A Method to measure the Suprathermal Density Distribution by Electron Cyclotron Emission

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Suggestion for Measurement of Suprathermal Density Distribution by Electron Cyclotron Emission

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Abstract

Electron cyclotron emission spectra of suprathermal electrons in a thermal main plasma are calculated. It is shown that for direction of observation oblique to the magnetic field, which decays in direction to the receiver, one may obtain information on the spatial density distribution of the suprathermal electrons from those spectra.

Introduction:

Radial electron temperature profiles are routinely measured in toroidal fusion experiments via electron cyclotron emission (ECE). Observation of cyclotron emission is usually within the horizontal midplane perpendicular to the main magnetic field.

Characteristic deviations of the temperature profiles are observed compared to those measured by other diagnostics e.g. Thomson scattering. The deviations can be partly attributed to the presence of a certain amount of suprathermal electrons within the thermalized main bulk:

Due to the relativistic change of mass, the emission frequency of a suprathermal electron is downshifted below the gyrofrequency at the point of emission. So the point of emission cannot be deduced directly from the emission frequency.

The paper shows that this lack of information can be overcome by observation in an oblique dierction with respect to the main magnetic field. The loci of emission can be deduced from the emission spectrum under these conditions, and the radial density distribution of the suprathermal electrons can be derived.

The plasma is considered with respect to the electrons as consisting of a thermal main part and a small optically thin suprathermal part.

The thermal part is described on the basis of temperature and density profiles as measured via Thomson scattering.

Because there is still no reference from other diagnostics on energy and density distribution of suprathermal electrons, certain assumptions had to be made.The following ones seemed most practicable [1] :

(1) Exponential shape of energy distribution function, being zero below a certain energy (see Fig.1),
(2) ratio $v_{\|}/v_{\perp} = const$,
(3) arbitrary density profile.

The emission of the thermal main part is calculated by means of the optical depth and Kirchhoff's law,that of the suprathermal part by means of the single particle radiation formula. The latter emission is partly absorbed in the thermal background on its way to the observer. For the optical depth τ of the thermal plasma the following expressions, calculated by several authors, are used :

At direction of observation $\perp B_0$ (ordinary (o) and extraordinary (x) mode):

$$\tau_{o,1} = \pi^2 \left(1 - \frac{\omega_p^2}{\omega_b^2}\right)^{\frac{1}{2}} \frac{\omega_p^2}{\omega_b^2} \frac{kT}{m_0 c^2} \frac{R}{\lambda} \qquad [2]$$

$$\tau_{x,1} = \begin{cases} \frac{2\pi}{e}\left(\frac{3\pi}{e}\right)^{\frac{1}{2}} \frac{\omega_p^2}{\omega_b^2} \frac{R}{\lambda} & \text{for } \frac{1}{2}\frac{\omega_p^2}{\omega_b^2}\frac{c^2}{v_t^2} < 1 \\ \frac{4\pi}{e}\left(\frac{3\pi}{2e}\right)^{\frac{1}{2}} \left(2 - \frac{\omega_p^2}{\omega_b^2}\right)^{\frac{3}{2}} \frac{\omega_b^2}{\omega_p^2} \left(\frac{kT}{m_0 c^2}\right)^2 \frac{R}{\lambda} & \text{for } \frac{1}{2}\frac{\omega_p^2}{\omega_b^2}\frac{c^2}{v_t^2} > 1 \end{cases} \qquad [3,4]$$

$$\tau_{o,2} = \pi^{\frac{3}{2}} \left(1 - \frac{1}{4}\frac{\omega_p^2}{\omega_b^2}\right) \left(\frac{kT}{m_0 c^2}\right)^2 \frac{R}{\lambda} \qquad [2]$$

$$\tau_{x,2} = 2\pi^2 \left| \frac{6 - \frac{\omega_p^2}{\omega_b^2}}{6 - 2\frac{\omega_p^2}{\omega_b^2}} \right|^2 \left| \frac{4\left(1 - \frac{\omega_p^2}{4\omega_b^2}\right)^2 - 1}{3 - \frac{\omega_p^2}{\omega_b^2}} \right|^{\frac{1}{2}} \frac{\omega_p^2}{\omega_b^2} \frac{kT}{m_0 c^2} \frac{R}{\lambda} \qquad [5]$$

$$\tau_{o,l>2} = .15 \frac{kT}{m_0 c^2} \tau_{x,l>2} \qquad [6]$$

$$\tau_{x,l>2} = \frac{\pi}{2c} \frac{\omega_p^2}{\omega_c} \frac{l^{2l-2}}{(l-1)!} \left(\frac{kT}{2 m_0 c^2}\right)^{l-1} R \qquad [6]$$

($\lambda = 2\pi c/\omega_b$, $\omega_b = eB/m_0$, $R =$ big radius of the torus)

At direction of observation Θ to B_0 :

$$\tau \rightarrow \tau \cdot (1 + \cos^2\Theta)(\sin\Theta)^{2l-2} \qquad [6]$$

The calculation is done onedimensional with sheath model.

Calculation of suprathermal emission:

The total radiation of an electron per unit of solid angle into direction Θ is [7]:

$$\xi_{o,x} = \frac{e^2\omega^2}{8\pi^2\epsilon_0 c}\sum_{l=1}^{\infty}\left\{ \begin{matrix} (\frac{cos\Theta-\beta_{\|}}{sin\Theta})^2 J_l^2(x) \\ \beta_\perp^2 J_l'^2(x) \end{matrix} \right. [W] \quad , \quad x = \frac{l\beta_\perp sin\Theta}{1-\beta_{\|}cos\Theta},$$

where the upper term in the bracket belongs to the o-mode, the lower to the x-mode. The distribution function as assumed in (1) is :

$$f(E) = \frac{1}{E_O}\cdot e^{-\frac{E-E_{CO}}{E_O}} \quad \text{for} \quad E \geq E_{CO},$$

