STABILITY INVESTIGATIONS OF THE ASDEX FEED-BACK SYSTEM WITH FILTERS FOR REDUCING THYRISTOR NOISE

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Abstract

A computer program for analysing the absolute and relative stabilities of any complex system by the root-locus method was developed. It is used to reanalyse the present horizontal position feed-back control in the ASDEX tokamak and to select the optimum parameters for this system with RCL filters for reducing thyristor noise.

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Introduction

Since a toroidal current loop tends to expand, it is necessary to obtain equilibrium by using external inward forces. Passive elements such as a copper shell or external windings can produce a vertical field suitable for balancing the internal magnetic pressure. In order to adjust the plasma position, there is a tendency in modern tokamaks to reduce passive effects. This makes it useful to work with feed-back systems because the interaction between the plasma current and the equilibrium magnetic field depends on the energy distribution within the plasma.

When feed-back control apparatus is used, particular attention must be paid in order that the response to an input signal comply with a few requirements. It should be as fast as possible, should not have too high overshoot and should not contain a steady-state error. Of course, these are contradictory requirements and a compromise must be obtained.

In modern control theory there are some techniques for studying these problems from different points of view. In some tokamaks, for instance, investigation of the control circuits has been done in the frequency domain /1,2/. This method permits the use of models to measure the characteristics of the system and the use of graphic techniques (Bode plots or Nyquist and Nichols diagrams). In other cases, for instance with multivariable systems or when it is not possible to make a model, it is convenient to work in the time domain using a matrix formalism to solve the differential equations which characterize the system /3,4/; this technique, known as "optimal feed-back control theory", has the advantage that the whole control apparatus can be investigated, including all nonlinear effects at large amplitudes.

The first part of this paper briefly describes the root-locus method, showing how it can be used to analyse the response of a feed-back system and its relative or absolute stability. The second part describes a flexible computer program that finds and plots the root-loci for any complex system. In the third part this program is used to reanalyse the horizontal position control in ASDEX, allowance also being made for the effects of the eddy currents in the vacuum vessel, measured with a 1:10 model. In addition, results are reported for the closed loop improved by means of an RCL filter in order to cut off high-frequency thyristor noise present in the system. The last section contains the conclusions.

Root-Locus Method

It is always possible to describe linear systems by means of a set of simultaneous differential equations, which can be Laplace transformed to yield a set of linear algebraic equations establishing a relation between input and the output of the system; with a block diagram this relation can be described as

$$G(p) = \frac{V(p)}{I(p)}$$

where I and V, respectively, are the input and the output of the system, G(p) is the transfer function and p is a complex frequency. The block diagram for a feed-back control is described in Fig. 1 and its new full transfer function assumes the form

$$T(p) = \frac{G(p)}{1 + G(p)H(p)}$$

It is obvious from the Laplace transformation theory that the time dependence of the system response is driven from the position, in the p plane, of the denominator roots in the transfer function T(p); studying this part of the problem is therefore equivalent to finding the zeros of the "characteristic equation" $1 + GH = 0$. Since a control system can be very complex and can have a lot of parameters that must be adjusted, it is obvious that in order to optimize the response, it must be known how the zeros of the characteristic equation migrate in the p plane by changing each parameter. The curves drawn from the zero migrations in this plane are referred to as root-loci, and from them one can decide whether the system response becomes faster or slower, whether it remains stable, how large the overshoot in the response is and so on. For instance, when the characteristic equation describes a second-order (two-zero) system or when there are two dominant roots (much closer to the imaginary axis than the others), the system can be characterised directly by using the literature /5/.

When it is possible to write the closed-loop characteristic equation as $1 + k\,GH = 0$, where k is one of the parameters that we are varying to optimize the response, just a few rules allow us to get a qualitative idea of the root-locus curve, for the parameter k, when we only know the zeros and the poles in the open-loop function GH. This is one of the greatest advantages of the root-locus method because it can be difficult in practice to find the zeros of high-order polynomials; these rules, however give an approximate graphic result rather quickly. Unfortunately, this procedure can only be easily applied to not very complex systems because we have to write the characteristic equation as $1 + kGH = 0$ and because, in the case of a system with high-order polynomials, we can have many roots close to each other. When we want the root-locus exactly or when we have a large system with many parameters, we are therefore compelled to use a computer to analyse our control apparatus.

Computer Program Description

As we already explained, the basic idea of the root-locus method is to find the zero migrations in the characteristic equation $1 + GH = 0$. Since we are working with linear systems, the solution of this equation can always be reduced to finding the zeros of a general polynomial of suitable degree. The literature contains a lot of numerical subroutines capable of finding real and complex roots of polynomials /6/. The main problem therefore was to use these subroutines for the more general situation of a feed-back control, avoiding any possible mistake introduced by handling polynomials. In Fig. 1 general feed-back system is shown where $G_1 \ldots G_h$ are the transfer functions of the single parts (note that G_h can also be a very complex closed internal loop) and H is the controller transfer function. In linear approximation each transfer function G and H can be written, respectively, as $G = \frac{A_h}{B_h}$ and $H = \frac{H_1}{H_2}$, where A_h, B_h, H_1, H_2 are elementary polynomials or, for complex systems (e.g. when G describes a complicate internal loop), are sums of products of elementary polynomials. It is therefore easy to see that finding the zeros of the characteristic equation is equivalent to finding the roots of $B\,H_2 + H_1\,A = 0$ (1), where $B = \prod_{h=1}^{N} B_n$ and $A = \prod_{h=1}^{N} A_h$; it is also clear that equation (1) is always represented by a sum of groups of products of polynomials, where each single polynomial describes a well identified part of the system. The basic idea of the program is to use as input only the attributes of these elementary polynomials (the degree, the coefficient values and the groups of the sum in which they are present) and to manipulate them in order to obtain the final larger polynomial describing the characteristic equation of the present system. This method allows one to obtain the root-loci for any complex apparatus while maintaining the individuality of each single part of the apparatus; indeed, each parameter is described by a coefficient of our elementary polynomial and, since each polynomial has its own individuality, the same happens for the element we are varying. This makes it easy to use as parameter in the root-locus determination whichever single element we want without manipulating the whole system; for the same reason it is also easy to have the root-loci in series for different parameters, thus obtaining a complete description for the whole system in a single graph.

