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# NEOCLASSICAL IMPURITY TRANSPORT IN THE PRESENCE OF TOROIDAL AND POLOIDAL ROTATION

W. Feneberg

**Abstract**

This paper presents an extended theory of neoclassical impurity transport, starting from the parameters of bulk plasma toroidal and poloidal rotation. Analytic expressions resulting from the influence of a compressible flow on the perpendicular momentum balance and on the neoclassical Braginskii parallel viscosity are derived. The predicted impurity transport is extensively compared with that in earlier papers.

**Neoclassical Impurity Transport in the Presence of Toroidal and Poloidal Rotation**

## I. Introduction

It is still an open question whether neoclassical theory is able to explain impurity transport in tokamaks. In particular to understand the impurity accumulation often observed in experiments, it is necessary to look for hitherto disregarded effects which could open up the possibility of classical flow reversal.

There have been numerous publications that study neoclassical transport in plasma discharges for zero toroidal rotation and take into account the influence of inertial forces arising with rotation in neutral-beam-heated tokamaks.

In addition to earlier work, this investigation presents an extended theory of neoclassical impurity transport valid for all cases of background plasma rotation poloidal as well as toroidal, whose parameters are considered as given. As will be shown later, the method of expansion into the inverse aspect ratio $\epsilon$ used here restricts us to a toroidal rotation velocity $V_{i,\varphi} < C_{S,Z}$, where $C_{S,Z}$ is the impurity sound velocity.

Especially the poloidal rotation is an important parameter which could allow classical flow reversal [1]. Even if one leaves aside the practical questions of how to create a desired amount of poloidal rotation in the presence of the strong damping associated with the inhomogeneity of the toroidal magnetic field, and how to maintain such rotation at all toroidal azimuths, this is still a meaningful investigation owing to the fact that the background plasma might not strictly adhere to neoclassical theory.

We treat here the case of a two-ion mixture (bulk ions and impurities) where impurities are taken into account in a trace amount with a concentration $\alpha = Z^2 \cdot n_Z/n_i \ll 1$ ( $Z$ and $n_Z$ being the impurity charge and density, $n_i$ the bulk ion density),and so collisions between impurities of different charges can be neglected as compared to collisions of impurities with bulk ions. While the impurities are calculated within the collisional regime, the light ion regime is arbitrary, this being expressed by an additional friction term depending on the temperature gradient [2].

The parallel viscous force acting on the impurities is determined by the Braginskii viscous tensor [3], which is used here in the formulation given by Stacey and Sigmar [4]. The gyro-viscous force is one order of magnitude smaller than the parallel stress in the $\omega_Z/\nu_Z$ expansion and is therefore neglected ($\omega_Z/\nu_Z$ being the impurity gyro frequency and collision frequency, respectively).

This viscous tensor contains the effect of magnetic pumping and also the contribution of terms describing a compressible flow, which has hitherto been neglected in the theory. An important feature of this Braginskii viscosity is that it strongly influences the poloidal impurity variation, and hence the Pfirsch-Schlüter expression for the transport, by a term of first order in $\epsilon$, which is also within the theory of Callen et al. [5] but was not taken into account in earlier papers [6, 7]. Especially at the transition from the collisonal to plateau regime, where we fit the viscosity coefficient so that the average value of the parallel force agrees with that published elsewhere [7], the impurity transport is found to be dominated by the viscosity which depends mainly on the poloidal rotation velocity. Flow reversal is therefore obtained as a result of viscosity forces for the case where background ions rotate poloidally in the direction opposite to the poloidal magnetic field by an amount larger than the bulk ion diamagnetic velocity.

In the same way as carried out by Stacey et al. [7] in the equation for the momentum balance perpendicular to the magnetic field, the poloidal flow of the impurity ions is considered to be compressible. This yields expressions for the impurity transport which strongly differ from those given by earlier work [6, 8], where this quantity was treated as a surface function according to a first-order Larmor radius expansion, which we do not use here. Our calculations deviate from the earlier ones mainly in the region of high collisionality, this being a realistic regime for medium and high-Z impurities under a lot of experimental conditions. The inward drift is found to be a factor of between two and three as small as in the old approximations based on the assumption of incompressible flow.

## II. Basic Equations

We start with the impurity momentum equation as given by Braginskii for the collisional regime:

$$n_Z m_Z(\vec{V}_Z \cdot \nabla)\vec{V}_Z + \nabla P_Z + \nabla\cdot \overleftrightarrow{\pi}_{Z,\parallel} + n_Z e_Z \nabla\Phi - n_Z e_Z(\vec{V}_Z \times \vec{B}) = \vec{R}_{Z,i} \qquad (1)$$

In this equation, $n_Z, m_Z, e_Z$ and $\vec{V}_Z$ denote the impurity density, mass, charge and flow velocity; $P_Z$ is the pressure, $\overleftrightarrow{\pi}_{Z,\parallel}$ the anisotropic parallel Braginskii stress tensor, $\phi$ the electrostatic potential, and $\vec{B}$ the magnetic field.

