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FAST DETERMINATION OF PLASMA PARAMETERS THROUGH FUNCTION PARAMETRIZATION

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ABSTRACT

The method of function parametrization, developed by H. Wind for fast data evaluation in high energy physics, is demonstrated in the context of controlled fusion research. This method relies on a statistical analysis of a large data base of simulated experiments in order to obtain a functional representation for intrinsic physical parameters of a system in terms of the values of the measurements. Rapid determination of characteristic equilibrium parameters of a tokamak discharge is shown to be a particularly indicated application. The method is employed on the ASDEX experiment to determine the following parameters of the plasma: position of the magnetic axis, geometric center, and current center; minor radius, elongation, and area of the plasma column; a normalized safety factor at the plasma boundary; the Shafranov parameter $\beta_p + l_i/2$; the flux difference between the plasma boundary and an external reference value; the position of the lower and upper saddle points, and the intersections of the separatrix with the four divertor plates. The relevant measurements consist of three differential poloidal flux measurements, four poloidal field measurements, the current through the multipole shaping coils, and the total plasma current. Function parametrization supplies a very accurate interpretation of these data, which is now used for online data analysis, and is also sufficiently fast to be suitable for real-time control of the plasma.

INTRODUCTION

In the interpretation of tokamak diagnostics the amount of experimental information that is utilized is often not limited by the rate at which measurements can be made, but more by the rate at which the raw diagnostics can be interpreted. Clearly then, efficient methods of data analysis are highly desirable. A very efficient procedure, function parametrization, was developed by H. Wind (CERN) for the purpose of momentum determination from spark chamber data [1], [2], and was recently proposed for tokamak physics applications [3]. Although this method has not previously been noticed outside the high energy physics field, it has a much wider range of applicability, and can be considered whenever many measurements are to be made with the same diagnostic setup. The application described in this paper concerns the determination of characteristic equilibrium parameters from magnetic measurements on a tokamak.

Function parametrization relies on an analysis of a large data set of simulated experiments, aiming to obtain an optimal representation of some simple functional form for intrinsic physical parameters of a system in terms of the values of the measurements. Statistical techniques for dimension reduction and multiple regression are used in the analysis. The resulting function may be chosen to involve only low-order polynomials in only a few linear combinations of the original measurements; this function can therefore be evaluated very rapidly, and needs only minimal hardware facilities.

Three steps have to be made for this method of experimental data evaluation. (1) A numerical model of the experiment is used to generate a data base of simulated states of the physical system, in which each state is represented by the values of the relevant physical parameters and of the associated measurements. (2) This data base is made the object of a statistical analysis, with the aim to provide a relatively simple function that expresses the physical parameters in terms of the measurements. (3) The resulting function is then employed for the fast interpretation of real measurements.

Determination of characteristic equilibrium parameters of a magnetically confined plasma is a particularly indicated application. The MHD model provides a well defined and generally accepted connection between the unknown intrinsic plasma parameters, the externally applied fields, and the magnetic measurements. Identification of the position and the profile of the plasma column is the basis for interpretation of practically all other diagnostics, and therefore requires an efficient algorithm. Here we describe a successful application of function parametrization for the interpretation of magnetic measurements on the ASDEX tokamak. It is shown that the method provides simple and very accurate approximations for a range of geometric and other parameters characterising the equilibrium configuration. These approximations are presently in use for

fast data analysis between discharges, but they are also suitable for real-time control of the experiment.

The paper is organized into three main Sections. A mathematical description of the method is given in Section 1, followed by a report on the ASDEX study in Section 2. In Section 3 a proposal for future developments is outlined. Preliminary versions of this work have appeared in an internal report [3] and in conference contributions [4]–[6].

1. Mathematical Description

Preliminaries. A classical physical system is considered, of which $\mathcal{P}$ denotes a typical state. The system may have any number of degrees of freedom, but interest will be restricted to a (partial) characterization by n intrinsic real parameters, represented collectively by a point $\mathbf{p} \in \mathbf{R}^n$. In the experimental situation $\mathbf{p}$ is to be estimated from the readings of m measurements, represented by a point $\mathbf{q} \in \mathbf{R}^m$. It is assumed that $\mathbf{p}$ is completely specified by $\mathcal{P}$, but that $\mathbf{q}$ may be a stochastic function of $\mathcal{P}$, the stochasticity being due to random errors in the measurement process. We will write $\mathbf{p} = \mathbf{p}(\mathcal{P})$ and $\mathbf{q} = \mathbf{q}(\mathcal{P})$.

