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INTRODUCTION TO THE THEORY OF RF PLASMA HEATING AND CURRENT DRIVE

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1. Macroscopic conservation theorems

We consider time periodic em - pumps acting upon a stable plasma, so that all field quantities except the internal energy density are supposed to be periodic functions of time.

Irreversible heating (or cooling) is represented by the time average of the rate of change of the internal energy density:

$$\mathcal{U} \equiv 3p/2 = \sum_s \left(\mathcal{U}_s + n_s m_s (\vec{v}_s - \vec{v})^2/2 \right) \tag{1}$$
$$\mathcal{U}_s \equiv 3p_s/2 = \int d^3\vec{w}\, f_s\, m_s (\vec{w} - \vec{v}_s)^2/2 ,$$

where subscript s refers to the particle species, f_s, n_s, m_s and $\vec{v}_s$ are respectively, the distribution function, the number density, mass, and bulk velocity of the s-particles, $\vec{v} = \sum_s n_s m_s \vec{v}_s/\rho$ is the center of mass fluid velocity, $\rho = \sum_s n_s m_s$ is the mass density of the plasma, and $\vec{w}$ is the coordinate of the velocity space.

From Poynting theorem

$$\partial/\partial t (E^2+B^2)/8\pi + \vec{j}\cdot\vec{E} + \mathrm{div}\left(c\vec{E}\times\vec{B}/4\pi\right) = 0, \tag{2}$$

and conservation of energy

$$\partial/\partial t\left(\varepsilon_{kin} + (E^2+B^2)/8\pi\right) + \mathrm{div}\left(\vec{Q}_{kin} + c\vec{E}\times\vec{B}/4\pi\right) = 0, \tag{3}$$

where ε_{kin} is the total kinetic energy of the plasma and $\vec{Q}_{kin}$ the total kinetic energy flow, it follows that

$$\int dV \vec{j}\cdot\vec{E} = (d/dt)\sum_s \int dV \left(n_s m_s v_s^2/2 + U_s\right) \quad (4)$$

provided the integration volume V - the plasma or a periodicity volume - be enclosed by a surface through which $\vec{Q}_{kin}$ vanishes. In time average

$$\int dV \overline{\vec{j}\cdot\vec{E}} = \sum_s \int dV \,\overline{dU_s/dt} \quad (5)$$

while the local $\overline{\vec{j}\cdot\vec{E}}$-value may be different from the local $\sum_s \overline{dU_s/dt}$ value.

Notice that Eqs. (2) to (5), applied to the system consisting of the RF source, the transmission line and the plasma, state that the heating power deposited into the plasma is some fraction of the integral $\int dV \overline{\vec{j}\cdot\vec{E}}$ extended to the volume of the RF source.

It can be shown /1/ that the quantity $p\varrho^{-5/3}$ which to within an unimportant constant represents the entropy per particle (which is constant during adiabatic compression) is given by the equation

$$(3p/2)\, d(p\varrho^{-5/3})/dt = -\operatorname{div}\vec{q}^{\,*} + \vec{j}^{\,*}\cdot(\vec{E}+\vec{v}\times\vec{B}/c) - \pi_{\alpha\beta}\,\partial v_\alpha/\partial x_\beta \;. \quad (6)$$

Here $\vec{q}^{\,*}$ is the flux density of kinetic energy in a frame moving with the fluid (the heat flow vector), $\vec{j}^{\,*}$ is the conduction current density (in a quasi-neutral plasma $\vec{j}^{\,*} = \vec{j}$) and $\overleftrightarrow{\pi}$ is the trace-less stress tensor ari ing as a result of the deviation of f_s from spherical symmetry:

$$\Pi_{\alpha\beta} = \sum_s \int d^3\vec{w}\, m_s f_s \{(w_\alpha - v_\alpha)(w_\beta - v_\beta) - \\ -(\vec{w} - \vec{v})^2 \delta_{\alpha\beta}/3\} . \tag{7}$$

Equation (6) shows that at low frequencies when the plasma is frozen to the $\vec{B}$-lines of force, heating can either be produced Ohmically, by the $\vec{E}$-component parallel to $\vec{B}$ or by magnetic pumping through the viscous term $-\Pi_{\alpha\beta}\partial v_\alpha/\partial x_\beta$. Assuming in slab geometry $\vec{v} = \vec{v}_x(x)$, the latter term is essentially $\eta_0(dv_x/dx)^2/3$ with $\eta_0 \simeq nT\nu/(\nu^2+\omega^2)$, where ν is the collision frequency and ω the wave angular frequency. This can be easily proven by solving the simplified Boltzmann equation

$$\partial f/\partial t + \mathrm{div}(\vec{w}f) \simeq -\nu(f - f_M) , \tag{8}$$

where f_M is the Maxwellian, with the Ansatz $f \propto \exp(-i\omega t)$. Since $\mathrm{div}\,\vec{v} = -\dot{n}/n = 0$, the $\vec{B}$-lines are also compressed and $B \propto n$. The RHS of Eq. (6) then becomes

$$\frac{1}{3} mT \frac{\nu}{\nu^2+\omega^2} (\dot{B}/B)^2 . \tag{9}$$

For a small amplitude periodic pump

$$B/B_0 = 1 + b\cos\omega t \quad , \quad |b| < 1 , \tag{10}$$

Eq. (6) is a linear ordinary differential equation with a time periodic coefficient of the Hill-Mathieu type. It has "unstable" solutions of the form $p(\omega t) \cdot \exp \gamma t$ where $p(\omega t)$ is a periodic function with the same period as the pump and the constant growth rate γ provides the gyrorelaxation heating rate /2/

$$\gamma \simeq (b/3)^2 \nu/(1+\nu^2/\omega^2) \leq b^2\omega/18 . \qquad (11)$$

In this case heating relies upon inducing a pressure anisotropy due to the constancy of the magnetic moment $\mu_s = m_s w_{\perp s}^2/2B$, between two consecutive collisions, which is then relaxed by the Coulomb collisions.

Higher heating rates require essentially space dependent pumps with frequencies well above the collision frequencies.

2. Wave-Particle Resonant Interactions

2.1 Single particle aspects

Single particle motion in a toroidal configuration can be decomposed into two parts, average motion at a uniform speed along certain trajectories and oscillatory motion about this average. The first includes the motion of passing particles along $\vec{B}_o$ and the vertical $\vec{B}_o$-curvature and grad $\vec{B}_o$ drift. The second includes Larmor gyration, trapped particle bounching along $\vec{B}_o$ in the $B_o \propto R^{-1}$ well, and the passing particle oscillations produced by this same B_o modulation.

Resonance occurs when the wave-frequency perceived in a frame performing the average particle motion is commensurable with at least one of the eigenfrequencies $\omega_{c\ell}$ of the periodic oscillations:

$$M(\omega - \vec{k}\cdot\overline{\vec{w}}) = \sum_\ell N_\ell \omega_{c\ell} . \qquad (12)$$

Obviously not all integers M and N_e are equally important. The harmonics of the Doppler shifted applied frequency are unimportant if the single particle displacements driven by the pump are small compared with the local wave length of the em-field, while the harmonics of the eigenfrequencies are unimportant if the elongation of the unperturbed single particle oscillations is small compared with the wave length. This statements follow from the Bessel function identity

$$e^{i\lambda\cos x} = \sum_m J_m(\lambda) e^{imx} \tag{13}$$

applied to the space dependence of the em-pump: $\exp ik\zeta = \exp i(k\bar{\zeta} + kz_{osz})$. In the former case $z_{osc.} \propto \cos(\omega t - k\bar{\zeta})$, in the latter $z_{osc} \propto \cos \omega_{cl} t$.

For vanishing N's, Eq. (12) is Cerenkov condition, which is exploited in Landau-damping. When $\vec{k}\cdot\vec{w} \simeq k_{\parallel} w_{\parallel}$, the longitudinal adiabatic invariant $J_{\parallel}$ is no longer a constant, while when $\vec{k}\cdot\vec{w} \simeq \vec{k}_{\perp}\cdot\vec{w}_{\perp}$ (subscript $_{\perp}$ refers to a direction perpendicular to $\vec{B}_0$), the third adiabatic invariant is destroyed /3/.