The friction $\vec{R}_{Z,i}$ between impurities and bulk plasma ions depends besides the relative velocities $\vec{V}_Z, \vec{V}_i$ corresponding to a shifted Maxwellian distribution on the influence of the parallel heat flux on the bulk ion distribution function, which stays in an arbitrarily collisional regime. According to our basic assumption $\alpha << 1$, the bulk ion distribution function is the same as calculated for a pure plasma. According to Ref. [2] this yields

$$\vec{R}_{Z,i} = \alpha_{Z,i}\Big(\vec{V}_i - \vec{V}_Z - 2\vec{b}\xi\epsilon\frac{T_i'}{eB_\Theta^o}cos\Theta\Big) \qquad (2)$$

and $\xi = 1; 1.5; -0.75$, respectively, in the collisional, plateau and collisionless regimes of the bulk ions. We have $\alpha_{Z,i} = m_i n_i \nu_{i,Z}$, where $m_i, n_i$ is the bulk ion density, and the Braginskii collision frequency $\nu_{i,Z}$ [3] is defined as

$$\nu_{i,Z} = \frac{4\sqrt{2\pi} n_Z e_i^2 e_Z^2 \ell n\Lambda}{3\sqrt{m_i} T_i^{3/2}}. \tag{3}$$

The unit vector along the magnetic field is $\vec{b} = \vec{B}/B$, and $T_i = \frac{dT_i}{dr}$ is the gradient of the background ion temperature.

To obtain explicit expressions for the impurity flux and density variation in a flux surface, we specialize to circular cross-section geometry with $B_\Theta = B_\Theta^o R_o/R$ the poloidal and $B_\varphi = B_\varphi^o R_o/R$ the toroidal magnetic field, where we use $R = R_o - r\cos\Theta, Z = r\sin\Theta$. ($\Theta$ being the poloidal angle, and r the minor radius of the flux surface)

The system of equations for the impurities terminates with the equation of continuity

$$\nabla(n_Z \vec{V}_Z) = 0, \tag{4}$$

where the source term is neglected, and the ansatz of the temperature $T_Z$ is constant on the magnetic surfaces.

The bulk ion density $n_i$ and their temperature $T_i$ are regarded as surface quantities consistently with our ordering into small rotation velocities and small $\alpha$: From the equation of quasi-neutrality $n_e = n_i + Zn_Z$ it is concluded that the influence of the poloidal impurity variation on the background ions can be neglected for $\alpha \ll 1$:

$$\delta n_i/n_i = \delta n_e/n_i - \frac{\alpha}{Z}\delta n_Z/n_Z.$$

In keeping with our concept of assuming the bulk plasma flow as given, we need only consider two equations for the backbround ions, viz. the equation of continuity

$$\nabla(n_i V_i) = 0 \tag{5}$$

and the radial momentum balance

$$P_i' - n_i e(E_r + (\vec{V_i} \times \vec{B})_r) = 0, \tag{6}$$

where $E_r = -\phi'(r)$ is the radial electric field and the electric potential is a surface function.

The expression for the perpendicular flux $\Gamma_Z = < n_Z V_{Z,r} >$ is derived as usual from the toroidal component of the momentum equation (1) as

$$\Gamma_Z = - < R_{Z,i\varphi}/ZeB_\Theta > . \tag{7}$$

An equivalent relation which gives some insight in the transport mechanism can be found by combining this eq.(7) with the parallel component of eq. (1), where the flux can be split into three parts:

$$\Gamma_Z = \Gamma_Z^{CL} + \Gamma_Z^{PS} + \Gamma_Z^V. \tag{8}$$

The first term $\Gamma_Z^{CL}$ is the classical flux

$$\Gamma_Z^{CL} = < \alpha_{Z,i}(V_{i,\Theta} - V_{Z,\Theta})/ZeB_\varphi > \tag{9}$$

due to the cross field flow $\vec{V}_\Theta \times \vec{B}_\varphi$ in a straight cylinder.

The second term is the so-called neoclassical Pfirsch-Schlüter term arising from the forces associated with the curvature of the bulk magnetic field:

$$\Gamma_Z^{PS} = - < (\vec{b}\nabla P_Z + n_Z e_Z \vec{b}\nabla\phi + n_Z m_Z \vec{b}(\vec{V}_Z \cdot \nabla)\vec{V}_Z)/ZeB_\Theta) >, \tag{10}$$

and the last term calculates the direct contribution of the parallel viscosity:

$$\Gamma_Z^V = - < \vec{b}\nabla \cdot \overleftrightarrow{\pi}_{Z,\parallel} /ZeB_\Theta > . \tag{11}$$

The parallel electric field will be neglected in eq. (10) and in the parallel component of eq. (1). Considering it would introduce into the calculations the inertial term of the plasma ions, which can be neglected in relation to the inertial term of the impurities, ordering to heavy ions so that $m_i/Zm_Z \ll 1$.

Only the radial electric field therefore enters the perpendicular momentum impurity balance, where inertial and friction forces are unimportant in relation to the radial pressure gradient corresponding to our ordering $V_{i,\varphi} < C_{S,Z}$:

$$P'_Z - n_Z e_Z(E_r + (\vec{V}_Z \times \vec{B})_r) = 0. \tag{12}$$

Eliminating the radial electric field from eqs. (6) and (12), we find the impurity flow to be always coupled to the bulk plasma ions by

$$(V_{Z,\varphi} - V_{i,\varphi}) - f_p^{-1}(V_{Z,\Theta} - V_{i\Theta}) = P_i'/en_iB_\Theta - P_Z'/Zen_ZB_\Theta \qquad (13)$$

(here $V_{Z,\varphi}, V_{i\varphi}$ is the toroidal component of the velocity, $f_p = B_\Theta/B_\varphi$).

