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NATURAL CURRENT PROFILES IN TOKAMAKS

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Abstract

It is proposed that a certain class of equilibrium solutions, which follow from an elementary variational principle, are the natural current profiles in tokamaks, to which actual discharge profiles tend to relax.

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

Recently, the idea of profile consistency[1] in tokamaks has attracted considerable interest. Loosely speaking, this means that tokamak plasmas have a tendency to set up certain natural profiles of current density $j(r)$ and temperature $T_e(r)$. Transport processes, in particular χ_e, are essentially determined by the energy deposition profiles. If the latter are such as to support the natural profiles, cross-field transport is minimal, becoming the stronger the more the actual method of plasma heating tends to generate profiles largely different from the natural ones. The idea of profile consistency is very attractive for a number of reasons. It would, for instance, identify the sources of free energy driving anomalous transport and thus give a more unifying direction to the rather diverging tendencies of present tokamak transport theory. The fundamental question, however, is what determines the natural profiles and what are they like. In Ref.1 it has been inferred from experimental observations that $T_e(r)$ and $j(r)$ have Gaussian shapes $\exp\{-\alpha r^2/a^2\}$, where α depends only on the ratio q_a/q_0. Measurements of $T_e(r)$ are, however, not accurate enough to discriminate between different bell-shaped distributions, in particular since the central part and the edge region seem to be dominated by processes not included in the simple concept of natural profiles. In addition, the current density, the fundamental profile, has practically not been measured at all. Hence a theory of the natural tokamak profiles would certainly be very useful. In a recent investigation of magnetic reconnection[2] it was found that the shape of the perpendicular current distribution in a current sheet observed in a series of numerical simulations could be explained by a simple variational principle. In this letter we would like to give a somewhat more general formulation of this theory and its application to the problem of natural current profiles in tokamaks.

The electric current in a magnetized plasma consists of a parallel and a perpendicular component with respect to the magnetic field, $\vec{j} = \vec{j}_{\parallel} + \vec{j}_{\perp}$. While the latter is fixed by the plasma pressure distribution - it represents the diamagnetic property of the plasma -, $j_{\parallel}$ is a priori undetermined. Only the total current I can be controlled. Since in a plasma in a strong external magnetic field parallel currents are easily induced, one usually has $j_{\parallel} >> j_{\perp}$, while on the other hand $j_{\parallel}$ is small enough, so that $\vec{B}_{\perp}$, the magnetic field generated by the parallel current, is small compared with the external potential field $\vec{B}_0$ and $\vec{j}_{\parallel}$ is essentially parallel to $\vec{B}_0$.

We consider an arbitrary low-β plasma configuration confined to a certain region V and carrying an externally controlled current I. In the presence of an effective magnetic reconnection process in the plasma one could expect the current distribution to relax to a state of minimum magnetic energy under the constraint of constant total current, i.e. obey the following variational principle

$$\delta\left\{\int\limits_V \frac{B_\perp^2}{2}dV + \alpha\left(I - \int\limits_F \vec{j}\cdot d\vec{F}\right)\right\} = 0 \tag{1}$$

where α is a Lagrange multiplier. (To make the second integral independant of the particular choice of F, V should be bounded by a magnetic surface.) It can, however, easily be shown, that the solution of (1) with unrestrained variation is singular, $B_\perp = 0$ in V with the current flowing on the bounding surface, which corresponds to the absolute minimum of the magnetic energy and is obviously not of much interest for tokamaks. To obtain a larger class of solutions the variation in (1) has to be performed in a different way. First we note that the actual relaxation process is slow and therefore constitutes a sequence of MHD equilibria. Furthermore energy and particle transport times are shorter than the current profile relaxation time. Hence there is a continuous pressure redistribution between different magnetic surfaces, so that neighboring states should not be connected by any dynamical constraints.

In the following we restrict ourselves to two-dimensional configurations which are described by a single function, the flux function ψ. Consider a geometry with the coordinate system ξ, η, ς, where ς is the ignorable coordinate. The external field $\vec{B}_0$ is assumed to be in the ς-direction and the field generated by the parallel current is $\vec{B}_\perp = \nabla\psi \times \nabla\varsigma$. For $B_\perp << B_0$ one has $j_{//} \simeq j_\varsigma/|\nabla\varsigma|$ with $j_\varsigma = \nabla\varsigma \cdot \nabla \times \vec{B}_\perp$. The equilibrium equation tells us that j_ς is a function of ψ and say ξ, where the dependence on ψ is the important one, which essentially determines the current distribution, while the ξ-dependence is a geometry effect vanishing in the case of plane geometry. Hence in the second integral in (1) we have to insert the general equilibrium current distribution $j_\varsigma(\psi, \xi)$, using $\int \vec{j} \cdot d\vec{F} = \int \vec{j} \cdot d\vec{F} d\varsigma = \int j_\varsigma dV$.

