FIRST RESULTS OF TIME SERIES ANALYSIS OF MHD FLUCTUATIONS IN THE ASDEX TOKAMAK

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IPP III/91

April 1984

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Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

Table of Contents

Abstract

A. Introduction

B. Power Spectra and Wave Identifications

C. Digital Complex Demodulation and Time Dependent Spectra

D. Experimental Results and Program Descriptions

References

Figure Captions

IPP III/91

DIGITAL ANALYSIS OF MHD FLUCTUATIONS IN ASDEX TOKAMAKS

L. Khadra

Abstract

Basic time series analysis techniques are used to investigate MHD fluctuations in ASDEX. In particular, Auto- and cross-power spectra are utilized to determine amplitude, frequency, and mode number of the magnetic field fluctuations and soft-X-ray signals. Moreover, time dependent spectra programs are used to follow the frequency evolution of various MHD-modes with time. (Some experimental results are presented.)

A. Introduction

With the introduction of the fast Fourier transform (FFT), the computer time required for Fouriert transform calculations was so sharply reduced that power and cross-spectra could be efficiently estimated from direct Fourier transform of time history records. The fast Fourier transform is merely an algorithm for computing the discrete Fourier transform (DFT) of a data series at all of the Fourier frequencies using relatively few arithmetic operations.

Computations of auto and corss-power spectra using FFT algorithm have been discussed extensively in reference /1/. Time series analysis, as applied to plasma fluctuations, has been discussed in detail in references /2,3/. The basic procedure as outlined in reference /2/ is to pass the time series through a window to reduce the effects of finite data length, to compute the FFT of the data, and then to compute auto and cross-power spectra. The spectra are then smoothed to reduce the statistical errors of the estimate.

The objective of these notes is to utilize basic time series analysis to analyze MHD fluctuations in the ASDEX tokamak. Section B will provide a brief review of some basic principles for the computation of auto, cross-power spectra, and coherency spectra. In section C we discuss the principle of digital complex demodulation procedure and how to use it to compute time dependent spectra. Section D presents some experimental results and descriptions of the output of FORTRAN programs.

B. Power-Spectra and Waves Identifications

For a real sampled time data function $f(t_i)$ the discrete Fourier transform (DFT) is defined as

$$F(f_k) = \frac{1}{N} \sum_{i=o}^{N-1} f(t_i) e^{-j2\pi ik/N} \tag{1}$$

and the inverse DFT is given by

$$f(t_i) = \sum_{k=o}^{N-1} F(f_k) e^{j2\pi ik/N} \tag{2}$$

where $f_k = k.\Delta f$ and $t_i = i.\Delta t$. The sampled time series consists of N discrete data points spaced Δt sec apart. In the frequency domain the transformed timed series consists of N discrete values spaced Δf Hz apart. The quantity Δf is called the elementary bandwidth and is related to Δt by the formula

$$\Delta f = \frac{1}{N.\Delta t} = \frac{1}{T} = \frac{fs}{N} \tag{3}$$

where T is the duration of the "raw" time series in sec, and f_s is the sampling frequency.

For a sampled data from a time record f(t) which is stationary, an estimate of a true power spectrum is defined as /1/

$$p(f_k) = \frac{2}{M} \sum_{m=o}^{M-1} F(f_{k+m})^2 \tag{4}$$

where k = 1,2,..., N/2M. To reduce the random error of the estimate, a smoothing operation indicated in eq. (4) is introduced. This smoothing operation is carried out by averaging over M adjacent spectral components.

Another way of smoothing the estimate is to average over an ensemble of estimates. For the programs used to analyze ASDEX data the first smoothing operation indicated in eq. (4) has been used.

To reduce the effect of leakage, which is a direct consequence of estimating a Fourrier transform of a finite data length, Hanning window has been used in the computation of power spectra programs.

The cross-power spectrum of two sampled time functions, which are stationary and measured at two different spatial points can be computed as

$$P_{12}(f) = \frac{2}{M} \sum_{m=o}^{M-1} F_1^{*}(f_{k+m}) F_2(f_{k+m}) \qquad (5)$$

where * denotes conjugate complex.
Since the cross-power spectrum is in general a complex quantity, it can be expressed as

$$P_{12}(f) = P_{12}(f) \cdot e^{i\theta_{12}(f)} \qquad (6)$$

where $P_{12}(f)$ is the cross amplitude spectrum and where $\theta_{12}(f)$ is the phase spectrum given by

$$\theta_{12}(f) = \theta_2(f) - \theta_1(f) \qquad (7)$$

As equation (7) indicates, the phase shift undergone by each spectral component is expressed in terms of the phase difference between the phase of $F_1(f)$ and the phase of $F_2(f)$. It is this property which enables us to determine the wave number of each of several modes present in the plasma.

Another important quantity which is of practical importance in analyzing plasma data is the squared coherency spectrum defined as

$$\gamma^2_{12}(f) = \frac{P_{12}(f)^{\;2}}{P_1(f).\, P_2(f)} \qquad (8)$$

The coherency spectrum measures the statistical correlation between two measured time series on a spectral basis. For further details of the phase and coherency spectrum we refer to reference /3/.