E is the kinetic energy of the electron. The emission of an electron gas of density n (radiation power per unit of volume, frequency, and solid angle into direction Θ) is :

$$j(\omega) = n\int_{E=0}^{\infty} \xi(\beta_\perp,\beta_{\|},\Theta)\cdot\delta(\frac{l\omega_b}{\gamma(1-\beta_{\|}cos\Theta)}-\omega)\cdot f(E)dE,$$

$$E = m_0c^2(\gamma-1) \quad , \quad \beta_{\|}^2+\beta_\perp^2 = 1-\frac{1}{\gamma^2} \quad , \quad \beta_{\|} = q\beta_\perp = \sqrt{\frac{1-\frac{1}{\gamma^2}}{1+\frac{1}{q^2}}}$$

Taking the inverse of the relativistic γ as new variable we get

$$u = \frac{1}{\gamma} \quad , \quad dE = -\frac{m_0c^2}{u^2}du \quad , \quad \alpha(u) = 1-\beta_{\|}cos\Theta = 1-\sqrt{\frac{1-u^2}{1+\frac{1}{q^2}}}cos\Theta$$

$$j(\omega) = -n\frac{m_0c^2}{l\omega_b}\int_{u_c}^{0}\xi\cdot\delta(u-F\alpha(u))\cdot\frac{f(u)du}{u^2} \quad , \quad F = \frac{\omega}{l\omega_b}$$

$$= n\frac{m_0c^2}{l\omega_b}\int_{0}^{u_c}\xi\cdot\delta(g(u))\cdot\frac{f(u)du}{u^2}$$

$$u_c = 1/(1+E_{CO}/m_Oc^2), \quad u_1 : \text{root of } g(u) = 0$$

$$\delta(g(u)) = \frac{\delta(u-u_1)}{g'(u_1)}$$

$$j(\omega) = n\frac{m_0c^2}{l\omega_b}\cdot\frac{1}{g'(u_1)}\left(\frac{\xi\cdot f}{u^2}\right)_{u=u_1} \quad \text{for} \quad u_1 < u_c$$

$$u_1 = F\cdot\frac{1+q^2-q\cos\Theta\sqrt{1+q^2-F^2(1+q^2\sin^2\Theta)}}{1+q^2+F^2q^2\cos^2\Theta}$$

For $\Theta = \pi/2$ $\alpha(u) = 1$, $u_1 = F$ and $g' = 1$. Fig.2 shows the first harmonics (x-mode upper, o-mode lower curve) calculated in this way, at $\Theta = 90^0$, fig.3 at $\Theta = 110^0$ (=the gyrating electron removes from the observer).

For $\Theta < \pi/2$ this method of integration fails , because for

$$F = \sqrt{\frac{1+q^2}{1+q^2\sin^2\Theta}} = F_R(\text{see below})$$

the root in the expression for u_1 gets imaginary, $u_1 = 1/F$ and $g'(u_1) = 0$. $j(\omega)$ is then calculated directly from the line radiation :

$$j(\omega) = \xi\cdot\left|\frac{dn}{d\omega}\right| \quad , \quad \left|\frac{dn}{d\omega}\right| = \frac{dn}{dE}\cdot\left|\frac{dE}{d\omega}\right| = n\cdot f(E)\cdot\left|\frac{dE}{d\omega}\right|,$$

$$\omega = \frac{l\omega_b}{\gamma - a\sqrt{\gamma^2-1}} \quad , \quad a = \frac{\cos\Theta}{\sqrt{1+\frac{1}{q^2}}},$$

$$\left|\frac{d\omega}{dE}\right| = \frac{l\omega_b}{m_0c^2}\frac{\pm 1}{\gamma^2(1-\beta_\parallel\cos\Theta)^2}\left(1-\frac{a\gamma}{\sqrt{\gamma^2-1}}\right) \quad \text{for} \quad \begin{cases}\gamma < F_R\\ \gamma > F_R\end{cases}.$$

Fig.4 shows the lines ξ for the o-mode, calculated in this latter way, for growing energy E starting at $E_{CO} = 0$): at first the positiv Dopplershift dominates (the gyrating electrons approach the observer) the relativistic decrease of frequency, it is $\omega/\omega_b > 1$. For

$$E > E_R = m_0c^2\cdot(F_R - 1)$$

the resulting increase of frequency decreases again (in order to reach F_R, E_{CO} must be not greater than E_R, see.fig.6). E_R is the energy, where $g' = 0$ holds. The line density there tends to infinity, caused by the assumption $q = const$.

For

$$E > m_0 c^2 \cdot \frac{2a^2}{1-a^2}$$

$\omega/\omega_b < 1$ holds.The curves show $j(\omega)$ for both modes. The curve for the o-mode shows a zero at

$$F = \frac{\sqrt{1-(1+\frac{1}{q^2})cos^2\Theta}}{sin^2\Theta} = F_O(\text{see fig.5})$$

($\beta_{\|} = cos\Theta$ in the formula for ξ_o).

Fig.5 shows F_R and F_O as functions of q for $\Theta = 70^0$. From $q \approx 2$ on F_R is already near its asymptotic value $1/sin\Theta$; so even if electrons with a distribution in pitchangle are present, a pronounced maximum of emission near $1/sin\Theta$ can be expected.

This maximum may play an important role in the experimental investigation of the radial distribution of suprathermal electrons: because its frequency is greater than $l\omega_b$ at the point of emission , no reabsorption by the thermal background plasma takes place on its way outside to the observer across the decaying B-field, and as its frequency has a nearly constant distance to $l\omega_b(r)$ for $q > 2$ (for $q = const$ the distance is exactly constant),one can get an image of the suprathermal density distribution superposed to the thermal temperature profile obtained from the measured spectrum, at least in the case when its energy distribution is independent of radius.

Fig.7 to 10 show computed profiles (by means of $r \sim 1/F$ from the frequency spectra). The lower curves represent the shape of the input suprathermal density distribution, the upper show the addition of the undamped suprathermal emission upon the thermal one (dashed). The deviation from the thermal profile in fig.9 and the maximum in fig.10 at $r \approx 10cm$ are caused by decreased-frequency suprathermal emission, which is absorbed in the outer region at low enough B-field by the thermal plasma, but because of the low density and temperature at those radii only to a small extent. It is this activity at the righthand profile slope which can be measured alone in the case $\Theta = \pi/2$.

