FISIC - A Full-wave Code to Model Ion Cyclotron Resonance Heating of Tokamak Plasmas

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Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

ABSTRACT

We present a user manual for the FISIC code which solves the integrodifferential wave equation in the finite Larmor radius approximation in fully toroidal geometry to simulate ICRF heating experiments. The code models the electromagnetic wave field as well as antenna coupling and power deposition profiles in axisymmetric plasmas.

1. Introduction.

The FISIC code solves Maxwell equations in a tokamak plasma in the ion cyclotron frequency domain. The present version assumes circular concentric magnetic flux surfaces. A generalisation to arbitrary MHD equilibrium configurations is under way.

The presented report is intended as a "user manual" for the code. A detailed description of the physical model and the numerical method including a discussion of the validity of the approximations which have been made and of the convergence properties can be found in [1] and [2]. Therefore we shall give in sections 2 and 3 only a brief summary with special emphasis on those features that are important for the user.

Since the code requires a large amount of CPU–time and memory space unsuccessful runs should be avoided. Therefore we have written a seperate test program to facilitate the adjustment of certain parameters. This program should be run before every new run of the FISIC code itself. In Chapter 4 we shall explain how to use this test code. This chapter contains also a discussion of the input quantities which are, up to those parameters controlling the diagnostic of the results, the same as for the main code.

Chapter 5 contains then a description of the FISIC code which is a package consisting of three sequential sets of routines between which auxiliary data sets are exchanged. In this chapter we shall also explain the output and plots which can be obtained to interpret the results.

In the appendices A and B we present the calling trees for the essential parts of this code.

2. The Wave Equation.

The FISIC code solves Maxwell equations in a tokamak plasma in the ion cyclotron frequency domain, taking into account:

a) full tokamak geometry, i.e. both toroidal and poloidal magnetic field (magnetic surfaces have circular cross-section);

b) finite Larmor radius effects describing mode conversion to Bernstein waves;

c) kinetic damping: ion cyclotron damping at the fundamental and first cyclotron harmonics, electron transit time and Landau damping.

The wave equations are then a set of partial integro–differential equations,

$$\text{rot}\,\text{rot}\,\vec{E} = \frac{\omega^2}{c^2}\left(\vec{E} + \frac{4\pi i}{\omega}\vec{J}^{(0)} + \sum_{ions}\vec{J}_i^{(2)} + \vec{J}_e^{(2)}\right) \tag{2.1}$$

Here the zero order current is given by:

$$\vec{E} + \frac{4\pi i}{\omega}\vec{J}^{(0)} = \hat{L}E_+\vec{u}_+ + \hat{R}E_-\vec{u}_- + \hat{P}E_\parallel\vec{u}_\parallel \tag{2.2}$$

the FLR ion current is:

$$\frac{4\pi i}{\omega}\vec{J}_i^{(2)} = 2\frac{c^2}{\omega^2}\vec{u}_+(\vec{u}_+\cdot\vec{\nabla})\left[\hat{\lambda}_{2,i}\vec{\nabla}\cdot(E_+\vec{u}_+^*)\right] \tag{2.3}$$

and the FLR electron contribution to the current is:

$$\frac{4\pi i}{\omega}\vec{J}_e^{(2)} = -2\frac{c^2}{\omega^2}\vec{\nabla}_\perp\times\left[\hat{\lambda}_{0,e}\left(\vec{\nabla}_\perp\times\vec{E}_\perp\right)\right] \tag{2.4}$$

Parallel and perpendicular refer to the local direction of the static magnetic field; $E_\pm$, $E_\parallel$ are the left and right circularly polarised and the parallel components of $\vec{E}$, respectively, and $\vec{u}_\pm$, $\vec{u}_\parallel$ the corresponding unit vectors. Moreover, we have introduced a number of operators which are orbit integrals along magnetic field lines:

$$\tilde{R}E_-(\vec{r}) = \left[1 - \frac{\omega_{pe}^2}{\omega^2}\frac{\omega}{\omega-\Omega_{ce}} - \sum_{ions}\frac{\omega_{pi}^2}{\omega^2}\frac{\omega}{\omega+\Omega_{ci}}\right]E_-(\vec{r}) \tag{2.5}$$

$$\begin{aligned}\hat{L}E_+(\vec{r}) = &\left(1 - \frac{\omega_{pe}^2}{\omega^2}\frac{\omega}{\omega+\Omega_{ce}}\right)E_+(\vec{r}) \\ &+ \sum_{ions}\frac{\omega_{pi}^2}{\omega^2}\int dv_\parallel\frac{e^{-v_\parallel^2/v_{thi}^2}}{\sqrt{\pi}v_{thi}}\left[i\omega\int_0^\infty d\tau\, e^{i\int^\tau(\omega-\Omega_{ci})\,d\tau}E_+(\vec{r}'(\tau))\right]\end{aligned} \tag{2.6}$$