C. Digital Complex Demodulation and Time Dependent Spectra

Digital Complex Demodulation

Digital complex demodulation, the digital equivalent of heterodyning, is a digital method by which one can obtain the time traces of amplitude and phase modulation of a carrier wave with a center frequency ω_0 /4/. In the following we outline the key ideas with the aid of Fig. 1. In Fig. 1, ϕ (x,t) denotes the signal measured at point x. Without any loss of generality we can set x = o. To illustrate the philosophy of the approach we assume that $\phi(t)$ is an amplitude and phase modulated wave described by the following equation:

$$\begin{aligned}\phi(t) &= a(t) \cos\left[\omega_0 t + p(t)\right] \qquad (9)\\ &= \frac{a(t)}{2} \{\exp\left[i\omega_0 t + ip(t)\right] + \exp\left[-i\omega_0 t - ip(t)\right]\}\end{aligned}$$

where a(t), p(t) and ω_0 denote the amplitude modulation, phase modulation, and carrier frequency, respectively. Following Fig. 1, we multiply Eq. (9) by an exponential term $2\exp(-i\omega_d t)$, where ω_d is the demodulation frequency, to obtain

$$\begin{aligned}2\phi(t)\exp(-i\omega_d t) = a(t) \{&\exp\left[i(\omega_0 - \omega_d)t + ip(t)\right] \qquad (10)\\ &+ \exp\left[-i(\omega_0 + \omega_d)t - ip(t)\right]\}\end{aligned}$$

If we let $\omega_d = \omega_0$ and run the result into a digital lowpass filter, with a cut-off frequency less than $2\omega_0$, we get the complex demodulate c(t),

$$c(t) = a(t)\exp[ip(t)] \tag{11}$$

Actually, the results of carrying out these operations in the computer are expressed in terms of $c_r(t)$ and $c_i(t)$, i.e., the real and imaginary parts of c(t), respectively. Consequently, the instantaneous amplitude and phase modulation are found by computing

$$a(t) = \left[c_r^2(t) + c_i^2(t)\right]^{1/2} \tag{12}$$

$$p(t) = \tan^{-1} \frac{c_i(t)}{c_r(t)}$$

Time Dependent Spectra

The basic assumption of classical time series analysis is that the time series under study is stationary. That is, the statistical properties of the time series are invariant with translations in time.

In practice, however, the assumption of stationarity is often violated and the classical approach to spectral analysis will lead to artificially broadenend features in the spectrum. One such situation is the magnetic field fluctuations in tokamaks. These fluctuations exhibits marked changes, in amplitude and frequency, with time. The nonstationary character of such time series requires the study of the energy distribution in the time frequency plane.

Time dependent spectra can be computed in one of two ways. The first way is based upon the computation of short time spectra. That is, the time series understudy is subdivided into a number of small records and it is assumed that each record is stationary. Each subrecord is then multiplied by an appropriate window and then FFT-algorithm is applied on each subrecord. Although an analysis of this type can produce useful qualitative features, care must be taken in interpreting the quantitative feature of the plot. Dividing up the time series into smaller records results in poor spectral resolution, and since there is no averaging a large random error results /1/. The second way is to use the principle of digital complex demoduation procedures. Since complex demodulation uses a digital filter to recover the complex demodulate, a form of averaging will be introduced. Another advantage in using complex demodulate is that we do not need to subdivide the time series into smaller records, hence, better spectral resolution is obtained. We note, however, that the procedure of complex demodulation requires longer computational time. Our computational procedure /5/ is performed by complex demodulating a certain number of frequencies between zero frequency and the Nyquist frequency. For each demodulate one decimates at a rate of one point in an interval given by the inverse of the filter bandwidth f_p. This reduces the computational cost by the factor $2 f_N/f_p$, where f_N is the Nyquist frequency.

D. Experimental Results

Program FFTASD

This program is used to compute auto-power spectra, cross-power spectrum, and coherency spectrum of two different time series. Frequency averaging procedures, as outlined in the previous pages, are used to smooth the estimates. Program FFTASD requires the following INPUTS: shot and diagnostics numbers, name of the two time series, starting time of the time series in sec, and number of data points to be transformed.

Figure 2 demonstrates an example of the output of this program as applied to the magnetic field fluctuations. The two time series are obtained from two magnetic probes placed $\Delta\theta = 14.6^{o}$ apart. The signals are band-passed with passband f_p = 100 Hz - 40 KHz and digitize with a sampling interval Δt = 25 μsec. Each time series consists of 1024 data points. The spectra are smoothed over 5 adjacent spectral components. This leads to a frequency resolution of

$$\Delta f = M/N\Delta t = 0.195 \text{ KHz}.$$

As it is apparent in Figure 2 the auto power spectra of the two channels A5WNW and A7WNW demonstrate a coherent peak at about 18 kHz. At this frequency the phase spectrum has a value of about 30^{o}. Thus, the mode number can be determined as

$$m = \frac{\theta_{12(f)}}{\Delta\theta} \cong 2$$

At the bottom of Figure 2 the squared coherency spectrum is plotted. The coherency, which measures the statistical correlation between two time series, shows a high degree of coherency ($\cong 0.95$) at the frequency 18 kHz. This is expected since the two probes measure the same physical quantity.

The broadenend feature of the peak at 18 kHz is due to the nonstationary character of the magnetic field fluctuations. To investigate the evolution of frequency with time a time dependent spectrum is needed.

Another interesting feature of the phase spectrum is its indication of the direction of propagation. As demonstrated in Figures 3a and 3b the phase spectrum shows a change in sign for the mode present at 19 kHz.