An easier program was also developed to study the system stability. This program looks for the migration of the nearest maximum to the imaginary axis, in the complete closed loop transfer function $T = \frac{G}{1 + GH}$, changing the selected parameter; it is obvious that this maximum corresponds to one zero of the characteristic equation $1 + GH = 0$. This program has the drawback that it only finds one root, but it is useful for fast preliminary analysis.

Results for Horizontal Position Control in the ASDEX Tokamak

a)

Figure 2 shows the block diagram for the present horizontal position control in the ASDEX tokamak, without internal plasma effects being taken into account. The block G_1 is an RC filter for cutting off high-frequency noise on the signal line produced by the plasma; the time constant for this filter is about $\tau_1 = 3 \times 1^{-3}$ s and its transfer function is of the form

$$G_1 = \frac{1}{1 + p\tau_1} = \frac{1}{A}$$

The block G_2 describes the electrical response of the subsystem composed of the vertical field windings and of the vacuum vessel; its transfer function has already been measured /7/ on a 1:10 ASDEX model and is of the form

$$G_2 = \frac{1 + p\,\tau_4}{(1 + p\,\tau_w)(1 + p\,\tau_v)}$$

where the time constants have the values $\tau_H = 10^{-2}$ s, $\tau_w = 10^{-1}$ s, $\tau_v = 5 \times 10^{-3}$ s. The block G_3 describes the vertical field penetration in the vacuum vessel; it was also measured with the model, yielding, for the transfer function

$$G_3 = \frac{1 + p\,10^{-3}\,\mathrm{s}}{1 + p\,10^{-2}\,\mathrm{s}} = \frac{D}{E}$$

The open-loop gain for this system is given from the thyristor amplification, from the flux difference measured in the plasma boundary with unity current in the vertical field windings and from a factor converting this flux difference into volts; multiplying all these factors, we obtain for the open-loop gain the value $\alpha = 13$.

The block H describes the controller, which consists of a proportional component plus a derivative action; its transfer function is given by

$$H = \frac{K + \tau_H\, p(K+K')}{1 + \tau_H\, p} = \frac{M}{N}$$

Here K is the proportional gain, K' is the gain for the derivative component, and τ_H is the differentiator time constant. It is easy to see that finding the zeros of the characteristic equation $1 + GH = 0$ for this system means finding the roots for the polynomial $P(p) = ACNE + \alpha DMB$, where A, C, N... are the elementary polynomial input in the root-locus computer program.

Figure 3 shows the root-locus curves obtained by varying the controller parameters; here the arrows indicate the direction of rising parameter values. The plot number /1/ is the root-locus using the proportional gain as parameter and without the derivative component. It is possible to observe that, when the gain is increased, the system becomes faster but less stable until, for K = 10, it is fully oscillating. Fixing K = 6 and introducing the derivative component with $\tau_H = 10^{-3}$ s, the curve /2/ gives the root-locus for the derivative gain K'; this locus shows that with rising K' the system becomes more stable. The third plot is obtained by varying the time constant τ_H and keeping K(=6) and k'(=20) fixed. It is interesting to note that there is optimum stability with $\tau_H = 10^{-3}$ s.

Figure 4 shows the system, inclusive eddy currents caused by the plasma current. The new full block $\frac{\Delta\Psi}{\Delta R}$ describes what the flux difference in the plasma boundary is when the external vertical field changes; $\frac{\Delta\Psi}{\Delta R}$ (where R is a plasma major radius displacement) is only a factor and was computed with an equilibrium code /7/; $\frac{\Delta R}{\Delta B}$ is a closed internal loop; indeed, when the vertical field changes, the plasma moves and this movement produces eddy currents in the vacuum vessel whose field must be added to the external field.

In order to know the value and the time dependence of the vertical field produced from these eddy currents, a measurement was made in the old ASDEX model, yielding the following transfer function

$$\frac{\Delta B}{\Delta R} = \frac{p\, \beta\, \tau_s}{1 + p\tau_s}$$

where $\tau_s = 10^{-2}$ s is the vacuum vessel time constant and β is the mirror field produced from the eddy currents.

The upper part of the internal loop $\frac{\Delta R}{\Delta B}$ is simply described by a numerical factor obtained by means of the Shafranov equilibrium together with the flux conservation /1/, and so the transfer function for the whole block $\frac{\Delta \Psi}{\Delta B}$ is of the form

$$\frac{\Delta \Psi}{\Delta B} = \frac{\gamma(1 + p\ \tau_s)}{1 + \tau_s(1 + \eta)}$$

where $\gamma = 45$ Vs/T, $\eta = 0.9$ and the new full open-loop gain assumes the value $\alpha = 75$. Figure 5 shows the root-locus for the complete system, the curve numbers having the same meaning as in Fig. 3. It is possible to observe how the system becomes globally a little less stable and less fast, but it again remains stable until a proportional gain of K = 2. It is to be noted that the theoretical steady-state error /5/ for this system,

$$e_{ss} = \lim_{p \to 0} \frac{1 + GH - G}{1 + GH}\ I(p)$$

assuming as input I(p) a unit step function and using a porportional gain K = 0.8 (typical of ASDEX shots), is $e_{ss} \simeq 0.2$, whereas during ASDEX operation a steady-state error of about 10 % was observed.

b)

In order to eliminate appreciably the high-frequency noise the ASDEX horizontal position control was improved by inserting an RCL filter behind the thyristor. Figure 6 shows the block diagram system for this situation, where the symbols have the usual meaning. The reference values for the filter components used for studying the controller were $R_F = 10^{-2}\,\Omega$, $C_F = 2 \times 10^{-2}$ F and $L_1 = 1.3 \times 10^{-4}$ H. Figure 7 shows the root-locus curves used as parameters: 1) the proportional gain without the derivative component; 2) the derivative component gain, keeping $K = 0.8$ and $\tau_H = 10^{-3}$ s fixed; the time constant τ_H with $K = 0.8$ and $K' = 4$ fixed; the arrows on the curves always indicate the locus direction of rising parameter values.

It is seen that the situation is a little more complicated than in Fig. 5; the RCL filter introduces a stable but fast oscillating damped root with a period of about $T = 10^{-3}$ s and a damping time of about $t = 2 \times 10^{-2}$ s; in addition, since the angle between the root position and the imaginary axis is too small, we can expect a high overshoot in the response. Preliminary measurements made during a few ASDEX shots seem to confirm these results. Observing the curve (2), we can see that in this situation it is not convenient to use a very high derivative component gain; in fact, the lower root becomes more stable, but the upper one moves towards the imaginary axis, making it necessary to strike a suitable compromise. We can state the same for the parameter τ_H, indeed the optimum for the lower curve being worst for the upper curve; the plot (1) is exactly the opposite, high proportional gain making the new root more stable and the old one more unstable.