$$\hat{P}E_\parallel(\vec{r})\vec{u}_\parallel = \frac{\omega_{pe}^2}{\omega^2}\int dv_\parallel\frac{e^{-v_\parallel^2/v_{the}}}{\sqrt{\pi}v_{the}}2\frac{v_\parallel^2}{v_{the}^2}\left[i\omega\int_0^\infty d\tau e^{i\omega\tau}E_\parallel\left(\vec{r}'(\tau)\right)\vec{u}_\parallel\right] \tag{2.7}$$

$$\hat{\lambda}_{n,s}\vec{E}_\perp(\vec{r}) = -\frac{1}{2}\frac{\omega_{ps}^2}{\Omega_{cs}^2}\frac{v_{ths}^2}{c^2}\int dv_\parallel \frac{e^{-v_\parallel^2/v_{ths}^2}}{\sqrt{\pi}v_{ths}} \left[i\omega \int_0^\infty d\tau\, e^{i\int^\tau(\omega - n\Omega_{cs})d\tau}\, \vec{E}_\perp\left(\vec{r}'(\tau)\right)\right] \tag{2.8}$$

Here ω_{ps}, Ω_{cs} are the plasma and cyclotron frequencies, respectively, $v_{ths} = \sqrt{2T_s/m_s}$ the thermal velocity, and

$$\vec{r}'(\tau) = \vec{r} - \int^\tau \vec{u}_\parallel v'_\parallel\, d\tau \tag{2.9}$$

Collisional damping can be included by giving ω in the exponents in the orbit integrals a small but nonvanishing imaginary part, $\omega = \mathrm{Re}(\omega) + i\nu$, where ν stands for an effective collision frequency.

These equations are the generalisation to tokamak geometry of those obtained by Swanson [3] and Colestock and Kasuba [4] assuming a plane–stratified geometry. Details of their derivation [5] and on the associated power balance [6] are presented elsewhere.

In certain ICRH scenarios the "quasielectrostatic" ion Bernstein wave is excited due to linear mode conversion near the plasma center and propagates then with increasing refractive index to the high magnetic field side. The only collisionless damping affecting these waves is electron Landau damping. It can be shown, however, that for $k_\perp \cdot \rho_i \lesssim 1$, ρ_i denotes the ion Larmor radius, the parallel component of the electric field of the IBW is much smaller than for a truely electrostatic wave and electron Landau damping becomes almost negligible [7,8]. As these waves propagate to the plasma periphery with rapidly decreasing wavelength, it becomes often difficult to maintain enough numerical resolution. To minimise this problem, and to take into account experimental indications that IBW are subject to efficient parametric or stochastic absorption well before the condition $k_\perp \cdot \rho_i \lesssim 1$ is violated [9,10], we introduce a somewhat artificial damping by multiplying $\hat{\lambda}_2$ by the factor

$$1 + i\, e^{-\epsilon^{-1}(\Omega_{ci}/k_{\perp BW}\, v_{thi})^2} \tag{2.10}$$

where ϵ is an adjustable parameter of order unity, and $k_{\perp BW}$ is estimated from the local dispersion relation. This hardly affects the fast magnetosonic wave, and does

not alter the amount of energy converted into IBW. It is omitted where the IBW is evanescent. The introduction of this damping also allows the direct diagnosis of the mode conversion efficiency from the code.

We are looking for forced solutions of the wave equation driven by an externally prescribed antenna current. The antenna is simulated as a sheath of current flowing perpendicularly to the static magnetic field. The central conductor and the Faraday screen are assumed to lie in same magnetic flux surface. The currents in the feeders and shorts are not taken into account.

The plasma is assumed to extend up to the perfectly conducting wall ("diffuse" plasma edge), which is also parallel to a magnetic flux surface. The resulting boundary conditions at the wall and matching conditions at the antenna can be found in ref. [1] and [2].

3. Numerical Method.

3.1. The Spectral Ansatz.

The wave equations (2.1—2.9) are solved with a semispectral discretisation [11, 12]: expansion in Fourier modes in the poloidal direction, finite elements with cubic Hermite interpolation radially. For this purpose, the electric field is expanded in toroidal and poloidal Fourier modes:

$$\vec{E} = \sum_{m,n_\varphi} \vec{E}^{m,n_\varphi}(r) e^{i(m\vartheta + n_\varphi \varphi)} \tag{3.1}$$

and the wave equation is cast in a weak variational (Galerkin) form by multiplying with the test functions

$$\left(\vec{F}^{m'}(\vec{r})\right)^* = e^{-im'\vartheta}\vec{G}^*(r) \ (\cdot e^{-in_\varphi \varphi}) \tag{3.2}$$

and integrating over the plasma volume. Each n_φ mode can be treated separately, the power deposition profiles and the antenna resistance R_A being additive in n_φ. Therefore we shall omitt in the following equations the ignorable coordinate φ and the good quantum number n_φ whenever possible. On the other hand poloidal Fourier modes are not even approximate eigenmodes of the wave equations, so that a large number is often required for an adequate representation of the solution.