In this form the variational process is, however, not unambiguously defined. The general variation of j_ς contains two contributions, one due to the variation of ψ, the other due to a change of the functional dependence on ψ, $\delta j_\varsigma = (\partial j/\partial\psi)\delta\psi + \hat{\delta} j_\varsigma$. The usual minimization principle implied in eq.(1) would require to choose $\hat{\delta} j_\varsigma$ in such a way that δj_ς is a real current variation satisfying Ampère's law $\delta j_\varsigma = \nabla\varsigma \cdot \nabla \times (\nabla\delta\psi \times \nabla\varsigma)$, which yields the minimum energy state $\psi = const$ mentioned above. By contrast we suggest the following extremum principle, where we choose $\hat{\delta} j_\varsigma = 0$, i.e. assume the functional dependence to be fixed during the variation, which yields an equation for $j_\varsigma(\psi)$. From the variational equation

$$\frac{\delta}{\delta\psi}\left\{\frac{1}{2}\int\limits_V (\nabla\psi \times \nabla\varsigma)^2 dV + \alpha\left(I - \int\limits_V j_\varsigma(\psi,\xi)dV\right)\right\} = 0 \tag{2}$$

we immediately obtain the Euler equation, using $\delta\psi = 0$ at the boundary,

$$\nabla\varsigma \cdot \nabla \times (\nabla\psi \times \nabla\varsigma) = \alpha\frac{\partial j_\varsigma}{\partial\psi}. \tag{3}$$

The solution of (3) by satisfying the condition $\int j_\varsigma(\psi)dV = I$ does in general not satisfy the condition that the total current equals I, since the corresponding current density is $\alpha\partial j_\varsigma/\partial\psi$, except for j_ς such that

$$j_\varsigma = \alpha\frac{\partial j_\varsigma}{\partial\psi}, \tag{4}$$

which gives

$$j_\varsigma = f(\xi)e^{\psi/\alpha}. \tag{5}$$

(The dependence on ξ is determined by the equilibrium equation as we shall discuss below for the case of axisymmetry.) In this sense solutions of the equilibrium equation with the current profile (5) correspond to states of minimum magnetic energy subject to the constraint of given total current. In contrast to the original variational principle (1), however, this modified principle does not seem to allow a simple physical interpretation.

Let us first discuss the case of plane geometry, x, y, z, where $j_z = j_z(\psi)$. Here eqs. (3), (5) become

$$\nabla^2\psi = -\lambda e^{\psi/\alpha}. \tag{6}$$

There are two parameters α and λ. While α is an amplitude scale factor of ψ and is hence essentially determined by the condition that the total current be equal to I, λ, which is related to the current profile width, is still free and should be determined by additional physics requirements. We thus find a one parameter family of profiles satisfying the extremum principle (2). For a one-dimensional current sheet, as discussed in Ref. 2, the solution of (5) is, with $\partial\psi/\partial x = -I$ for $x \to \infty$,

$$\psi = \frac{I^2}{\lambda} \ln \frac{1}{\cosh \frac{\lambda}{I} x}. \tag{7}$$

and the current profile

$$j_z = \frac{\lambda}{\cosh^2 \frac{\lambda}{I} x}. \tag{8}$$

This is the well-known solution for a collisionless current sheet, where the exponential in eq.(6) arises from the choice of a Maxwellian particle distribution. Here we now find that it may be more generally valid, being the profile to which any current sheet tends to relax. In fact, the numerical simulations of current sheet formation presented in Ref.2 show profiles that are approximated by eq.(8) with surprising accuracy.

