MAXIMUM BOOTSTRAP CURRENT COMPATIBLE WITH BALLOONING-STABLE PRESSURE PROFILES

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Abstract:

Bei festen, angenommenen Verhältnis zwischen logarithmischen Temperatur- und Dichtegradient ist die lokale Stromdichte des neoklassischen Bootstrap-Stromes im Tokamak proportional zum Druckgradient und ist daher durch ideale Ballooning-Moden nach oben begrenzt. Gleichzeitig hängt die Stabilitätsgrenze für letztere Moden von der Stromverteilung ab. Unter der Annahme eines überall marginal stabilen Druckgradienten wird jene Verteilung eines von außen durch nicht-induktiven Stromtrieb aufgeprägten Saatstromes errechnet, bei der der Bootstrap-Strom den maximalen Bruchteil des Gesamtstromes trägt. Dabei wird das sogenannte s-α Modell eines kreisförmigen Tokamaks für die Auswertung des Ballooning-Kriteriums benützt. Unter der zusätzlichen Nebenbedingung $q \geq 1$ ergibt sich dafür unter günstigen Annahmen für Dichte- und Temperaturprofile ($d\,log\,T / d\,log\,n = 1$) für ein Aspektverhältnis von 3 ein maximaler Anteil des Bootstrap-Stromes von 50% bei q_a=2 bzw. 66% bei q_a=3.

Maximum Bootstrap Current Compatible with Ballooning-stable Pressure Profiles

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1. Introduction

Steady-state operation is a desirable feature for the Next Step Machines and possibly necessary for a tokamak fusion reactor/1/, /2/. For the foreseeable device dimensions and the expected operating ranges of plasma density, non-inductive drive of the total plasma current by HF waves or beam injection would claim, however, a substantial fraction of the total electric power generated. The best hope at present therefore lies in a combination of diffusion-driven bootstrap current /3/, /4/ and non-inductively driven seed current.

The toroidal bootstrap current density on a given flux surface is composed of contributions depending on the gradients of T_i, T_e and n_e. For $T_e = T_i = T$ and $\eta = n\, dT / (T\, dn) \geq 0$ it increases proportionally to the pressure gradient dp / dr and hence will be limited by pressure-driven MHD instabilities. It is therefore of interest to investigate the maximum contribution to the total plasma current that can be expected from the bootstrap current under the condition of MHD stability.

In the following we consider only ballooning modes, since the limits imposed by them on the total toroidal beta $<\beta_t>$ are in reasonable agreement with experimental observations /5/ and since reactor operation close to these limits promises intrinsic stabilization of thermonuclear burn without large-scale fluctuations in the plasma parameters. The evaluation of the ballooning mode criterion conforms to that in ref. /6/ by being restricted to a geometry of circular flux surfaces and by taking into account other modes only through limiting the safety factor q everywhere to values ≥ 1.

2. Physical Model and Mathematical Analysis

Following Rosenbluth et al. /7/, we can write the bootstrap current density j_{bt} in the banana regime as

$$j_{bt}(r) = -\frac{1}{B_\Theta} \sqrt{r/R} \frac{dp}{dr} \left(2{,}44 \frac{1}{1+\eta} + 0{,}13 \frac{\eta}{1+\eta} \right) \qquad \text{... (1)}$$

on the assumptions described in the introduction. The restriction to the banana regime is justified since we are interested in the reactor parameter range, and since we shall show later that for realistic values of q_a the optimum current density distributions make a vanishing bootstrap contribution in the core region.

To obtain a significant bootstrap current, it is obviously important that the fraction $1/(1+\eta)$ of the pressure gradient which is due to the density gradient be as large as possible. Usual experimental conditions in strongly heated tokamaks do not satisfy this requirement. Very peaked density profiles have been obtained, however, in the S-regime shots of TFTR /8/, which have in fact furnished the most conclusive evidence so far of the existence of the bootstrap current. Recently, strongly peaked density profiles during beam injection were also realized by continuous pellet injection in ASDEX /9/. Both of these experimental situations also showed significant improvement over the usual L-regime energy confinement values, which indicates that the realization of η-values $\approx$ 1 is also conducive for the attainment of breakeven conditions.