The advantage of the spectral method over a direct discretisation in the poloidal coordinate ϑ is that it allows the explicit evaluation of the orbit integrals in the constitutive relations,

$$\hat{Z}_s^n(\vec{E}) = \int_{-\infty}^{+\infty} dv_\parallel \frac{e^{-v_\parallel^2/v_{ths}^2}}{\sqrt{\pi} v_{ths}} \left\{ i\omega \int_0^\infty d\tau \, e^{i\int_0^\tau (\omega - n\Omega_{cs}(r,\vartheta'))d\tau} \vec{E}(r,\vartheta') \right\}$$
$$= \sum_m \vec{E}^m(r) e^{im\vartheta} \int_{-\infty}^{+\infty} dv_\parallel \frac{e^{-v_\parallel^2/v_{ths}^2}}{\sqrt{\pi} v_{ths}} \left\{ i\omega \int_0^\infty d\tau \, e^{i\int_0^\tau (\omega - n\Omega_{cs}(r,\vartheta') - k_\parallel^{m,n_\varphi} v_\parallel)d\tau} \right\} \tag{3.3}$$

provided that the contribution from trapped particles with reflection point close to resonance can be neglected. Here we have defined for each mode the effective parallel wavenumber

$$k_\parallel^{m,n_\varphi} = m\frac{\sin\Theta}{r} + n_\varphi \frac{\cos\Theta}{R} \simeq \frac{(n_\varphi + m/q)}{R} \tag{3.4}$$

with

$$\tan\Theta = B_\vartheta / B_\varphi$$

and q denotes the local safety factor. Under rather unrestrictive conditions it is then possible to approximate $\hat{Z}_s^n$ through the familiar Fried–Conte Z function [13] :

$$\hat{Z}_s^n(\vec{E}) \simeq -\sum_m x_{0,s}^m Z(x_{n,s}^m) \vec{E}^m(r) e^{im\vartheta}$$

where

$$x_{n,s}^m = \frac{\omega - n\Omega_{c,s}(r,\vartheta)}{k_\parallel^m v_{ths}}$$

Note that for nonvanishing collisions the argument of the Z function is complex. A more accurate evaluation of $\hat{Z}_s^n$ could be easily incorporated if a sufficiently fast algorithm becomes available.

The spectral Ansatz (3.1) converts for each poloidal mode m the integral operators $\hat{L}$, $\hat{P}$ and $\hat{\lambda}_{n,s}$ into the algebraic operators $\tilde{L}(r,\vartheta;k_\parallel^m)$, $\tilde{P}(r;k_\parallel^m)$ and $\tilde{\lambda}_{n,s}(r,\vartheta;k_\parallel^m)$ and the FLR wave equations into a (large) set of coupled ordinary differential equations in the radial variable only. The number N_m of modes required for an adequate representation of the solution can be estimated from the shortest wavelength to be resolved, usually the IBW: N_m is roughly the same as the number of points which would be require in a conventional discretisation in ϑ.

Due to the small electron inertia, $E_\parallel$ is efficiently screened at ion frequencies. An appreciable saving of CPU time and memory can then be achieved by solving the parallel component of the wave equation as

$$E_\parallel^m \simeq \frac{c^2}{\omega^2}\,\tilde{P}^{-1}(r;k_\parallel^m)\Big\{\vec{u}_\parallel\cdot\vec{\nabla}^m\times\big(\vec{\nabla}^m\times\vec{E}_\perp^m\big)\Big\} \simeq \frac{c^2}{\omega^2}\frac{ik_\parallel^m}{\tilde{P}(r;k_\parallel^m)}\vec{\nabla}^m\cdot\vec{E}_\perp^m \qquad (3.5)$$

3.2. The Spatial Integration

The variational form of the equations reads then after integration by parts of terms containing second derivatives :

$$\begin{aligned}
&\sum_m\int dV\, e^{i(m-m')\vartheta}\Big\{(\vec{\nabla}^{m'}\times\vec{G})^*\cdot(\vec{\nabla}^m\times\vec{E}^m)\\
&\qquad-\frac{\omega^2}{c^2}\begin{pmatrix}G_+^*\\G_-^*\end{pmatrix}\begin{pmatrix}\tilde{L}(r,\vartheta;k_\parallel^m) & 0\\ 0 & \tilde{R}(r,\vartheta)\end{pmatrix}\begin{pmatrix}E_+^m\\E_-^m\end{pmatrix}\\
&\qquad-\frac{c^2}{\omega^2}\vec{\nabla}^{m'}\cdot\vec{G}_\perp^*\frac{\big(k_\parallel^m\big)^2\tilde{P}(r;k_\parallel^m)}{|\tilde{P}(r;k_\parallel^m)|^2}\vec{\nabla}^m\cdot\vec{E}_\perp\\
&\qquad+2\big[\vec{\nabla}^{m'}\cdot(\vec{u}_+^*G_+)\big]^*\tilde{\lambda}_2(r,\vartheta;k_\parallel^m)\,\vec{\nabla}^m\cdot(\vec{u}_+^*E_+^m)\\
&\qquad+2(\vec{\nabla}_\perp^{m'}\times\vec{G}_\perp)^*\tilde{\lambda}_0(r,\vartheta;k_\parallel^m)(\vec{\nabla}_\perp^m\times\vec{E}_\perp^m)\Big\}\\
=&\sum_m\int d\vec{S}\, e^{i(m-m')\vartheta}\Big\{\vec{G}_\perp^*\times(\vec{\nabla}_\perp^m\times\vec{E}_\perp^m)+2\vec{u}_+G_+^*\,\tilde{\lambda}_2(r,\vartheta;k_\parallel^m)\,\vec{\nabla}^m\cdot(\vec{u}_+^*E_+^m)\Big\}.
\end{aligned} \qquad (3.6)$$

