Information on Z_{eff} from the Sawtooth-Performances in the Center of Ohmic Tokamak Discharges

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Abstract

Achievement of information on the mean effective ion charge in the center of ohmic tokamak discharges from sawtooth-relaxations of the plasma is considered. This method is found to supply trustworthy results for usual tokamak parameters. While its application requires some effort in data analysis, it can provide a valuable determination of Z_{eff}-data, independent of the information from bremsstrahlung radiation losses of the plasma.

The mean effective ion charge, Z_{eff}, is an important parameter in the detailed description of the plasma performance in tokamak discharges. This quantity is often derived from the bremsstrahlung radiation losses of the plasma, but may also be obtained from information on the electrical plasma resistivity (see, for example, page 15 - 30 of ref [1]), with some discrimination to be made in the results of these two different methods (see appendix 1). Here the latter procedure is applied in considering the relaxations of the central plasma region of tokamak discharges immediately after sawtooth crashes[2]. Phenomenologically, relatively peaked radial profiles of the plasma temperature (or density) are suddenly flattened and then recover slowly again to the state before the crash, successively followed by the next sawtooth cycle (see fig. 1).

The power balance of the plasma electrons about the plasma center for the initial period of plasma recovery after the sawtooth collapse (see, e.g. ref.[3]), assuming an isotropic velocity distribution function of the electrons) is:

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_e kT_e\right) + div\left(\frac{3}{2} n_e \cdot kT_e \cdot \vec{v}\right) + (n_e \cdot kT_e) \cdot div\vec{v} + div\vec{q}_e = P_\Omega^{el} - P_{ei} - P_{rad} \qquad (1)$$

or (applying the continuity equation $\left(\frac{\partial n_e}{\partial t} + div(n_e \cdot \vec{v}) = 0\right)$):

$$\boxed{\frac{3}{2} n_e \cdot \frac{d(kT_e)}{dt} + (n_e \cdot kT_e) \cdot div\vec{v} + div\vec{q}_e = P_\Omega^{el} - P_{ei} - P_{rad}} \qquad (1a)$$

with the heat transport terms of the electrons on the left-hand side:

$$P^{el}_{conv.} = (n_e \cdot kT_e) \cdot div\,\vec{v} \quad \equiv: \quad \text{Convectively transported power density}$$

$$P^{el}_{cond.} = div\,\vec{q}_e \quad \equiv: \quad \text{Conductively transported power density}$$

and the heating terms appearing on the right-hand side:

P^{Ω}_{el} ≡: Ohmic power density deposition to the electrons

P_{ei} ≡: Power density transfer from the electrons to the ions

P_{rad} ≡: Power density losses of the electrons by radiation.

We now replace the individual terms of eq.(1a) with the following approximate forms and absolute units:

I.)

$$\boxed{\frac{3}{2} n_e \cdot \frac{d(kT_e)}{dt} \xrightarrow[(r\to 0)]{} 2.4 \times 10^{-19} \cdot n_{eo}(m^{-3}) \cdot T_{eo}(eV) \cdot \frac{\Delta T_{eo}(eV)/T_{eo}(eV)}{\Delta t\,(sec)} \left(\frac{Watt}{m^3}\right)} \quad (2)$$

where:

T_{eo} ≡: Electron temperature in the plasma center $(r\to 0)$

$\Delta T_{eo}/\Delta t$ ≡: See fig. 1

n_{eo} ≡: Electron density in the plasma center, usually derived from the interferometrically determined line-integrated density:

$$\overline{n_e} = \left(\frac{\int_{-a}^{+a} n_e(r) \cdot dr}{2a} \right) \qquad (a \equiv: \text{plasma radius})$$

through the experimentally determined relation[4]:

$$n_{eo} = (a_1 + a_2 \cdot q_a) \cdot \overline{n_e} \quad (2a)$$

where q_a is the cylindrical q given by:

$$q_a = \frac{B_{To}}{B_{pa}} \cdot \frac{a}{R_o} = \frac{10^7}{2} \cdot \frac{a^2(m)}{R_o(m)} \cdot \frac{B_{To}(T)}{I_{pl}(A)}$$

and the coefficients a_1 and a_2 are (for ASDEX):

	Hydrogen	Deuterium	(Helium)
a_1	0.89	0.62	0.675 for $\bar{n}_e \approx 1.5\text{x}10^{19}$ (m^{-3}) 0.70 for $\bar{n}_e \approx 3\ \text{x}10^{19}$ (m^{-3})
a_2	0.075	0.195	0.139 for $\bar{n}_e \approx 1.5\text{x}10^{19}$ (m^{-3}) 0.159 for $\bar{n}_e \approx 3\ \text{x}10^{19}$ (m^{-3})

II.)
$$P^{el}_{conv.} = (n_e kT_e)\cdot \mathrm{div}\,\vec{v} \rightarrow (n_e kT_e)\cdot(\mathrm{div}\,\vec{v})_r = (n_e kT_e)\cdot\left(\frac{v_r}{r} + \frac{\partial v_r}{\partial r}\right)$$

Applying the continuity equation again:

$$-\frac{\partial n_e}{\partial t} = \mathrm{div}(n_e\cdot\vec{v})_r = \frac{1}{r}\frac{\partial}{\partial r}(r\cdot n_e\cdot v_r) \tag{3a}$$

results in:

$$P^{el}_{conv.} \rightarrow -kT_e\left(\frac{\partial n_e}{\partial t} + v_r\cdot\frac{\partial n_e}{\partial r}\right) \tag{3b}$$

and also:

$$r\cdot n_e\cdot v_r = -\frac{r^2}{2}\cdot\frac{\partial n_e}{\partial t} + \int\frac{r}{2}\cdot\frac{\partial^2 n_e}{\partial t\cdot\partial r}\cdot dr \quad . \tag{3c}$$