In reality the energy distribution of the suprathermals will depend more or less on the radius; but a measurement at $\Theta \neq \pi/2$ should at least give an indication on the presence of such electrons with $v_{\|} \neq 0$.

Result:

Observing electron cyclotron emission of electrons having equal pitch angles (i.e. $q = v_{\|}/v_{\perp} = const$) at directions deviating from 90 degrees with respect to the magnetic field the spectral intensity gets infinity at that kinetic electron energy, where the increase of the positive Dopplershift is compensated by the relativistic decrease of the gyro-frequency. Radiation of this frequency(F_R) is not reabsorbed on its way outside across decreasing B-field by the thermal background plasma because of $F_R > 1$. F_R depends only on Θ and q. As it approaches its asymptotic value $1/sin\Theta$ already near $q \approx 2$, one can obtain the emission radius from the emission maximum on the frequency axis and get an image of the suprathermal density distribution superposed to the thermal temperature profile .

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[7] G.Bekefi, Radiation Prcesses in Plasmas J.Wiley,N.Y.1966

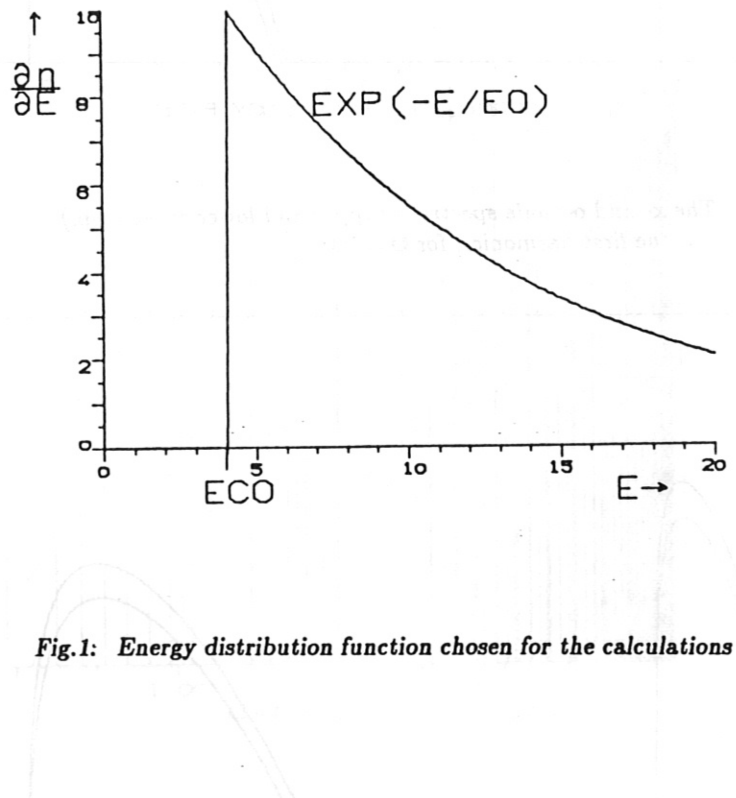


Fig.1: Energy distribution function chosen for the calculations