If only the coefficient of the gyrofrequency is different from zero, Eq. (12) reduces to the gyroresonance condition

$$\omega - \vec{k}\cdot\vec{w} = N\omega_{cs} \quad (\omega_{cs} = e_s B_0 / m_s c).$$

Waves satisfying this condition destroy the invariance of μs and are used in cyclotron heating. Notice that N can be positive or negative. The latter case is referred to as anomalous Doppler effect: the absorption of a wave by a particle is accompanied by an increase in the longitudinal (parallel to $\vec{B}_0$) energy of the particle and by a decrease in its transverse energy (nearly elastic scattering).

2.2 Velocity Space Aspects: qualitative description

When $\omega \gg \nu_{coll}$, an irreversible and steady energy flow from a monochromatic em-wave to the resonant plasma particles occurs as long as there are more particles capable of absorbing energy from the wave then particles giving energy to the wave. This is the case when Coulomb collisions are frequent enough to prevent the formation of a plateau in the time-averaged velocity distribution function, $\langle f \rangle$, in the phase-space regions where particles are trapped in the through of the wave (which we suppose of small but finite amplitude). For instance, because of the constancy of μ, a wave with a $\vec{B}$-component parallel to $\vec{B}_0$, $B_{1\parallel} = - bB_0 \cos(kz - \omega t)$, traps a particle with $\omega_c \gg \omega$ in the low $|B_0 + B_{1\parallel}|$ - regions when $|w^*_\parallel - \omega/k|^2 < 2bw^{*2}_\perp$: here the star indicates values at the points where $|B_0 + B_{1\parallel}|$ is minimum. In the phase plane of Fig.1 the trapped particles trajectories appear as nested closed orbits while the untrapped particle trajectories appear as long wobbly lines. The border between trapped and untrapped trajectories is called the separatrix. The phase-plane region enclosed by the separatrix is called island. In the absence of collisions a steady-state $\langle f \rangle$ must be constant along particle trajectories. The time scale of the plateau formation is the bounce time τ_B of the trapped particles. In the above example $\tau_B \simeq 2\pi/|\omega - k_\parallel w_\parallel| \simeq 2\pi/k_\parallel v_t \sqrt{2b}$.*) On the other hand, the time for restoring a Maxwellian slope over the velocity range $\Delta w_\parallel \simeq v_t\sqrt{2b} \ll v_t$ is the reciprocal of the effective collision frequency, $\nu_{eff} \equiv \nu_{\Delta w_\parallel}$, for scattering particles out for $\Delta w_\parallel$. Since the dominant contribution to scattering in velocity space is made by the small-angle distant encounters rather than by close encounters which completely change the particle velocity but are much less frequent, the collision operator $(\delta f/\delta t)_{coll}$ has the Fokker-Planck form describing drag and diffusion in velocity space /4/ as

$$(\delta f/\delta t)_{coll} = - \operatorname{div}_{\vec{w}} (d\vec{w}/dt\, f - \overleftrightarrow{D}_\nu \operatorname{grad}_{\vec{w}} f) \qquad (14).$$

Here $\overline{d\vec{w}/dt} \simeq \nu_{coll} \cdot \vec{w}$ and $\overleftrightarrow{D}_\nu \simeq \nu_{\Delta\vec{w}} \overline{(\Delta\vec{w})(\Delta\vec{w})}/2 \simeq \nu_{coll}\, v_t^2/2$, where ν_{coll} is the frequency of a 90° deflection resulting from long range Coulomb encounters. Thus, with $\nu_{eff} \simeq \nu_{coll}(v_t/\Delta w_\parallel)^2$ the time-average distribution function $\langle f \rangle$ of the previous example will remain close to a Maxwellian (even when Eq.(12) holds true) provided that $\tau_B \nu_{eff} > 1$.i.e. $\nu_{coll} > k_\parallel v_t b^{3/2} \equiv \nu^*$. Then, if the wave amplitude is small, $(f - \langle f \rangle)$ is small quantity changing rapidly compared to τ_{coll} : it obeys, therefore, a formally collisionless linearized equation.

In the limit $\nu_{coll} < \nu^*$, i.e. when resonant particles remain trapped in the potential well of the wave, collisions have to be retained explicitly in the equation. Indeed, if there would be no collisional detrapping at all, the distribution function of the resonant particles along $w_\parallel$ would be perfectly symmetric with respect

*) where $v_t^2 = 2\ T/m$.

to the $w_{\parallel}$-value given by Eq. (12) and there would be no undirectional energy flow from the pump wave to the plasma. The quantitative treatment of the $\nu_{coll} < \nu^{x}$ case was carried out for the first time by ZAKHAROV and KARPMAN in a fundamental paper /5/. The resulting behaviour of the heating rate as a function of ν_{coll} is plotted schematically in Fig. 2 where the three regimes, collisional-nonlinear, collisionless-linear, and collisional-linear are exhibited as ν_{coll} increases.

An important aspect of the wave-particle interaction when wave trapping is negligible is that $<f>$ evolves according to a diffusion equation of the Fokker-Planck type also for what concerns the interaction with the em-field /6/:

$$\partial <f>/\partial t = \mathrm{div}_{\vec{w}} \{ \overleftrightarrow{D}(A^2, \vec{w})\, \mathrm{grad}_{\vec{w}} <f> + \\ + e E_{DC} <f>/m \} + (\delta <f>/\delta t)_{coll} \tag{15}$$

where the tensor $\overleftrightarrow{D}$, the so-called quasilinear diffusion coefficient, is proportional to A^2, the square of the amplitude of the em-wave /7/. This is because in the presence of a wave packet, the particles experience a non-vanishing force only on those parts of their unperturbed trajectories where the wave phase they see is slowly varying (or stationary) and because collisions not only contribute to control the resonance duration but also ensure that the particles "forget" the wave-phase when leaving resonance. If τ_T is the time between two successive resonances of a particle and $\Delta\tau_T$ is the irreproducibility in τ_T due to Coulomb scattering, requiring that the particles forget the wave phase just says that $\Delta\tau_T \cdot \omega_{res} \gg 1$.
This randomisation criterion ensures that at each resonance the particle receives incoherent velocity increments.

A remarkable new result is that an equation of type (15) also describes ion heating by a coherent Lower-Hybrid wave ($\omega \gg \omega_{ci}$). propagating perpendicularly to a uniform magnetic field if the amplitude of the wave exceeds a threshold /8/. On a time scale between the wave period and the cyclotron period the ion behaves as though in a zero magnetic field so that the wave-particle resonance condition is essentially $\omega = \vec{k}.\ \vec{w}$. Supposing $\vec{k} = \vec{k}_y$, since $0 \leq |v_y| \leq v_\perp$, only ions with $v_\perp > \omega/k_y$ (i.e. those with sufficiently high perpendicular energy) pass through resonance, $v_y = \omega/k_y$ (twice per cyclotron orbit). The force they receive here can be approximated by a δ-function (the resonance duration is much shorter than ω_{ci}^{-1}). Now any mechanism which is capable of decorrelating the ions and the wave at least once per cyclotron period causes the wave to be ion Landau damped (notice, incidentally, that here the resonant particles are much more numerous than those satisfying the Landau condition along y in the absence of $\vec{B}_0$, because the magnetic field sweeps the vector $\vec{w}_\perp$ through all angles). Collisions are unsufficient to destroy phase coherence at these high frequencies. Instead, it has been found that phase coherence is destroyed when the electric field is so large that the kick received by the ion on one transit through resonance is sufficient to change the phase that the particle sees when next in resonance on the average by at least $\pi/2$. This is so since the magnitude of the kicks received at resonance is a sensitive function of the phase at the beginning of the resonance.

If we look at the phase space we discover that a such field amplitudes (which are easily encountered in heating experiments) a particle's orbit wanders over most of phase space, roughly spending equal amounts of time in equal areas (i.e. the particle's orbit is approximately ergodic). Indeed the phase space is no longer characterized by a single island centered around one resonant point, $\omega - \vec{k}.\vec{w} = 0$, but by the presence of an infinity of higher order islands centered around the points defined by Eq. (12). For fields above threshold, these islands overlap thus allowing almost unrestricted motion in $\vec{w}_\perp$.