Approximating the radial profiles of n_e and T_e about the plasma center after the sawtooth crash by:

$$n_e \approx n_{eo}\cdot\left(1-\left(\frac{r}{a}\right)^2\right)^{\alpha_n^{low}} \rightarrow n_{eo}\cdot\left(1-\alpha_n^{low}\cdot\left(\frac{r}{a}\right)^2\right)$$

$$T_e \approx T_{eo}\cdot\left(1-\left(\frac{r}{a}\right)^2\right)^{\alpha_T^{low}} \rightarrow T_{eo}\cdot\left(1-\alpha_T^{low}\cdot\left(\frac{r}{a}\right)^2\right)$$

and making use of eq. (3c) yields:

$$v_r \cdot \frac{\partial n_e}{\partial r} \approx \frac{\alpha_n^{low}}{\left(1-\alpha_n^{low}\cdot\left(\frac{r}{a}\right)^2\right)}\left[\frac{\partial n_{eo}}{\partial t}\left(\frac{r}{a}\right)^2 + \frac{3}{2}\left(\alpha_n^{low}\cdot\frac{\partial n_{eo}}{\partial t} + n_{eo}\cdot\frac{\partial \alpha_n^{low}}{\partial t}\right)\cdot\left(\frac{r}{a}\right)^4\right]$$

i.e.:

$$P^{el}_{conv.} \longrightarrow -kT_{eo}\cdot\left(1-\alpha_T^{low}\cdot\left(\frac{r}{a}\right)^2\right)\left[\frac{\partial n_{eo}}{\partial t} - n_{eo}\cdot\frac{\partial \alpha_n^{low}}{\partial t}\cdot\left(\frac{r}{a}\right)^2 + (\ldots.)\cdot\left(\frac{r}{a}\right)^4 \ldots\right]$$

For the central plasma region:

$$\boxed{P^{el}_{conv.} \xrightarrow[(r\to 0)]{} -kT_{eo}\cdot\frac{\partial n_{eo}}{\partial t} \longrightarrow -1.6\times 10^{-19}\cdot T_{eo}(eV)\cdot n_{eo}(m^{-3})\cdot\frac{\Delta n_{eo}/n_{eo}}{\Delta t(sec)}\left(\frac{Watt}{m^3}\right)} \qquad (3)$$

(with ASDEX-experiments suggesting:

$$0.1\times\frac{\Delta T_{eo}/T_{eo}}{\Delta t(sec)} \lesssim \frac{\Delta n_{eo}/n_{eo}}{\Delta t(sec)} \lesssim 0.5\times\frac{\Delta T_{eo}/T_{eo}}{\Delta t(sec)}$$

, however, the experimental $\left(\frac{\Delta n_{eo}/n_{eo}}{\Delta t}\right)$ -data sometimes exhibit inconveniently large scatter.)

III.)

$$P^{el}_{cond.} = \operatorname{div}\vec{q}_e \rightarrow (\operatorname{div}\vec{q}_e)_r = \frac{(q_e)_r}{r} + \frac{\partial}{\partial r}(q_e)_r$$

Substituting for the radial part of the electron heat flux density:

$$(q_e)_r = -\varkappa^e_\perp \cdot \frac{\partial (kT_e)}{\partial r} = -n_e \cdot \chi_e \cdot \frac{\partial (kT_e)}{\partial r} \quad ,$$

where: χ_e =: Electron heat diffusivity,
results in:

$$P^{el}_{cond.} \rightarrow -\frac{n_e \cdot \chi_e}{r} \cdot \frac{\partial (kT_e)}{\partial r} - \frac{\partial (n_e \cdot \chi_e)}{\partial r} \cdot \frac{\partial (kT_e)}{\partial r} - n_e \cdot \chi_e \cdot \frac{\partial^2 (kT_e)}{\partial r^2}$$

and with the radial profile approximation after the sawtooth crash:

$$n_e \rightarrow n_{eo} \cdot \left(1 - \alpha_n^{low} \cdot \left(\frac{r}{a}\right)^2\right)$$

$$T_e \rightarrow T_{eo} \cdot \left(1 - \alpha_T^{low} \cdot \left(\frac{r}{a}\right)^2\right)$$

$$\chi_e \rightarrow \chi_{eo} \cdot \left(1 + b \cdot \left(\frac{r}{a}\right)^2\right)$$

we obtain:

$$P^{el}_{cond} \rightarrow 4 \cdot n_{eo} \cdot \chi_{eo} \cdot (kT_{eo}) \cdot \frac{\alpha_T^{low}}{a^2}\left[1 + 2(b - \alpha_n^{low}) \cdot \left(\frac{r}{a}\right)^2 + (\ldots)\left(\frac{r}{a}\right)^4 \ldots\right].$$

For the central plasma in absolute units:

$$\boxed{P^{el}_{cond.} \xrightarrow{(r \to 0)} 6.4 \times 10^{-19} \cdot \frac{n_{eo}(m^{-3}) \cdot \chi_{eo}\left(\frac{m^2}{sec}\right) \cdot T_{eo}(eV) \cdot \alpha_T^{low}}{a^2(m)} \left(\frac{Watt}{m^3}\right)} \qquad (4)$$

where (assuming q=1 in plasma center after the sawtooth crash):

$$\boxed{\chi_{eo} = 1.61 \times 10^{22} \cdot \frac{B_{To}(T)}{n_{eo}(m^{-3}) \cdot T_{eo}(eV) \cdot A^{1/2}} \left(\frac{m^2}{sec}\right)} \qquad (4a)$$