The aim of the function parametrization is to obtain some reasonably simple function, $\mathbf{F} : \mathbf{R}^m \to \mathbf{R}^n$, such that for any state $\mathcal{P}$ the associated $\mathbf{p}(\mathcal{P})$ and $\mathbf{q}(\mathcal{P})$ satisfy $\mathbf{p} = \mathbf{F}(\mathbf{q}) + \mathbf{e}$ for a sufficiently small error term $\mathbf{e}$. The functional form of $\mathbf{F}$ may typically be chosen as a low-order polynomial in only a few linear combinations of the components of $\mathbf{q}$. The unknown coefficients in $\mathbf{F}$ are then determined by analysis of a data base containing the values of the parameters $\mathbf{p}_\alpha$ and of the measurements $\mathbf{q}_\alpha$ corresponding to N simulated states $\mathcal{P}_\alpha$ $(1 \leq \alpha \leq N)$. This is a problem of function fitting over scattered data in the high-dimensional space $\mathbf{R}^m$, for which techniques from multivariate statistical analysis are appropriate. To a statistician the $\mathbf{q}_\alpha$ are the 'conditions', the $\mathbf{p}_\alpha$ are the 'responses', $\mathbf{F}$ is a 'regression', and function fitting is called regression analysis. The terminology of conditions and responses is very unnatural in the present context, and we will refer to these instead as the independent and the dependent variables, or as the measurements and the physical parameters. Otherwise the terminology from statistical analysis is retained.

The n physical parameters need not all be independent, and even if no exact relations exist between them, the nature of the measurements may in practice allow only a limited number of combinations to be determined independently. Let n_0 denote that number of independently determinable combinations. Then it must be that $m \geq n_0$, and for a well diagnosed experiment in fact $m \gg n_0$, while for an accurate statistical analysis it is furthermore required that $N \gg m$. For the applications that we have in mind, $\mathcal{P}$ formally has infinitely many degrees of freedom, $n_0 < 10$, n is arbitrary, $m \sim 10 - 100$, and $N \sim 10^3 - 10^4$.

The data base. In the first stage a code is employed to generate a data base. This code must be suited to compute possible states of the physical system over the whole of the system's regime, and must also contain a model for the measurements. The code will take certain numerically convenient parameters as input, and produce $\mathbf{p}$ and $\mathbf{q}$ as

results. The input parameters are varied, and for each successful calculation, indexed by α, the values $\mathbf{p}_\alpha$ and $\mathbf{q}_\alpha$ are saved. As the subsequent automatic analysis will only detect dependencies that are reflected in the data base, one must ensure that every parameter or combination of parameters that can vary in the actual experiment is also varied when generating these data. At best, one employs a suitable pseudo-random variation of the code parameters in order to generate the data base.

Dimension reduction. Since the dimensionality m of the space of the measurements may be of the order of several tens in many cases, and since a linear representation for $\mathbf{p}$ in terms of $\mathbf{q}$ is not expected to suffice, the dimensionality of the space of trial functions with which the physical parameters will be fitted can be very large. A polynomial model of degree l, for instance, has $\sim m^l/l!$ degrees of freedom for each physical parameter. It is therefore necessary to first reduce the number of independent variables (the components of $\mathbf{q}$) by means of a transformation to a lower-dimensional space. A second aim for this transformation of variables must be to eliminate or reduce multicollinearity (near linear dependencies) between the data points, and thus to improve the conditioning of the regression problem [**7**, ch. 8]. Multicollinearity is likely to be present whenever the number of measurements is much larger than the number of independently measurable physical parameters; specific causes may be some underlying smoothness in the data, or any explicit physical constraint that relates different measurements.

A method for dimension reduction and elimination of multicollinearity that is widely used in statistics is based on principal component analysis (PCA) [**7**, ch. 8], [**8**, ch. 8]. From the N suitably scaled pseudo-measurements $\mathbf{q}_\alpha$, each of which is a point in $\mathbf{R}^m$, the sample mean, $\bar{\mathbf{q}} = N^{-1}\sum_\alpha \mathbf{q}_\alpha$, and the $m \times m$ sample dispersion matrix,

$$\boldsymbol{S} = N^{-1}\sum_{\alpha=1}^{N}(\mathbf{q}_\alpha - \bar{\mathbf{q}})(\mathbf{q}_\alpha - \bar{\mathbf{q}})^T, \tag{1}$$

are calculated. $\boldsymbol{S}$ is symmetric and positive semi-definite. An eigenanalysis yields m eigenvalues, $\lambda_1^2 \geq \ldots \geq \lambda_m^2 \geq 0$, with corresponding orthonormal eigenvectors $\mathbf{a}_1, \ldots, \mathbf{a}_m$. Any measurement vector $\mathbf{q}$ may be resolved along these eigenvectors to obtain a set of transformed measurements, $x_i = \mathbf{a}_i \cdot (\mathbf{q} - \bar{\mathbf{q}})$. This is the principal component transformation, and the x_i are the principal components of the measurement vector $\mathbf{q}$.