2.3 Velocity Space Aspects: quantitative analysis

Because of the fundamental role played by the Quasi-Linear Fokker-Planck (QLFP) equation in the theory of RF plasma heating and current drive, it may be useful to give here a simple derivation of it. We start from the QL diffusion term which we want to derive in the simplest case of a travelling electrostatic wave

$$E(z,t) = \int dk\, E_k \cos(kz - \omega t)\,, \tag{16}$$

with E_K kept constant in space and time by external sources which compensate for the absorption by the plasma. Writing $f = \langle f\rangle + f_1$, where f_1 is the fast-time-dependent part of the distribution function, the Boltzmann equation splits into the following two equations:

$$\partial\langle f\rangle/\partial t + \partial/\partial v_z \langle eE(z,t) f_1/m\rangle = C(\langle f\rangle), \quad (a)$$

(17)

$$\partial f_1/\partial t + v_z \partial f_1/\partial z + \partial/\partial v_z \left(eE(z,t)\langle f\rangle/m\right) \simeq -\nu f_1, \quad (b)$$

thus

$$f_1 = \int dk \left(\frac{-eE_k}{2m}\right) \frac{\partial\langle f\rangle}{\partial v_z} \left\{ \frac{e^{i(kz-\omega t)}}{\nu + i(kv_z - \omega)} + \text{c.c.} \right\}$$

and

$$\langle eE(z,t) f_1/m\rangle = -\int dk \int dk' \left(\frac{e}{2m}\right)^2 E_k E_{k'} \frac{\partial\langle f\rangle}{\partial v_z} \cdot$$
$$\cdot \left\langle \left(e^{i(k'z-\omega t)} + \text{c.c.}\right)\left(\frac{e^{i(kz-\omega t)}}{\nu + i(kv_z - \omega)} + \text{c.c.}\right)\right\rangle . \tag{18}$$

As the average of $\exp(\pm 2i\omega t)$ is zero and, by definition $\langle e^{\pm i(k-k')z}\rangle_z = \delta(k-k')$, Eq. (18) becomes

$$\langle eE(z,t) f_1/m\rangle = -\int dk \left(\frac{eE_k}{2m}\right)^2 \frac{\partial\langle f\rangle}{\partial v_z} \left\{ \frac{2\nu}{\nu^2 + (kv_z - \omega)^2} \right\} \tag{19}$$

where $\lim_{\nu\to 0} \{2\nu/(\nu^2 + (kv_z-\omega)^2)\} \to 2\pi\,\delta(kv_z - \omega)$.
Thus for a rectangular wave spectrum, using Heaviside step functions $\Theta(x)$,

$$E_k^2 = E_0^2\{\theta(k-k_2)-\theta(k-k_1)\}/(k_1-k_2) \equiv E_0^2\hat{\theta}_k/\Delta k, \tag{20}$$

we have

$$\begin{aligned} &\langle eE(z,t)f_1/m\rangle = -\frac{D_0\hat{\theta}}{v_z}\frac{\partial\langle f\rangle}{\partial v_z}, \\ &D_0 = \frac{\pi}{2}\left(\frac{eE_0}{m}\right)^2/\Delta k, \\ &\hat{\theta} \equiv \theta(v_z-\omega/k_2)-\theta(v_z-\omega/k_1). \end{aligned} \tag{21}$$

Now we consider the collision term $C(\langle f\rangle) = \mathrm{div}_{\vec{w}}(\vec{j}_w)$. In spherical coordinates (w,θ,ϕ) with ϕ ignorable:

$$C = \frac{1}{w^2}\frac{\partial}{\partial w}(w^2 j_w) + \frac{1}{w\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, j_\theta). \tag{22}$$

For the electrons, the only species we consider explicitly, we assume

$$\begin{aligned} &j_w = \nu(w)\left(w\langle f\rangle + \tfrac{1}{2}v_t^2\,\partial\langle f\rangle/\partial w\right), \\ &\left(v_t^2 = 2T_e/m_e\right) \\ &j_\theta = \tfrac{1}{2}(1+Z)\nu(w)\,w\,\partial\langle f\rangle/\partial\theta. \end{aligned} \tag{23}$$

The term proportional to the ion charge state Z accounts for the collisions with ions and is called the Lorentz term, all other terms describe electron-electron collisions. The j_w term describes energy diffusion and Maxwellisation, the j_θ term describes pitch angle scattering and spherical symmetrisation. Taking for $\nu(w)$ the high speed limit

$$\nu(w) = \omega_{pe}^4 \ln\Lambda / 4\pi m_e w^{3} \equiv A/w^3 \tag{24}$$

and writing $\xi = \cos\theta$, Eq. (22) becomes, dropping the angle brackets for brevity,

$$C(f) \equiv C_E(f) + C_S(f),$$

$$C_E = \frac{A}{w^2}\frac{\partial}{\partial w}\left(f + \frac{1}{2} v_t^2 \frac{1}{w}\frac{\partial f}{\partial w}\right), \tag{25}$$

$$C_S = \frac{1}{2}(1+Z)\frac{A}{w^3}\frac{\partial}{\partial \xi}\left((1-\xi^2)\frac{\partial f}{\partial \xi}\right). \tag{26}$$

Equation (25) conserves the electron number density but not their energy. This drawback is the result of an oversimplification in j_W. An ad hoc energy diffusion term conserving also energy is

$$C_E^* = \frac{A}{w^2}\frac{\partial}{\partial w}\left(\frac{\partial}{\partial w^2}\left(w^2(f_M - f)\right)\right). \tag{27}$$

The collision term has a remarkable non-vanishing momentum: the collisional rate of change of the particle free-path parallel to a given direction:

$$\vec{\lambda} = \lambda \vec{e}_{\parallel} = \vec{e}_{\parallel}\int d^3\vec{w}\,(w_{\parallel}/\nu(w))\,C(f). \tag{28}$$

With $d^3\vec{w} = 2\pi\, w^2\, dw\, d\xi$ and Eqs. (26) and (27), we have

$$\lambda = 2\pi\int dw \int d\xi\, \xi \left\{ w\frac{\partial}{\partial w}\frac{\partial}{\partial w^2}\left(w^2(f_M - f)\right) + \frac{1}{2}(1+Z)\frac{\partial}{\partial \xi}\left((1-\xi^2)\frac{\partial f}{\partial \xi}\right)\right\},$$

and repetedly integrating by parts

$$\lambda = -(5+Z)\int d^3\vec{w}\; w_{\parallel} f \, . \tag{29}$$

(Using Eq. (25) would have given the same result to within a correction $O(v_t^2/w^2)$). Thus

$$j_{\parallel} = -e\int d^3\vec{w}\; w_{\parallel} f = \frac{e}{5+Z}\int d^3\vec{w}\; \frac{w_{\parallel}}{\nu(w)} C(f) \tag{30}$$

Since, on the other hand,

$$\partial f/\partial t + \mathrm{div}_{\vec{w}}(\vec{\Gamma}) = C(f) \, , \tag{31}$$

where $\vec{\Gamma}$ is the flux of particles in velocity space including, but not restricted to, QL diffusion, DC-electric field acceleration, etc., we have

$$j_{\parallel} = \frac{e}{5+Z}\int d^3\vec{w}\,(w_{\parallel}/\nu(w))\,\mathrm{div}_{\vec{w}}\,\vec{\Gamma} \quad , \tag{32}$$

$$= \frac{-e}{5+Z}\int d^3\vec{w}\,(\vec{\Gamma}\cdot\partial/\partial\vec{w})(w_{\parallel}/\nu(w)) \, . \tag{33}$$

This is a special case of a remarkably general result derived by Antonsen and Chu /9/ who expressed $w_{\parallel}/\nu(w)$ in terms of the Spitzer-Härm distribution function f_{SH}, i.e. the solution of Eq. (31) when $\vec{\Gamma}$ is solely the result of a DC-electric field, weak enough so that f departs only slightly from f_M at all energies (i.e. there is negligible runaway electron production):

$$\partial f_{SH}/\partial t + f_M e E_{DC} w_{\parallel}/T = C(f_{SH})$$

then
$$j_{SH} = -e\int d^3\vec{w}\, w_{\parallel} f_{SH}$$

and from Eq. (32)

$$j_{SH} = \frac{-e}{5+Z}\int d^3\vec{w}\, w_{\parallel} f_M e E_{DC} w_{\parallel}/\nu(w)T. \tag{34}$$

Thus we have

$$f_{SH} = f_M w_{\parallel} e E_{DC}/T(Z+5)\nu(w) \tag{35}$$

Equation (33) then becomes

$$j_{\parallel} = -e\int d^3\vec{w}\,(\vec{\Gamma}\cdot\partial/\partial\vec{v})(T f_{SH}/f_M e E_{DC}), \tag{36}$$

which is essentially Eq. (7) of Ref. /9/.