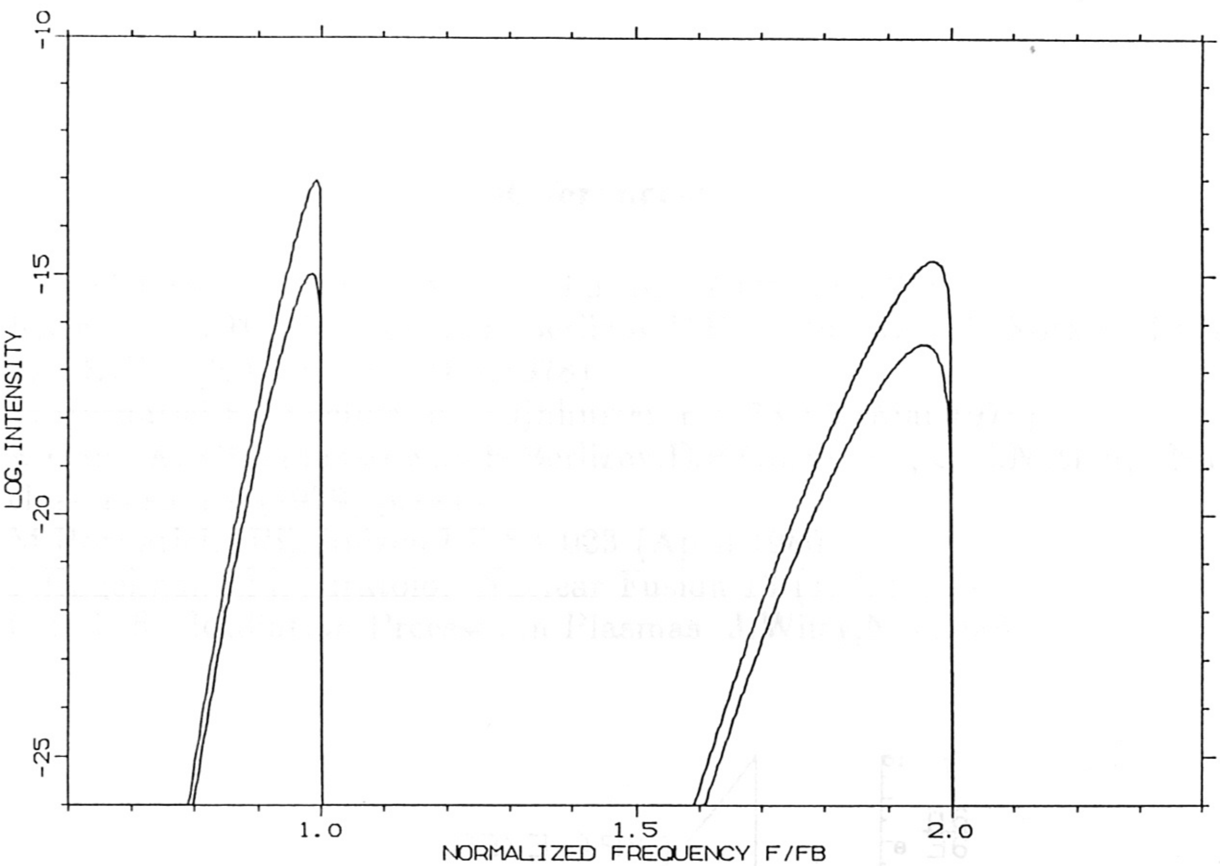


***Fig.2:* The x- and o-mode spectrum (upper and lower curve resp.) at the first harmonics, for $\Theta = 90^0$**

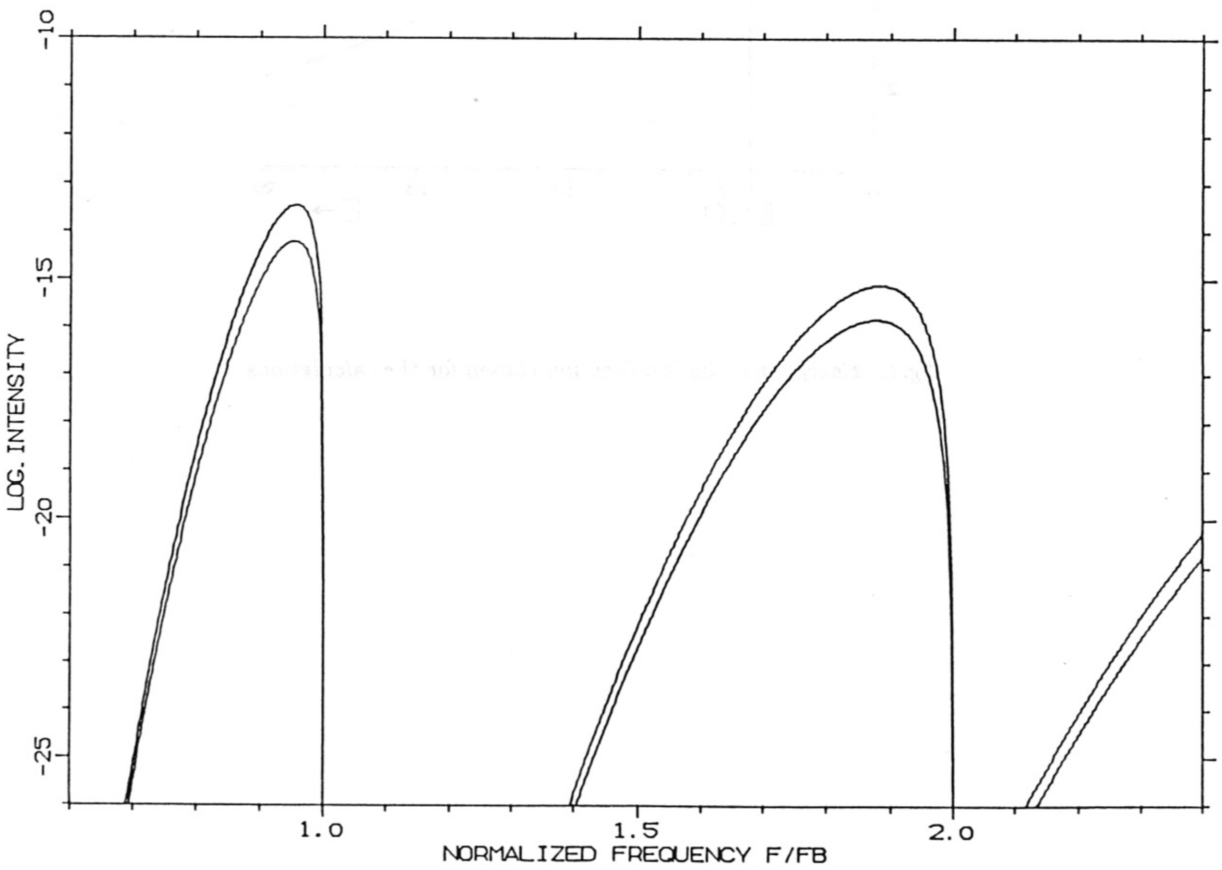


***Fig.3:* as fig.2, but for $\Theta = 110^0$**

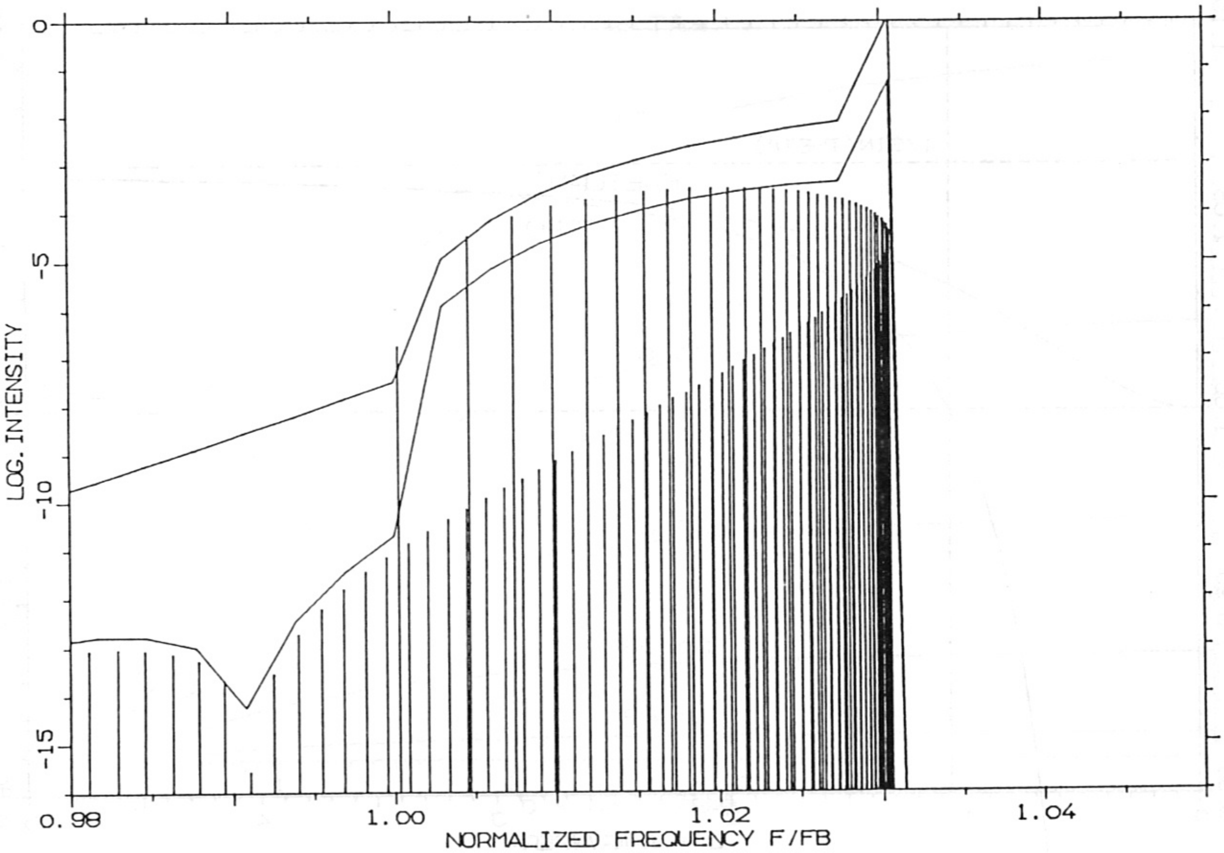


Fig.4: Illustration of the calculation of the spectrum (upper curve x-mode, lower one o-mode) from the spectral lines ξ, which are shown for the o-mode; $\Theta = 70^0$

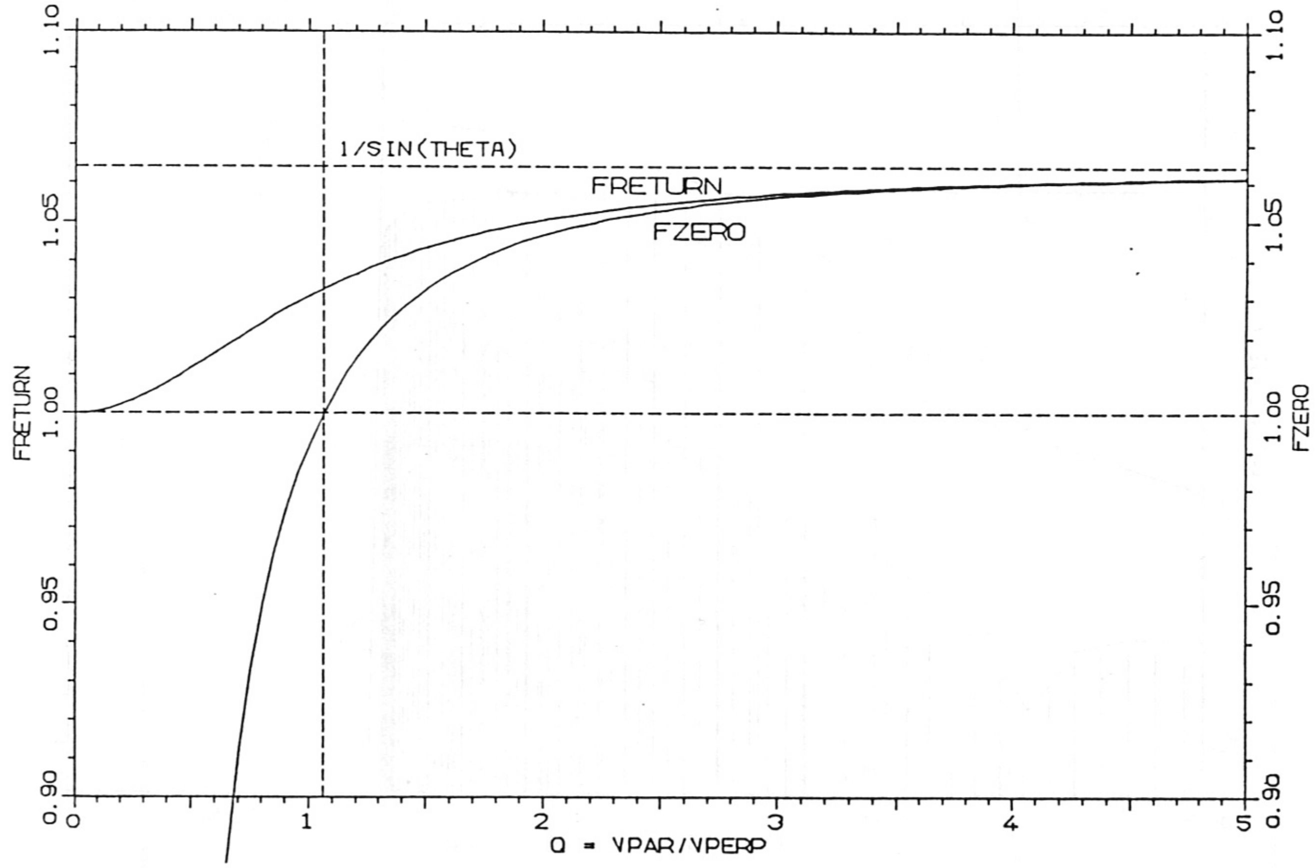


Fig.5: $F_R (= F_{RETURN})$ *and* $F_O (= F_{ZERO})$ *as functions of* $v_{\|}/v_{\perp}$ *for* $\Theta = 70^0$