To lowest order in an aspect ratio expansion the limit on the pressure gradient given by ideal ballooning modes can be expressed as

$$\left(\frac{dp}{dr}\right) = -\frac{B_t^2}{2\mu_0 R q^2} \cdot \alpha(s) \qquad \text{... (2)}$$

in terms of the safety factor $q = B_t r / (R B_\theta)$, the minor and major radii r and R of the flux surface considered and the toroidal and poloidal magnetic field strengths B_t and B_θ. Following ref. /6/, we approximate the function α /10/ of the shear parameter $s = r\,dq / (q\,dr)$ by $\alpha = s/1.67$.

The bootstrap current density for a pressure gradient at the ballooning mode limit thus becomes

$$j_{bt}(r) = \frac{\gamma B_t}{2 \mu_0 q^2} \sqrt{r/R} \frac{dq}{dr} \qquad \ldots (3)$$

with $\gamma = (1.46 + 0.08\,\eta) / (1 + \eta)$. The profile of the safety factor q(r) itself is in turn determined by the total current density, which is composed of the bootstrap current j_{bt} and the externally driven seed current j_s. It is convenient to introduce the average total current density within the radius r as new dependent variable

$$K(r) = \frac{2}{r^2} \int_0^r r' (j_{bt} + j_s)\, dr' ,$$

in terms of which the bootstrap current distribution at marginal ballooning stability is determined by the linear differential equation

$$\frac{dK}{dr} = 2 \frac{j_s - K}{r\left(1 + \frac{\gamma}{2}\sqrt{R/r}\right)} . \qquad \ldots (4)$$

We seek, among all the distributions compatible with equ. (4), that which corresponds to the smallest fraction of non-bootstrap driven current

$$\kappa_s = \int_0^a j_s(r)\, r\, dr \Big/ \int_0^a K(r)\, r\, dr$$

and satisfies the requirement $q \geq 1$ everywhere. The latter restriction, in fact, introduces into this problem a dependence on q_a, as $q = 2 B_t/(\mu_0 R\, K(r))$, so that $K(r) \leq K(a)/q_a$. The function to be varied when seeking the minimum of κ_s is the spatial distribution of the seed current density $j_s(r)$.

The solution to equ. (4) is given by

$$K^* = e^{-F} \left(j_s^*(0) + \int_0^r \frac{2 j_s^*(r)}{r\left(1 + \frac{\gamma}{2}\sqrt{\frac{R}{r}}\right)} e^F dr \right) \qquad \ldots (5)$$

with

$$F(r) = \int_0^r \frac{2\,dr}{r\left(1 + \frac{\gamma}{2}\sqrt{\frac{R}{r}}\right)}$$

and with asterisks being used to designate current densities normalized by $2\,B_t/(\mu_0 R)$. According to equ. 5, the contribution $dI_s\,/\,dr$ to the seed current at a radius r_s is amplified by a factor

$$a^2 \pi\, d\,K(a)\,/\,d\,I_s = \frac{a^2\,/\,r_s^2}{\left(1 + \frac{\gamma}{2}\sqrt{\frac{R}{r_s}}\right)} \exp\left\{-\int_{r_s}^{a} \frac{2\,dr}{r\left(1 + \frac{\gamma}{2}\sqrt{\frac{R}{r}}\right)}\right\}$$

which, for spatially constant γ, increases monotonically with decreasing r_s. Optimum use of the seed current is thus made if it is concentrated as close as possible to the axis. In compatibility with the limitation $q\,(r) \geq 1$ we obtain, for a given q_a, as seed current distribution minimizing κ_s

$$j_s^*(r) \equiv 1 \text{ for } 0 \leq r \leq r_a\,,$$

$$j_s^*(r) \equiv 0 \text{ for } r_a < r \leq a\,.$$

As, according to equs. (1) and (2), no pressure gradient and hence no bootstrap current either is possible over a shear-free region, also the total dimensionless current density is also constant and equal to 1 for $0 \leq r \leq r_a$.