Figure 4 demonstrates the power spectra of one soft-X-ray array. The array consists of 20 detectors. The viewing geometry of the soft-X-ray array is shown in Fig. 5. The signals are band-passed with passband 100 Hz - 60 kHz, and digitized with a sampling interval of 10 μsec.

The spectra of the detectors show a well-defined peak at 27.450 kHz. This peak can be seen by all detectors. The mode number, as determined from the phase spectrum, is m = 1. As the viewing of the detectors becomes closer to the center two more coherent peaks at 16.67 and 44.12 kHz can be seen. Since the frquency matching condition is satisfied, i.e. 16.67 + 27.45 = 44.12, the quesion arises of whether the waves present at the frequencies 16.67, 27.45, and 44.12 KHz are nonlinearly coupled. In order to answer this question we need to apply bispectral analysis techniques on the soft-X-ray signals. Bispectral analysis technique, which has been developed in reference /6/, enables us to discriminate between spontaneously excited modes and those which are coupled by nonlinear wave-wave-interactions by examining the phase coherency between them.

PROGRAM PHKAN

This program computes the phase shifts of a certain mode for all soft-X-ray detectors and with reference to an arbitrary detector. This programm requires the following INPUTS: shotnumber, starting time of the soft-X-ray signals, the value of frequency for which the phase shift to be computed, and the reference detector.
In Fig. 6 the phase shift of the mode present at 27.450 kHz with reference to detector A is computed.

Program ZEITSP

This program is used to compute time dependent spectra of nonstationary time series. The computational procedures are outlined in section C. Program ZEITSP requires the following INPUTS: shot and diagnostic numbers, name of channel, starting time of the time series in sec, and number of data points to be transformed.

Figure 7 shows an example of the output of this program as applied to magnetic field fluctuations obtained from one probe. The measured time series consists of 4096 data points and is chopped into 32 subintervals. The frequency resolution is then given by $1/N\Delta t = 0.312$ kHz. The spectrum is plotted on a linear axis in power to emphasize the evolution of large-amplitude modes. Figure 7 clearly demonstrates the variation of frequency which leads to broad peaks in the smoothed spectra.

To determine time evolutions of various modes simultaneously present in plasma, the computation of a time-dependent mode spectrum is required. Furthermore, we note that mode analysis, as discussed in section B, was based on two probes and the interference effect between various modes has not been taken into account. In order to accurately determine poloidal mode hierarchy in ASDEX, the techniques discussed in sections B and C should be further developed. This will be studied in the near future.

Acknowledgement

I would like to acknowledge the many contributions through fruitful discussions by F. Karger, M. Kornherr and J. Gernhardt. This work was performed in a relatively short time, thanks to the professional assistance of Miss E. Reimann, who prepared the computer programs. A special appreciation is also given to the DAAD for the scholarship I received to visit the Max-Planck-Institut.

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/6/ Y.C. Kim and E.J. Powers, "Digital Bispectral Analysis and its Applications to Nonlinear Wave Interactions", IEEE Trans. Plasma Sci., PS-7, 1979, page 120.

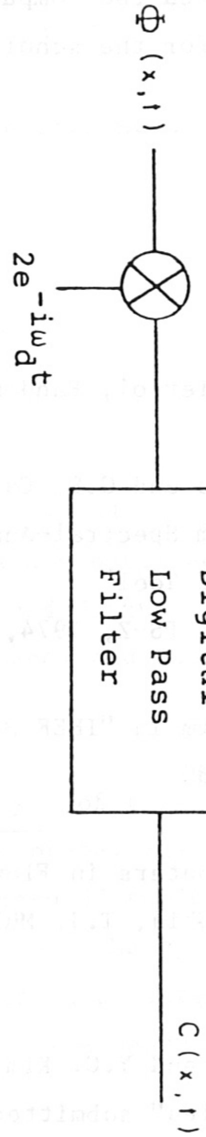


FIG.1 A BLOCK DIAGRAM ILLUSTRATING COMPLEX DEMODULATION.