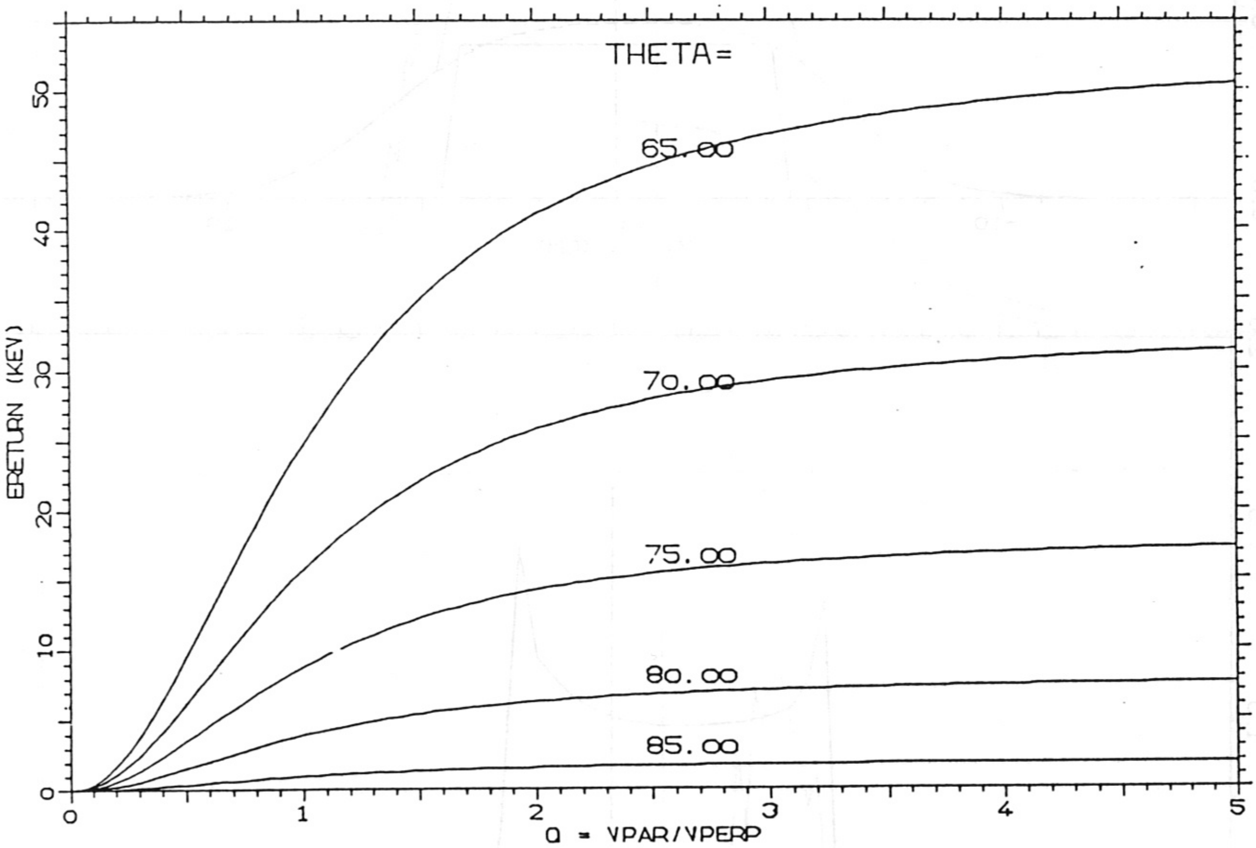
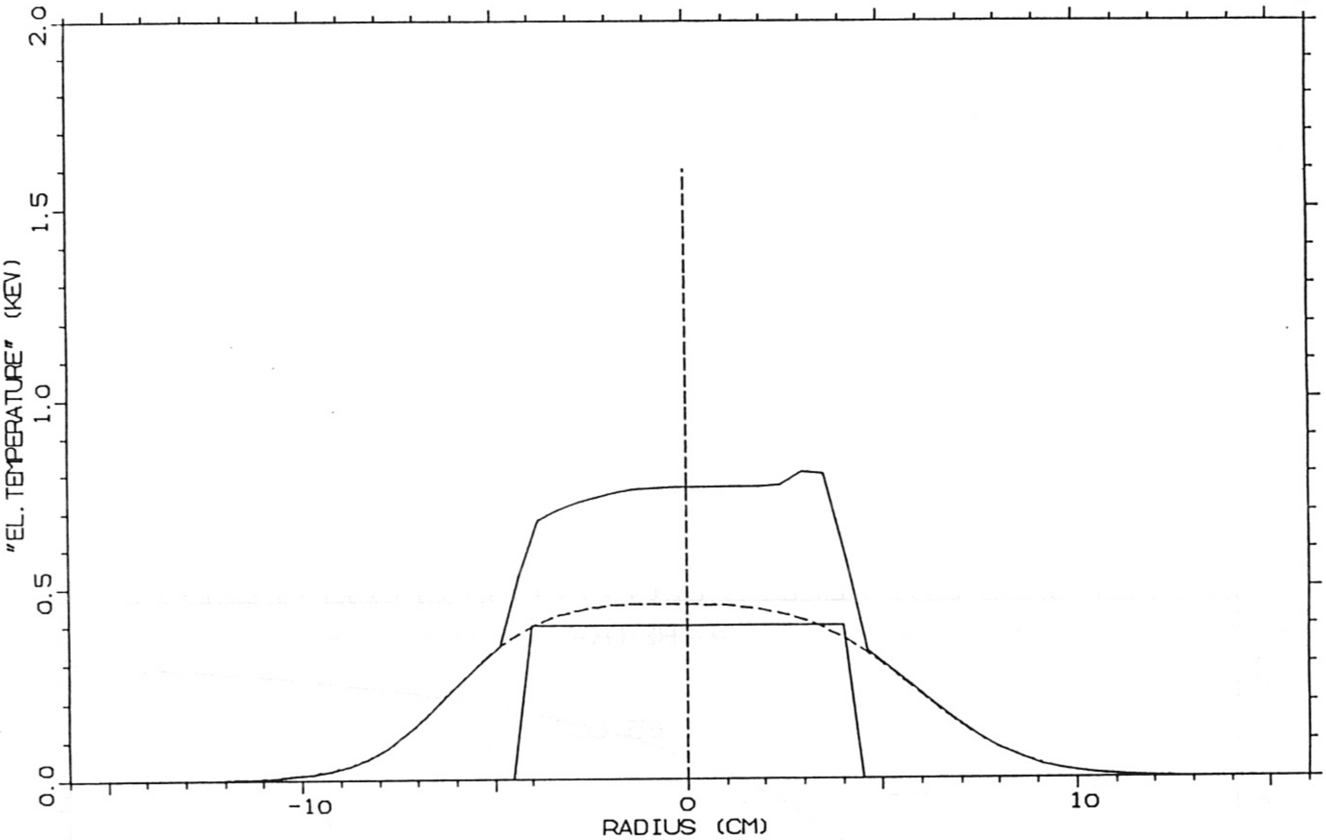