Since the electron FLR corrections are always very small at the antenna or wall, their contributions have been neglected in the boundary terms. This is not the case for the contributions due to the ion FLR current if a second harmonic cyclotron resonance lies within the plasma.

The poloidal integration is done by means of the Fast Fourier Transformation (FFT) [14] on an equidistant mesh of N_{ps} gridpoints for the nonresonant electron terms and the terms involving only differential operators. For the terms describing fundamental and harmonic ion cyclotron resonances a denser mesh of N_p gridpoints is used. To make the FFT most efficient N_{ps} and N_p should be potencies of 2. For $k_\parallel^m = 0$ the upper limit of the widths of cyclotron resonances depends only on the value of the effective collisional frequency ν. To ensure a satisfactory poloidal

resolution, we choose

$$\nu/\omega = \frac{\chi 2\pi r}{N_p \cdot R_T}, \tag{3.7}$$

where χ is an adjustable parameter of order unity. Eq. (3.7) guaranties that at least χ poloidal gripoints lie within the halfwidth of a resonance. It corresponds typically to values $\frac{\nu}{\omega} \simeq 10^{-4} - 10^{-3}$, which is well within the usually assumed range and much smaller than what is commonly used in cold plasma codes.

The elementwise radial integration is done by means of a Gauss–sheme with N_g points per finite element. The criteria used to determine the radial mesh have been discussed in ref. [2].

In principle arbitrary toroidal spectra and poloidal current distributions of the antenna can be chosen.

4. Parameters, Input Variables and the Test Program FTEST

With the exception of those variables, which are needed to control the plotting of certain quantities and the diagnostic, the input variables of *FTEST* and *FISIC* are the same and are read as NAMELIST *INPUT*. Temperature and density profiles are evaluated in the routine *PROFTD*, the antenna current in the routine *ANTC*. Moreover some integer numbers used to dimension arrays in the test program *FTEST* as well as in the various routines of the code *FISIC* must be specified as Fortran parameters.

4.1. Input Parameters and Variables.

The following Fortran parameters have to be specified first:

NM Number of poloidal modes N_m; The largest prime factor of *NM* must not exceed 19. This condition arises from the FFT–Routine used to reconstruct the electromagnetic field after its Fourier modes have been evaluated. On the Cray X–MP memory space sets an upper limit of $NM \leq 108$. Odd numbers might be preferred so that there are as many positive as negative modes in the sample.

NTT Number of poloidal gridpoints N_p used for the poloidal integration of resonant terms by FFT. *NTT* must be a potency of 2 and larger than $2 \cdot NM$, a typical value is $NTT = 124$.

NTTS Number of poloidal gridpoints N_{ps} used for the poloidal integration of nonresonant terms by FFT.*NTTS* must be a potency of 2, *NTTS*=8 or 16 should ensure sufficient resolution.

NELM Parameter used for dimensioning of radial arrays. Must be equal or larger than the actually used number of finite elements N_{elem} which is evaluated in the routine *MESH*.

NGAUS Number of Gauss points N_g used in the radial integration per finite element. A typical value is $NGAUS = 3$.

NVRB Number of variables N_{vrb}; $NVRB = 2$ unless one changes the code so that all three components of the electric wavefield are explicit variables.

The input variables specifying basic plasma and Tokamak properties as well as some adjustable parameters are read as NAMELIST *INPUT* :