The time average of the absorbed power density is

$$W_s = \int d^3\vec{w}\,\tfrac{1}{2}m_s w^2 \partial f_s/\partial t = \int d^3\vec{w}\,(\vec{\Gamma}\cdot\partial/\partial\vec{v})\tfrac{1}{2}m_s w^2. \tag{37}$$

As an example, consider the case of a monochromatic pump $E(z,t) = E_0 \cos(k_0 z-\omega t)$ with sufficiently small E_0 to be allowed to set $\partial f/\partial w_{\parallel} = -2w_{\parallel} f_M/v_t^2$ in Γ_{QL}. Then Eq. (37) provides the classic Landau-damping result

$$W_s = \{\omega_{ps}^2 E_o^2/\omega 8\pi\} 2\sqrt{\pi}\, w_o^3 e^{-w_o^2}, \tag{38}$$

where the expression in curly brackets is the reactive electrostatic power density, ω_{ps}^2 is the plasma frequency and $w_0 = \omega/k_0 v_t >> 1$. An analogous expression can be found for $j_\parallel$ from Eq. (32) or (33) in the same limit. The ratio

$$-j_\parallel/\dot{W} = \frac{e\, 2 w_o^2}{m_e v_t A(5+Z)}, \tag{39}$$

is usually referred to as the figure of merit /10-12/.

In the case that $\vec{\Gamma}$ is very localized in velocity space we can write for $-j_\parallel/w$

$$-j_\parallel/\dot{W} = \frac{e}{5+Z}\,\frac{(\vec{\Gamma}\cdot\partial/\partial\vec{w})(w_\parallel/\nu(w))}{(\vec{\Gamma}\cdot\partial/\partial\vec{w})\frac{1}{2}mw^2} \tag{40}$$

an interesting result, independent of the form of the distribution function, first derived (in a different way) by Fisch & Boozer /12/. The surprise in Eqs. (33) to (40) is that $\vec{j}_\parallel$ can be generated even inducing a purely perpendicular particle flux. Following Ohkawa /13/ a cyclotron wave travelling in one direction along the toroidal coordinate can be used to selectively increase the perpendicular energy of the resonant electron population, in order to asymmetrically trapping some passing particles. At the same time, trapped particles are symmetrically detrapped by Coulomb collisions. As a result, there is a net increase in the electron toroidal angular momentum in the direction opposite to the propagation of the wave. This momentum is dissipated by the ions to generate a toroidal current. The fact that a net momentum is created opposite to the wave momentum is not surprising. For instance, in axisymmetric geometry only the canonical momentum of a particle is conserved: if there is radial displacement of the particle orbit or a driven radial flow, there is indeed creation of toroidal angular momentum, mRv_ϕ

$$\frac{d}{dt}(mRv_\phi) = -e\frac{d}{dt}(RA_\phi) = ev_{dr}RB_\theta \tag{41}$$

where v_{dr} is the electron drift velocity along the minor radius of a plasma embedded in concentric magnetic surfaces.

The Ohkawa effect is described by Eq. (36) by taking for the Spitzer-Härm distribution the expression appropriate to passing particles in a torus, which has a loss cone defined by $|\xi|<|\xi_{Mirror}|$.

Finally we want to prove the formal similarity of Landau and cyclotron damping in the collisionless linear regime. Assuming for the unperturbed state a uniform velocity distribution f_0 which is a function of the kinetic energy, ε, and for the pump wave the usual form $\exp i(\vec{k}.\vec{r} - \omega t)$, the Vlasov equation for the perturbed part of the distribution function f_1, Eq. (17)b, for $\omega \ll \omega_c$ becomes

$$[i(\vec{k}.\vec{V}_0 - \omega) + \nu]f_1 = -(d\varepsilon/dt)_1 \, df_0/d\varepsilon \, , \tag{42}$$

where $(d\varepsilon/dt)_1$ is the perturbation of the rate of change of the single particle kinetic energy in the drift approximation

$$(d\varepsilon/dt)_1 = e\vec{V}_0 \cdot \vec{E}_1 + \mu \partial B_1/\partial t \, , \tag{43}$$

and $\vec{V}_0$ is the unperturbed guiding center velocity which we assume, for simplicity, to be independent of space and time. The last term in (43) is the induction effect of a time dependent $\vec{B}$-field and is due to the curl of $\vec{E}$ acting about the circle of gyration. Obviously this term has to be dropped in the case $\vec{B}_0 = 0$.

The solution of Eq. (42) describes Landau damping provided that when integrating over velocity space the quantity $[(\vec{k}\cdot\vec{V}_0 - \omega) - i\nu]^{-1}$

be interpreted as

$$\lim_{\nu \to 0} [(\vec{k}.\vec{V}_0 - \omega) - i\nu]^{-1} = P\{(\vec{k}.\vec{V}_0 - \omega)^{-1}\} + \\ + i\pi\delta(\vec{k}.\vec{V}_0 - \omega) , \tag{44}$$

where P indicates that the Cauchy principal value has to be taken and $\delta(x)$ is the Dirac function.

The simplest case of cyclotron damping occurs with a circularly polarized wave propagating along a uniform B_0-field: $E_1(r,t) = E_1 \cdot (e_x \cos(kz - \omega t) + e_y \sin(kz - \omega t))$ where we use Cartesian coordinates with z taken along $\vec{B}_0$. By writing $x_x = w_\perp \sin\phi$ and $w_y = w_\perp \cos\phi$, where $d\phi/dt = \omega_c$, for the velocity components and the corresponding equations for the particle coordinates x and y, we obtain

$$(d\varepsilon/dt)_1 = e w_\perp E_1 \sin(kz + \phi - \omega t) \tag{45}$$

and for $f_1 \propto \cos(kz + \phi - \omega t)$ the following Vlasov equation

$$(k w_z + \omega_c - \omega) f_1 = (d\varepsilon/dt)_1 \, df_0/d\varepsilon \, . \tag{46}$$

Harmonic cyclotron damping occurs if the wave vector $\vec{k}$ has a component say along the x-direction. In this case, the Jacobi identity, Eq.(13), can be used to write the Vlasov equation in the form

$$(k w_z + n\omega_c - \omega) f_1 = (d\varepsilon/dt)_1 \, |J_{n-1}(k_x w_\perp/\omega_c)| \, df_0/d\varepsilon \tag{47}$$

where $(d\varepsilon/dt)_1$ can again be given by Eq. (45). Equations (42) to (47) show the basic similarity of Landau and cyclotron damping.

The time average of the absorbed power density is then given as

$$W = \frac{1}{2} \mathrm{Re} \left\{ \int d^3\vec{w} f_1^* (d\mathcal{E}/dt)_1 \right\} , \tag{48}$$

where the star indicates complex conjugate, $d^3\vec{w} = 2\pi dw_\parallel d(w_\perp^2/2)$ in the Landau-damping case, and $d^3\vec{w} = d\phi dw_\parallel d(w_\perp^2/2)$ in the cyclotron damping cases. Notice, in conclusion, that Eqs. (43) and (45) give the kinetic energy excursions of a non-colliding single particle, and that the plasma temperature increase may well be much larger than these excursions: the latter are nothing more than the basic steps of the "random walk" process in energy space to which power absorption is ultimately due (even though this process is not explicitly exploited in the actual calculation of W).