## III. Ordering into Inverse Aspect Ratio

Analytic expressions for the impurity transport and the poloidal asymmetric distributions of impurity ions are derived in this section. In solving the equations for the unknown quantities G, where G stands for $V_{Z\varphi}, V_{Z,\Theta}, n_Z$, an expansion into the inverse aspect ratio $\epsilon$ was carried out with

$$G = G^o(r)(1 + \epsilon\tilde{G}_c cos\Theta + \epsilon\tilde{G}_s sin\Theta). \qquad (14)$$

It will only be necessary to work to first order in $\epsilon$, so that $R, B \tilde{=} 1 - \epsilon cos\Theta$.

The equation of continuity (4) is treated in the usual way on the assumption that the flow comes into equilibrium along the magnetic field lines on a time scale much shorter than it does perpendicularly to field lines, which implies that

$$Rn_ZV_{Z,\Theta} = K_Z(r), \qquad (15)$$

where $K_Z$ is a surface function.

It is worth noting that this time scale model is not at all trivial: Let us estimate the radial transport with $n_ZV_{Z,r} \approx \frac{T}{ZeB_\varphi r}\frac{\partial n_Z}{\partial\Theta}$ and the poloidal impurity rotation $V_{Z,\Theta}^o$ with eq. (16), but for $V_{i,\Theta}^o = 0$; it is then clear that comparing terms coming from $\frac{\partial}{\partial r}(n_ZV_{Z,r})$ with those from $\frac{\partial}{\partial\Theta}(n_ZV_{Z,\Theta})$ we find the condition to hold for the neglect of the radial velocity in the continuity equation to be

$$Tn_Z^{o\prime}/Zn_Z^o \gg P_i'/n_i,$$

which means that only cases far away from accumulation equilibrium can be investigated with the simplified model of eq. (15).

To lowest order $\epsilon^o$ we have, as published elsewhere [8],

$$V_{i,\varphi}^o - V_{Z,\varphi}^o = 0$$

and (16)

$$V_{i,\Theta}^o - V_{Z,\Theta}^o = \frac{1}{eB_\varphi^o}(P_i'/n_i - P_Z^{o\prime}/Zn_Z^o).$$

To first order in $\epsilon$ eq. (13) reads, when the first-order terms of $V_{Z,\Theta}$ are computed with eq. (15),

$$\begin{aligned} V_{i\varphi} - V_{Z,\varphi} = {} & \epsilon\, cos\Theta f_p^{-1}\Big(\frac{2}{eB_\varphi^o}(P_i'/n_i - P_Z^{o\prime}/Zn_Z^o) + \tilde{n}_{Z,c}V_{Z,\Theta}^o + \frac{T}{eB_\varphi^o Z}\tilde{n}'_{Z,c}\Big) \\ & + \epsilon sin\Theta f_p^{-1}\Big(V_{Z,\Theta}^o\tilde{n}_{Z,S} + \frac{T}{eB_\varphi^o Z}\tilde{n}'_{Z,S}\Big). \end{aligned} \quad (17)$$

Comparing this expression for the differences in the toroidal velocities with Ref. [8], we observe, in addition to the first term, terms caused by the compressibility of flow, which we take into account here. To neglect these quantities arising from the poloidal density variation is not allowed, because they are of the same order in $\epsilon$ and they are responsible for the existence of neoclassical flow reversal with poloidal rotation, which is missing if the transport is only calculated with the often used approximation of incompressibility here.

Unfortunately, the analytical result for the impurity flux now depends nonlinearly on the difference $(P_i'/n_i - P_Z^{o\prime}/Zn_Z^o)$ in the pressure gradients and can no longer be divided into convective and diffusive components.

In leading order, the averaged flux $\Gamma_Z$ is $\epsilon^2$ and we have for the classical part (see eq. (9))

$$\Gamma_Z^{CL} = -n_Z^o \frac{\epsilon^2 T}{ZerB_\varphi^o}\Big(\frac{P_i'}{n_i} - \frac{P_Z^{o\prime}}{Zn_Z^o}\Big)/q^2 \Omega e B_\varphi^o V^*, \tag{18}$$

with the definition of a parameter of collisionality

$$\Omega = -(\omega_i/\nu_{i,i}) f_p^2 Z^{-2} P_i / r P_i' \tag{19}$$

and the ion diamagnetic drift velocity

$$V^* = P_i'/eB_\varphi^o n_i \tag{20}$$

($\omega_i$ is the bulk ion gyro frequency).

The Pfirsch-Schlüter flux $\Gamma_Z^{PS}$ is given by

$$\Gamma_Z^{PS} = n_Z^o \frac{\epsilon^2 T}{ZerB_\varphi^o} \tilde{n}_{Z,S}\Big(1 + \frac{m_Z V_{i,\varphi}^{o2}}{2T}\big(1 - 2f_p^{-1} V_{i,\varphi}^{o-1}\big(V_{Z,\Theta}^o + \frac{T}{eB_\varphi^o Z}\tilde{n}_S'\big)\big)\Big). \tag{21}$$

It is this term that depends linearly on the up-down asymmetry and can change sign with the sign of $\tilde{n}_{Z,S}$. But for this purpose it must dominate over the classical flux, which makes a minimum of up-down asymmetry necessary. At the plasma edge, where the parameter of collisionality is $\Omega \ll 1$, the classical flux always dominates over $\Gamma_Z^{PS}$ and the viscosity contribution $\Gamma_Z^V$.