NSPEC	Number of ion species, must be smaller than 5.
DENEC	Central electron density $n_e(0)$, in electrons per cubic cm.
$DENIC(I),\ I = 1, NSPEC$	Relative densities of the ion species in %, must add up to 100.
$ATM(I),\ I = 1, NSPEC$	Atomic mass number of the ion species.
$AZI(I),\ I = 1, NSPEC$	Atomic charge number of the ion species.
TEMPEC	Central electron temperature $T_e(0)$, in eV.
$TEMPIC(I),\ I = 1, NSPEC$	Central ion temperatures $T_i(0)$, in eV.
RT	R_T, distance plasma center–vertical axis (large radius) in cm.
RW	r_W, distance plasma center–wall, in cm.
RANT	r_{ANT}, distance plasma center–antenna, in cm.
RPLAS	a, distance plasma center–plasma edge, in cm.
WANT	Toroidal width w_{ANT} of the (dipole) antenna, in cm.
ALC	Effective poloidal propagation length $\sqrt{LC}$ of the antenna.
BZERO	Strength of the static magnetic field $\vec{B}_0$ at the plasma center, in Gauss.
AJTOR	Toroidal plasma current I_{tor}, in Ampère.
FREQCY	Frequency of the excited radiation, in $\sec^{-1}$.
NPHI	Toroidal mode n_φ.
EPS	Parameter ϵ to adjust the "stoch." IBW damping, cfr. eq. (2.10), should be of order 1.
XSI	Parameter χ to determine the effective collisional frequency so that sufficient resolution of ion cyclotron resonances is ensured, cfr. eq. (3.7), should be of order 1.

Moreover the NAMELIST *INPUT* contains the following integer variables to

control different options :

$IRES1 = 1/0$	Fundamental cyclotron resonance/ no fundamenta cyclotron resonance of any ion species lies in the plasma.
$IRES2 = 1/0$	Harmonic cyclotron resonance/ no harmonic cyclotron resonance of any ion species lies in the plasma.
$ISTOCH = 1/0$	"Stochastic" IBW damping included/ not included, should be 1 if an IBW might be excited.
$IBND = 1/0$	Contributions due to the ion FLR current in the boundary terms at the antenna included/not included. The difference between these two options is very small, the first one is the correct one from the physical point of view, the latter is simpler.
$IFLR = 1/0$	Ion FLR current included/ not included.

4.2. Temperature and Density Profiles.

Radial temperature and density profiles are evaluated in the routine *PROFTD*. They must be chosen in such a way that their derivatives are continuous. In the present version the following form for the various radial profiles $f(r)$ has been assumed:

$$f(r) = \begin{cases} f(0) - (f(0) - f(a))\,(r/a)^{\gamma}, & \text{if } r < a \\ f(r_W) + \left(\frac{r_W - r}{r_W - a}\right)^{\xi} (f(a) - f(r_W)), & \text{if } r > a \end{cases}$$

where ξ is evaluated in such way that the profiles and their derivatives are continuous at the plasma edge $r = a$ and $f(r_W) = f(a) \cdot e^{-(r_W - a)/\lambda}$. The following quantities must be specified:

$RDPL$; $RTEPL$; $RTIPL(I)$, $I = 1, NSPEC$	$f(a)/f(0)$, relative values of the density, electron temperature and ion temperatures at $r = a$ compared to the central values.
$DEXP$; $TEEXP$; $TIEXP(I)$, $I = 1, NSPEC$	Exponents γ of the various profiles for $r \leq a$.
$EDFLD$; $ETEFLD$; $ETIFLD(I)$, $I = 1, NSPEC$	Exponential decay lengths λ in the scrape off layer, in cm.

4.3. ANTENNA CURRENT.

The central conductor of the antenna is assumed to lie in a magnetic flux surface at $r = r_{ANT}$ and the current flows perpendicular to the static magnetic field. Its toroidal spectrum and poloidal distribution are specified in the routine $ANTC$. We assume that the poloidal current distribution has the same form for all values of n_φ. It is specified by means of the array CFJ which is proportional to the antenna current at the N_p poloidal gridpoints used for the ϑ–integration. The relative weight of the toroidal mode n_φ is proportional to $g(n_\varphi)$. For example for a dipole antenna with toroidal width w_{ANT} located at the low field side one would have $g(n_\varphi) = \sin(x \cdot n_\varphi)/(x \cdot n_\varphi)$ with $x = w_{ANT}/2 \cdot (r_{ANT} + R_T)$ Moreover one has to specify S_{n_φ} which is defined as

$$S_{n_\varphi} = \sum_{n_\varphi} \int_0^{2\pi} d\varphi \, g(n_\varphi) \, e^{i \cdot n_\varphi \cdot \varphi}$$

For the dipole antenne with the toroidal spectrum given above one obtaines for instance $S_{n_\varphi} = 2\pi$. The Fortran names for these quantities are $GNPHI$ and $SNPHI$.

4.4. THE TEST PROGRAM FTEST.

Using the input described above, the test routine $FTEST$ evaluates :

– The radial mesh; The criteria which are applied to ensure sufficient resolution of the cyclotron resonances as well as of the electromagnetic wavefield and other properties of the radial mesh have been described in ref. [2]. The user should make shure that the calculated number of actually used finite elements N_{elem} does not exceed the parameter $NELM$ which dimensions radial arrays.

– Some central plasma parameters and properties of the incoming radiation like ω_{ps}, Ω_{cs}, v_{ths}, $P = 1 - \frac{\omega_{pe}^2}{\omega^2}$, $n_\parallel^0 = \frac{n_\varphi}{\frac{\omega}{c} R_T}$, $\frac{\omega}{c} R_T$.