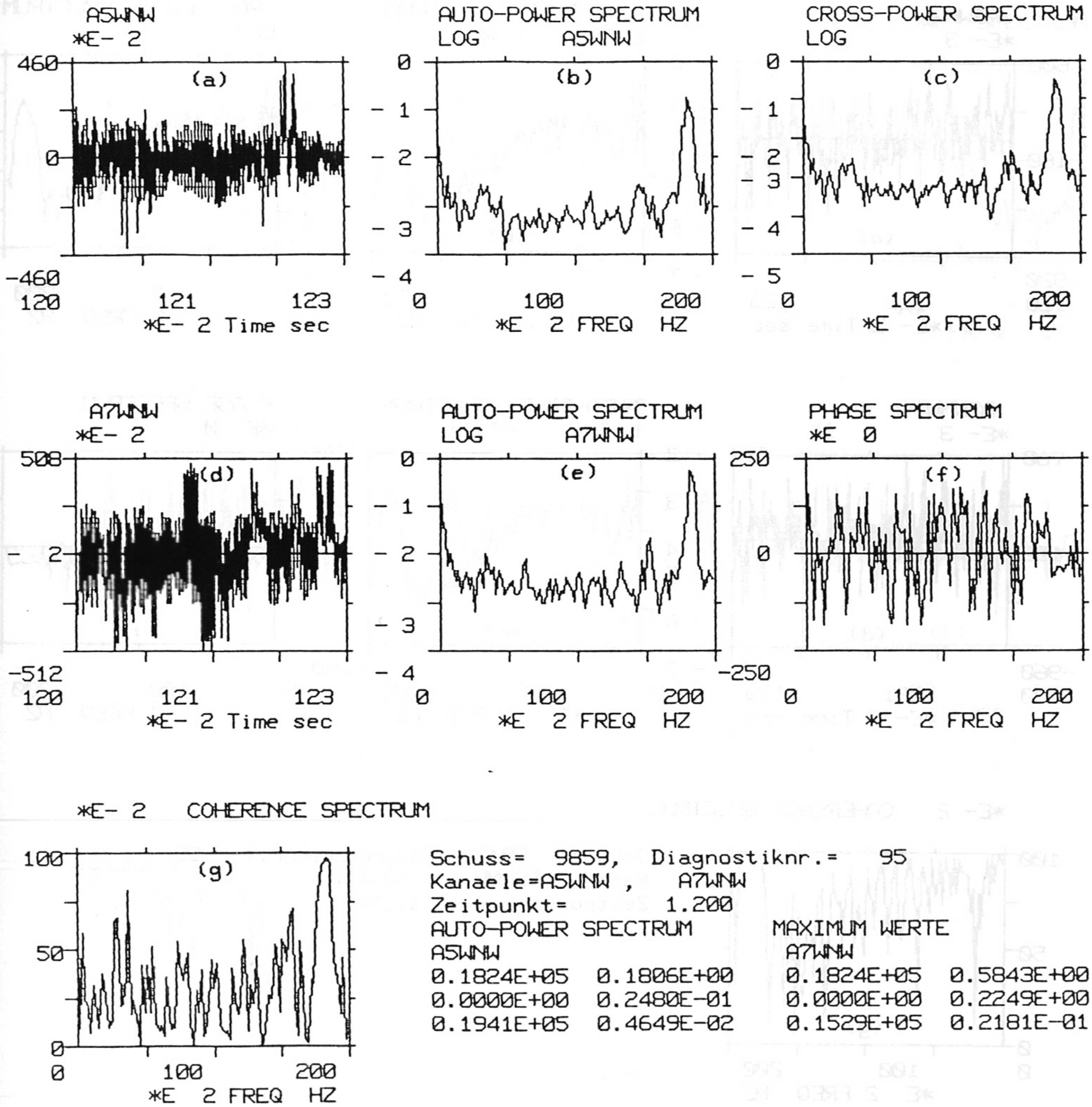


FIG.2 AUTO-POWER SPECTRA AND CROSS-POWER SPECTRUM
OF TWO SIGNALS