3. Wave propagation in non uniform plasmas

In this section we briefly discuss the main properties of wave propagation in a magnetically confined plasma. In such a radially stratified medium, not only the frequency of the wave but also the two wave numbers corresponding to the toroidal and poloidal directions of the torus can, at least in principle, be thought of as being determined by the external launching structure. Then the radial dependence of the wave quantities is determined by a system of ordinary differential equations. In a warm collisionless plasma with fixed temperature, this system is equivalent to a single sixth order equation in one of the field components, whose solutions correspond to three kinds of waves (there are two of each kind differing only in the (opposite) direction of propagation). These three waves correspond to the existence of three different interaction forces. One force derives from an electrostatic potential Φ, the other two from the two independent components of a vector potential $\vec{A}$. These three forces are related to the kinetic pressure, the electro - magnetic pressure, and the electro-magnetic tension. However, we shall see below that if $\omega \ll \omega_{ci}$ the x-dependence of the wave is given by a single 2nd order equation. This is because the $\vec{B}$-tension force along x does not depend on x, and because across $\vec{B}_0$ the kinetic and magnetic pressures appear always combined in the single quantity $(p + B^2/8\pi)_1$. In the opposite limit of very high frequency, charge separation and electric forces are dominant. The necessary inertia is provided by the ion (electron) mass if the frequency is very low (high). In general, however, the effective inertia can involve both m_e and m_i and is a sensitive function of the ratios $\omega/\omega_{c\alpha}$ and of the nature of the interaction force.

In a <u>hot</u> collisionless plasma there is virtually an infinity of waves (the dispersion relation involves transcendental functions of the wave number vector $\vec{k}$, instead of the first three powers of k^2 as in the warm plasma case) although only some of them will be observable and even fewer will be relevant to plasma heating: e.g. the so-called Bernstein modes which are very slow waves propagating almost across $\vec{B}_0$ at $\omega \simeq n\omega_c$. Transcendental functions enter for two reasons. (1) Only particles whose velocity along $\vec{B}_0$ is sufficiently close to a resonant value (12) are sensibly affected by a very-low-amplitude wave with given ω and $k_\parallel$. Since the strength of such an interaction is controlled by the slope of f_0 at the resonant velocity - a transcendental function of $\omega/k_\parallel$ - this introduces the Fried and Conte plasma dispersion function. (2) Because of the Larmor excursions of the particles across $\vec{B}_0$, the Jacobi identity (13) introduces into the dielectric tensor series of Bessel functions of argument $(k_\perp v_{ts}/\omega_{cs})^2/2$.

If the temperature is neglected there are only two kinds of waves in an essentially collisionless plasma. Taking all wave amplitudes $\propto e^{-i\omega t}$, and normalizing lengths to c/ω, Maxwell equations read

$$\text{curl } \vec{E} = i\vec{B} \tag{49}$$

$$\text{curl } \vec{B} = -i\vec{D} \tag{50}$$

$$\vec{D} = \vec{\varepsilon}\,\vec{E} \tag{51}$$

With orthogonal coordinates (x_1, x_2, x_3) having $\vec{x}_3$ along $\vec{B}_0$, the non-vanishing components of the cold-plasma dielectric tensor $\vec{\varepsilon}$ are

$$\begin{aligned} \varepsilon_{11} = \varepsilon_{22} \equiv \varepsilon_1 &= 1 - \sum_s \omega_{ps}^2/(\omega^2 - \omega_{cs}^2), \\ \varepsilon_{12} = -\varepsilon_{21} \equiv \varepsilon_2 &= -i\sum_s \omega_{ps}^2\omega_{cs}/\omega(\omega^2 - \omega_{cs}^2), \\ \varepsilon_{33} \equiv \varepsilon_3 &= 1 - \sum_s \omega_{ps}^2/\omega^2 . \end{aligned} \tag{52}$$

(neglecting unity on the RHS of Eqs. (52) is neglecting displacement currents).

First of all we consider the case where the space dependence of the plasma parameters is negligible and assume wave amplitudes $\propto e^{i\vec{n}_\perp \cdot \vec{x}_\perp + i\, n_\parallel x_\parallel}$ (as usual subscripts $\perp$ and $\parallel$ refer to the $\vec{B}_0$ direction). Then the solubility of the linear system (49)-(51) is the cold plasma dispersion relation (DR)

$$\varepsilon_1 n_\perp^4 - b n_\perp^2 + c = 0 \tag{53}$$

with

$$b = (\varepsilon_1 + \varepsilon_3)(\varepsilon_1 - n_\parallel^2) + \varepsilon_2^2 \; ; \; c = \varepsilon_3 \left[(\varepsilon_1 - n_\parallel^2)^2 + \varepsilon_2^2 \right]$$

and $$\Delta \equiv b^2 - 4\varepsilon_1 c =$$

$$= \left[\varepsilon_1(\varepsilon_1 - \varepsilon_3 - n_\parallel^2) + \varepsilon_2 + n_\parallel \sqrt{\varepsilon_3} \right] \left[\varepsilon_1(\varepsilon_1 - \varepsilon_3 - n_\parallel^2) + \varepsilon_2 - n_\parallel \sqrt{\varepsilon_3} \right] .$$

When $b^2 \gg 4\varepsilon_1 c$ i.e.

$$\left| (\varepsilon_1 - \varepsilon_3)(\varepsilon_1 - n_\parallel^2) + \varepsilon_2^2 \right| \gg 2 \left| n_\parallel \varepsilon_2 \sqrt{\varepsilon_3} \right| , \tag{54}$$

the solutions of the DR are the so-called extraordinary (X), and ordinary (O) wave

$$n_{\perp X}^2 \simeq (\varepsilon_1 - n_\parallel^2) + \varepsilon_2^2 / (\varepsilon_1 - n_\parallel^2) ,$$

$$n_{\perp O}^2 \simeq \varepsilon_3 (\varepsilon_1 - n_\parallel^2) / \varepsilon_1 . \tag{55}$$

In the special case of perpendicular propagation Eqs. (55) hold strictly and with no restrictions on $\overleftrightarrow{\varepsilon}$.

From Eqs. (49) to (51) one can deduce that the extraordinary wave has $E_\parallel = 0$, while the ordinary wave has $B_\parallel = 0$. These facts can be exploited to derive the separate equations for the X and O waves in an inhomogeneous plasma, when the WKB approximation can be made along $\vec{B}_0$:

$$\vec{B}_0 \cdot \mathrm{grad}\, f \simeq i n_\parallel B_0 f \,. \tag{56}$$

For the X wave, from $E_\parallel = 0$ and from the x_1 and x_2 components of Eq. (49) we obtain

$$B_1 = -n_\parallel E_2 \;\; ; \;\; B_2 = n_\parallel E_1 \tag{57}$$

Then the x_1 and x_2 components of Eq. (50) become

$$\begin{cases} (\varepsilon_1 - n_\parallel^2) E_1 + \varepsilon_2 E_2 = i \partial_{x_2} B_\parallel \,, \\ -\varepsilon_2 E_1 + (\varepsilon_1 - n_\parallel^2) E_1 = -i \partial_{x_1} B_\parallel \,, \end{cases}$$

or

$$\begin{aligned} \left[(\varepsilon_1 - n_\parallel^2)^2 + \varepsilon_2^2\right] E_1 &= i\left(-\varepsilon_2 \partial_{x_1} + (\varepsilon_1 - n_\parallel^2)\partial_{x_2}\right) B_\parallel \,, \\ \left[(\varepsilon_1 - n_\parallel^2)^2 + \varepsilon_2^2\right] E_2 &= -i\left((\varepsilon_1 - n_\parallel^2)\partial_{x_1} - \varepsilon_2 \partial_{x_2}\right) B_\parallel \,. \end{aligned} \tag{58}$$