– Positions of fundamental and harmonic ion cyclotron resonances with their Doppler widths in the equatorial plane for m=0.

– Positions of confluences, cut–offs or wave resonances.

– The poloidal antenna current profile and its Fourier Transform. The latter provides some information if N_p is large enough to resolve the current profile.

Moreover the some plots can be obtained (x denotes here the horizontal coordinate in the equatorial plane). They are controlled by a set of variables which is read as NAMELIST *IPLOT* :

$ITEPL \neq 0$	$T_e(x)$.
$ITIPL = I$	$T_i(x)$ for the I–th ion species.
$IDPL \neq 0$	$n_e(x)$.
$IQPL \neq 0$	$q(r)$; This plot can be used to estimate for which poloidal modes $k_{\parallel}^{m,n_\varphi} \simeq (n_\varphi + m/q)/R_T$ is equal to zero.
$IXEPL \neq 0$	$x_{0,e}^{m=0}(x)$; This quantity is important to estimate kinetic damping through electrons.
$INB \neq 0$	$k_{\perp BW} \cdot \rho_i(r)$ at the high field side; This plot should facilitate the adjustment of the parameter ϵ in the "stoch." IBW damping term: The IBW is usually completely absorbed before $\epsilon(k_{\perp BW}\rho_i(r))^2 \gtrsim 1.3$.
$ISPR \neq 0$	$\tilde{S}(x)$; This plot shows if and where an ion–ion hybrid resonance or an Alfvèn resonance $n_{\parallel}^2 = \tilde{S}$ lie within the plasma.
$ILPR \neq 0$	$\tilde{L}(x)$; This plot shows the position of ion cyclotron resonances as well as of the cut–off $n_{\parallel}^2 = \tilde{L}$ associated with an ion–ion hybrid resonance.
$IRPR \neq 0$	$\tilde{R}(x)$; This plot can be used to estimate the position of the low density cut–off $n_{\parallel}^2 = \tilde{R}$.
$ILAMPR \neq 0$	$\tilde{\lambda}_2(x)$, shows the position of second harmonic ion cyclotron resonances.
$ILPT \neq 0$	$\tilde{L}(\vartheta)$ at $r = RTNPLT$ and its Fourier transform.
$ILAMPT \neq 0$	$\tilde{\lambda}_2(\vartheta)$ at $r = RTNPLT$ and its Fourier transform.

Some of the quantities are evaluated assuming $k_{\parallel} = n_\varphi/R$, i.e. for the poloidal mode $m = 0$. The last two plots give also some information if the poloidal mesh is dense enough to resolve the ion cyclotron resonances and can therefore be used to adjust the parameter χ, eq. (3.7).

There are no data exchanged between this test routine and the main code.

5. The Code FISIC.

The code *FISIC* is a package of three sets of routines between which data sets are transferred. The first and main part which is called *MATSOL* evaluates the stiffness matrix and solves the linear system for a given toroidal mode n_φ using a frontal method. In appendix A the calling tree for this program is given. The input to this routines and the evaluated poloidal spectrum of the electromagnetic field are, together with the radial mesh, stored externally (on Cray–disc). The unit identifier for this data file is 9. Upon reading these data the second set of routines called *BALANCE* containes the diagnostic of these results and evaluates the power deposition profiles and the contribution to the antenna load due to this toroidal mode. These quantities are then, together with the radial and poloidal mesh and a few other quantities which will be needed, stored externally (on Cray disc). The unit identifier of this data set is 11. The calling tree for the program *BALANCE* can be found in appendix B.is The third set of routines called *DPROF* finally reads these data for various toroidal modes and evaluates the total power deposition profiles and the radiative resistance of the antenna.

5.1. Evaluation of the Stiffness Matrix and Solution of the Linear System.

Since this part of the code produces with the exception of the number of radial finite elements no written output and no plots, those variables of the NAMELIST *IPLOT* are not needed here. Otherwise the input parameters and variables for this routine are the same as for *FTEST* and have already been explained in the previous chapter. These variables and the radial mesh are stored externally and will be read by the following part of the code, the program *BALANCE*. The external unit identifier for this data file is 9.

The stiffness matrix is blocktridiagonal. Each block has a size of $(2 \cdot N_{vrb} \cdot N_m)^2$ complex elements. There are as many block rows as radial gridpoints. Since the whole stiffness matrix would be in many cases much too large for the central memory of the Cray X-MP, a frontal method which combines the evaluation of the stiffness matrix and the solution of the linear system using auxiliary storage has been developed: After a block row has been fully assembled its L/U decomposition

is evaluated and stored externally. This is done very efficiently by a code kindly provided by Dr. W. Kerner [15] which saves time as is works simultaneously on the L/U decomposition of one block row and I/O of another. Later on these auxiliary data are used together with the right hand side of the linear system to solve the matrix equation. The solution, i.e. the poloidal Fourier spectrum of the perpendicular components of the electromagnetic wavefield and their radial derivatives at the radial meshpoints are then added to the data file where the input variables and the radial mesh have already been stored before (external unit identifier 9). Note that due to the structure of the linear solving routine the solution is stored "backwards", i.e. the quantities associated with the last radial gridpoint are stored first. The interface between the linear solver, which also handles the I/O operations of the auxiliary data sets and the external storage of the results, and the routines evaluating the block rows of the stiffness matrix and the right hand side are called *BLOX* and *RHS*, respectively. Otherwise there is no connection between these parts of the program. This modular structure would make it very easy to incorporate another solving routine.