The transformed measurements $(x_i)_{1\leq i\leq m}$ are linearly independent within the sample, have zero mean, and variance λ_i. One of the aims, the reduction of multicollinearity, has therefore been achieved, but if all m components x_i are retained, the dimensionality of the problem is not reduced. The assumption underlying a regression analysis

based on PCA is that the most significant information will be contained in the first few principal components, $(x_i)_{1\le i\le m_0}$, where $m_0 \le m$, and preferably $m_0 \ll m$. These m_0 components are called the 'significant components', and the associated first s eigenvectors $\mathbf{a}_i$ are the 'significant variables'. The desired dimension reduction is thus achieved through the transformation $\mathbf{R}^m \to \mathbf{R}^{m_0}$ defined by $\mathbf{x} = \boldsymbol{A}^T \cdot (\mathbf{q} - \bar{\mathbf{q}})$, where $\boldsymbol{A}$ is the matrix that has columns $\mathbf{a}_i$ $(1 \le i \le m_0)$. The selection of m_0 will be based on an inspection of the behaviour of the sequence of eigenvalues, but no universally accepted prescription exists. One hard criterion is that one must choose $m_0 \ge n_0$. On the other hand the principal components corresponding to smaller eigenvalues are more difficult to measure accurately, and a study of the effect of measurement errors will provide an upper bound on the reasonable values for m_0.

The use of PCA for dimension reduction is motivated by the idea that those linear combinations of the measurements ('affine' combinations in mathematical terminology) that display the largest variance are also the best suited for the interpretation of the data. In practice, however, significant information may well be concealed in linear combinations of the measurements that show relatively little variation, or alternatively, some measurements may vary widely without much correlation with the physical parameters that are to be determined. A preliminary linear or nonlinear transformation of variables on the basis of physical insight will then be beneficial. PCA is invariant only under orthogonal transformations; it is not invariant under scaling of variables, or under more general linear transformations.

We may remark that principal component analysis is not the only possible procedure for dimension reduction. Perhaps even more popular, but less satisfactory for our type of application, is simple discarding of variables. The use of canonical correlation analysis [**8**, ch. 10] was discussed in our earlier work [**3**].

Regression analysis. Having defined the linear transformation $\mathbf{q} \to \mathbf{x}$ it is next necessary to face the task of fitting the, in general nonlinear, relation between $\mathbf{x}$ and $\mathbf{p}$. The problem has been simplified by the dimension reduction obtained with the transformation $\mathbf{q} \to \mathbf{x}$, and it is expected to be better conditioned through the elimination of multicollinearity.

It is desired to find for each component p_j $(1 \le j \le n)$ a regression, $p_j = f_j(\mathbf{x}) + \epsilon_j$, to fit the data $(\mathbf{x}_\alpha, \mathbf{p}_\alpha)_{1\le\alpha\le N}$. A polynomial model, of the form

$$p_j = \sum_{\mathbf{k}} c_{\mathbf{k}j} \cdot \prod_{i=1}^{m_0} \phi_{k_i}(x_i/r_i) + \epsilon_j, \tag{2}$$

is suitable. Here, the multi-index $\mathbf{k}$ has m_0 components $k_1, \ldots, k_{m_0}$ in the nonnegative integers, the $c_{\mathbf{k}j}$ are the unknown regression coefficients, which are determined by a least-squares fitting procedure over the data base, $(\phi_k)_{k\geq 0}$ is some family of polynomials, r_i is a suitable scaling factor for the component x_i, and ϵ_j is the error term.

An upper bound on some norm of $\mathbf{k}$ must be supplied in order to make the model finite, and in addition one can employ with the above model some form of subset regression, the objective being to retain in the final expression only the terms which make a significant contribution to the goodness-of-fit. A variety of algorithms exists for deciding which terms to retain and which to discard; see for instance [7, ch. 7], or [9, ch. 6].

This completes the construction of the function parametrization, $\mathbf{p} = \mathbf{F}(\mathbf{q}) + \mathbf{e}$, which is thus given by Eq. (2) together with the relation $\mathbf{x} = \boldsymbol{A}^T \cdot (\mathbf{q} - \bar{\mathbf{q}})$. In order to test the adequacy of the regression function it is proper to generate a new, independent collection of simulated experiments, and to evaluate the magnitude of the error term $\mathbf{e}$ from this second set of data.

We remark again that a significant effort may be involved in generating and analyzing the data base, but that the evaluation of the final function — and this is the operation that has to be performed many times — is almost trivial.

Treatment of erroneous signals. An attractive feature of the principal component analysis, also pointed out by Wind [1], [2], is that the least significant components can be used to provide constraints on the data. These constraints make it possible to test whether actual measurements are consistent with the model that was used to generate the data base, and also to automatically correct failing signals.

Specifically, let us assume that the principal component analysis has been performed on simulated measurements that have been scaled and transformed in such a way that they are assumed in the experiment to suffer independent random errors coming from a normal distribution having mean 0 and width σ. Now for each actual measurement $\mathbf{q}$ we define

$$\chi^2(\mathbf{q};\sigma) = \sum_{i=1}^{m} \frac{x_i^2}{\lambda_i^2 + \sigma^2}, \tag{3}$$

where $(x_i)_{1\leq i\leq m}$ is the complete set of principal components (significant and insignificant) associated with $\mathbf{q}$.

In this case, χ^2 will have average value $\sim m$, and much larger values of χ^2 indicate either an error in the measurements, or a violation of some assumption that was employed in generating the data base. If it is known that one or more specific components

of the measured **q** are in error, then these components can be restored to that set of values by which the quadratic form χ^2 is minimized. This requires only the solution of a system of linear equations having dimension equal to the number of failing signals.