It should be mentioned here that in other devices, such as stellarators, the geometry-dependent flux $\Gamma_Z^{PS}$ might be found to be different from that in tokamaks, e.g. much smaller when the return currents are minimized, but the inward drift of the classical term is expected to be always the same.

In order to calculate the viscosity-dependent part $\Gamma_Z^V$ (eq. 11) of the flux, we worked with the Braginskii expression in the formulation of Stacey-Sigmar (4) and found the following expansion in $\epsilon$:

$$\vec{b}\nabla\cdot\overleftrightarrow{\pi}_{Z,\|} = \frac{2f_O\eta_Z^o}{r^2}\epsilon cos\Theta\Big(V_{Z,\Theta}^o - \frac{2}{3}\tilde{n}_{Z,C}V_{Z,\Theta}^o - \frac{T}{eB_oZ}\tilde{n}'_{Z,C}\Big)$$
$$-\frac{2f_p\eta_Z^o}{r}\epsilon\ sin\Theta\Big(\frac{2}{3}\tilde{n}_{Z,S}V_{Z\Theta}^o + \frac{T}{eB_oZ}\tilde{n}'_{Z,S}\Big) \tag{22}$$
$$+\frac{3f_p\eta_Z^o}{2r^2}\epsilon^2\Big(V_{Z,\Theta}^o - \frac{2}{3}\tilde{n}_{Z,C}V_{Z,\Theta}^o - \frac{T}{eB_oZ}\tilde{n}'_{Z,C}\Big)$$

$$+\text{Terms of order } \epsilon^2 sin2\Theta, \epsilon^2 cos2\Theta. \tag{22}$$

**We retained one second order $\epsilon^2$ term, which contributes to the flux, while the periodic terms with $\epsilon^2 sin2\Theta$ or $\epsilon^2 cos2\Theta$ would only contribute with $\epsilon^4$ and can therefore be neglected.**

**As already mentioned in the introduction, the first order $\epsilon$ terms are important for their strong influence on the poloidal impurity variation and hence on the Pfirsch-Schlüter flux (eq. 21). The first term proportional to $V_{Z,\Theta}^o$ in the equation above describes the magnetic pumping present when impurities rotate poloidally and is also consistent with the theory of Callen et al. [5]. But the compressible parts are of the same order, especially the inside-outside variation $\tilde{n}_{Z,C}$, which rapidly grows in the case of toroidal plasma rotation due to the centrifugal force.**

**The Braginskii coefficient of viscosity depends on the total collision time $\tau_{Z,i}$ between impurities and bulk plasma ions [6] and scales as $T^{5/2}$. It therefore becomes unrealistically large at the transition between the collisional and plateau regimes. For this reason the viscosity coefficient $\eta_Z^o$ was corrected so that the average value $< \vec{b}\nabla\cdot\pi_{Z,\|} >$ for the incompressible term proportional to $V_{Z,\Theta}^o$ in eq. (22) fits to that given by Stacey et al. [7]:**

$$\eta_Z^o = \gamma n_Z^o T\tau_{Z,i}, \text{with}\gamma = \nu_{*,Z}^2\epsilon^{3/2}/\Big(\big(1+\nu_{*,Z}\big)\big(1+\epsilon^{3/2}\nu_{*,Z}\big)\Big) \tag{23}$$

**and $\nu_{*,Z} = \nu_{Z,i}qR_o/(v_{th,Z}\epsilon^{3/2})$.**

**This expansion of the viscosity (eq. 22) now yields the following result for $\Gamma_Z^V$:**

$$\Gamma_Z^V = \frac{2\epsilon^2 f_p \eta_Z^o}{ZeB_\varphi^o}\Big(1 - \frac{3}{8\pi}\Big)\Big(V_{Z,\Theta}^o - \frac{2}{3}V_{Z,\Theta}^o \tilde{n}_{Z,C} - \frac{T}{eB_o Z}\tilde{n}'_{Z,C}\Big). \tag{24}$$

The final result for the impurity variation is obtained from the parallel component of the momentum equation (1), the parallel electric field being neglected as previously.

The driving forces for the parallel pressure gradient are friction due to eqs. (2) and (17), the above-treated parallel viscous force due to the first-order $\epsilon$ part of eq. (22) and the inertia, which also was taken fully into account. The poloidal and toroidal zero-order rotations enter in addition to the perpendicular pressure gradients of bulk plasma ions and impurities, as two further parameters for defining the solution completely. They are connected with the background ion flow through eq.(16). As is well known, using a complete neoclassical model fixes the poloidal rotation at [2]:

$$V_{i,\Theta}^o = -gT_i'/eB_o; g = 1.7;\ 0.5;\ -1.17, \tag{25}$$

respectively, in the collisional, plateau and banana regimes.