In general the program *MATSOL* requires most of the CPU time of the *FISIC* code. For not too small N_m it is dominated by the time needed to solve the linear system, which is proportional to $N_{elem} \cdot N_m^3$, whereas the time for the evaluation of the stiffness matrix scales roughly like $N_{elem} \cdot N_g \cdot N_m \cdot N_p \cdot \ln N_p$.

5.2. Diagnostic and Power Balance for a Single Toroidal Mode.

After reading the data written previously by the program *MATSOL*, i.e. the values of the input variables, the radial mesh and the poloidal Fourier spectrum of the perpendicular components of the electric wave field and their radial derivatives at the radial gridpoints, the program *BALANCE* consists of two parts: one reconstructs the wave field and visualises it in various plots, the other evaluates the power balance, i.e. deposition profiles and the contribution to the antenna load due to the toroidal mode n_φ. Moreover the same output and plots that have been described in the section about the test routine *FTEST* and are controlled by the variables in the NAMELIST *IPLOT* can be obtained again. To control the diagnostic and which plots of the wavefield and the deposition profiles shall be drawn, the following variables are read as NAMELIST *DIAG* :

$NTEPL,\ NTEPL < 5$	Number of plots of $\vec{E}_\perp(r,\vartheta)$ for fixed ϑ to be specified by $ITEPL$;
$ITEPL(I),\ I = 1, NTEPL$	$\vartheta = 2\pi \cdot (ITEPL(I) - 1)/NM$
$NMPL,\ NMPL < 5$	Number of plots of the poloidal spectrum $E_+(m)$ and its Fourier transform $E_+(\vartheta)$ for fixed radii $r = RMPL$ to be specified. These plots are particularly useful to check the convergence in the poloidal direction, i.e. if NM has been chosen large enough.
$RMPL(I),\ I = 1, NMPL$	radii for the above plots in cm.
$IHL(IC) = 0/1$	Determines if the contour lines of the component specified by IC shall be plotted.
$IC = 1$	$\mathrm{Re}(E_x)$
$IC = 2$	$\mathrm{Im}(E_x)$
$IC = 3$	$\mathrm{Re}(E_y)$
$IC = 4$	$\mathrm{Im}(E_y)$
$IC = 5$	$\mathrm{Re}(E_+)$
$IC = 6$	$\mathrm{Im}(E_+)$
$IC = 7$	$\mathrm{Re}(E_-)$
$IC = 8$	$\mathrm{Im}(E_-)$
$IC = 9$	$\lvert E_\perp \rvert$
$IC = 10$	$\lvert E_x \rvert$
$IC = 11$	$\lvert E_y \rvert$
$IC = 12$	$\lvert E_+ \rvert$
$IC = 13$	$\lvert E_- \rvert$
	Plotted is the region $XMIHL < x < XMAHL$, $YMIHL < y < YMAHL$, in cm.

E_x and E_y denote here the horizontal and vertical component of the perpendicular electric field $E_\perp$. The units of the electric field is V/cm, normalised to an antenna current of 1 Ampère.

$IBAL = 0/1$ If $IBAL = 0$ the part of the program which evaluates the deposition profiles and requires much more CPU–time than the other parts, will not be executed. In this case the variables discussed in the following need not to be specified.

$ITABL = 0/1$ No plot/plot of $dP_{ABS}(x)/(r\,dr\,d\vartheta)$ in the equatorial plane.

$IABPL = 0/1$ No plot/plot of $dP_{ABS}(r)/(r\,dr)$, integrated over magnetic flux surfaces.

$I3DI1 = 0/1$ No 3d–plot/3d–plot of the deposition profile for ion cyclotron damping at the fundamental resonance.

$I3DI2 = 0/1$ No 3d–plot/3d–plot of the deposition profile for ion cyclotron damping at the harmonic resonance.

$I3DET = 0/1$ No 3d–plot/3d–plot of the deposition profile for electron TTMP.

$I3DEL = 0/1$ No 3d–plot/3d–plot of the deposition profile for electron Landau damping.

$I3DST = 0/1$ No 3d–plot/3d–plot of the deposition profile for "stochastic" damping of the ion Bernstein wave.

$ISYM$ should be equal 0. $ISYM \neq 0$ causes a slightly different form of the local power deposition due to parallel kinetic flux terms.

$IDSAV$ If $IDSAV \neq 0$ the total antenna load and deposition profiles due to this toroidal modes are, together with the radial mesh and a few other parameters, stored externally for the program $DPROF$. The unit number for this data set is 11.