(see eq.(7) of ref.[5])

and:

$$\boxed{\alpha_T^{low} \approx \alpha_T^{av} - \frac{1}{2}\left[\left(\frac{a}{r_i}\right)^2 - \alpha_T^{av}\right] \cdot \left(\frac{T_{eo}^{top} - T_{eo}^{low}}{T_{eo}^{av}}\right)} \qquad (4b)$$

(see appendix 2)

where: r_i =: Sawtooth inversion radius

α_T^{av} =: Time-averaged T_e-profile factor

IV.)

$$\boxed{P_{\Omega}^{el} \xrightarrow[(r \to 0)]{} \frac{j_o^2}{\sigma_{\parallel}} = 1.40 \times 10^8 \, \frac{Z_{eff} \cdot B_{To}^2(T) \cdot \ln\Lambda}{R_o^2(m) \cdot T_{eo}^{3/2}(eV)} \left(\frac{Watt}{m^3}\right)} \qquad (5)$$

where:

$$\sigma_{\parallel} = 1.81 \times 10^4 \cdot \frac{T_e^{3/2}(eV)}{Z_{eff} \cdot \ln\Lambda} \quad (Ohm \cdot m)^{-1} \qquad (5a)$$

$$\Lambda = 1.55 \times 10^{13} \cdot \frac{T_e^{3/2}(eV)}{Z^2 \cdot n_e^{1/2}(m^{-3})} \qquad (5b)$$

$$j_o = \frac{2 \cdot B_p(r \to 0)}{\mu_o \cdot (r \to 0)} = \frac{2 \cdot B_T(r \to 0)}{\mu_o \cdot R_o \cdot q(r \to 0)}$$

$$q(r) = \frac{r}{R} \cdot \frac{B_T}{B_p} \xrightarrow[(r \to 0)]{} 1$$ (after sawtooth crash)

V.)

$$P_{ei} = \sum_{\nu} \frac{\frac{3}{2} \cdot n_e \cdot k(T_e - T_i)}{\tau_\nu^{eq}}$$

With the equipartition time for the ion species ν (density n_i^ν):

$$\tau_\nu^{eq} = 3.14 \times 10^{14} \cdot \frac{A_\nu \cdot T_e^{3/2}(eV)}{Z_\nu^2 \cdot n_i^\nu \cdot (\ln\Lambda)_\nu} \quad (sec)$$

the power density transfer from electrons to plasma ions is approximated by:

$$\boxed{P_{ei} \xrightarrow[(r\to 0)]{} 7.65 \times 10^{-34} \cdot \frac{n_{eo}^2(m^{-3}) \cdot (T_{eo}(eV) - T_{io}(eV)) \cdot \ln\Lambda}{T_{eo}^{3/2}(eV)} \cdot F \left(\frac{Watt}{m^3}\right)} \quad (6)$$

where for deuterium or helium discharges with fully ionized ion species in the plasma center the correction factor F is approximately (see appendix 3):

$$F \equiv: \sum_{\nu} \left(\frac{n_{io}^\nu \cdot Z_\nu}{n_{eo}}\right) \cdot \frac{Z_\nu}{A_\nu} \longrightarrow \frac{1}{2} \sum_{\nu} \left(\frac{n_{io}^\nu \cdot Z_\nu}{n_{eo}}\right) = \frac{1}{2} \quad (6a)$$

i.e.:

$$\boxed{P_{ei}^{D/He} \xrightarrow[(r\to 0)]{} 3.82 \times 10^{-34} \cdot \frac{n_{eo}^2(m^{-3}) \cdot \ln\Lambda}{T_{eo}^{1/2}(eV)} \cdot \left(\frac{T_{eo} - T_{io}}{T_{eo}}\right) \left(\frac{Watt}{m^3}\right)} \quad (6b)$$

(For hydrogen discharges see appendix 3,

and for approximate evaluation of $\left(\frac{T_{eo} - T_{io}}{T_{eo}}\right)$ see eq.(12) of appendix 4.)

VI.) $P_{rad} = P_{ff} + P_{fb} + P_{bb} + \dots$ (7)

with:

P_{ff} ≡: (free-free)-bremsstrahlung radiation losses

P_{fb} ≡: (free-bound)-recombination radiation losses

P_{bb} ≡: (bound-bound)-line radiation losses.

Information on P_{rad} may be derived experimentally from the bolometric measurements, but completely general statements on the plasma radiation losses are hard to make analytically, except for the contribution:

$$P_{ff} \xrightarrow{(r\to 0)} 1.71 \times 10^{-38} \cdot n_{eo}^{2}(m^{-3}) \cdot T_{eo}^{1/2}(eV) \cdot \sum_{\nu} \left(\frac{Z_{\nu}^{2} \cdot n_{io}^{\nu}}{n_{eo}}\right) \left(\frac{Watt}{m^{3}}\right) . \quad (7a)$$

For T_{eo}-values of present day tokamaks this is, however, a small quantity as compared with P_{ei} (see eq. (6)):

$$\frac{P_{ff}}{P_{ei}} \xrightarrow{(r\to 0)} 2.2 \times 10^{-5} \cdot \frac{T_{eo}(eV)}{\ln\Lambda} \cdot \left(\frac{T_{eo}-T_{io}}{T_{eo}}\right)^{-1} \cdot \frac{\sum_{\nu}\left(\frac{Z_{\nu}^{2} \cdot n_{io}^{\nu}}{n_{eo}}\right)}{\sum_{\nu}\left(\frac{Z_{\nu}^{2} \cdot n_{io}^{\nu}}{n_{eo} \cdot A_{\nu}}\right)} \approx 10^{-5} \cdot T_{eo}(eV) \ll 1 \quad (7b)$$

(for representative values: $\ln\Lambda \approx 17$; $(T_{eo}-T_{io}) \approx 0.25 \times T_{eo}$; $A \approx 2$).