***Fig.6:** The kinetic energy $E_R (= E_{RETURN})$ corresponding to F_R as a function of $v_\parallel / v_\perp$*



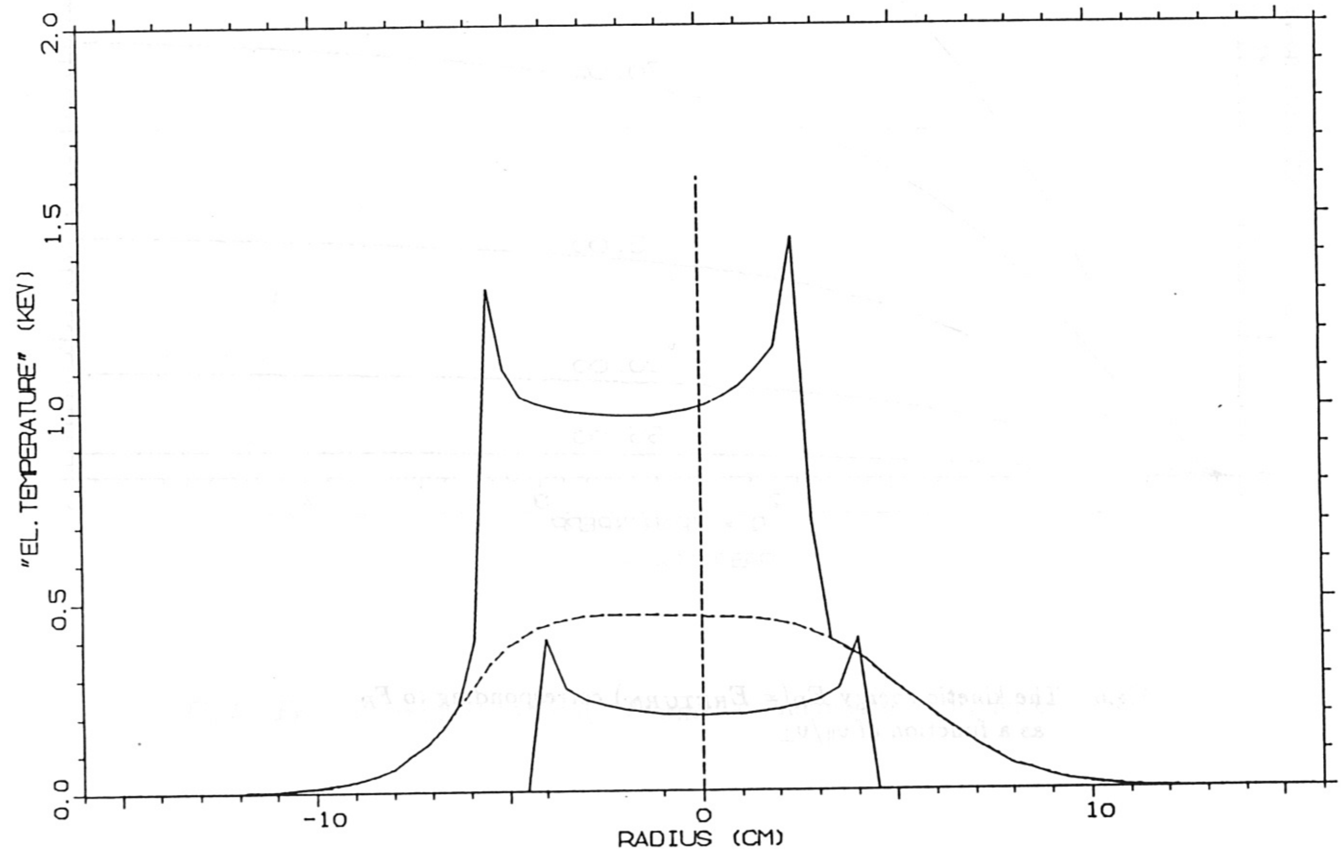
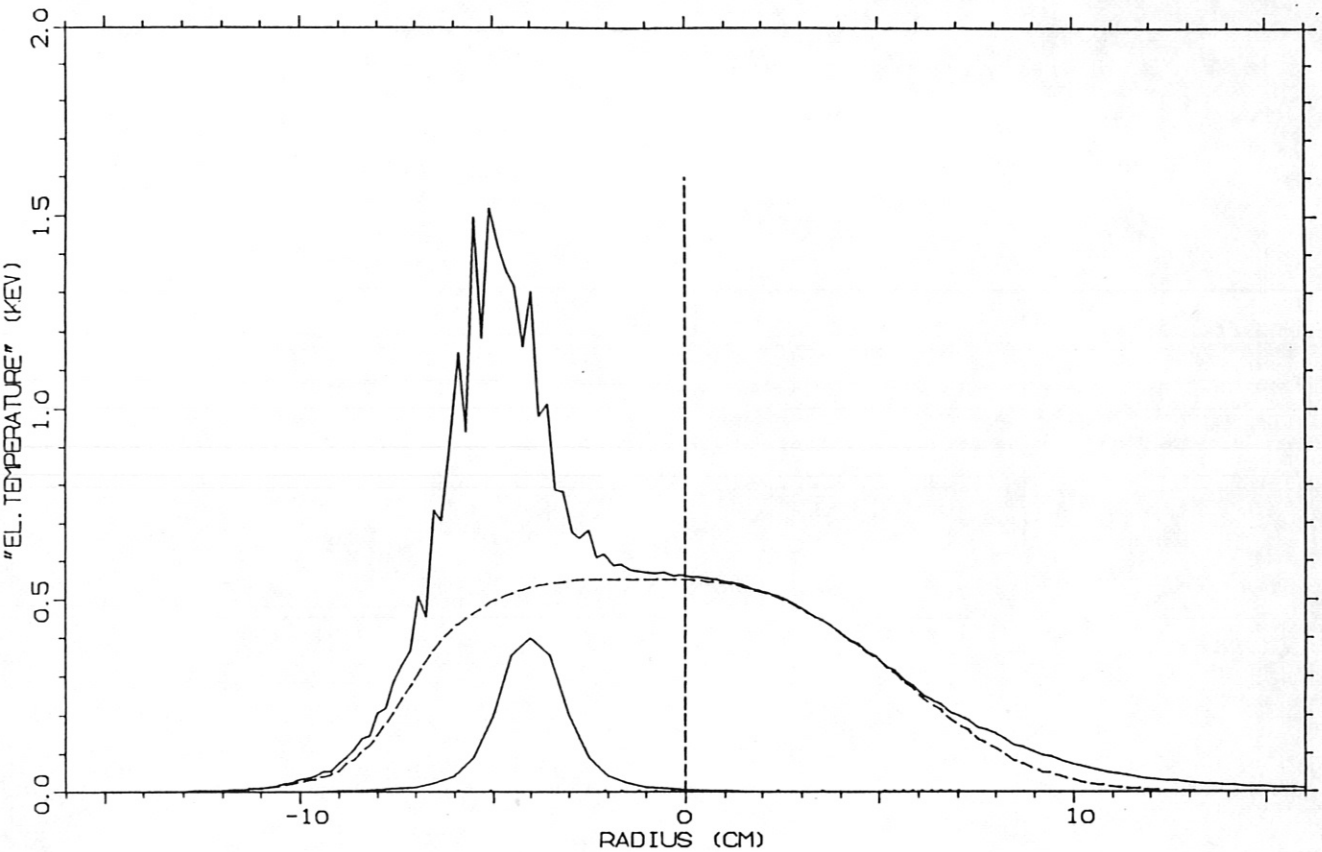
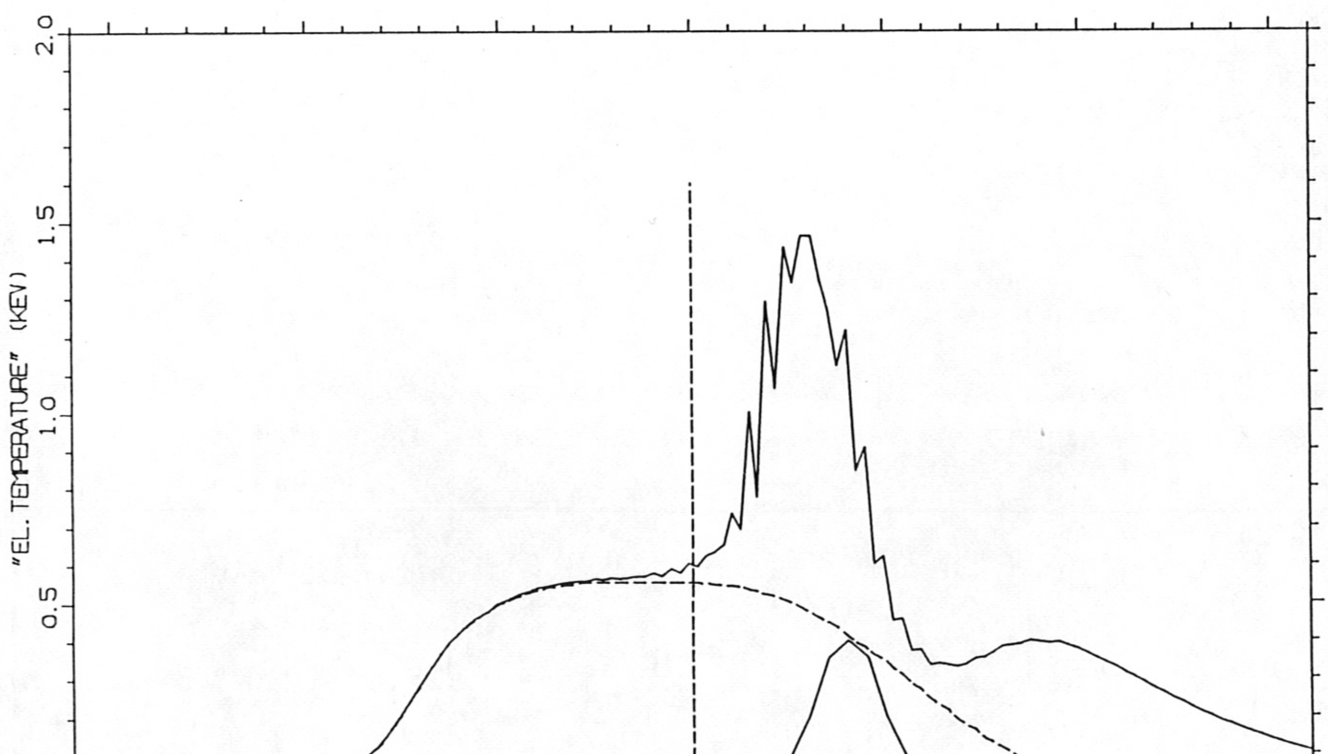


***Fig.7 to 10:** Profiles calculated from the corresponding spectra for several arbitrarily chosen suprathermal density distributions (lower curve, rel.units). The dashed curve is the profile got from the thermal part alone.*



2.0
1.5
1.0
0.5
0.0
"EL. TEMPERATURE" (KEV)
-10
0
10
RADIUS (CM)



2.0
1.5
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"EL. TEMPERATURE" (KEV)
-10
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RADIUS (CM)