2. Application to the ASDEX Magnetic Data Analysis

Description. As an initial study we applied function parametrization to the determination of a limited set of characteristic equilibrium parameters for the ASDEX experiment, using only magnetic signals measured outside the plasma. Important features of the geometry of ASDEX are displayed in Fig. 1, which shows the location of the vacuum vessel, the divertor plates, the poloidal field and flux measuring coils, and an example equilibrium plasma configuration.

The relevant measurements for our study consist of three differential flux measurements, four measurements of the component of the poloidal field tangential to the measuring contour, the current through the multipole shaping coils, and the toroidal plasma current. However, the plasma current is scaled out of the problem, so that 8 independent measurements remain. With reference to Fig. 1 these are $\psi_3 - \psi_1$, $\psi_4 - \psi_2$, $\psi_3 - \psi_4$, B_1, B_2, B_3, B_4, and I_{mpc}, in each case normalized to unit plasma current, $I_t = 1$.

The physical parameters to be determined are

(r_{axis}, z_{axis})	position of the magnetic axis
(r_{curr}, z_{curr})	position of the current center
(r_{geom}, z_{geom})	position of the geometric center
a, b	horizontal and vertical minor radius
b/a	elongation
A	area of the poloidal cross-section
$\beta_p + l_i/2$	Shafranov parameter
q_{95n}	normalized safety factor
$\psi_b - \psi_r$	relative flux value at the plasma boundary
(r_{x1}, z_{x1})	position of the lower saddle point
(r_{x2}, z_{x2})	position of the upper saddle point
$\psi_{x2} - \psi_{x1}$	flux difference between the two saddle points
z_{bi}, z_{bo}, z_{ti}, z_{to}	intersections of separatrix and divertor plates

The current center is defined by $r_{curr}^2 I_t = \int r^2 j_t \, dS$ and $z_{curr} I_t = \int z j_t \, dS$, where the integrals are taken over the poloidal cross section of the plasma. These integrals can be rigorously evaluated from the external magnetic measurements [**10**]. Furthermore, ψ_b is the flux at the plasma boundary and $\psi_r = \frac{1}{4}(\psi_1 + \psi_2 + \psi_3 + \psi_4)$. These fluxes, and also ψ_{x1} and ψ_{x2}, have been scaled to correspond to unit plasma current. The normalized safety factor q_{95n} is the value of the safety factor at the contour $\psi - \psi_a = 0.95 \cdot (\psi_b - \psi_a)$, scaled to unit plasma current and unit toroidal field coil current (ψ_a denotes the flux on the magnetic axis). The subscripts indicating the divertor plates are mnemonic for bottom-inner, bottom-outer, top-inner, and top-outer.

The data base was generated using the Garching equilibrium code [11], which after significant optimization in view of these calculations now computes an equilibrium on our 64×128 grid in ~ 0.6 sec. on the Cray 1. The free parameters of the code were randomly varied in order to cover the operating regime of the ASDEX experiment in the divertor mode, and for each of the ~ 4000 calculated equilibria the corresponding magnetic measurements and physical parameters were recorded.

Results. A principal component analysis of the correlation matrix obtained from the simulated measurements gave the following sequence of eigenvalues: 3.67, 1.91, 1.39, 0.99, $2.01 \cdot 10^{-2}$, $1.12 \cdot 10^{-2}$, $1.90 \cdot 10^{-3}$, $8.93 \cdot 10^{-4}$. (Using the correlation matrix instead of the dispersion matrix is equivalent to scaling all measurements in such a way that they have unit variance).

It thus appeared that 4 significant variables should be retained for the regression analysis. Of these four, the first two are even under up-down reflection of the equilibrium, and together already provide a measure of the radial position and of $\beta_p + l_i/2$. The third significant variable is odd under reflection, and is related to the vertical position of the plasma, and the fourth one (again even under reflection) is essentially the multipole current. The four least significant components cannot be recovered accurately from beneath the error level of realistic measurements.

Further analysis showed that the measurement of the multipole current is of little overall importance, but is mainly relevant for the determination of the intersection of the separatrix with the divertor plates. After experimenting with various possible regression models we selected a model that is second order in the first three significant variables, and first order in the fourth: For each of the physical parameters p,

$$\begin{aligned} p = {} & c_0 + c_1 x_1' + c_2 x_2' + c_3 x_3' + c_4 x_4' + c_5 H_2(x_1') \\ & + c_6 x_1' x_2' + c_7 x_1' x_3' + c_8 H_2(x_2') + c_9 x_2' x_3' + c_{10} H_2(x_3') + \epsilon \end{aligned} \tag{4}$$

where $x_i' = x_i/\lambda_i$ and $H_2(x) = (x^2 - 1)/\sqrt{2}$. The coefficients $c_0, \ldots, c_{10}$ are to be determined, and ϵ is the error term. These particular basis functions have been chosen in order to obtain an approximately orthonormal family.