In discharges with high levels of impurities, the power losses by recombination plus line radiation may exceed the bremsstrahlung contribution by orders of magnitude (see appendix 5) - and the P_{rad}-contribution must definetively be taken into account in eq.(1). For simplicity, we will neglect P_{rad} in the following for tokamak discharges of interest.

Summarizing the results of eqs.(2) - (7), we obtain from eq. (1a) after few transformations:

$$\boxed{Z_{eff} = 1.714\times10^{-27}\cdot\left(\frac{R_o}{B_{To}}\right)^2\cdot n_{eo}\cdot T_{eo}\left[\frac{T_e^{3/2}}{\ln\Lambda}\left\{\left(\frac{\Delta T_{eo}/T_{eo}}{\Delta t}-\frac{2}{3}\cdot\frac{\Delta n_{eo}/n_{eo}}{\Delta t}\right)+2.67\cdot\frac{\chi_{eo}\cdot\alpha_T^{low}}{a^2}\right\}+3.19\times10^{-15}\cdot F\cdot n_{eo}\left(\frac{T_{eo}-T_{io}}{T_{eo}}\right)\right]} \tag{8}$$

with: B_{To} ≡: B_{To}(Tesla)

n_{eo}; n_{io} ≡: $n_{eo}(m^{-3})$; $n_{io}(m^{-3})$

T_{eo}; T_{io} ≡: T_{eo}(eV); T_{eo}(eV)

R_o ; a ≡: R_o(m); a(m)

Δt ≡: Δt(sec)

and: $\Lambda = 1.55\times10^{13}\cdot\frac{T_{eo}^{3/2}}{Z^2\cdot n_{eo}^{1/2}}$ (see eq.(5b))

$\chi_{eo} = 1.61\times10^{22}\cdot\frac{B_{To}}{n_{eo}\cdot T_{eo}\cdot A^{1/2}}\left(\frac{m^2}{sec}\right)$ (see eq.(4a))

$\alpha_T^{low} = \alpha_T^{av}-\frac{1}{2}\left[\left(\frac{a}{r_i}\right)^2-\alpha_T^{av}\right]\cdot\left(\frac{T_{eo}^{top}-T_{eo}^{low}}{T_{eo}^{av}}\right)$ (see appendix 2)

$$F = \begin{cases} 1/2 & \text{for deuterium/helium discharges} \\ 1\ (\to 1/2) & \text{for hydrogen discharges} \end{cases}$$ (see appendix 3)

$$\left(\frac{T_{eo}-T_{io}}{T_{eo}}\right) = \begin{cases} \leq \left[1+n_{eo}(\times10^{-19}\ m^3)\right]^{-1} \\ \left(\geq \left[1+(n_{eo}(\times10^{-19}\ m^3))^2\right]^{-1}\right) \end{cases}$$ (see eq.(10) of appendix 4)

i.e. for <u>deuterium/helium discharges in the ASDEX-tokamak</u> (R_o = 1.65m; a = 0.4m):

$$\boxed{Z_{eff} = 4.67\times10^{-27}\cdot\frac{n_{eo}\cdot T_{eo}}{B_{To}^2}\left[\frac{T_{eo}^{3/2}}{\ln\Lambda}\left\{\left(\frac{\Delta T_{eo}/T_{eo}}{\Delta t}-\frac{2}{3}\cdot\frac{\Delta n_{eo}/n_{eo}}{\Delta t}\right)+16.67\cdot\chi_{eo}\cdot\alpha_T^{low}\right\}+1.594\times10^{-15}\cdot n_{eo}\left(\frac{T_{eo}-T_{io}}{T_{eo}}\right)\right]} \tag{8a}$$

Examples of Z_{eff}-results derived with the discussed method from several ASDEX-discharges are presented in Table I and compared with corresponding values ($Z^* = \sum_{\nu}\left(\frac{Z_\nu^2 \cdot n_{io}^\nu}{n_{eo}}\right)$) from bremsstrahlung measurements[6] (see eq.(7a) and appendix 1) and Z_{eff}^{PS} from computer simulations of the plasma performance[7]. Discharges with high levels of impurities have been selected on purpose for these comparisons and the residual dependences of the results on Z and A occurring by the terms $\ln\Lambda$ and χ_{eo} were considered iteratively (Z_{eff}-values in brackets refer to the lower limits of α_T^{low} and $\left[(T_{eo}-T_{io})/T_{eo}\right]$).

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Appendix 1: Mean effective ion charge and plasma impurity content

According to the accessibility to diagnostic methods, two different definitions are in use for specifying the mean effective ion charge in plasmas:

a) from bremsstrahlung radiation losses

$$P_{ff} = 1.71 \times 10^{-38} \cdot n_e^2 (m^{-3}) \cdot T_e^{1/2} (eV) \cdot \sum_\nu \left[\frac{Z_\nu^2 \cdot n_i^\nu}{n_e}\right] \left(\frac{Watt}{m^3}\right) \qquad (7a)$$

an average effective ion charge may be specified as (see, for example, page 18 of ref.[1]):

$$\boxed{Z^* \equiv: \frac{\sum_\nu Z_\nu^2 \cdot n_i^\nu}{\sum_\nu Z_\nu \cdot n_i^\nu} = \frac{\sum_\nu Z_\nu^2 \cdot n_i^\nu}{n_e}} \qquad (9a)$$

and

b) from the electrical resistivity of the plasma (according to Spitzer and Härm; see, e.g., ref.[8] or page 20/21 of ref.[1]):

$$\sigma_{\parallel}^{-1} = 5.23 \times 10^{-5} \cdot \frac{\ln \Lambda}{T_e^{3/2} (eV)} \cdot Z_{eff} \; (Ohm \cdot m) \quad . \qquad (5a)$$

Taking electron-electron encounters into account one may obtain:

$$\boxed{Z_{eff} = Z^* \cdot \frac{\alpha}{0.51}}$$

where:

Z_ν	1	2	4	16	($Z_\nu \to \infty$)
α	0.51	0.44	0.38	0.32	0.30

.