In solving the parallel momentum equation it was necessary, for simplicity,to work with the ansatz for the radial derivates of the Fourier components:

$$\tilde{n}'_{Z,S,C}/\tilde{n}_{Z,S,C} \approx n_Z^{o\prime}/n_Z^o.$$

After some tedious calculations consisting of collecting all quantities proportional to $sin\Theta, cos\Theta$, we got the result for $\tilde{n}_{Z,S}$, and $\tilde{n}_{Z,C}$:

$$\tilde{n}_{Z,S} = -\Omega(2\alpha\bar{G} - \delta_Z U^2)/(\Omega^2\bar{G}^2 + \delta_Z^2) \tag{26}$$

and

$$\tilde{n}_{Z,C} = -(2\alpha\delta_Z + \Omega^2 U^2 \bar{G})/(\Omega^2\bar{G}^2 + \delta_Z^2). \tag{27}$$

The following definitions are used:

$$\alpha = \Big(\frac{1}{eB_\varphi^o}\Big(\frac{P_i'}{n_i} - \frac{P_Z^{o\prime}}{Zn_Z^o}\Big) - \frac{\xi}{eB_\varphi^o}T_i' - \frac{\tilde{\gamma}}{2}V_{Z,\Theta}^o\Big)/V^*, \tag{28}$$

$$V^* = P_i'/eB_\varphi^0 n_i; \quad \tilde{\gamma} = \epsilon^{-3/2}\left(\frac{m_Z}{m_i}\right)^{1/2} / \left((1+\nu_{*,Z})(1+\epsilon^{3/2}\nu_{*,Z})\right),$$

$$\delta_Z = \left(V_{Z,\Theta}^o\left(1+\frac{2}{3}\tilde{\gamma}\right) + \frac{Tn_Z^{o\prime}}{eB_o Z n_Z^o}(1+\tilde{\gamma})\right)/V^*, \tag{29}$$

$$\bar{G} = 1 - \left(V_{Z,\Theta}^{o2} + V_{Z,\Theta}^o \frac{Tn_Z^{o\prime}}{eB_\varphi^o Z n_Z^o}\right)/\left(\frac{f_p^2 T}{m_Z}\right), \tag{30}$$

$$U^2 = \left(\left(f_p V_{i,\varphi}^o - V_{Z,\Theta}^o\right)^2 + V_{Z,\Theta}^{o2}\right)/\left(\frac{f_p^2 T}{m_Z}\right). \tag{31}$$

## IV. Discussion of Results and Conclusions

**Figure 1 shows the evaluation of the inward drift velocity with the formulas already given (curve (1)) for a typical ASDEX pellet discharge at a time where accumulation equilibrium has not been reached ($Tn'_{zo}/Zn_{z,o} << P'_i/n_i$), this condition also applying to Figs. 2 and 3. The background plasma (peak density $n_{e,o} = 1.5 \cdot 10^{20} m^{-3}$, peak temperature $T_{e,o} = 780\ eV$) is mainly in the plateau region. The dominating metal is iron. Viscosity effects could be neglected here and the toroidal rotation is zero ($V_{i,\varphi}^o = o$), but inertial terms due to the poloidal impurity rotation, which are important for heavy impurities in smaller devices when**

$$\left(\frac{P_i'}{eB_\theta^o n_i}\right)^2 / \left(\frac{T}{m_Z}\right) > 0.4 \div 0.5,$$

**have been retained. For comparison, we present in curves (2) the result from calculations with the formulas for the Pfirsch-Schlüter flux as published by the TFR group [9] (solid line), and [8] (dashed line), which are based on the usual neoclassicl Larmor radius expansion, while the classical flux $\Gamma_Z^{CL}$ to be added to the Pfirsch-Schlüter contribution was calculated according to eq. (18) of this paper. The solid curves result from the complete neoclassical theory including the $\nabla_\perp T$ contribution with $\xi = 1.5$; $g = 0.5$ (see eqs. (2) and (28)), whereas this contribution is omitted in the dashed curves.**

The inward drift of the "old" theory is found to be much too high: This effect is due to a difference in the scaling with the collisionality parameter $\Omega$, which in Fig. 1 here is always in the range $\Omega \leq 1$. In the theory of Rutherford the flux scales as

$$\Gamma_Z \sim \Omega^{-1},$$

while we found

$$\Gamma_Z \sim \Omega/(1+\Omega^2) + (q^2\Omega)^{-1},$$

where the last term is the classical flux always dominating in the region $\Omega << 1$. The predicted flux of curve (1) fits well to the bolometric measurements [10].

Temperature screening has very little effect: In studying cases with lower collisionality $\Omega > 1$, the viscosity entering with the coefficient $\tilde{\gamma}$ into eqs. (28) and (29) becomes important, growing to $\tilde{\gamma} = \frac{1}{2}(m_Z/m_i)^{1/2}$ at the transition from the collisional to the plateau regime, and so always overcomes the $T_i'$ term. There is no flow reversal due to the temperature screening.

The only classical flow reversal we have found is caused by poloidal rotation of the bulk ions, as shown in the example in Fig. 2: When background ions are at rest ($V_{i,\Theta}^o = o$) then the radial electric field acting on the impurities produces a rotation of the impurities with $V_{Z,\Theta}^o = -P_i'/eB_\varphi^o n_i$ (eq. (16)) in the direction of the poloidal magnetic field, shifting the impurities downwards so that the coefficient $\tilde{n}_{Z,S}$ for up-down asymmetry becomes negative, which always leads to inward drift (see eq. (21)). This effect is further increased in the presence of toroidal rotation. On a rotation of the background plasma with $V_{i,\Theta}^o = 2P_i'/eB_\varphi^o n_i$ the impurity rotation is reversed and the flux is directed outwards. This result shows the importance of neoclassical effects for understanding impurity transport. When strong plasma rotation is induced by the momentum transferred by neutral injection, a neoclassical transport is observed to be highly dominant over the anomalous one. Besides toroidal rotation, we found poloidal rotation here as a second parameter, which not only influences the asymmetries but also significantly defines the transport. Inward drifts of impurities as measured in the burst-free H-mode of ASDEX can be interpreted according to neoclassical theory as an effect of the toroidal rotation velocity in conjunction with poloidal rotation of the bulk ions that is smaller than the ion diamagnetic drift motion. Under this condition there is an explosive increase in the central impurity concentration that culminates in radiation collapse. This result emerges out from calculations even for co-injection and seems at first glance, contradictory to that reported in the work of Stacey et al. [17], who predict that there is always flow reversal with co-injection. We studied these differences to our work and found that it is due to the ansatz of highly anomalous drag frequency, which is not introduced into the momentum balance equation presented here:

The anomalous drag frequency has two important features from which it could be measured in principle.

Firstly it describes radial momentum loss of impurities and could give rise to a considerable shift between impurity and bulk plasma rotation. With the drag frequency $\nu_{d,Z}$ defined in the same way as was done by the authors already mentioned, the following relation for the zero-order quantities was found from the parallel momentum balance neglecting the parallel electric field and the source term, which is small in relation to the friction forces:

$$V_{i,\varphi}^{o} - V_{Z,\varphi}^{o} = \alpha V_{Z,\varphi}^{o} \text{with } \alpha = \frac{m_Z \nu_{d,Z}}{m_i Z^2 \nu_{i,i}}. \tag{32}$$

In order to explain shifts between impurities and bulk ions of the order of 50 %, one needs, for values typical of JET or ASDEX, anomalous impurity confinement times about two orders of magnitudes smaller than the plasma energy confinement time, this being consistent with the drag frequency values used by Stacey et al. [7].

Secondly, studying the radial momentum balance (eq. (13)) we obtain in zero order

$$V_{i,\varphi}^{o} - V_{Z,\varphi}^{o} = -P_i' f_p^{-1}/en_i B_\varphi^0 + f_p^{-1}(V_{i,\Theta}^{o} - V_{Z,\Theta}^{o}). \tag{33}$$

From this and eq. (32) we find the interesting result

$$V^o_{Z,\Theta} = V^o_{i,\Theta} - P'_i/en_iB^o_\varphi - V^0_{i,\varphi}f_p^{-1}\alpha/(1+\alpha), \qquad (34)$$

from which it is concluded that an anomalous drag frequency produces impurity poloidal rotation $V^d_{Z,\Theta} = -V^o_{i,\varphi}f_p^{-1}\alpha/(1+\alpha)$ such as is always necessary for outward drift with co-injection ($V^o_{i,\varphi} > o$) or inward drift with counter-injection ($V_{i,\varphi} < o$) in keeping with our statements on poloidal rotation.

But in H-mode discharges with improved confinement time the anomalous drag frequency is too small to reverse the flux; we therefore did not include this effect in the theory.

In Fig. 3 we show the calculation for the coefficient $\tilde{n}_{Z,C}$ of the inside-outside asymmetry in the case of zero toroidal rotation. It is interesting to see poloidal rotation of bulk ions causing a change in sign here, thus opening the possibility of measuring the important parameter of poloidal rotation by measuring the inside-outside variation.

In the presence of toroidal rotation such a measurement is not so informative owing to the fact that the centrifugal force always leads to impurity accumulation on the torus outside.

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**Figure Captions:**

Fig. 1: Prediction of neoclassical inward drift of iron impurity in a typical ASDEX pellet discharge.

Curves (1) and (2) refer to calculations presented here and elsewhere [8, 9], respectively.

Measured temperature and density profiles.

Fig. 2: Prediction of neoclassical impurity drift velocity of iron impurity in ASDEX with toroidal ($V_{i,\varphi} = V(1-\rho^2); \rho = r/a; V = 10^5 m/s$) and poloidal rotation. Peak temperature $T_o = 2\ keV$, peak density $n_{e,o} = 1.5 \times 10^{20}\ m^{-3}$. Co-injection. Calculation done with freely invented temperature and density profiles.

Fig. 3: Prediction for the inside-outside asymmetry of iron impurity in ASDEX as a function of poloidal rotation. Temperature and density profiles as in Fig. 2.