3. Results and Discussion

The distribution of the total current density $j\,(r) = j_s + j_{bt}$ following from the solution of equ. 4 for the optimal seed current distribution described above is shown in Fig. 1, together with the corresponding pressure distribution $p\,(r)$ for the case of an aspect ratio $A = R/a = 3$, $q_a = 2$ and $\gamma = 0.75$. The latter corresponds to an optimistic, though not unreasonable value of $\eta = 1$. The current densities obtained in this way have in general discontinuities at both the $q = 1$ surface and at $r = a$ and will therefore presumably be unstable to other modes - such as tearing or external kink modes - not discussed here.

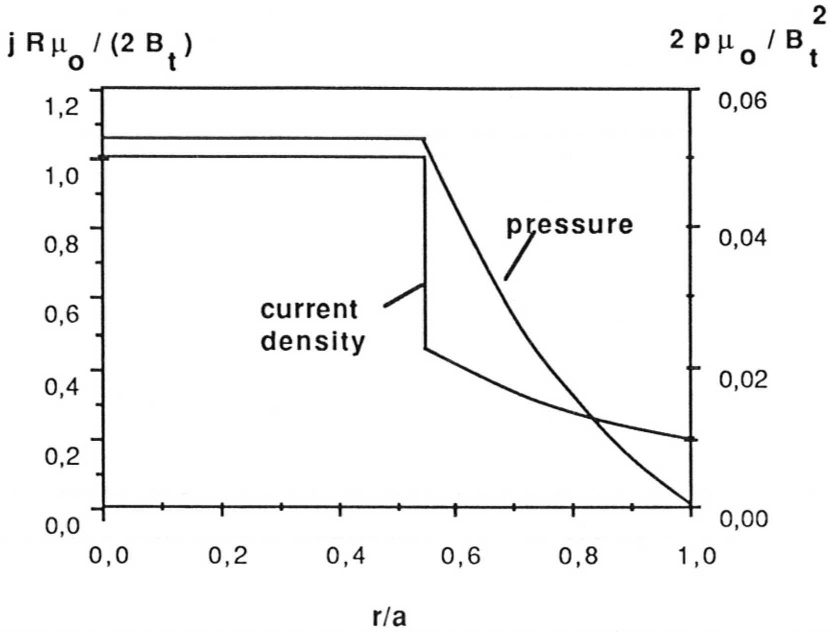

Fig.1: Pressure and current density profiles maximizing the bootstrap contribution to the total plasma current under the restriction of ballooning stability and $q \geq 1$. For A=3, $q_a = 2$, $\gamma = 0.75$.

The fraction I_s/I_p of externally applied seed current required according to the present criteria is a function of A, q_a and γ illustrated in Fig. 2. In particular it strongly decreases with increasing q_a, reaching values below 1/3 for A = 3, γ = 0.75 and $q_a \geq 3$. The even more favourable curve γ = 1.5 also shown corresponds to the unrealistic limiting case of constant plasma temperature.