Figures 8, 9 and 10 show, respectively, the root-locus having the resistance as parameter, the capacity and the inductance of the RCL filter with the controller values fixed at $K = 0.8$, $K' = 4$, and $\tau_H = 10^{-3}$ s. We observe in Fig. 8 that, in order to eliminate the oscillations experienced in ASDEX with the new filter, it might be sufficient to raise the value of the resistance R_F; this should decrease the overshoot and its damping time.

Conclusions

A computer program for investigating any complex system by the root-locus method was developed. It requires as input that the characteristic equation simply be formed with elementary polynomials describing the individual system components. Therefore, in order to have the root-locus for any parameter, it is not necessary to rearrange the characteristic equation as $1 + k\ GH = 0$.

This program was used to reanalyse the present horizontal position control in the ASDEX tokamak, with allowance also for the effects of eddy currents induced in the vacuum vessel by plasma displacement. This effect is represented in the block diagram by an internal closed loop, and its transfer function was measured in the old 1:10 ASDEX model.

The program was also used in an attempt to improve the system with an RCL filter in order to cut off high-frequency noise. The root-loci for the controller parameters showed that the filter introduces new roots and a compromise has to be reached in order to optimise the position of these new roots and of the old ones.

Furthermore, the root-locus shows that the filter introduces a fast oscillation with a relatively high overshoot in agreement with some experimental shots in ASDEX; in order to eliminate these oscillations, a root-locus for the filter parameters was made, demonstrating that this can be done by raising the value of the filter resistor.

As was done for the ASDEX horizontal position control, the same technique can be also used to analyse and plan different feed-back devices in future new tokamaks, readily allowing the controller characteristics to be optimized.

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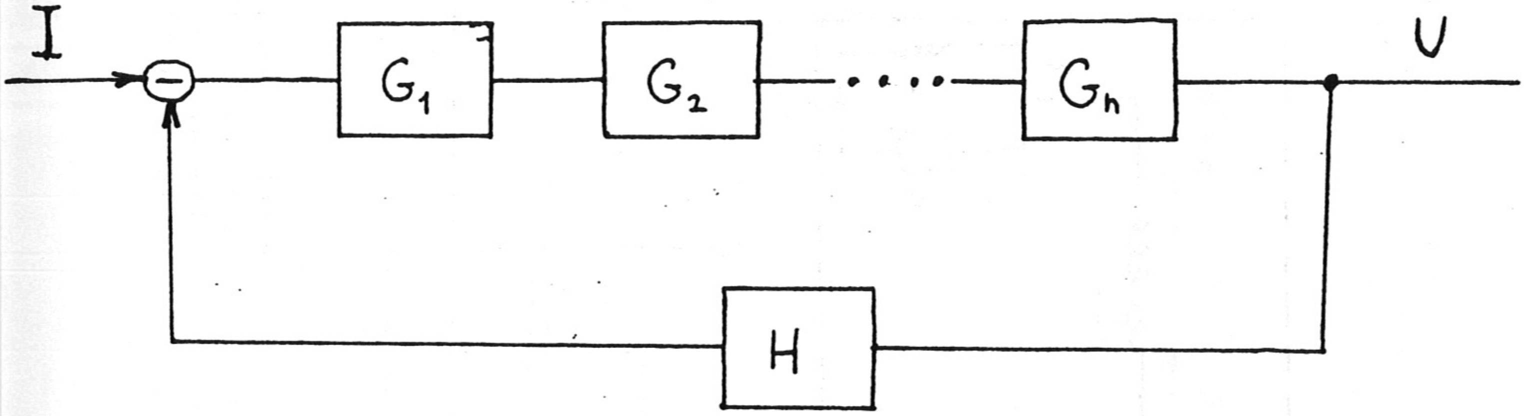
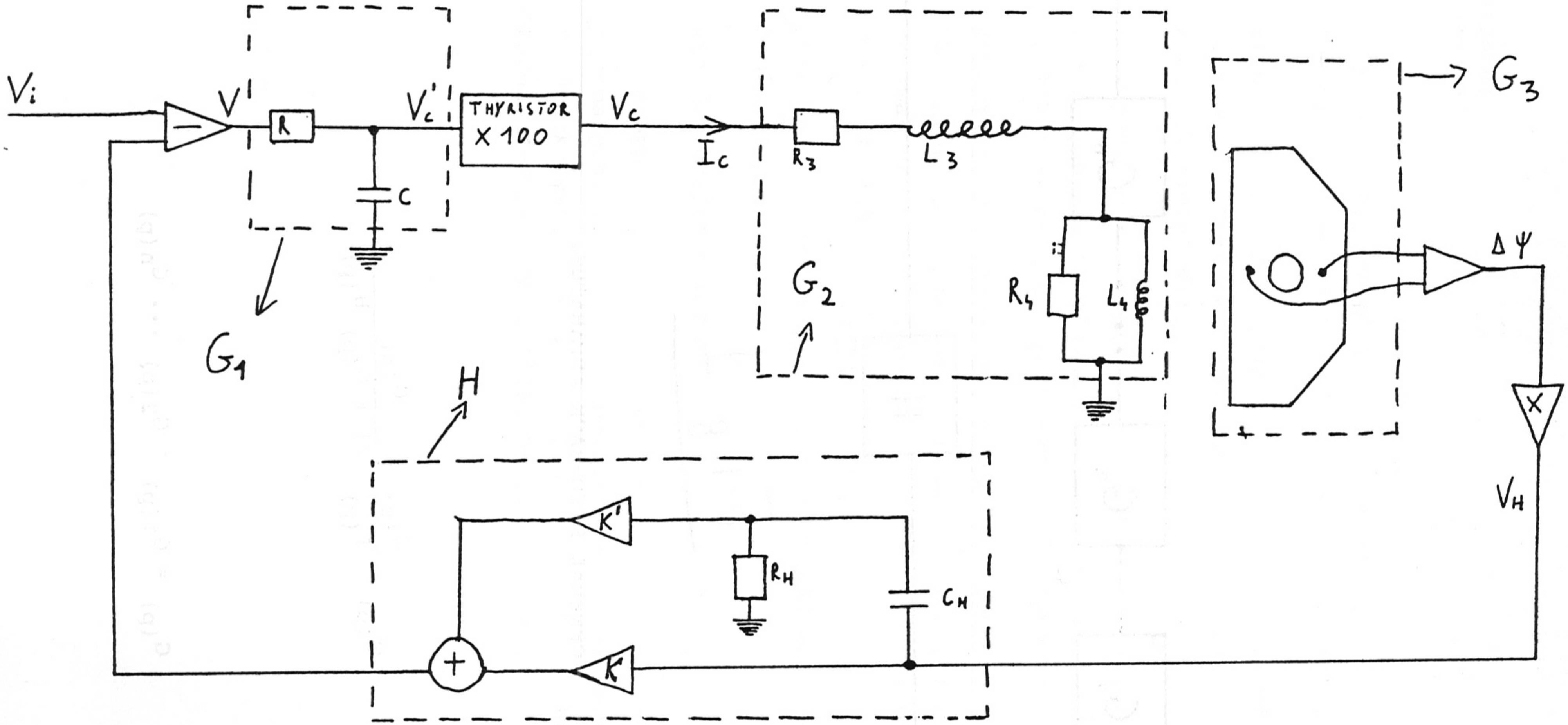