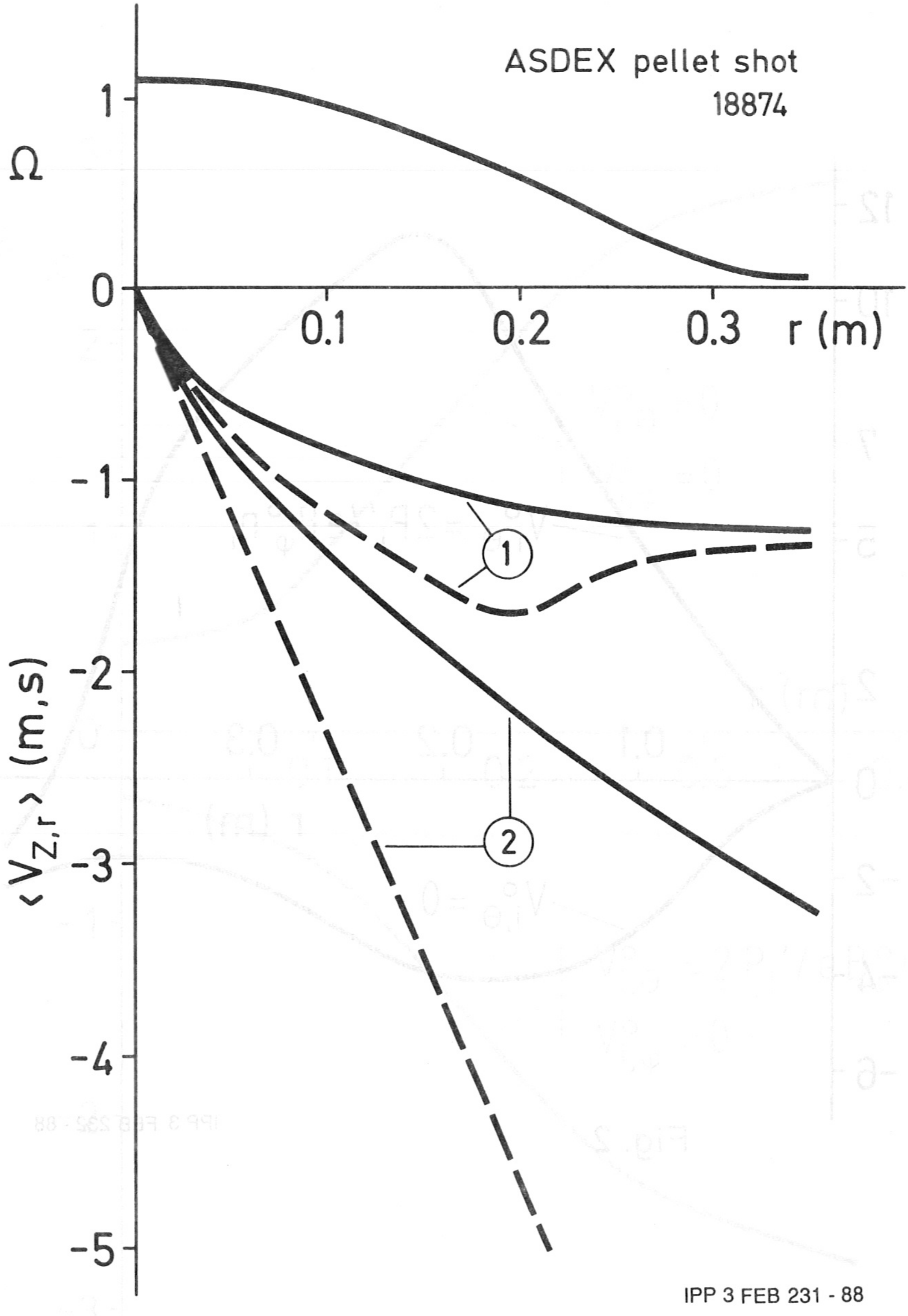


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Fig. 1

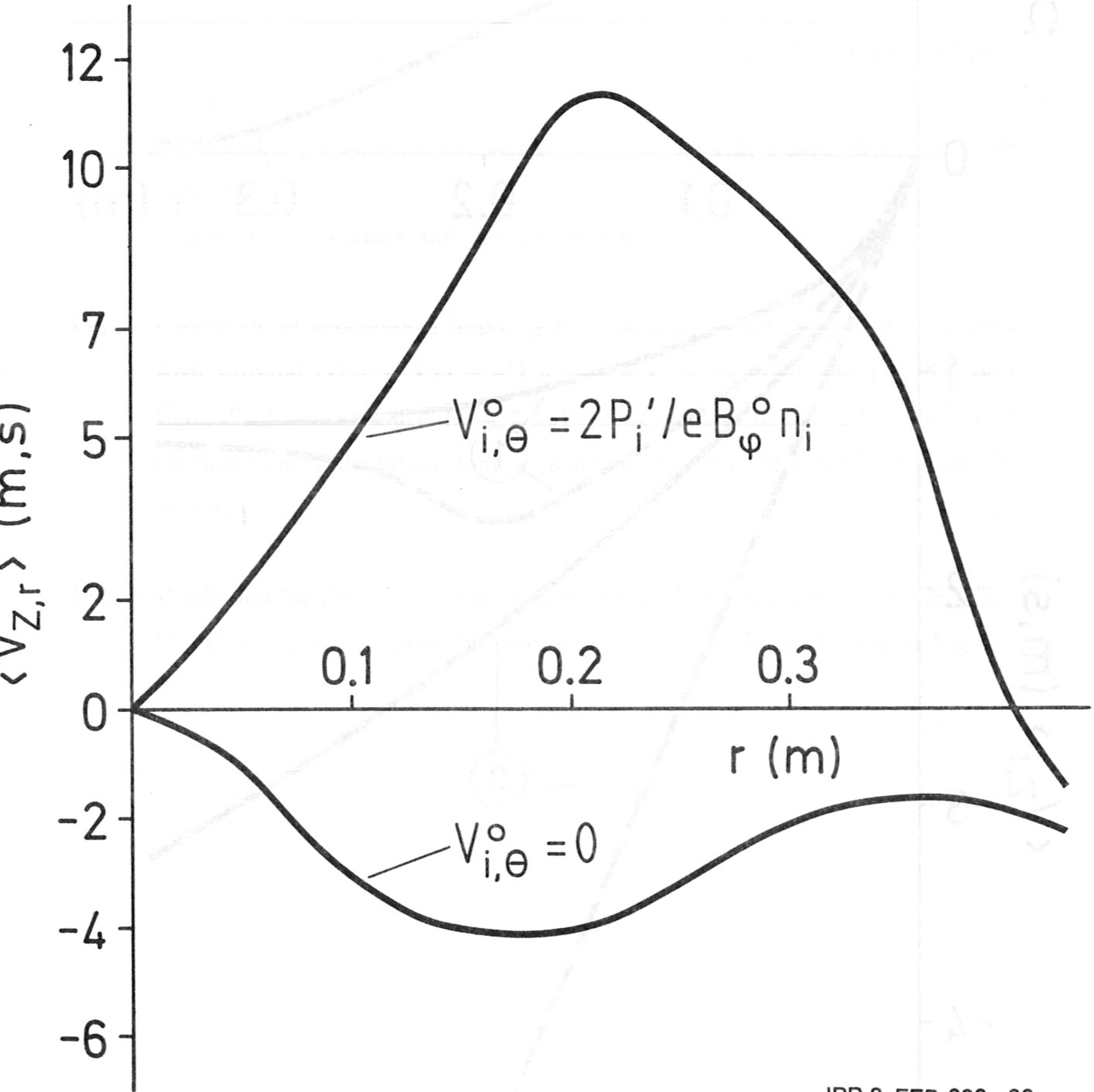


Fig. 2