Expressions (58) inserted into the x_3-component of Eq. (49) give

$$\partial_{x_1}\left\{\frac{(\varepsilon_1 - n_\parallel^2)\partial_{x_1} B_\parallel - \varepsilon_2 \partial_{x_2} B_\parallel}{(\varepsilon_1 - n_\parallel^2)^2 + \varepsilon_2^2}\right\} + \partial_{x_2}\left\{\frac{(\varepsilon_1 - n_\parallel^2)\partial_{x_2} B_\parallel + \varepsilon_2 \partial_{x_1} B_\parallel}{(\varepsilon_1 - n_\parallel^2)^2 + \varepsilon_2^2}\right\} + B_\parallel = 0 \tag{59}$$

Not all space derivatives of $\vec{\varepsilon}$ are equally important in such an equation. Keeping only the derivatives of $(\varepsilon_1 - n_\parallel^2)$ which can vanish, Eq. (59) takes the more transparent form

$$\partial_{x_1}\left((\varepsilon_1 - n_\parallel^2)\partial_{x_1}B_\parallel\right) + \partial_{x_2}\left((\varepsilon_1 - n_\parallel^2)\partial_{x_2}B_\parallel\right) + \left((\varepsilon_1 - n_\parallel^2)^2 + \varepsilon_2^2\right)B_\parallel = 0 . \tag{60}$$

Since in a uniform isotropic medium ($\varepsilon_2 = 0$) the equation has to be of the Helmboltz type, Eq. (50) can heuristically be put in the intrinsic form

$$\mathrm{div}_\perp\left\{(\varepsilon_1 - n_\parallel^2)\,\mathrm{grad}_\perp B_\parallel\right\} + \left[(\varepsilon_1 - n_\parallel^2)^2 + \varepsilon_2^2\right]B_\parallel = 0 \tag{61}$$

For the 0 wave, from $B_\parallel = 0$ and from the x_1 and x_2 components of Eq. (50) we obtain

$$\begin{aligned} \varepsilon_1 E_1 &= n_\parallel B_2 - \varepsilon_2 E_2 \\ \varepsilon_1 E_2 &= -n_\parallel B_1 + \varepsilon_2 E_1 \end{aligned} \tag{62}$$

and from the x_1 and x_2 components of Eq. (49)

$$\begin{aligned} iB_1 &= \partial_{x_2}E_\parallel - i n_\parallel E_2 \\ iB_2 &= -\partial_{x_1}E_\parallel + i n_\parallel E_1 \end{aligned} \tag{63}$$

so that

$$(\varepsilon_1 - n_\parallel^2)E_1 = i n_\parallel \partial_{x_1}E_\parallel - \varepsilon_2 E_2$$

$$(\varepsilon_1 - n_\parallel^2)E_2 = i n_\parallel \partial_{x_2}E_\parallel + \varepsilon_2 E_1$$

Inserted into the x_3-component of Eq. (50) they give

$$\varepsilon_3 E_\parallel = \partial_{x_1}\left\{-\partial_{x_1}E_\parallel + \frac{i n_\parallel}{\varepsilon_1 - n_\parallel^2}\left(i n_\parallel \partial_{x_1}E_\parallel - \varepsilon_2 E_2\right)\right\} - $$
$$- \partial_{x_2}\left\{\partial_{x_2}E_\parallel - \frac{i n_\parallel}{\varepsilon_1 - n_\parallel^2}\left(i n_\parallel \partial_{x_2}E_\parallel + \varepsilon_2 E_1\right)\right\}. \tag{64}$$

Disregarding the $x_{1,2}$ derivative of ε_2 and $(\varepsilon_1 - n_\parallel^2)$ and using again $B_\parallel = 0$, give finally

$$\partial_{x_1}\left(\varepsilon_1 \partial_{x_1}E_\parallel\right) + \partial_{x_2}\left(\varepsilon_1 \partial_{x_2}E_\parallel\right) + \varepsilon_3\left(\varepsilon_1 - n_\parallel^2\right)E_\parallel = 0 \tag{65}$$

which for $n_\parallel^2 \gg |\varepsilon_1|$, becomes the equation for an electrostatic wave. This suggests the following intrinsic form of Eq. (65):

$$\mathrm{div}_\perp\left(\varepsilon_1 \,\mathrm{grad}_\perp E_\parallel\right) + \varepsilon_3\left(\varepsilon_1 - n_\parallel^2\right)E_\parallel = 0. \tag{66}$$

When the WKB approximation is made also along the other coordinate - say x_2 - on the magnetic surfaces, Eqs. (61) and (66) become ordinary differential equations in the radial direction. They can always be put in the "canonical" form $d^2y/dx^2 + k^2(x)y = 0$, where $y(x)$ is the product of a properly chosen function of x and of the field component under consideration.

Points where $k^2 = 0$ separate a region of wave propagation ($k^2 > 0$) from a region of evanescence ($k^2 < 0$). They are called cut-offs (C-points) since they produce at least partial reflection of the wave energy flux. In a nonuniform plasma no general simple statement can be made about the number and position of the C-points as they depend upon the behaviour of the ε's as functions of x. The physical significance of the points where the coefficient in front of the $B_\parallel$- and $E_\parallel$-derivative in Eq. (59) and (64) goes to infinity is simply a statement of wave polarization: in space such points are usually quite close to

C-points. In a uniform plasma the C-points of Eq. (61) are given by equation $(\varepsilon_1-n_\parallel^2)\,(\varepsilon_1-(n_\parallel^2+n_2^2)) = \varepsilon_2^2$, those of Eq. (66) by equation $(\varepsilon_1-n_\parallel^2)\,\varepsilon_3 = n_2^2\,\varepsilon_1$. Energy transmission through an evanescence region of finite width is called tunnelling /5/. Points where $k^2 \to \infty$ are called resonances (R-points): they are the only places where energy can be absorbed in the limit of zero collision frequency. In the case of perpendicular propagation only the X-wave has R-points: when $\varepsilon_1 = 0$. When $n_\parallel \neq 0$, it is the O-wave which has R-points when $\varepsilon_1 = 0$, while now Eq. (61) has R-points when $\varepsilon_1 = n_\parallel^2 \neq 0$. However, a close examination of the validity condition (54), would reveal that Eq. (61) is correct when $\varepsilon_1 \to n_\parallel^2 \neq 0$ only in the limit $\varepsilon_3 \to \infty$ and that only in this limit, i.e. when ω is so low that the electron inertia is negligible, we can speak of a resonance at $\varepsilon_1 = n_\parallel^2 \neq 0$. In all the other cases, the points where $\varepsilon_1 = n_\parallel^2$ turn out be close to the points where $\varepsilon_1 = 0$ and the situation is such that there are no longer two decoupled waves as there the two phase-velocities would have comparable values. These physically significant points are called linear mode conversion or turning-points (T-points) since the rf-energy can pass from one wave to the other. We leave the discussion of the T-points for the next section and concentrate here on the R-points.

The resonances occuring for $\varepsilon_1 = 0$ are generally called hybrid resonances. There is one hybrid resonance frequency above ω_{ce} and one between every two consecutive gyrofrequencies of the various ion species in the plasma. Thus, in the important case of a two-ion plasma (for example D-T or H-D) there are three hybrid resonance frequencies /14/:

1. The upper hybrid resonance (UHR) frequency

$$\omega^2 = \omega_{UHR}^2 \simeq \omega_{pe}^2 + \omega_{ce}^2 \;, \tag{67}$$

2. The lower hybrid resonance (LHR) frequency

$$\omega^2 = \omega_{LHR}^2 \simeq \sum_i \omega_{pi}^2 / (1 + \omega_{pe}^2/\omega_{ce}^2) \,, \tag{68}$$

where we assumed $\omega_{ci}^2 << \omega_{pi}^2$, and, with the same assumption,

3. The ion-ion hybrid resonance (I^2HR) frequency

$$\omega^2 = \omega_{I^2HR}^2 \simeq \omega_{c1}\omega_{c2} \frac{n_1 m_1 + n_2 m_2}{n_1 m_2 + n_2 m_1} \,, \tag{69}$$

where $n_i(m_i)$ is the number density (mass) of the ion of species $i = 1,2$. Equation (69) depends on the ratio n_1/n_2, but is independent of plasma density.