Finally the code evaluates how the incoming power is distributed among the various damping processes for this toroidal mode. The CPU-time for the program $BALANCE$ scales like $N_{elem} \cdot N_g \cdot N_m \cdot N_p$ if $IBAL = 1$.

5.3. Evaluation of the total Power Balance.

Using the data sets provided by the program $BALANCE$ for various toroidal modes n_φ, $DPROF$ evaluates the total power balance, i.e. deposition profiles and antenna load. To save CPU–time one usually does not run the programs $MATSOL$ and $BALANCE$ for the complete toroidal spectrum launched by the antenna. Instead the total power balance is estimated as a weighted sum of the contributions from those toroidal modes which have been analysed. In the program $DPROF$ their number must be specified as parameter $NNPHI$. In the present version $NNPHI$ must be smaller than 10. The weights of the contributions from the individual n_φ's must be specified by the array $WNPHI(INPHI)$, $INPHI = 1, NNPHI$. For example if the power balance has been evaluated and stored for the toroidal modes n_φ = 0,4,8,12,16 and 20, $NNPHI$ would be 6 and $WNPHI(1) = 5$ (the five modes $n_\varphi = 0, \pm 1, \pm 2$ are assumed to give similar results), whereas $WNPHI(2-6) = 8$ since each of these contributions stands for 8 (positive and negative) n_φ's. Moreover the parameters NR, NT, NTS which are used to dimension certain arrays, must be specified in such a way that they are equal or larger than the greatest values of $N_{elem}+1$, N_p+1, $N_{ps}+1$, respectively.

The program $DPROF$ evaluates then the total antenna load and power deposition profiles including 3–d plots for the various damping mechanisms and a plot of $dP_{ABS}/r\,dr$. The CPU-time required by this part of the code is negligible.

Appendix A. Calling tree for the program *MATSOL*.

To initialize the program *FISIC* and the test program *FTEST* the following routines are called (underlined names refer to external routines; if nothing else is stated they are Cray routines):

PARA
BINI CUBSPL
PROFTD
MESH CL CZETA
CLAM CZETA
RTEN
RX
SURF
INIFFT CFFT2
GAUS
ANTC FFTDIR
SCNRM2
RX

Then the routine *MASTSOL* is called. We list the subroutines in the calling tree in alphabetic order:

```
MASTSOL CONX   RHS
        SOLV   ALU    BLOX   BNDAFP  CLAM     CZETA
                                     CLAMS
                                     FFTDIR
                                     RX
                                     SURF
                                     TENST    CTEN (d.a.)
                              OP     CL       CZETA
                                     CLAM     CZETA
                                     CLAM0
                                     CLAMS
                                     CP       CZETA
                                     DIFF
                                     FFTDIR
                                     FFTS
                                     OPADD    FFTS
                                     OPADDT   FFT (d.a.)
                                     RTEN
                                     RX
                                     SURF
                                     TENST    CTEN (d.a.)
                      CDOTU
                      CGEFAP ICAMAX
                      CGESLP
                RSOLV CCOPY
                      CDOTU
                      CGES2P CCOPY
                      SKPIPR
                VSOLV CCOPY
                      CDOTU
                      CGES2P CCOPY
```

Some of the routines listed here still refer to other subroutines:

FFTDIR	CFFT2		
FFTS	CFFT2		
CLAMS	CLAM	CZETA	
	CST	CL	CZETA
		RTEN	
SURF	BPOL	VALSPL	
	DTHETA	VALSPL	
	PROFTD		

Appendix B. Calling tree for the program *BALANCE*.

After the external data file provided previously by the program *MATSOL* has been read, the initialisation of the program *BALANCE* is, up to the routine *MESH* which is not called, the same as for the program *MATSOL*, which has been described in appendix A. Then the routines *DIAG*, *OUTPUT*, *ANTP* and, if $IBAL \neq 0$, *PABS* are called:

```
DIAG    CUBSPL
         EANT
         ERLHL
         FFTREV  CO6ECF
         GPLOT3
         MINMAX
         VALSPL
         ZREX    ZRT
OUTPUT   CL      CZETA
         CLAM    CZETA
         CLAMS
         FFTDIR
         GPLOT
         GPLOT3
         RTEN
         RX
         SURF
ANTP
```

PABS CL CZETA

CLAM CZETA

CLAM0

CP CZETA

CST CL CZETA

RTEN

ERL3D

MINMAX

PLOTTH

PPLOT

RX

SUMOD

SURF

ZEL ZRT

ZET ZRT

ZI1 ZRT

ZI2 ZRT

ZST ZRT

CO6ECF is a NAG Library, *MINMAX* a Erlgraf Library routine. The plotting routines *ERL3D*, *ERLHL*, *GPLOT*, *GPLOT*3, *PLOTTH* and *PPLOT* contain Erlgraf routines. See appendix A for the calling trees of *FFTDIR* and *SURF*. The routine *OUTPUT* is also called by the test program *FTEST*.

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