We now turn to the case of axisymmetry R, z, ϕ, with ϕ being the ignorable coordinate, where the equilibrium equation is

$$\frac{1}{R}\Delta^* \psi = -j_\phi = -Rp'(\psi) - \frac{1}{R}TT'(\psi), \tag{9}$$

with the modified Laplacian

$$\Delta^* = R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial}{\partial R} + \frac{\partial^2}{\partial z^2}. \tag{10}$$

When this is compared with eq. (5), it is found that the ratio p'/TT' is constant and hence eqs. (3) and (5) become

$$\Delta^* \psi = -\lambda(\mu R^2 + 1 - \mu)e^{\psi/\alpha}. \tag{11}$$

Here μ is essentially β_p, the poloidal β. We call the solution of eq. (11), supplemented by the condition $\psi = 0$ at the plasma boundary, the natural tokamak profile. Obviously, the profiles of j_ϕ and p' are very similar and therefore strongly differ from equilibria with peaked current and flat pressure profiles, which have recently been invoked in order to separate the effects of shear and pressure gradients and thus reach higher stable β-values.

To discuss the solutions of eq. (11) in more detail, let us consider the case of large aspect ratio with circular cross-section. Since $R \simeq R_0$, eq. (11) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \psi}{\partial r} = -\lambda e^{\psi/\alpha}, \tag{12}$$

where r is the radial variable within the torus, $R = R_0 + r\cos\theta$. The solution of (12) is

$$\psi = 2\alpha \ln \frac{1+\gamma}{1+\gamma\dfrac{r^2}{a^2}}, \tag{13}$$

with $8\alpha\gamma/(1+\gamma)^2 a^2 = \lambda$. Introducing the total current $I = 8\pi\alpha\gamma/(1+\gamma)$ to eliminate α, and $q_a/q_0 = \pi j_0(0)/I$, the current profile assumes the form

$$j_0 = \frac{I}{\pi a^2}\frac{\dfrac{q_a}{q_0}}{\left(1+\left(\dfrac{q_a}{q_0}-1\right)\dfrac{r^2}{a^2}\right)^2} \tag{14}$$

which is also a well-known profile, the "peaked" profile introduced in Ref.3. Since in a sawtoothing discharge we have $q_0 = 1$, the profile parameter $q_a/q_0 - 1$ in eq. (13) is fixed by prescribing I, and in this case the natural profile is uniquely determined. It should also be noted that the solution of the unrestricted variational principle (1) corresponds to the special case $q_a/q_0 \to 0$.

How do the natural profiles (14) relate to real current profiles in tokamaks, which are always driven systems? The conventional school of thought is that for given average values of T_e, n_e, etc. the heat conductivity χ_e is essentially fixed and the heat deposition determines the profile $T_e(r)$, which in resistive equilibrium then determines the current profile $j(r) \sim (T_e(r))^{3/2}$. By contrast, the hypothesis of profile consistency emphasizes the primary role of the current profile, which tends to have some natural shape $j_0(r)$, while $\chi_e(r)$ is such that, rather independently of the heat deposition, T_e is close to $(j_0(r))^{2/3}$.

Since in a tokamak $j(r)$ is usually affected by MHD activity in the center and strong cooling in the edge region enforcing $j(a) \simeq 0$, it is only for $q_a/q_0 >> 1$, where $j_0(a) << j_0(0)$, that j can be expected to be close to the corresponding natural profile. (In addition tearing modes with $m > 1$ may locally modify the current profile.) Since $j_0(r) \sim p_0(r)$, resistive equilibrium requires that $T_e \sim j_0^{2/3}$ and $n_e \sim T_e^{1/2}$, which is not inconsistent with experimental observations, at least for ohmic discharges.

Deposition profiles of auxiliary heating which are usually broader than for ohmic heating tend to drive the system away from the natural profiles and thus lead to stronger diffusion. Only for divertor discharges allowing high T_e at the plasma edge in the H-mode may broad current profiles with finite $j(a)$ be generated and approach the corresponding natural ones. The considerable reduction of χ_e observed in this case is consistent with this picture.

Finally, it should be recalled that equilibria in the reversed field pinch (RFP), where $B_\perp \sim B_0$, are determined by an energy principle[4] analogous to eq.(1). The difference is that for the RFP internal magnetic reconnection is much faster than in the tokamak and does not constitute a sequence of equilibria, and that the total helicity is the crucial conserved quantity. In tokamaks helicity conservation seems to be important only during rapid relaxation events such as sawtooth and major diruptions. It might also be interesting to note that replacing the last term in eq.(1) by a very similar expression, $\int \vec{B} \cdot \vec{j} dV$, the variational principle (1) yields a force free equilibrium just as in Taylor's energy principle[4]. In the tokamak approximation this solution corresponds to constant current density, which is not much more relevant for tokamaks than the surface current solution of (1).

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