Most of the physical parameters exhibit a first-derivative discontinuity in their dependence upon the measurements, which is due to the fact that the x-point defining the plasma boundary can lie either in the lower or in the upper half plane. One may visualize this first-derivative discontinuity by considering the behaviour of (for instance) the plasma minor radius, as the center of gravity of the plasma cross section is moved smoothly from a position in the lower half plane to the corresponding position in the

upper half. At first the plasma boundary is defined by the lower x-point, and the minor radius increases as the plasma center moves away from this x-point. Upon crossing the meridian plane, the plasma boundary becomes dependent upon the upper x-point, and the minor radius decreases as the plasma is shifted further upwards. The relation between plasma minor radius and vertical position is non-smooth as the meridian plane is crossed, and this lack of smoothness is also present in the relation between the minor radius and the measured data. Separate fits were therefore used for the two cases when the boundary x-point is located in the lower or in the upper half plane respectively. The fits were constrained to match continuously for up-down symmetric configurations. (A similar procedure would be required for modelling a limiter experiment, when an inside and an outside limiter are both present).

The results obtained with this model for some representative physical parameters are shown in Table I.

Table I

parameter	average	minimum	maximum	variance	$\bar{\epsilon}(\delta = 0.0)$	$\bar{\epsilon}(\delta = 0.1)$
r_{axis}	1.722	1.630	1.830	$5.6 \cdot 10^{-2}$	$2.2 \cdot 10^{-3}$	$4.1 \cdot 10^{-3}$
r_{curr}	1.710	1.612	1.824	$5.6 \cdot 10^{-2}$	$3.1 \cdot 10^{-4}$	$3.4 \cdot 10^{-3}$
r_{geom}	1.661	1.515	1.807	$5.7 \cdot 10^{-2}$	$2.8 \cdot 10^{-3}$	$5.1 \cdot 10^{-3}$
z_{axis}	0	−0.100	0.100	$5.4 \cdot 10^{-2}$	$2.3 \cdot 10^{-3}$	$5.8 \cdot 10^{-3}$
z_{curr}	0	−0.102	0.102	$5.4 \cdot 10^{-2}$	$2.5 \cdot 10^{-3}$	$6.0 \cdot 10^{-3}$
z_{geom}	0	−0.134	0.134	$7.3 \cdot 10^{-2}$	$9.8 \cdot 10^{-3}$	$1.2 \cdot 10^{-2}$
a	0.367	0.290	0.463	$3.0 \cdot 10^{-2}$	$3.8 \cdot 10^{-3}$	$6.2 \cdot 10^{-3}$
b	0.358	0.295	0.438	$2.5 \cdot 10^{-2}$	$1.9 \cdot 10^{-3}$	$4.7 \cdot 10^{-3}$
b/a	0.978	0.796	1.033	$3.2 \cdot 10^{-2}$	$8.1 \cdot 10^{-3}$	$8.5 \cdot 10^{-3}$
A	0.416	0.271	0.592	$6.1 \cdot 10^{-2}$	$5.5 \cdot 10^{-3}$	$1.2 \cdot 10^{-2}$
$\beta_p + l_i/2$	1.791	0.563	3.428	$6.3 \cdot 10^{-1}$	$1.2 \cdot 10^{-2}$	$4.4 \cdot 10^{-2}$
q_{95n}	$5.07 \cdot 10^{-2}$	$3.26 \cdot 10^{-2}$	$8.62 \cdot 10^{-2}$	$9.1 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$
$\psi_b - \psi_r$	$8.39 \cdot 10^{-7}$	$3.64 \cdot 10^{-7}$	$1.31 \cdot 10^{-6}$	$1.7 \cdot 10^{-7}$	$1.3 \cdot 10^{-8}$	$3.0 \cdot 10^{-8}$
r_{x2}	1.550	1.530	1.570	$1.1 \cdot 10^{-2}$	$5.3 \cdot 10^{-3}$	$5.4 \cdot 10^{-3}$
z_{x2}	0.447	0.417	0.483	$1.2 \cdot 10^{-2}$	$4.6 \cdot 10^{-3}$	$4.7 \cdot 10^{-3}$
z_{ti}	0.836	0.685	0.934	$4.6 \cdot 10^{-2}$	$3.2 \cdot 10^{-3}$	$7.1 \cdot 10^{-3}$
z_{to}	0.862	0.764	0.965	$4.1 \cdot 10^{-2}$	$3.3 \cdot 10^{-3}$	$6.4 \cdot 10^{-3}$

For each of the parameters, Table I shows first the average, minimum, and maximum values occurring in the data base, and the standard deviation about the average. The

last two columns show the standard error, $\bar{\epsilon}$, of the model in Eq (4), first for exact measurements ($\delta = 0.0$), and then for measurements that have been randomly perturbed by a term coming from a normal distribution of zero mean and width equal to a fraction $\delta = 0.1$ of the variance of the measurements. SI units are employed throughout.