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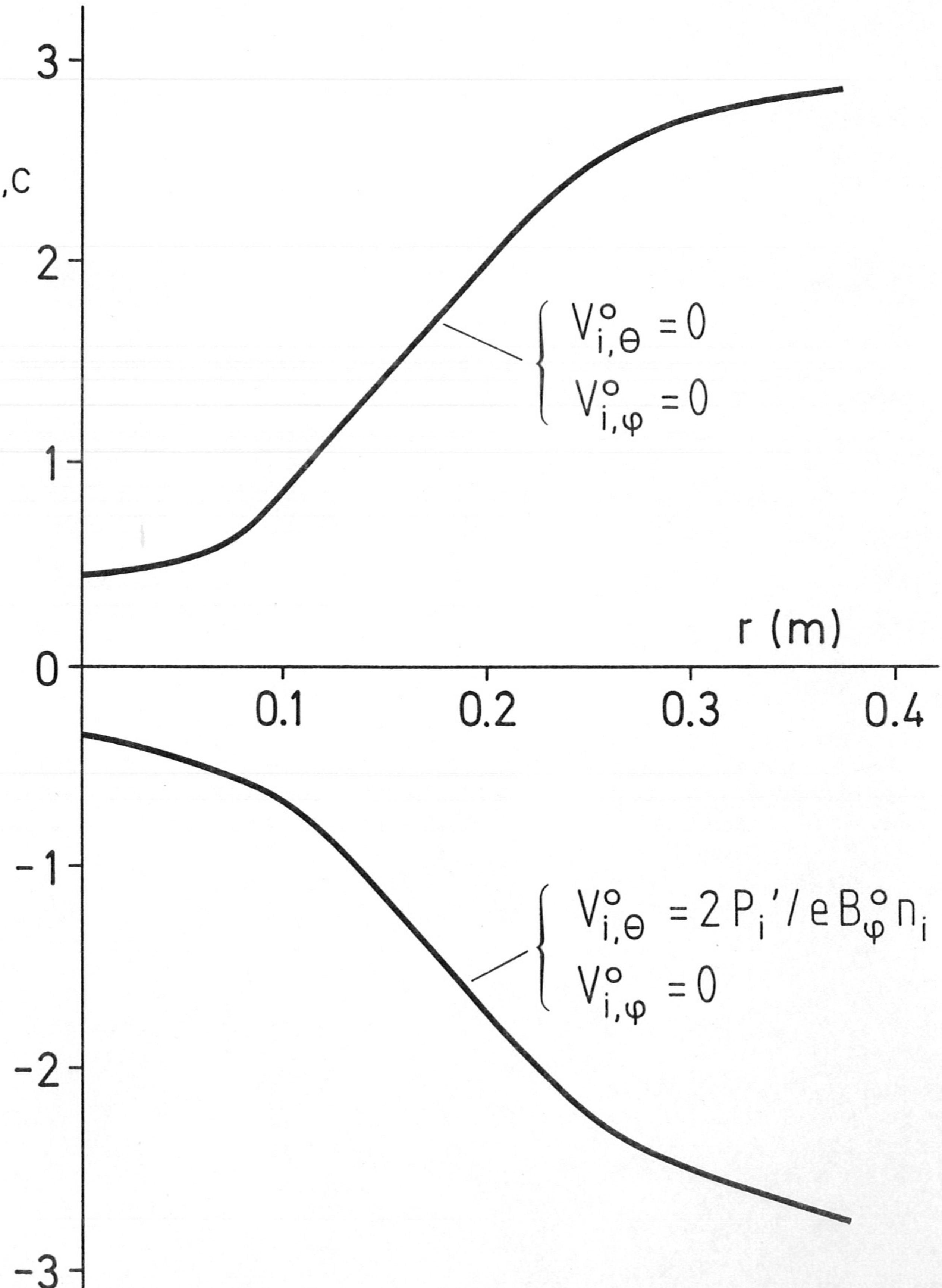


Fig. 3

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