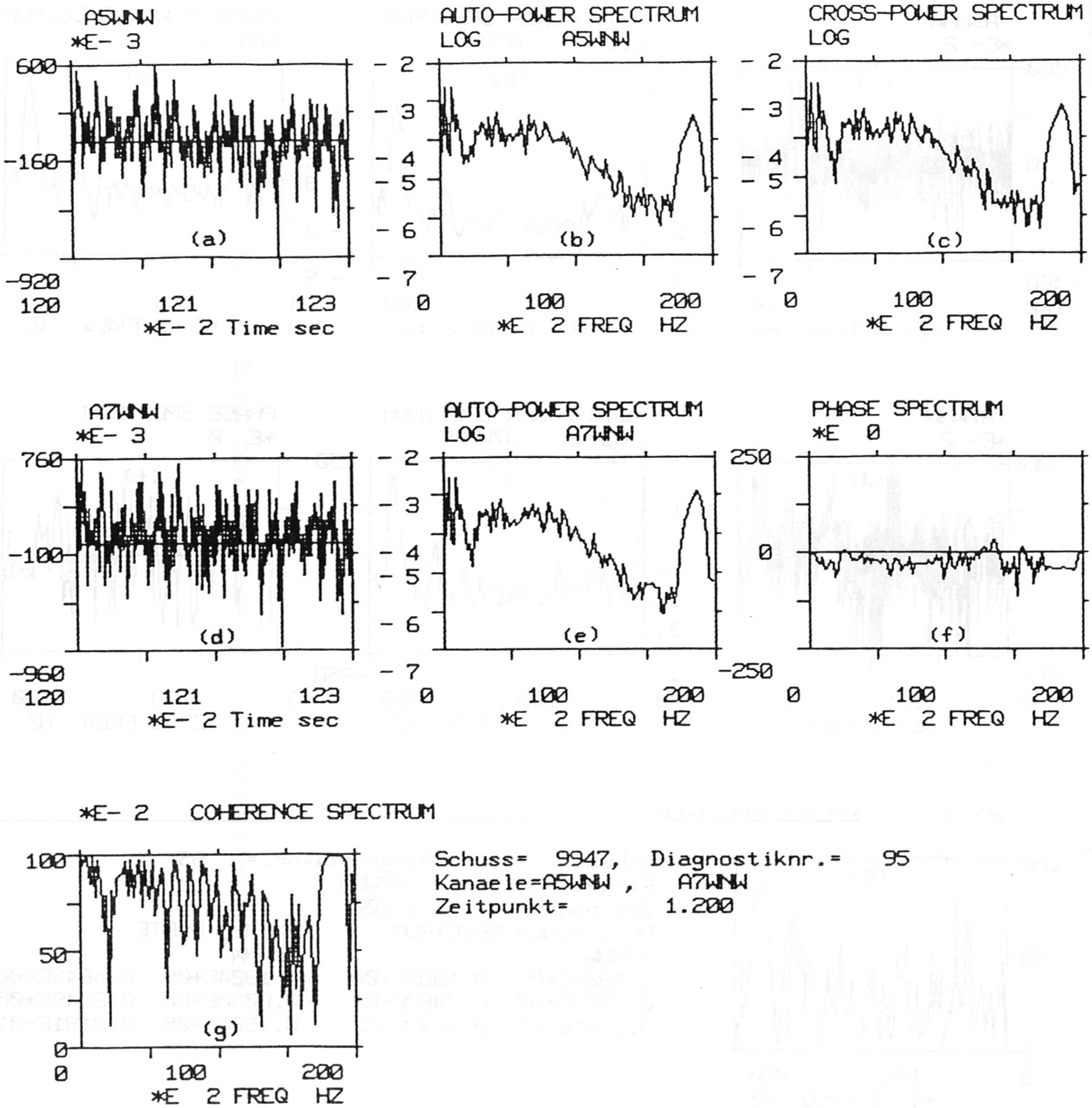


FIG.3A PHASE SPECTRUM SHOWS A NEGATIVE PHASE SHIFT