In a single-ion species plasma the non-electrostatic $\varepsilon_1 = n_\parallel^2 \neq 0$ resonance occurs at the frequency

$$\omega^2 = \omega_{ci}^2 / (1 + (\omega_{pi}/k_\parallel c)^2) \tag{70}$$

where we have assumed $\omega_{ci} << \omega_{pi}$. This is called (rather improperly) either the shear Alfvén resonance or the perpendicular ion-cyclotron resonance /4/. In a two-ion species plasma there are two solutions to Eq. $\varepsilon_1 = n_\parallel^2 \neq 0$ in the ionic frequency domain. If $\omega_{c1,2}^2 << \Sigma_i \omega_{pi}^2 << (k_\parallel c)^2$ they are close to ω_{c1}^2 and ω_{c2}^2. If $\Sigma_i\, \omega_{pi}^2 >> \{(k_\parallel c)^2, \omega_{c1,2}^2\}$ one solution corresponds to a resonance at the frequency

$$\omega^2 \simeq (k_\parallel c\, \omega_{c1}\omega_{c2})^2 / (\omega_{c1}^2 \omega_{p2}^2 + \omega_{c2}^2 \omega_{p1}^2) \,, \tag{71}$$

which is related to (70), the other being very close to the ω_{I^2HR}, Eq. (69), corresponds to a T-point.

4. rf-energy flow and accessibility

An important property of the resonance frequencies (67) to (71) is that they do not coincide with any of the eigenfrequencies ω_{cs} of the unperturbed single-particle motion. This is a direct consequence of the fact that at resonance the displacement currents are negligible /16/ (this is a consequence of Faraday's law) so that div $\vec{j} = 0$. Indeed the latter equation implies that at resonance the s-species mean velocity $\vec{v}_s$ has no component in the x-direction. This is possible only if the $\vec{E}$-component which rotates in the same sense as the s-particle unperturbed motion vanishes when $\omega \to \omega_{cs}$ so that v_{sx} (left) + v_{sx} (right) = 0. Frequencies (67) to (71) are the eigenfrequencies of the free oscillations of the plasma-field system which do not propagate energy in the x-direction : their group velocity must either be parallel to the resonance surface or vanish. It is only in such cases that resonance does occur at the points of a plasma profile where a local eigenfrequency equals the frequency of a propagating em-field. When thermal effects are neglected, the group velocity of the electrostatic modes $\varepsilon_1 = 0$ vanishes, while the group velocity of the electromagnetic modes $\varepsilon_1 = n_\parallel^2 \neq 0$ is along $\vec{B}_o$. For instance, when $\omega \ll \omega_{ci}$ the group velocity is equal to the Alfvên velocity vector $\vec{B}_o(4\pi\rho)^{-1/2}$: in this limit the free oscillations are essentially those of a collection of elastic strings stretched along $\vec{B}$ /17/.

We now consider the problem of power absorption in the case of a wave incident on a plasma profile where it encounters a R-point. The very fact that for real ω equations (61) and (66) are singular in real space at the R-points, implies that no dissipation mechanism is included in the equations. However, absorption is formally obtained from their singular solutions if, when integrating over real space, these are interpreted as generalized functions of $\left(\omega_{res}(x)-\omega\right)$. Thus as in the Landau damping case (44) the power absorbed is independent of ν_{coll}. The underlying physical picture is the following. If we examine the group velocity we discover that $t_{\to R}$, the time it takes the wave energy to approach the R-point starting from any point at a finite distance from it, is infinitely long /15/, in the sense that $t_{\to R} = \lim_{\nu_{coll} \to 0} \alpha/\nu_{coll}$ where $\alpha \lesssim 1$ is a constant. Therefore the rf-power density would become infinite at the R-point (both $\vec{j}$ and $\vec{E}$ tending to infinity) unless energy is dissipated by some mechanism. If $\alpha \approx 1$ Coulomb collisions in the immediate neighbourhood of the R-point provide the required mechanism. If $\alpha << 1$ a different effect must be looked for. This is provided by linear mode conversion to a warm (or hot) plasma wave since a R-point of a cold plasma wave is the point toward which the T-point of some warm (or hot) plasma wave tends when the temperature goes to zero. This is illustrated on Fig.3 which is a plot of $k_x^2 = k_x^2(k_y^2, k_t^2, \omega; x)$ as derived by a local dispersion relation (of course the WKB-approximation fails in the neighbourhood of a T-point). The important point here is that the square of the phase velocity of the warm (or hot) plasma wave steadily decreases when the distance from the T-point increases. Thus if the phase-velocity at the T-point is still too high to allow for substantial wave-energy dissipation there, the rf-energy is diverted away from the T-point until the wave reaches a plasma region where the conditions are met for efficient damping. A completely satisfactory understanding of the energy flow and absorption problem in such cases became possible after the appearance of a fundamental paper by ZASLAVSKII et al. /18/. They used the theory of the solutions of differential equations of the type $\alpha y^{IV}(x) + \beta(x)y''(x) + \gamma(x)y(x) = 0$ (primes denote x-derivatives) with a small parameter α preceding the highest-order derivative, to handle <u>quantitatively</u> the wave transformation which takes place at the T-point $\beta^2 = 4\alpha\gamma$. A direct application of these techniques to the hybrid resonances has been made by STIX in a frequently quoted letter /19/ where he showed that a cold-plasma wave moving inward towards the resonance can be completely converted into a warm-plasma wave (Fig. 3) well before the resonance point is reached.

Let us now briefly discuss power flow and absorption when C-points are presented on a plasma profile at some distance from the R-point. Figure (4a) is an example of a fairly compact C-R-C triplet where the R-point separates propagation from evanescence regions. This situation is encountered at low frequency ($\omega^2 << \omega_{ci}^2$, see Eqs. (69) and (70)) in the case of the fast wave (Eq. 61) when $n_2^2 < |\varepsilon_1 - n_\parallel^2|$. The energy flow in the case of a wave incident from the right is calculated in /20/.

The power absorption is found to vanish with the distance $c_1-c_2 \propto (\omega/\omega_{ci})$. If $n_2^2 \gg |\varepsilon_1-n_\parallel^2|$, the R-point is located inside the propagation region (Fig. 4b and /21/).

The situation corresponding to resonance (68) is recovered as the limit of the previous case for $c_1 \to \infty$ (Fig. 4c): this is treated in detail in Chapter 21 of Ref. /15/. A fast wave first encountering the C-point is to a large extent reflected there. Due to tunnelling through the evanescence region some energy is absorbed at the R-point and the rest is transmitted beyond this point. A fast wave first encountering the R-point experiences strong absorption, no reflection, and, again, some transmission through the evanescent region. These facts are exploited to calculate power absorption in a Tokamak /22/. A qualitative picture of the C- and R-surfaces on a Tokamak cross-section as derived from (61) when the geometrical optics approximation $\partial/\partial x = ik_x$ is made, is shown on Fig. 5. If $v_A \ll c$, i.e. neglecting displacement currents, the situation of a single-ion plasma is particularly simple /23/ being described by

$$(k_\perp v_A/\omega)^2\left[A - (k_{/\!/} v_A/\omega)^2\right] = \{A+(A^2-A)^{1/2}-(k_{/\!/} v_A/\omega)^2\}\{A-(A^2-A)^{1/2}-(k_{/\!/} v_A/\omega)^2\} \tag{72}$$

where v_A is the Alfven speed and $A = \omega_{ci}^2/(\omega_{ci}^2-\omega^2)$. A plot of $k_\perp v_A/\omega)^2$ versus $k_\parallel v_A/\omega)^2$ for two cases, $\omega<\omega_{ci}(A>1)$ and $\omega>\omega_{ci}(A<0)$ is shown on Fig. 6. Thus if $\omega<\omega_{ci}$ there are two C-points and one R-point, if $\omega>\omega_{ci}$ there is one C-point and no R-point. The existence of $k_\parallel^2$-intervals where $k_\perp^2 > 0$ offers the possibility of eigenmodes within the plasma torus. They occur when the toroidal wave number is such that

$$k_\phi R = n = 0,\ 1,\ 2,\ \dots \text{ and } J_m\,(k_r a) = 0 \tag{73}$$

where R and a are the major and minor radius of the plasma, respectively, and J_m is a Bessel function.