In case of a single ion species plasma ($Z^* = Z_\nu$) and disregarding any enhancement by trapped particle, bootstrap current etc. effects, the mean effective ion charge derived from bremsstrahlung losses Z*, therefore, compares with the Spitzer-Härm value Z_{eff} as:

Z^*	1	2	4	16	$(Z^* \to \infty)$
Z_{eff}	1	1.73	3.0	10	$0.59 \times (Z^* \to \infty)$

Correlated with the mean effective ion charge is the impurity content of the respective plasma. Since a general discussion is somewhat complex, the situation will only be illustrated by the example of a hydrogen ($Z_o = 1$; $n_o = n_i^1 = n_1$) and of a helium ($Z_o = 2$; $n_o = n_i^2 = n_2$) plasma contaminated by just one ionized impurity species (Z; $n_i^\nu = n_Z$), i.e.:

$$Z^* \cdot n_e = \sum_\nu Z_\nu^2 \cdot n_i^\nu = Z_o^2 \cdot n_o + Z^2 \cdot n_Z \quad ;$$

$$n_e = Z_o \cdot n_o + Z \cdot n_Z \quad .$$

The impurity content of the plasma may be indicated either by:

c) Relative number of impurity ions in the hydrogen or helium plasma:

$$\boxed{\alpha_Z = \frac{n_Z}{n_o + n_Z} = \frac{(Z^* - Z_o) \cdot Z_o}{(Z - Z_o) \cdot (Z + Z_o - Z^*)}} \qquad (9c)$$

or by:

d) Relative number of "impurity electrons" in the hydrogen or helium plasma:

$$\boxed{\beta_Z = \frac{Z \cdot n_Z}{n_e} = \left(\frac{Z^* - Z_0}{Z - Z_0}\right)} \qquad (9d)$$

For instance, in case of fully stripped (Z=8) oxygen (or alternatively F_e^{XXI} (Z=20)) ions being the only impurities in the hydrogen or helium plasma we have the intercorrelation:

Z*	$\alpha^{H}_{O^{8+}}$	$\beta^{H}_{O^{8+}}$	$\alpha^{He}_{O^{8+}}$	$\beta^{He}_{O^{8+}}$	$\alpha^{H}_{Fe^{20+}}$	$\beta^{H}_{Fe^{20+}}$	$\alpha^{He}_{Fe^{20+}}$	$\beta^{He}_{Fe^{20+}}$	Z_{eff}
1	0.000	0.000			0.000	0.000			1.0
2	0.020	0.143	0.000	0.000	0.003	0.053	0.000	0.000	1.7
3	0.048	0.286	0.048	0.167	0.006	0.105	0.006	0.056	2.4
4	0.086	0.429	0.111	0.333	0.009	0.158	0.012	0.111	3.0
5	0.143	0.571	0.200	0.500	0.013	0.211	0.020	0.167	3.6
6	0.238	0.714	0.333	0.667	0.017_5	0.263	0.028	0.222	4.2
7	0.429	0.857	0.556	0.833	0.023	0.316	0.037	0.278	4.8
8	1.000	1.000	1.000	1.000	0.028	0.368	0.048	0.333	5.3

Appendix 2: T_e-profile factor after sawtooth crash: α_T^{low}

Consideration of the conductively transported power density from the central plasma region after the sawtooth crash (eq.(4)) requires the knowledge of the relevant T_e-profile factor α_T^{low} at this time. Diagnostics for T_e-profiles with high time- and space-resolution (such as ECE-diagnostics) may allow the derivation of this quantity directly from experimental data. Sufficiently reliable results proved to be achievable also in the following way:

Approximating the T_e-profiles about the plasma center

before the crash: $$T_e^{top}(r) \approx T_{eo}^{top} \cdot \left(1 - \alpha_T^{top} \cdot \left(\frac{r}{a}\right)^2\right)$$

after the crash: $$T_e^{low}(r) \approx T_{eo}^{low} \cdot \left(1 - \alpha_T^{low} \cdot \left(\frac{r}{a}\right)^2\right)$$

(with T_{eo}^{top} and T_{eo}^{low} according to fig. 1),

defining the inversion radius r_i by:

$$T_e^{top}(r_i) \approx T_{eo}^{top} \cdot \left(1 - \alpha_T^{top} \cdot \left(\frac{r_i}{a}\right)^2\right) \equiv T_{eo}^{low} \cdot \left(1 - \alpha_T^{low} \cdot \left(\frac{r_i}{a}\right)^2\right) \approx T_e^{low}(r_i)$$

and introducing time averaged values of:

central electron temperature: $$T_{eo}^{av} \approx \frac{T_{eo}^{top} + T_{eo}^{low}}{2}$$

profile factor α_T : $$\alpha_T^{av} \approx \frac{\alpha_T^{top} + \alpha_T^{low}}{2}$$