AT ABOUT 19 KHZ

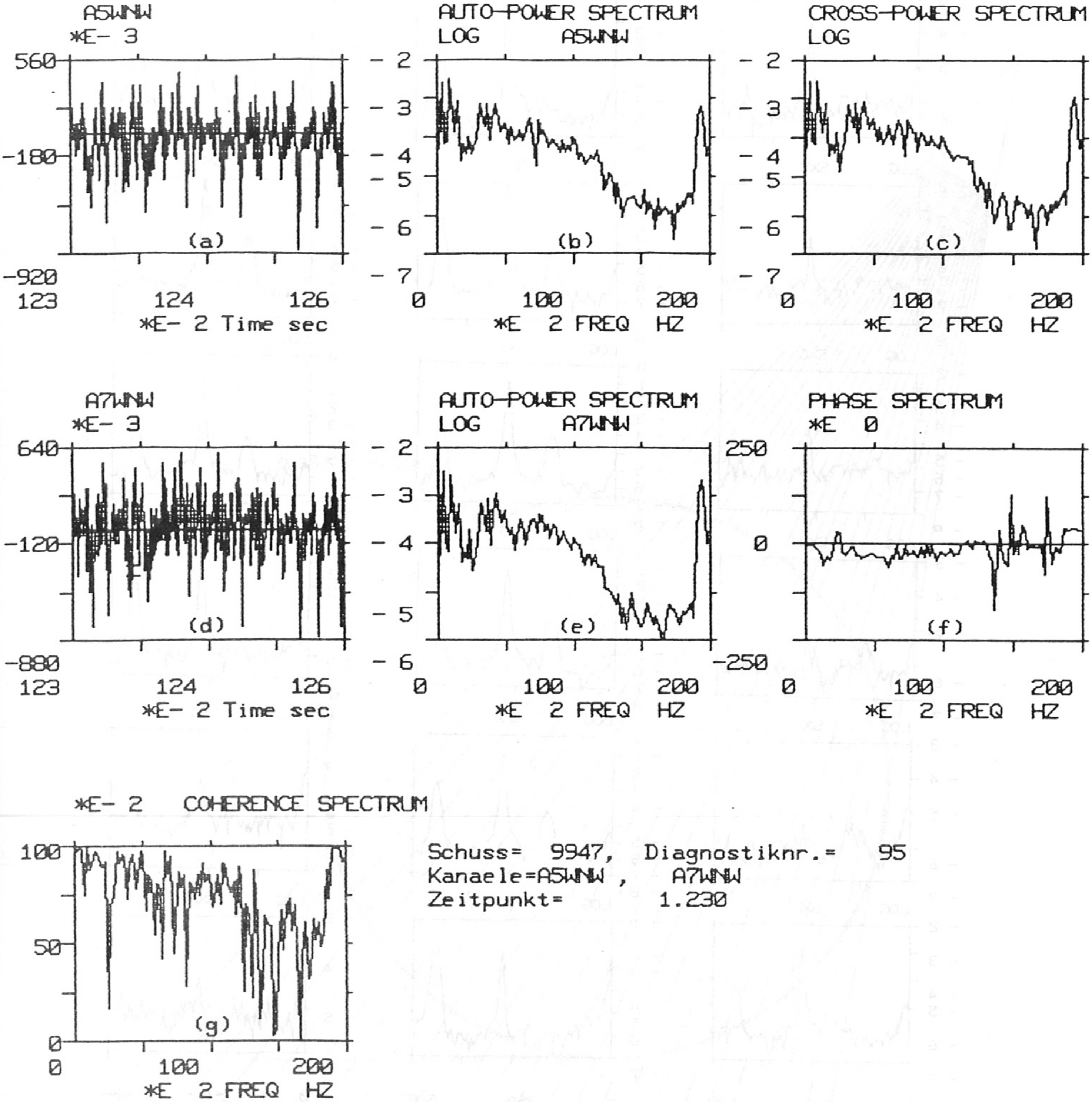
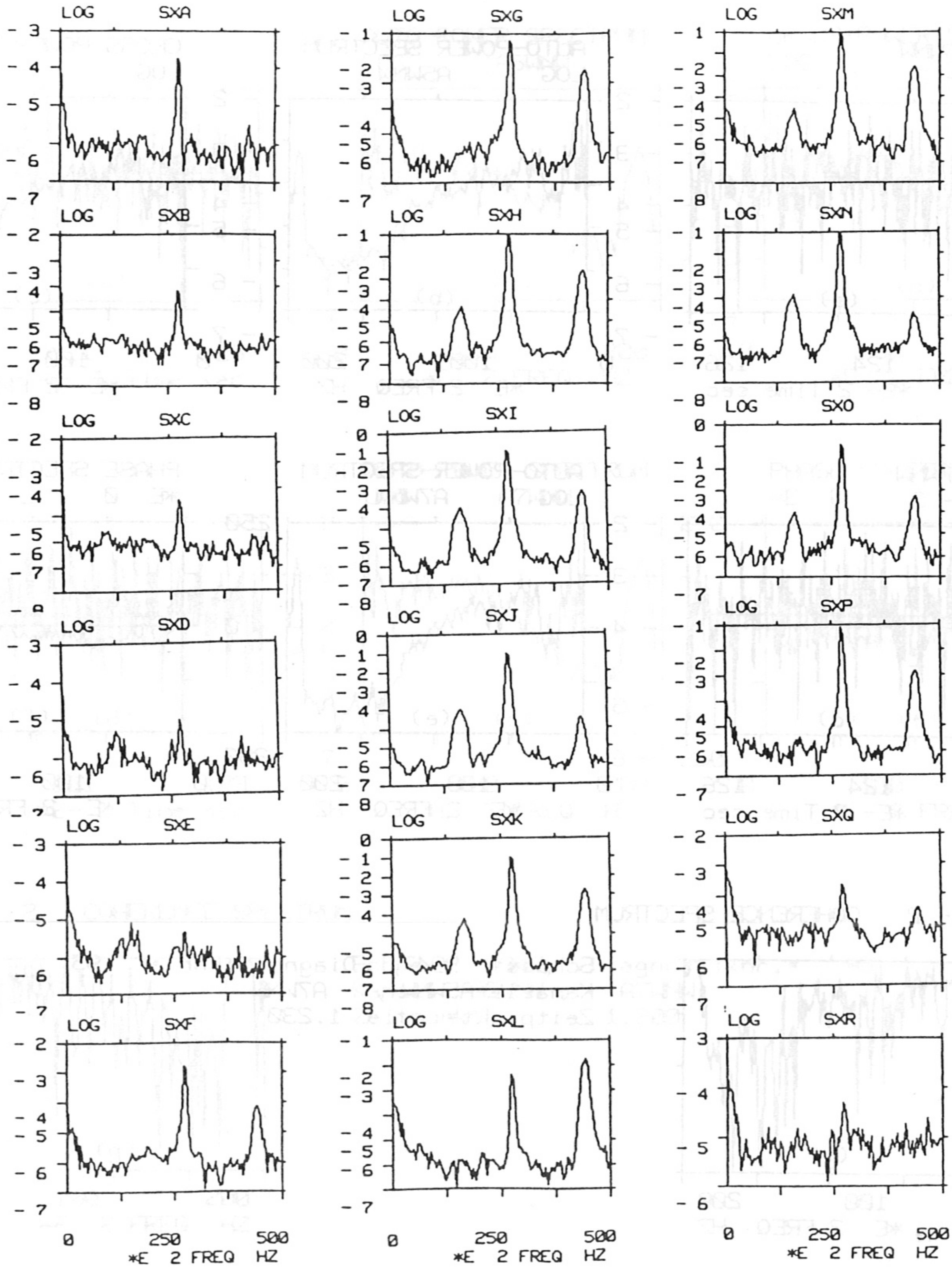