Fig 1

GENERAL FEED-BACK APPARATUS.

$$T_{(p)} = \frac{U_{(p)}}{I_{(p)}} = \frac{G_{(p)}}{1 + G_{(p)} \, H_{(p)}}$$

$$G_{(p)} = G_{1(p)} \cdot G_{2(p)} \cdots G_{n(p)}$$

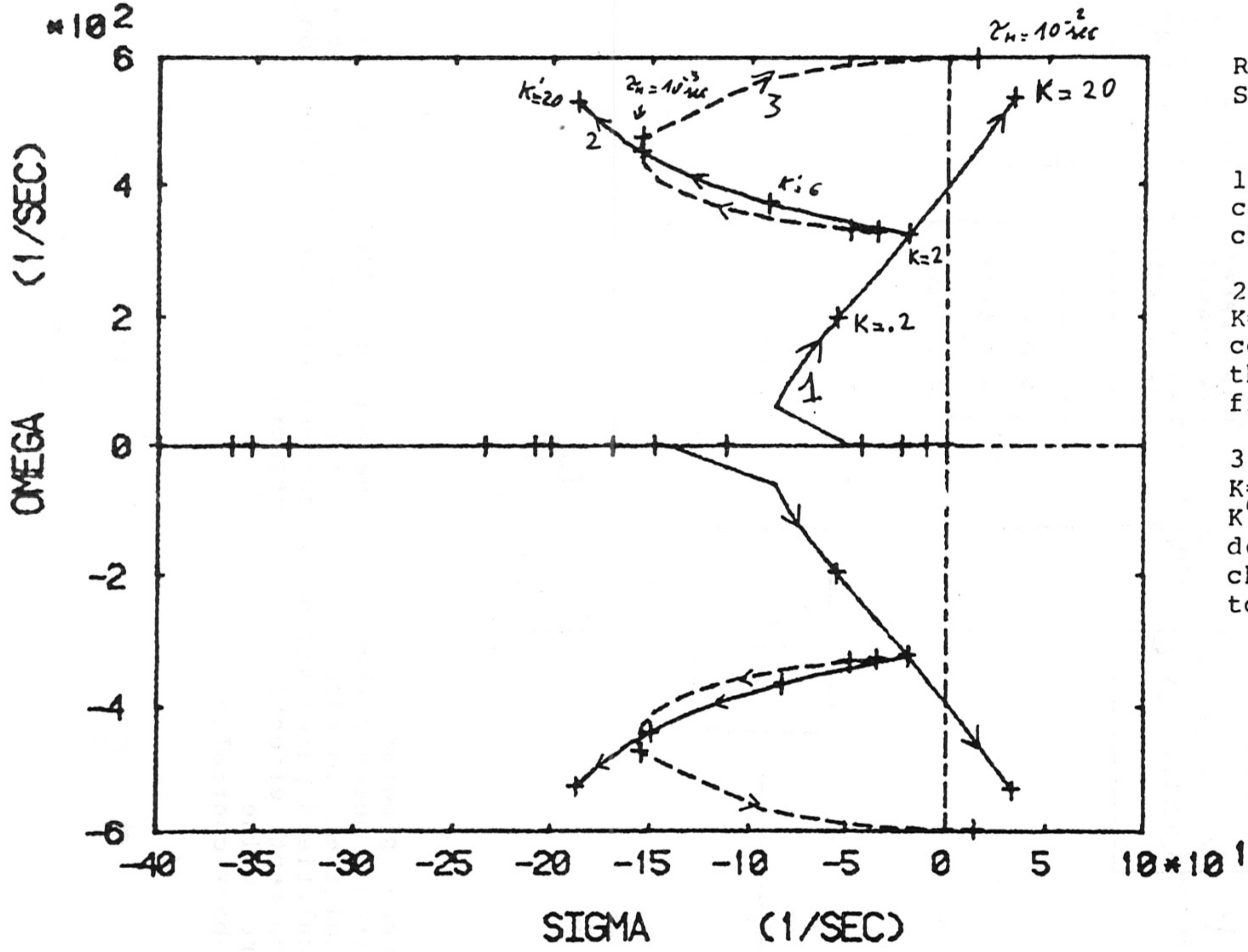


V_i = References program
G_1 = High-freq. noise filter
TH = Thyristor amplifier
G_2 = Vertical field impedance
G_3 = Vertical field penetration
$\Delta\Psi$ = Measured value
V_H x H= Feed-back controller

Fig. 2

Block diagram describing the present horizontal position control in ASDEX,without allowance for plasma current induction effects.

ASDEX ROOT-LOCUS



ROOT-LOCI FOR THE FIG.(2) SYSTEM

1) Curve : only proport. controller; the gain changes from K=Ø to K=2Ø.

2) Curve : proport. gain K=const=6; derivator time constant τ_H =const=1Ø$^{-3}$ s; the derivator gain changes from K' =Ø to K' =3Ø.

3) Curve : proport. gain K=const=6; derivator gain K' =const=2Ø; the derivator time constant changes from τ_H =1Ø$^{-4}$ s to τ_H =1Ø$^{-2}$ s.