(information on α_T^{av} may be gained with the help of a profile fitting program attached to the computerized ECE-data-conversion into the $T_e(r)$-representation, which allows for averaging about several sawtooth periods) leads, after little rearrangements, to the approximate representation:

$$\boxed{\alpha_T^{low} \approx \alpha_T^{av} - \frac{1}{2}\left[\left(\frac{a}{r_i}\right)^2 - \alpha_T^{av}\right] \cdot \left(\frac{T_{eo}^{top} - T_{eo}^{low}}{T_{eo}^{av}}\right)} \qquad (4b)$$

Appendix 3: Correction factor F in power density transfer from electrons to ions

For most ion species (charge Z_ν, density n_i^ν, mass number A_ν), present in the central region of the plasmas considered here, the introduction of:

$$\frac{Z_\nu}{A_\nu} \approx \frac{1}{2} \longrightarrow \boxed{F = \sum_\nu \left(\frac{n_i^\nu \cdot Z_\nu}{n_e}\right) \cdot \frac{Z_\nu}{A_\nu} \approx \frac{1}{2} \sum_\nu \left(\frac{n_i^\nu \cdot Z_\nu}{n_e}\right) = \frac{1}{2}} \qquad (6a)$$

is a good or, at least, fair approximation. The main exceptions are hydrogen ($Z_1 = A_1 = 1$) plasmas (density n_H), where:

$$\boxed{F^H = \sum_\nu \left(\frac{n_i^\nu \cdot Z_\nu}{n_e}\right) \cdot \frac{Z_\nu}{A_\nu} \approx \frac{n_H}{n_e} + \frac{1}{2} \underbrace{\sum_\nu \frac{n_i^\nu \cdot Z_\nu}{n_e}}_{\text{(except hydrogen)}} = \frac{n_H}{n_e} + \frac{1}{2} \cdot \frac{n_e - n_H}{n_e} = \frac{1}{2}\left(1 + \frac{n_H}{n_e}\right)}$$

i.e.:

$$F^H \approx \begin{cases} 1 \text{ for "clean" hydrogen discharges } (n_H \approx n_e) \\ 1/2 \text{ for highly contaminated hydrogen discharges } (n_H \ll n_e) \end{cases}$$

For illustration, a hydrogen plasma contaminated by just one ionized impurity species (for example oxygen: $Z = 8$, $n_Z = n_{O^{8+}}$) may be considered, i.e. (compare eq.(9d) of appendix 1):

$$\frac{n_H}{n_e} \longrightarrow \left(1 - \frac{Z \cdot n_Z}{n_e}\right) = \left(1 - \frac{Z^* - 1}{Z - 1}\right) \curvearrowright \boxed{F_Z^H \approx 1 - \frac{1}{2}\left(\frac{Z^* - 1}{Z - 1}\right)}$$

$$\left(F^H_{O^{8+}} \approx \frac{15 - Z^*}{14}\right)$$

Appendix 4: Approximate evaluation of $(T_{eo} - T_{io})/T_{eo}$

Due to scatter in the experimentally obtained electron and ion temperatures the error margins of directly derived $((T_{eo} - T_{io})/T_{eo})$-values are usually intolerable for reliable evaluations of the power density transfer from plasma electrons to ions (eq.(7)). On the other hand, compilations of extensive experimental data from former ASDEX-discharges[9)] suggest that most of the widely scattering $((T_{eo} - T_{io})/T_{eo})_{exp}$ - results are confined to about the bounds (compare "dots" in fig. 2):

$$\left[1+\left(n_{eo}\times 10^{-19}\,m^{3}\right)^{2}\right]^{-1} \lesssim \left(\frac{T_{eo}-T_{io}}{T_{eo}}\right) \lesssim \left[1+\left(n_{eo}\times 10^{-19}\,m^{3}\right)\right]^{-1}$$

and computer simulations of the plasma performance[7)] indicate, that $((T_{eo} - T_{io})/T_{eo})_{calc}$-values (compare "crosses" in fig. 2) tend to the upper rather than to the lower limit:

$$\boxed{\left(\left[1+\left(n_{eo}\times 10^{-19}\,m^{3}\right)^{2}\right]^{-1}\right) \lesssim \left(\frac{T_{eo}-T_{io}}{T_{eo}}\right) \lesssim \left[1+\left(n_{eo}\times 10^{-19}\,m^{3}\right)\right]^{-1}} \qquad (10)$$

Appendix 5: Brief comment on power losses by impurity radiation

While eq.(7b) indicates power losses of the plasma electrons by bremsstrahlung radiation about a factor of 10^2 (for $T_{eo} \approx 10^3$ eV) below the level for being substantial in the general heat-balance equation, this deficit may be removed in highly contaminated plasmas by recombination plus line radiation from impurity ions. Since a general treatment of the problem is rather complex, the situation may again be illustrated by the special case when the plasma impurities consist of oxygen or iron ions only. Comprehensive calculation[10)] of the plasma radiation losses indicate that for typical tokamak paramters ($T_{eo} \approx 10^3$ (eV); $n_{eo} = 0.5 - 5 \times 10^{19}$ (m^{-3})) the total plasma radiation exceeds the only bremsstrahlung radiation even in:

a) "pure oxygen plasmas" by a factor $\left(\frac{P_{rad}}{P_{ff}}\right)_{O^{8+}} \lesssim 10$

b) "pure iron plasmas" by a factor $\left(\frac{P_{rad}}{P_{ff}}\right)_{(Fe^{19+} \leftrightarrow Fe^{23+})} \lesssim 10^2$

This suggests that neglection of the P_{rad}-term in eq.(1) may, indeed, be justified in low-Z_{eff} tokamak discharges.