FIG.3B PHASE SPECTRUM SHOWS A POSITIVE PHASE SHIFT
AT ABOUT 19 KHZ



Schuss= 8459, Diagnostiknr.= 201

FIG.4 AUTO-POWER SPECTRA OF THE SOFT-X-RAY UNIT

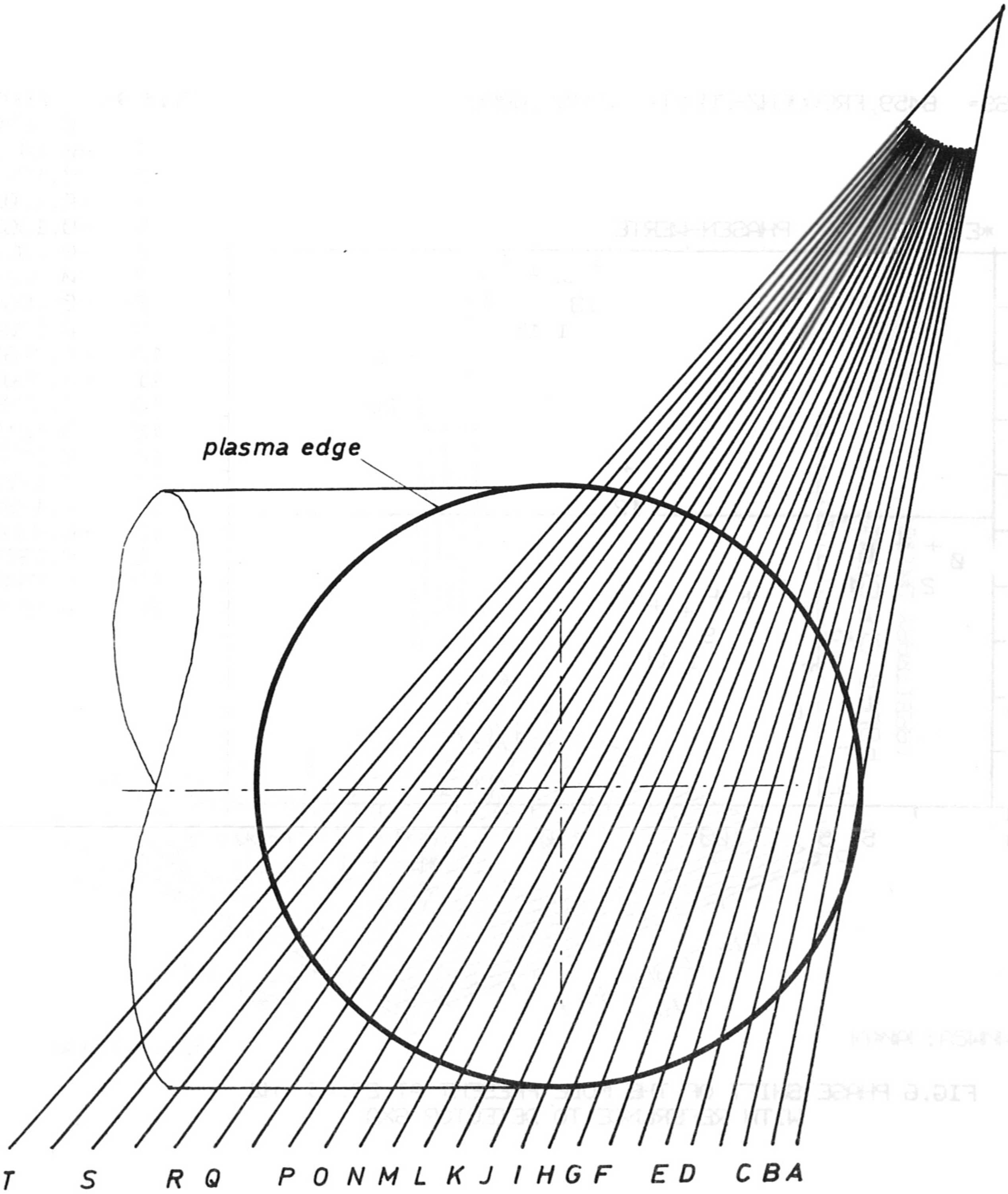
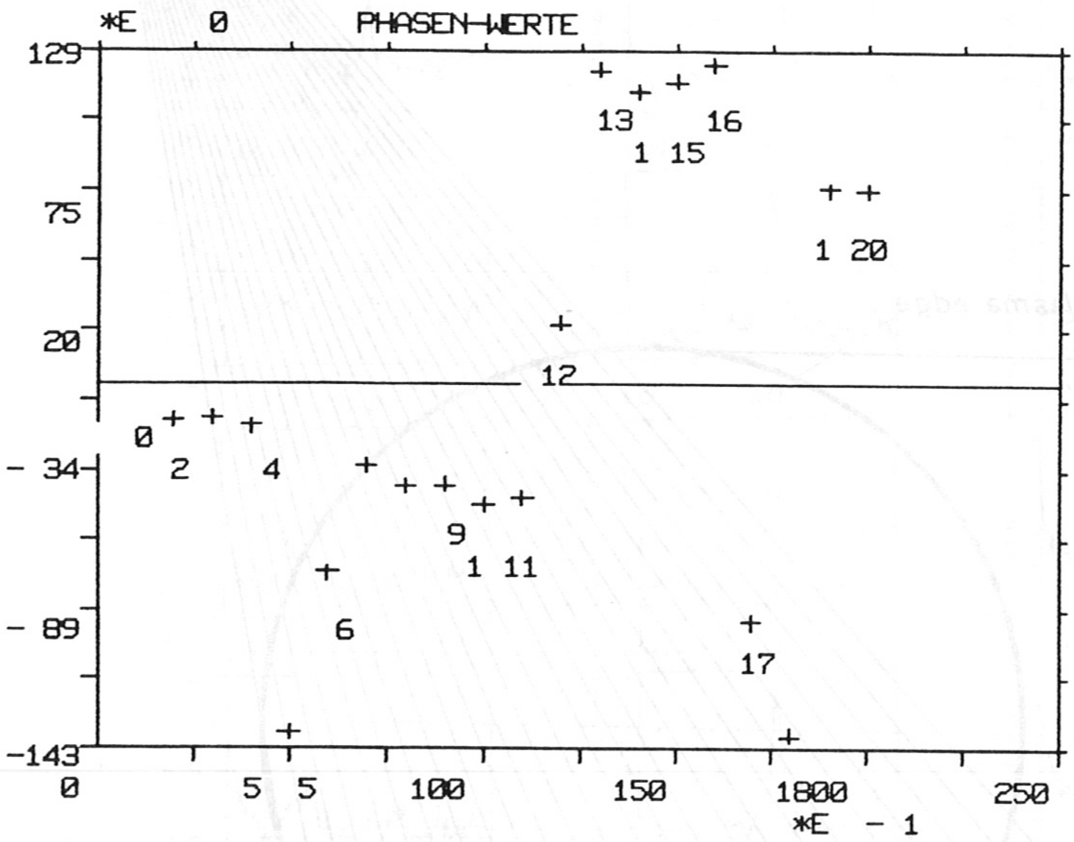