It is seen that the accuracy of the approximations obtained by function parametrization is indeed very good, not just for 'perfect' data, but also in the presence of measurement errors. Notice that the column labelled 'variance' shows the standard error associated with a very naive method of data interpretation, viz. ignoring the data and assuming that each physical parameter always has its average value. (The root mean square error, evaluated from the data base, of the assumption that any physical parameter always has its average value, is just the variance of that physical parameter). The quality of any other method of data interpretation may be evaluated not just in absolute terms, but also in relation to this naive procedure.

Besides Eq. (4) we tried several other regression models, up to a complete third order polynomial model in the first four principal comonents, and a complete quadratic model in six principal components. The third order polynomial model provides only marginally more accurate results than Eq. (4), whereas the model involving six principal components is more accurate for ideal measurements, but less accurate at realistic error levels. The choice of Eq. (4) was then motivated by its simplicity.

In the analysis of a different, related set of data we also tried an approach in which the (infinitely many) terms in a multivariate Taylor series model were ranked in a particular linear order, and added one by one to the model equation until an asymptotic error level was attained. The criterion for defining the linear order was the eigenvalue product associated with the Taylor series term; e.g. associated with the term $x_1^2 x_2$ is the eigenvalue product $\lambda_1^2 \lambda_2$. Clearly all eigenvectors must be < 1 for this ranking to make sense, but this is easily achieved by scaling. We envisage that this approach could be useful for situations in which more complicated models than Eq. (4) are required, in particular when it is used in conjunction with some form of stepwise regression.

As presently incorporated into the ASDEX data analysis, function parametrization is used to provide a complete picture of the time evolution of the quantities r_{geom}, z_{geom}, $r_{axis} - r_{geom}$, q_{95n} and $\beta_p + l_i/2$, immediately after every shot. Furthermore the time derivative of the quantity $\psi_b - \psi_r$ is used to correct the loop voltage measured by the flux loops to the actual value pervailing at the plasma surface: this is important for an accurate evaluation of the electromagnetic power flux into the plasma column during phases with strong dynamics, like during pellet injection.

Additional parameters characteristic of the field configuration and needed for the interpretation of other diagnostics can be evaluated at arbitrary instances, or their

evolution can be displayed. This group encompasses particularly the location of the intersection points between the divertor target plates and the separatrix, as well as the distance between the separatrix and antenna elements or probes in the plasma boundary region.

Comparison with other methods. Until now, the online analysis of magnetic measurements on ASDEX has been done using the FILM code, in which these measurements are fitted using a model that represents the plasma by a single wire current in an unknown location, and that contains two free parameters to describe the horizontal and vertical external fields.

The Shafranov parameter $\beta_p + l_i/2$ and the radial shift of the geometric center of the plasma with respect to the machine center ($\Delta = r_{geom} - r_{mach}$) have been computed on the basis of Shafranov's method [**12**], which relies on the assumptions of large aspect ratio, small radial shift, up-down symmetry, and circular cross section. Given an estimate of the plasma minor radius a, the theory of [**12**] provides linear expressions for $\beta_p + l_i/2$ and Δ in terms of the measurements. In the implementation on ASDEX this method employed a different set of diagnostics, so we do not have a direct comparison between the Shafranov theory and the new method. However, we have compared the *best possible linear* formula for $\beta_p + l_i/2$ and Δ (obtained from a regression analysis assuming a linear model in terms of all the original measurements) with the formula obtained by function parametrization as described earlier. The result is that for all of the parameters r_{axis}, r_{curr}, r_{geom}, and $\beta_p + l_i/2$, the optimal linear fit gives about twice as large an error as the fit obtained by function parametrization. This discrepancy is primarily due to the fact that in our application of function parametrization we have different fits depending upon the location of the separatrix x-point, and thereby obtain a more accurate representation for asymmetric configurations; the quadratic terms in Eq. (4) are small for these parameters.

Timing and storage requirements. In view of the limited number of magnetic probes installed on ASDEX and the simplicity of the model, Eq. (4), it is not surprising that the function parametrization method of data interpretation is very fast for the present application. In fact, the time required for the analysis is completely dominated by the time needed to retrieve the data from disk, and therefore we refrain from giving absolute timing figures for this study.

It is perhaps of more interest to estimate the required number of floating point operations (flops) for the analysis of one time slice on an experiment such as JET, which has many more magnetic diagnostics, and also more freedom in plasma shaping. To project m measurements onto 6 principal components (this number appears reasonable)

requires $\sim 12m$ flops. There are many different ways in which to code the algorithm for correction of failing signals, but $\sim 12m$ flops is easily achieved. A quadratic model in the 6 principal components has 28 terms, so to approximate n physical parameters using such a model requires $\sim 56n$ flops. Assuming then $m = 32$ and $n = 25$, and adding 30% overhead, we arrive at ~ 3000 flops, of which the majority occur in matrix-vector multiplication.

The storage requirement for the matrix defining the principal component transformation and the matrix of coefficients in the regression is $m^2 + 28n$ words, which is trivial.