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#	Gas	B_{To} (Tesla)	I_{Pl} (kA)	n_{eo} ($\times 10^{19} m^{-3}$)	T_{eo} (eV)	$\frac{\Delta T_{eo}}{T_{eo}\,\Delta t}$ (s^{-1})	$\frac{\Delta n_{eo}}{n_{eo}\,\Delta t}$ (s^{-1})	$\frac{T_{eo}^{top}-T_{eo}^{low}}{T_{eo}^{av}}$	r_i (m)	α_T^{av}	$\frac{T_{eo}-T_{io}}{T_{eo}}$	Z_{eff}	From other methods: Z^* (from P_{ff}^{el})	From other methods: Z_{eff}^{ps} (from plasma simulations)
20690	H_2	2.2	245	1.7	1250	17.2	(4.8)	0.081	0.06	4.4-(4.0)	0.37 -(0.26)	3.3 -(2.9)		
20688		2.2	315	1.6	1390	25.4	(6.2)	0.131	0.08_5	3.1-(2.9)	0.38_5-(0.28)	3.5 -(3.2)		
20695		2.2	375	1.5_5	1510	30.1	(3.7)	0.152	0.10	2.4-(2.2)	0.39 -(0.29)	3.9 -(3.6)		
20679		2.2	315	2.9	935	14.2	6.8	0.142	0.07_5	2.8-(2.6)	0.26 -(0.11)	1.7 -(1.2)		
20684		2.2	315	3.9	680	12.7	4.3	0.123	0.06_5	2.8-(2.6)	0.20_5-(0.06)	1.3 -(0.7)		
15580	D_2	2.18	310	1.3	1470	16.6	9.3	0.077	0.11_5	2.0-(1.9)	0.44 -(0.37)	3.0 -(2.8)	1.6	3.0
15593		2.18	410	3.1	940	13.0	2.7	0.082	0.14	1.4-(1.1)	0.25 -(0.10)	1.7 -(1.2)	2.8	1.7
18118		2.30	450	3.3	1160	12.5	2.1	0.092	0.13_5	1.2-(1.0)	0.23-(0.08_5)	2.0_5-(1.6)	2.0	
18118		2.30	250	4.5	890	5.6	1.6	0.083	0.10	1.9-(1.5)	0.18 -(0.05)	1.4_5-(0.8)	1.5 - 2.0	
15649		2.18	410	5.6	660	9.0	(1.2_5)	0.097	0.12_5	1.3-(1.2)	0.25 -(0.10)	1.1_5-(0.7)	1.4	1.2
17952	He	2.30	250	1.8	1190	11.8	(2.4)	0.050	0.07_5	2.6-(2.0)	0.36 -(0.23)	1.9 -(1.4)	1.0	
17952		2.30	320	1.6	1380	16.9	(1.6)	0.094	0.11_5	1.9-(1.5)	0.38 -(0.28)	2.2_5-(1.9)	2.0	
17952		2.30	390	1.5	1440	19.5	(1.9)	0.096	0.13_5	1.3-(0.9)	0.40 -(0.31)	2.1 -(1.8)	≥ 4	
17952		2.30	450	1.4	1480	25.7	(2.1)	0.133	0.15	0.9-(0.5)	0.41 -(0.33)	2.1 -(1.7)	≥ 4	

<u>Table I :</u> Examples of experimental Z_{eff}-results.

