B^2}{\langle B^2 \rangle} c^4 \left| \frac{dF_0}{dy} \right|^2 \right] dy = 0. \quad (6)$$

Equation (6) corresponds to equations (B.37) and (B.38) of [2].

The generalizations (5) and (6) allow relatively easy computation of growth rates for a wide class of tokamak and stellarator equilibria and are a natural complement of previous results.

References

- [1] D. Correa-Restrepo, Z. Naturforsch. **37a**, 848(1982).
- [2] D. Correa-Restrepo, Plasma Phys. **31**, 435 (1989).