For real-time use it will be desirable to evaluate the approximations in a time that is less than the skin-time of the vacuum vessel, although there can be no need to stay very far below that bound. The skin-time is typically 3–10 milliseconds (3 ms on JET). It is seen that a small processor, having a few kilowords of 32-bit store and a speed of ~ 5 Mflops/sec for modest size matrix-vector multiplication, will meet these requirements with ample room to spare. Evidently the function parametrization method of data interpretation is suitable for real-time control.

3. Further Prospects

The principal advantage of the use of function parametrization is that the computationally intensive part of the procedure — the elaborate model calculations and the statistical analysis — has to be carried out only once during the lifetime of a diagnostic set-up or device, whereas the calculations needed in the application involve only simple algebra. An additional advantage in comparison with more traditional methods for rapid data interpretation, relying on analytic approximations derived by consideration of exactly solvable special models, is that function parametrization is designed to find an optimal representation within a rather large class of algebraic functions, and may therefore well be more accurate than the traditional method. In situations where many measurements are made using the same experimental set-up the ultimate gain in diagnostic capabilities can obviously be very large.

It is planned to employ this method on the ASDEX Upgrade tokamak, planned to come into operation in late 1988, for real-time analysis of a wide spectrum of diagnostic signals whose use in a feedback loop for the control of discharge parameters is necessary or desirable. Present-day tokamak operation still proceeds in many parameters via trial and error, giving completely reproducible results and the quasistationary discharge behaviour required for a reactor only under rather narrowly defined conditions. Particularly the experience on ASDEX has shown that a dramatic improvement in this respect can be obtained by feedback control, at least for those parameters included in the control loop, like plasma current, position, and density. Evidently the number of additional knobs (plasma cross-section shaping, power and composition of additional heating, pellet injection, impurity injection, and pumping) has raised significantly the possibilities for active control, but has also dramatically underlined (e.g. by the failure so far to produce stationary discharge conditions near the ideal MHD β-limits) the actual need for it.

The principal obstacle to the use of additional diagnostic input at present is the impossibility to digest it *in real time* to give a more detailed picture of the discharge conditions. As ASDEX Upgrade makes a significant technical effort for similarity to a reactor in the machine aspects of the control problem (notably the choice of the poloidal field coils distant from the plasma) it is consistent to strive for a similar advance also in the other aspects (diagnostics, feedback system, software) of this problem.

For the application on ASDEX Upgrade it is planned to have an integrated package of diagnostics giving real-time information, under all discharge conditions, including the main option of strongly non-circular, high β_p equilibria, about

1) location of the plasma boundary,

2) intersections of the separatrix and the divertor target plates,

3) plasma energy content,

4) shape of interior flux surfaces,

5) electron temperature profile,

6) electron density profile,

7) current density profile,

8) impurity density distribution and plasma rotation.

Diagnostics to be used for this will be,

A) system of current, voltage and flux loops, and magnetic probes,

B) array of FIR interferometers,

C) electron cyclotron resonance measurements of electron temperature,

D) thermography camera observing the divertor plates,

E) soft X-ray pinhole cameras.

An inherent advantage of the function parametrization method of data analysis is its ability to mix information from different types of diagnostics. Important is that these diagnostics complement each other to give together a unique interpretation of the discharge conditions. That this is the case for the set (A)–(E) for a determination of (1)–(8) we will describe below. Obviously additional diagnostics (e.g. bolometry) can be added to give further real-time information for inclusion into feedback loops; their connection to the above package however will most probably occur only in the direction that they will use information (particularly the flux-surface topography) provided by the above system.

The physical model to be used for producing simulated diagnostic results is the axisymmetric MHD equilibrium equation. For each configuration of equilibrium flux surfaces we assume a range of profiles of electron temperature T_e, electron density n_e, and impurity radiation I_ν, using the facts that T_e and n_e should be functions of ψ only and that I_ν should be a function of ψ and R, where the R dependence arises from the possibility of rotation and the resulting accumulation of impurities at the large R side of a flux surface due to the effect of the centrifugal force. The latter effect (elegantly documented by recent tomography results of Smeulders on ASDEX [**13**]) leads to impurity radiation profiles of the form $I_\nu(\psi, R) = f(\psi)\exp(\alpha(\psi)R^2)$ (with $\alpha(\psi)$ depending on toroidal rotation frequency, dominating impurity mass, and ion temperature), so that only two functions of ψ are needed to prescribe it.

The flux functions n_e, T_e, f, and α (naturally parametrized so that they each contain only a few truly independent variables) yield distinctive signals in each of the diagnostics, which contain information both about the profiles of these flux functions, and also about

the flux surface topology. To the latter point the chosen diagnostic set in particular yields mutually complementing information:

A) contains good information about the flux surface shape at the boundary and in the outer zones of the discharge (besides I_t, $\beta_p + l_i/2$, and — for noncircular cross section — some information on l_i);

B) has information about the flux surface structure in the density gradient zone (apart from giving simultaneously the density profile);

C) contains information about the radial position of the magnetic axis and about the inside and outside intersection points of the near-axis flux surfaces with a horizontal plane;

D) improves the accuracy of the magnetic determination of the separatrix in the region where it is quantitatively most important;

E) contains information of the 2-d structure of the interior flux surfaces (by choice of appropriate filters it can be guaranteed that the radiation indeed is dominated by these interior zones — contrary to the situation with general bolometry).