Fig 3

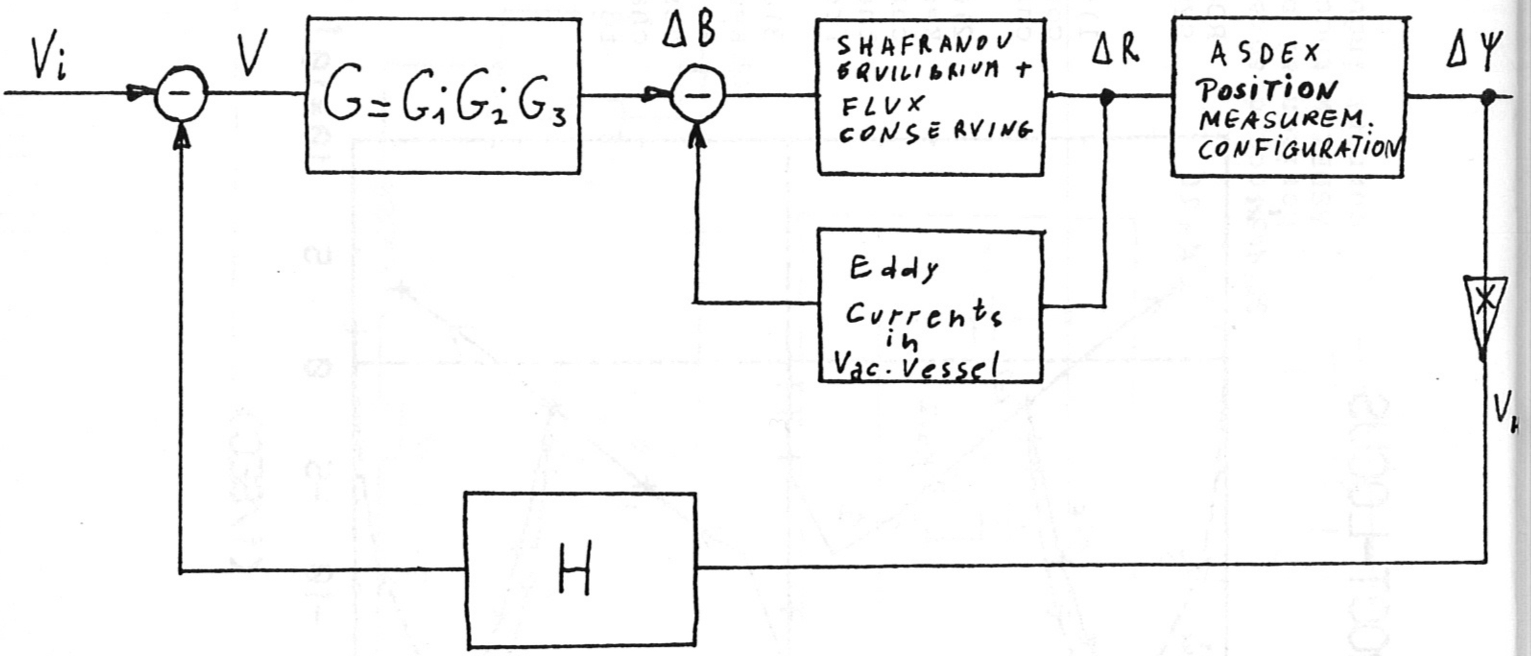
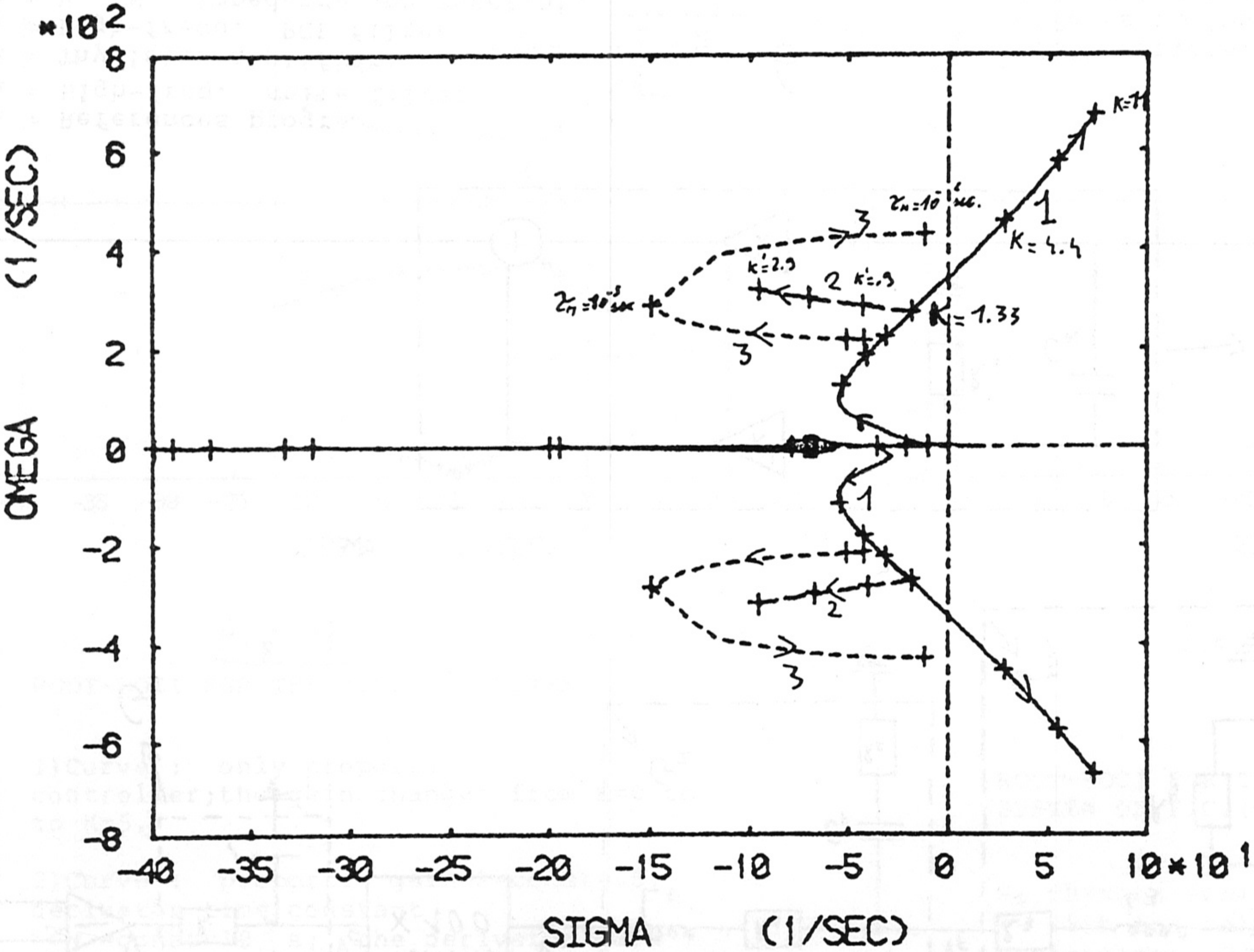