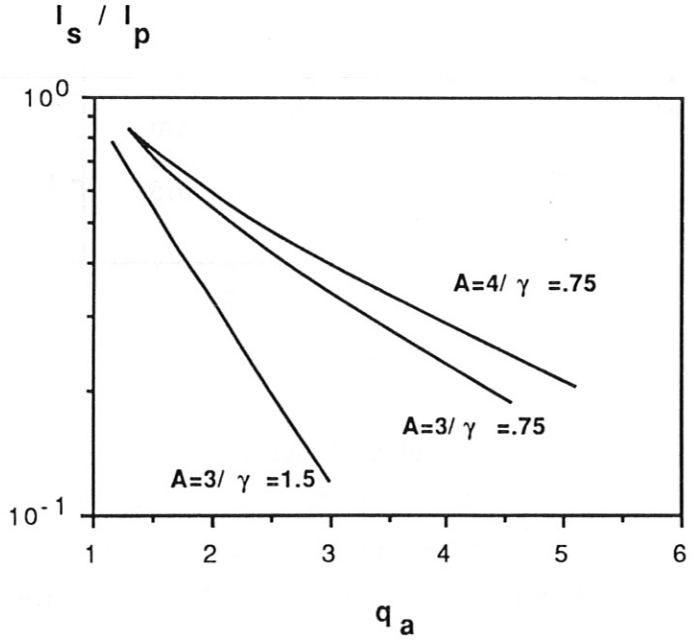

Fig.2: Required seed current contribution I_s to total plasma current I_p for optimized, ballooning mode stable current density and pressure profiles as a function of q_a , for different combinations of A and γ.

The current density distributions obtained in this way do not give absolutely the highest values of $\langle\beta_t\rangle$ compatible with ballooning mode stability and the limitation $q \geq 1$. The latter would be realized by a homogeneous distribution of the total plasma current inside the $q = 1$ surface /6/ and would therefore be even more prone to the other instabilities mentioned above. In fact, experimental data on $<\beta_t>$-limits are approximately 30% below this absolute optimum, fitting rather well the scaling predicted by /6/ and /11/

$$<\beta_t> = c_{Troyon} \, I_p/(a \, B_t) \quad [\%, MA, m, T] \quad \dots (6)$$

with a value of $c_{Troyon} \approx 2.8$ /5/. Figure 3 shows, for A=3 and $\gamma = 0.75$, the $<\beta_t>$ limits corresponding to the two cases of optimum current distribution for ballooning stability and of maximum fraction of bootstrap-driven current, together with the empirically confirmed predictions of equ. (6). Being closer to realistic distributions of j(r), stable against other modes as well, our bootstrap current distributions yield $<\beta_t>$-limits rather similar to those actually achieved in experiments.

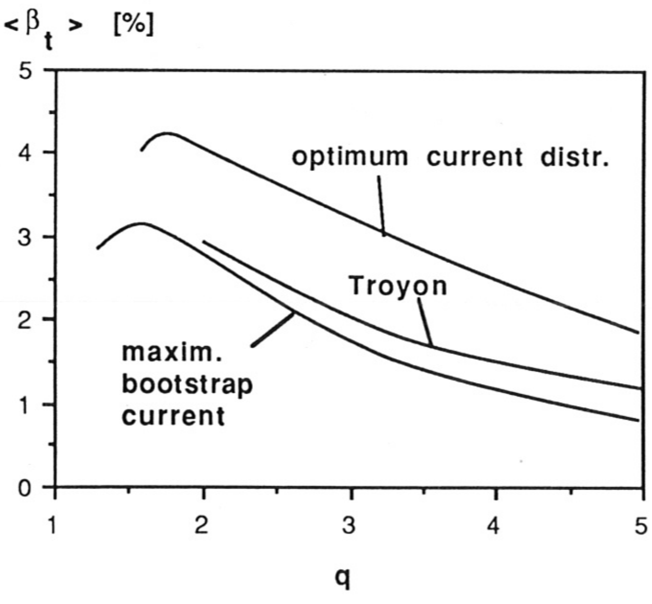

Fig.3: Limits on $<\beta_t>$ given by ideal ballooning modes as function of q_a for a value of A = 3. Also shown is the Troyon limit for a value of $c_{Troyon} = 2.8$.

According to the present model, we might therefore hope for a contribution from bootstrap currents in the range of 1/2 to 2/3 of the total plasma current, with rather modest losses in the maximum achievable $<\beta_t>$. This is conditional on realizing rather peaked density profiles. Even larger contributions from bootstrap currents (as well as larger stable values of $<\beta_t>$) could be hoped for if $q(0) < 1$ (as indicated by some experiments) or access to the second regime of stability could be achieved. Obviously, the above calculations will have to be extended to include the effects of noncircular plasma cross-sections and corrections of higher order in inverse aspect ratio.

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