Obviously the distinction of this method to previous approaches (apart from the speed in evaluation) is in the simultaneous use of information: e.g. the soft X-ray signals alone do not allow to determine the flux surface structure in the presence of unknown rotation, but can do it in combination with $T_e(R)$ signals in a single plane. At the same time profiles are determined for the important discharge parameters even in geometrically complex situations.

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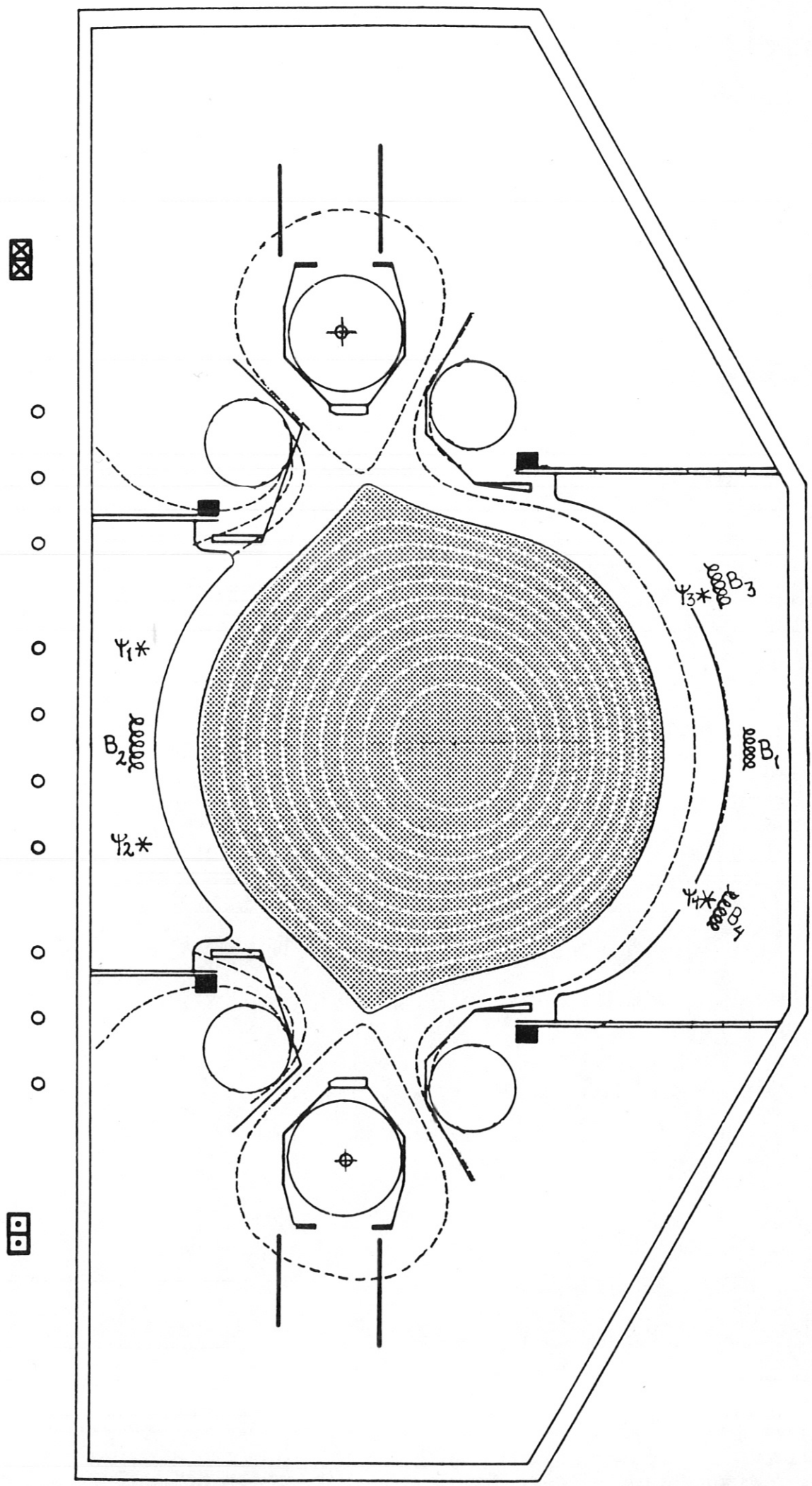
FIGURE CAPTIONS

Fig. 1. Poloidal cross-section through the ASDEX experiment. Measurements of the poloidal field and flux, $B_1, \ldots, B_4$ and $\psi_1, \ldots, \psi_4$ are made at the locations shown. The measurement I_{mpc} is the current through each of the two main multipole shaping coils, which are marked with the symbol $\oplus$. Each of the four smaller multipole coils next to the divertor entrances carries a return current $-\frac{1}{2} I_{mpc}$. Notice that the equilibria in our data base are in general not up-down symmetric.



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