Fig 4

V_i = Reference program
G_1 = High-freq. noise filter
G_2 = Vertical field impedance
G_3 = Vertical field penetration
G_4 = Eddy current effects
$\Delta\Psi$ = Measured value
$V_H \times H$ = Feed-back controller

The present ASDEX horizontal position control, with allowance for the plasma current and vacuum vessel eddy current effects.

ASDEX ROOT-LOCUS



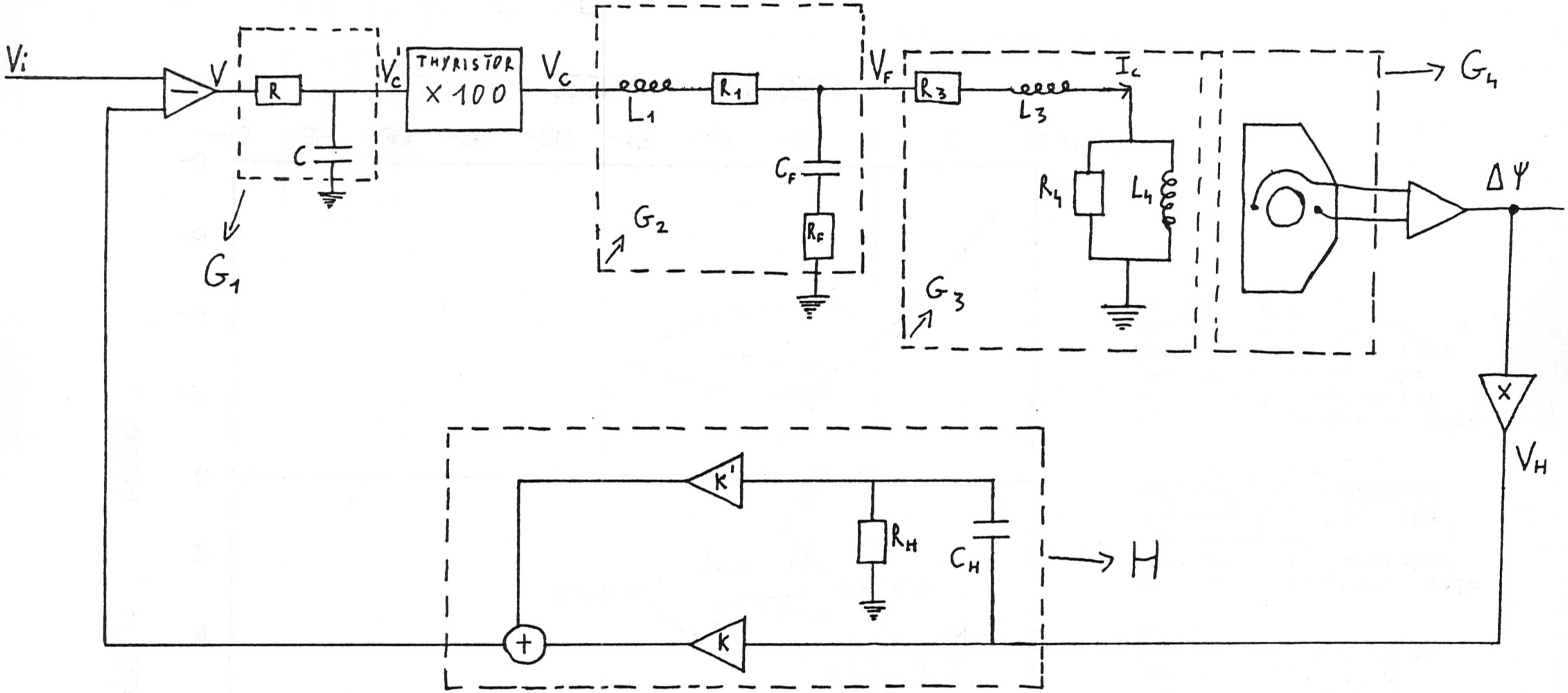
ROOT-LOCI FOR THE FIG.(4) SYSTEM

1) Curve : only proport. controller; the gain changes from K=Ø to to K=11.

2) Curve : proport. gain K=const=1.3; derivator time constant τ_H =const=1Ø$^{-3}$ s; the derivator gain changes from K'=Ø to K'=3.

3) Curve : proport. gain K=const=.8; derivator gain K'=const=3; The derivator time constant changes from τ_H =1Ø$^{-4}$ s to τ_H =1Ø$^{-2}$ s.

Fig 5



V_i = References program
G_1 = High-freq. noise filter
TH = Thyristor amplifier
G_2 = High-frequ. RCL filter
G_3 = V. F. impedance and penetrat.
G_4 = Eddy current effects
$\Delta\Psi$ = Measured value
$V_H \times H$= Feed-back controller

Fig 6

ASDEX horizontal position (current effects included) control with an RCL filter for reducing thyristor noise.