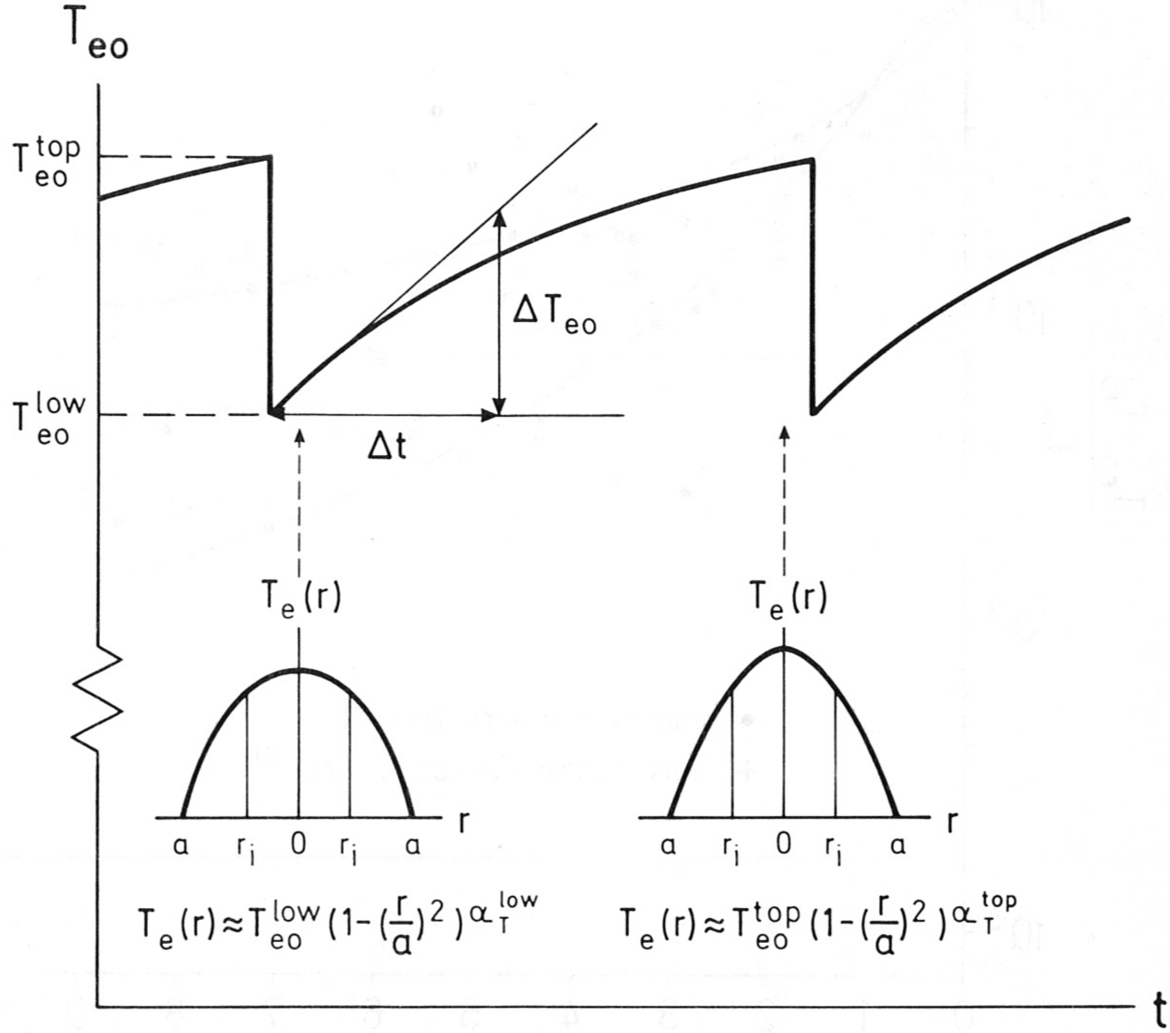

Fig. 1: T_e - Performance during Sawtooth-Period.

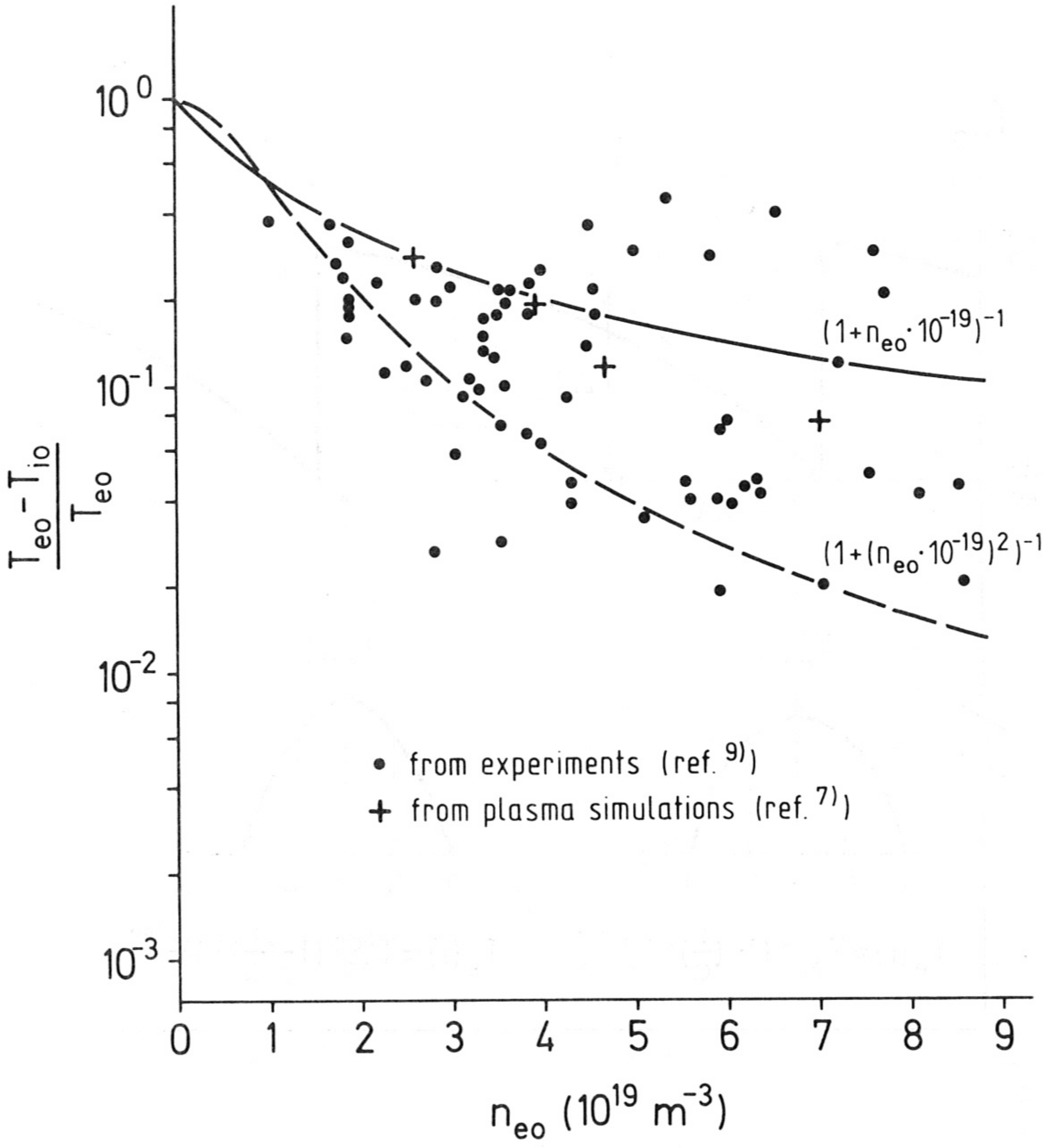

Fig. 2 : Approximate Representation of $((T_{eo} - T_{io}) / T_{eo})$ - Data.