Fig. 5 Viewing geometry of the soft X-ray unit (SXA...SXT)

SCHUSS= 8459,FREQUENZ-PUNKT= 27450.0000



KANALNR.	PHASE
0	0.0000E+00
2	-0.1443E+02
3	-0.1372E+02
4	-0.1702E+02
5	-0.1363E+03
6	-0.7377E+02
7	-0.3229E+02
8	-0.4066E+02
9	-0.3984E+02
10	-0.4767E+02
11	-0.4507E+02
12	0.2336E+02
13	0.1219E+03
14	0.1133E+03
15	0.1173E+03
16	0.1239E+03
17	-0.9307E+02
18	-0.1379E+03
19	0.7549E+02
20	0.7533E+02

FIG.6 PHASE SHIFT OF THE MODE PRESENT AT 27.45 KHZ
WITH REFERENCE TO DETECTOR SXA

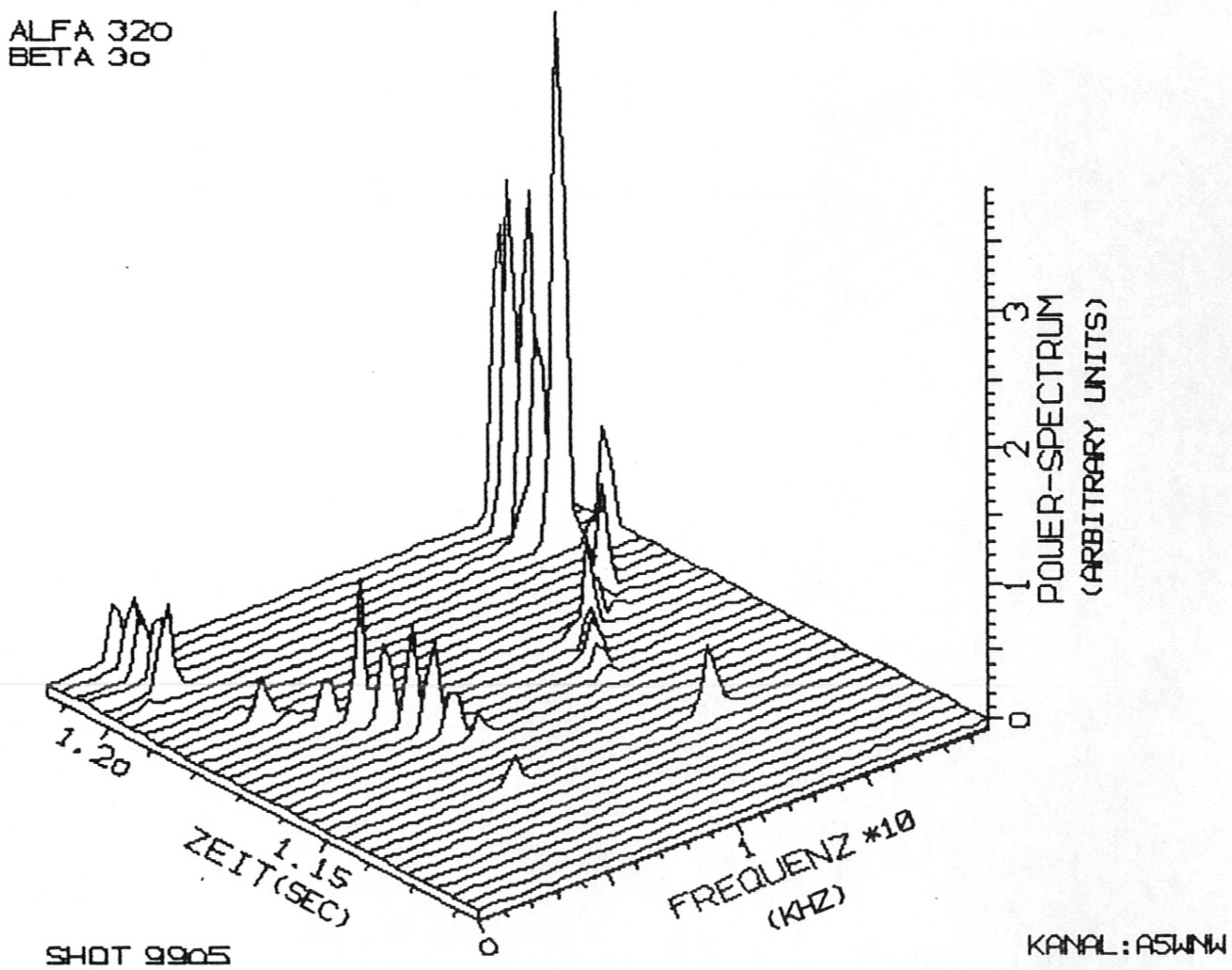


FIG.7 TIME-DEPENDENT SPECTRUM OF MAGNETIC FIELD FLUCTUATIONS
(STARTING TIME:1.115)