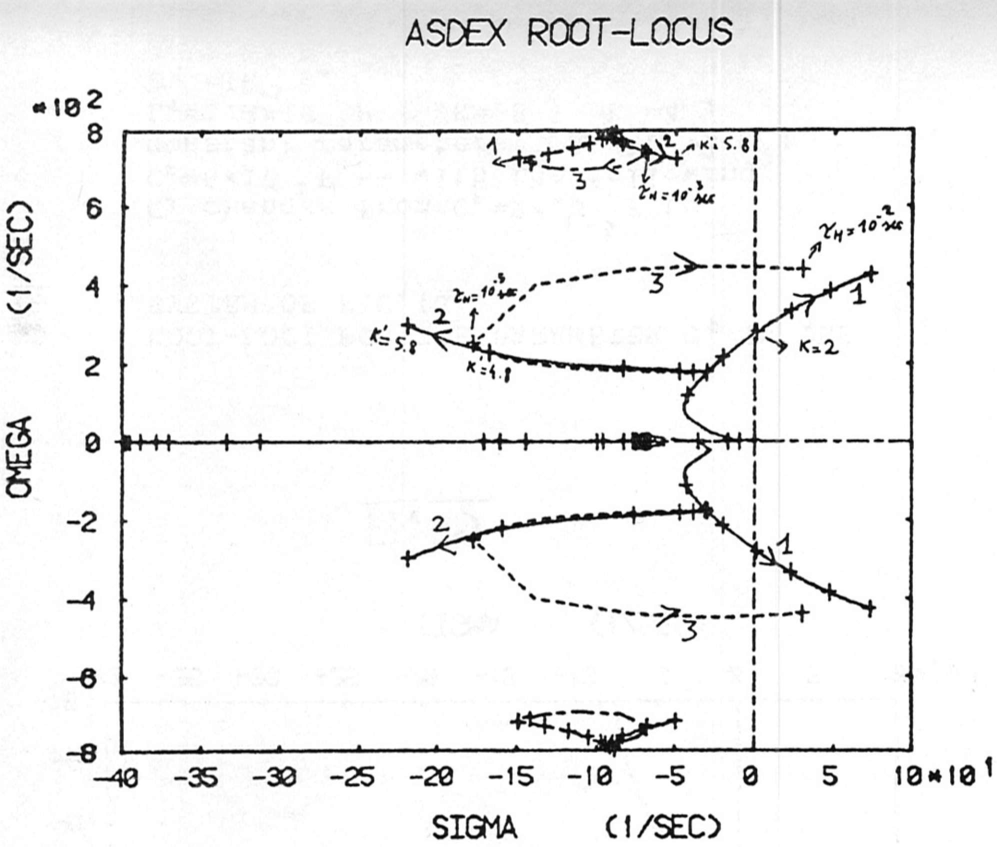


Fig 7

ROOT-LOCI FOR THE FIG.(6) SYSTEM

1) Curve : only proport. controller; the gain changes from K=0 to to K=5.

2) Curve : proport. gain K=const=.8; derivator time constant τ_H =const=10^{-3} s; the derivator gain changes from K'=0 to K'=6.

3) Curve : proport. gain K=const=.8; derivator gain K'=const=4; the derivator time constant changes from $\tau_H = 10^{-4}$ s to $\tau_H = 10^{-2}$ s.

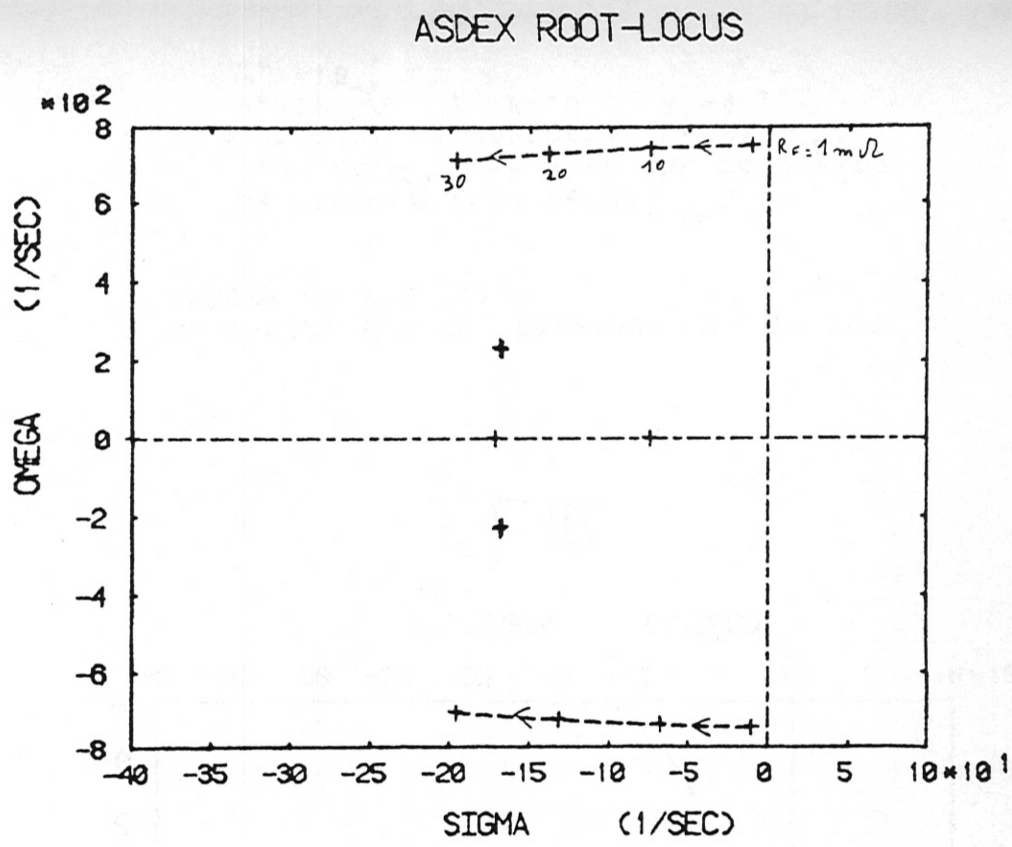


Fig 8

ROOT-LOCI FOR THE PARAMETER R_F IN THE SYSTEM OF FIG.(6)

R_F changes from $R_F = 10^{-3}\,\Omega$ to $R_F = 3\times10^{-2}\,\Omega$ -- with the following constant parameters: $C_F = 2\times10^{-2}$ F ; $L_1 = 1.3\times10^{-4}$ H; K=.8 ; K'=4 ; $\tau_H = 10^{-3}$ s.

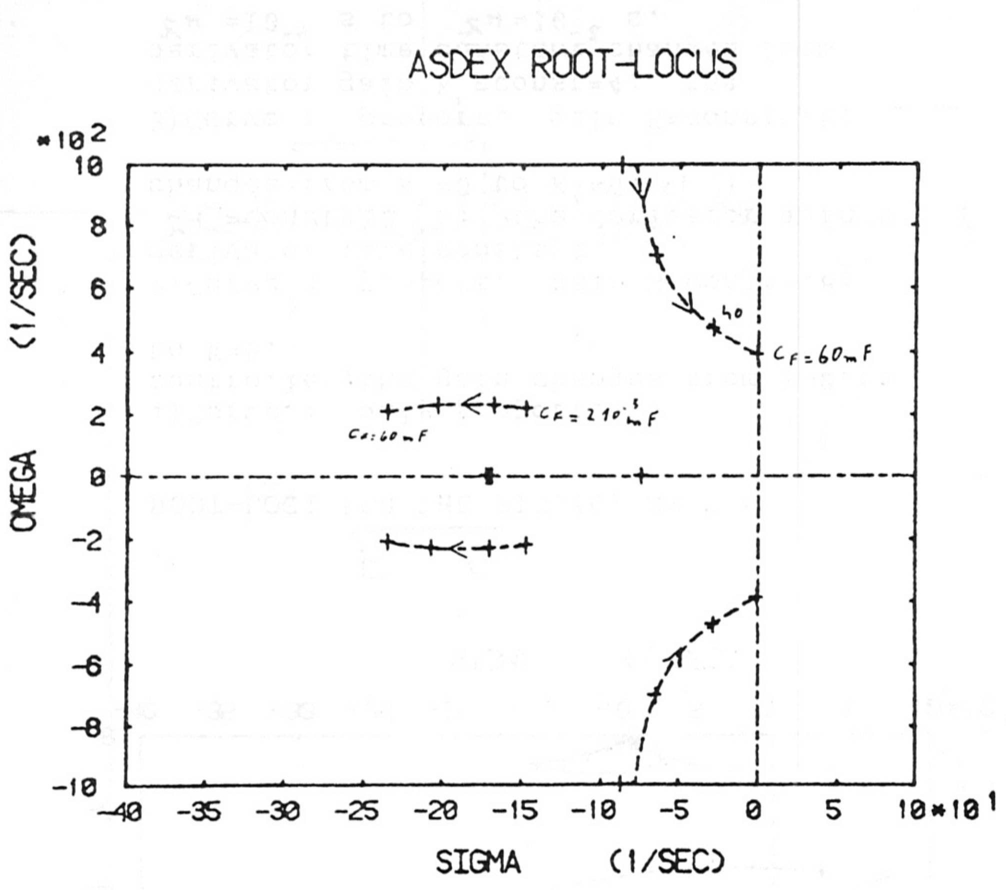


Fig 9

ROOT-LOCI FOR THE PARAMETER C_F IN THE SYSTEM OF FIG. (6)

C_F changes from $C_F = 2\times10^{-3}$ F to $C_F = 6\times10^{-2}$ F -- with the following constant parameters: $R_F = 10^{-2}\,\Omega$; $L_1 = 1.3\times10^{-4}$ H ; $K = .8$; $K' = 4$; $\tau_H = 10^{-3}$ s.

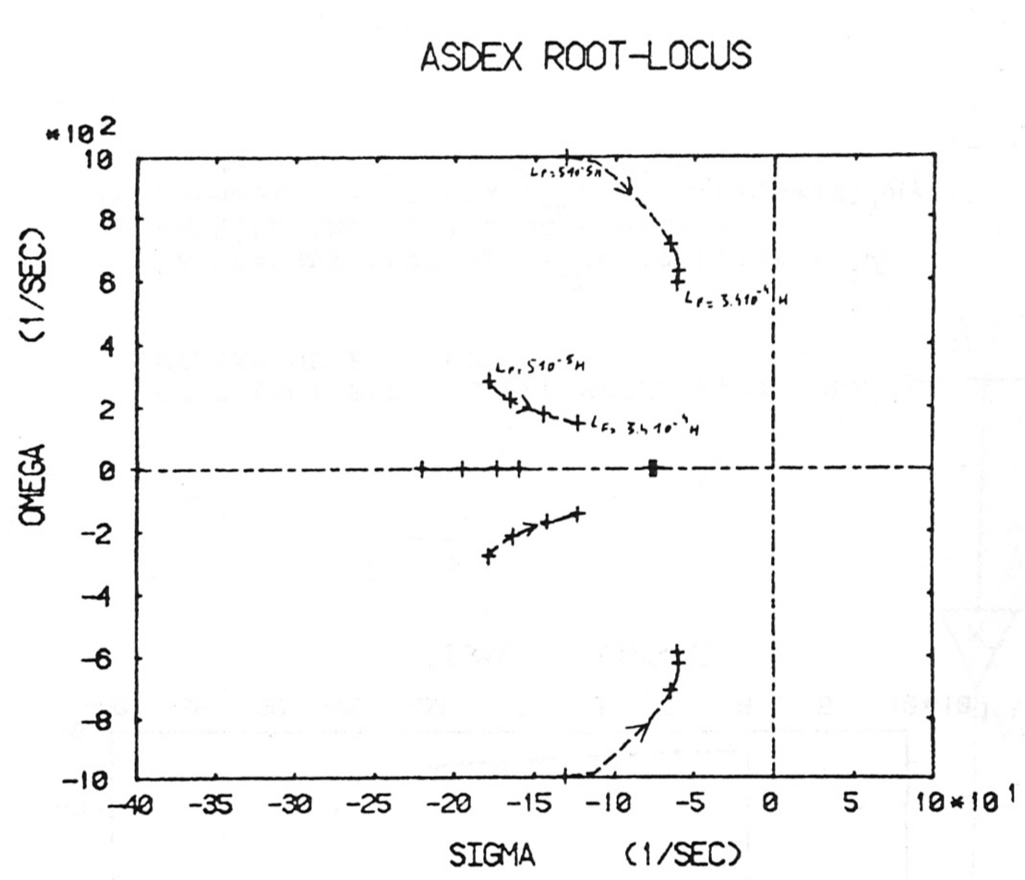


Fig 10

ROOT-LOCI FOR THE PARAMETER L_1 IN THE SYSTEM OF FIG. (6)

L_1 changes from $L_1 = 5\times10^{-5}$ H to $L_1 = 3.4\times10^{-4}$ H -- with the following constant parameters: $C_F = 2\times10^{-2}$ F ; $R_F = 10^{-2}\,\Omega$; $K = .8$; $K' = 4$; $\tau_H = 10^{-3}$ s.