As first emphasized by ADAM and SAMAIN /24/ heating in the ion cyclotron resonance (ICRH) domain in large (and dense) toroidal devices is based on the penetration and polarization properties of the fast wave (Eqs. (61) and (72)) with appropriate $(\omega, k_\theta, k_\phi)$ values (θ is the poloidal angle) and is very different from ICRH in the model-C Stellarator /25/. The latter was essentially based on the

the longitudinal propagation of the slow (torsional) wave with given (ω, k_r, k_θ) values towards Stix's magnetic beach. In the case of the fast wave, the left-hand $\vec{E}$-component, E_ℓ, which is the component rotating in the direction of the ion gyromotion, is strongly screened in a dense plasma ($\omega_{pi} \gtrsim k_\parallel c$) when $\omega > \omega_{ci}$, $E_\ell/|\vec{E}| \simeq k_\parallel r_{Li}/2\sqrt{\pi}$, and only moderately reduced when $\omega \to 2\omega_{ci}$, $E_\ell/|\vec{E}| \simeq 1/3$. Recent ICRH experiments in two-ion component plasmas where $\omega_{c2} = 2\omega_{c1}$ (e.g. H-D plasmas) have thrown new lights on the entire subject by demonstrating the importance of the relative concentration of the species on wave damping and particle heating /26/.

If the parallel wave number is chosen so as to satisfy the transit time resonance condition for thermal particles $\omega \simeq k_\parallel v_{ts} \ll \omega_{cs}$, then $|A| < 1$ and in a low β plasma $|K_\parallel v_A| \gg \omega$. Thus Eq. (72) gives $K_\perp^2 \simeq -K_\parallel^2$: the wave is radially evanescent but with an acceptably long evanescence length if

$$\mathrm{Im}\{k_r a\} \simeq ma/R \lesssim 1 . \tag{74}$$

Transit Time Magnetic Pumping (TTMP) has been proposed in various versions /21, 27/ depending primarily on the nature of the driving term in the energy equation (43) which reads

$$(d\varepsilon_s/dt)_1 \simeq e_s W_\parallel E_{1\parallel} + e_s(W_\parallel^2 + W_\perp^2/2)E_{1Z}/\omega_{cs}R + \mu_s \partial B_1/\partial t , \tag{75}$$

where Z is along the axis of a vertical cylindrical coordinate system (Fig. 7). The last term on the right hand side of (75) is the driving pump of the original TTMP version - the compressional version - which involves sinusoidal modulations in the ϕ-direction of the $\vec{B}$-field strength, $B_1 = |\vec{B}_1 \cdot \vec{B}_0/B_0|$, produced by ordinary m = 0 azimuthal coils. In this case the solenoidal part of $\vec{E}_1$ is mainly $\vec{E}_{1\theta}$ and the E_{1z} term of (75) can be neglected. On the other hand, there is in addition an irrotational $\vec{E}_{\parallel 1}$ component which ensures charge neutrality in spite of the preferential action of the pump on one of the plasma components, when $\omega \simeq \vec{k} \cdot V_{0s} \simeq k_\parallel V_{\parallel s}$. As first shown by KOECHLIN and SAMAIN /27/, TTMP can also occur with a torsional pump ($\vec{B}_1 \cdot \vec{B}_0 = 0$) characterized by a solenoidal E_{1z} component. This field is essentially constant along z, sinusoidal in ϕ, and weakly dependent on R, and can perform work on the particles thanks to the existence of the vertical drift velocity (again in the presence of an electrostatic $E_{\parallel 1}$-component). The most appropriate field structure has the wave numbers $|m| = |n| = 1$. An interesting coil system which can produce it (particularly easy in D-shaped vacuum vessels) is shown on Fig. 8.

The characteristic frequency of these two TTMP versions is between a few tens of kHz and about 200 kHz. Other TTMP versions have been found /27/ some of which involve $\omega \simeq k_\perp \cdot V_{0\perp s}(k_\parallel \to 0)$, corresponding to operating frequencies from a few kHz to a few tens of kHz.

The heating rate has been calculated in the various cases: roughly it has the form $\gamma_H = |\vec{k}.\vec{V}_{os}(w \simeq v_{ts})|.|\vec{B}_1/\vec{B}_o|^2$, so that in order of magnitude the power density absorbed is $\simeq \beta|\partial\{(E^2+B^2)/8\pi\}/\partial t|$. In a low-$\beta$ plasma this is unfortunately a small fraction of the available reactive power density. However, the thermonuclear prospects of TTMP (in all his versions) are poor because of a more practical reason. The first wall of a thermonuclear device is opaque to em fields with f $\gtrsim$ 100 Hz and the instantaneous RF $\vec{B}$-flux through any poloidal or equatorial cross-section of the vacuum vessel has to vanish (this follows from Faraday law, since the line integral of $\vec{E}$ all the way round any closed path on the conducting shell vanishes). Thus if the instantaneous RF $\vec{B}$-flux created by the RF coils through the poloidal or equatorial plasma cross-section does not vanish - as here with waves which do not oscillate in space (74) - eddy currents have to flow in the first wall. Then the heating efficiency is the result of a compromise between two contradictory requirements: diminishing the image currents in the wall while keeping the plasma cross-section as large as possible. One realizes that the efficiency of any ion TTMP version will remain disappointingly low even in large thermonuclear devices /28/. Therefore our conclusion is that the lowest practicable frequencies for RF-heating purposes in toroidal plasmas are the Alfvén frequencies $\omega > |\vec{k}.\vec{v}_A|$ at which $k_\perp^2 > 0$ at least on some readial extent within the plasma column /29/.

In a toroidal plasma the quantity $k_\parallel v_A$, which is proportional to

$$k_{/\!/} B_o \equiv \vec{k} \cdot \vec{B}_o \equiv (nB_\phi/R + mB_\theta/r) \equiv (nq+m)B_\theta/r \qquad (76)$$

(q is the usual safety factor or inverse rotational transform) is a function of r which vanishes at the MHD singular surfaces $q(r) = -m/n$.

Thus it may well happen that the R-point ($A = (k_{\parallel} v_A/\omega)^2$, occurs within the plasma even if at the plasma periphery $k_{\parallel}(a)v_A(a) \gg \omega$. Of course the R-point may occur within the plasma even if $k_{\parallel}$ never vanishes as in the original proposals of plasma heating by a resonant fast wave /29/. Theory predicts /30/ that as long as there is one R-surface or two R-surfaces well separated in space the fraction of the available reactive power which can be absorbed should be substantial.

It remains to consider the hybrid resonances (67) and (68). Both cases are recovered as the $c_2 \to -\infty$ limit of Fig. 4c. In this highly idealized model (Fig. 4d) the resonance is inaccessible to a wave launched from the left, while all the energy of a wave launched from the right is completely absorbed at $x = 0$.(in reality in the latter case the cold-plasma wave energy is linearly mode converted into a warm-plasma wave at a T-point located on the right of $x = 0$, see Fig.3). Let us first consider the L.H.R. frequency. It can be shown (1) that the resonance the X-wave has at the frequency (68) when $k_{\parallel}c/\omega \to 0$ is inaccessible from the low density side, and (2) that k_x^2, the square of the wave-number vector along x of the O-wave, is real and positive all the way from the C-point

$$(\omega_{pe}(x)/\omega)^2 \simeq 1 + k_y^2/(k_z^2 - (\omega/c)^2) \tag{77}$$

to the R-point (67), provided that the externally launched wave is slowed down along $\vec{B}_0$ so as to satisfy the Stix-Golant condition /31/

$$n_{\parallel}^2 > (1 - \omega^2/\omega_{ci}\omega_{ce})^{-1} = 1 + (\omega_{pe}/\omega_{ce})^2_{[\omega_{LHR}=\omega]}, \tag{78}$$

where $n_{\parallel} = k_{\parallel}c/\omega$ is the parallel index of refraction. The last subscript in (78) indicates evaluation at the point where the frequency (68) is equal to ω. A very successful LHR antenna is the Grill /32/ a phased array of wave guides (mounted flush on the liner) with their small side in the toroidal direction and excited in the fundamental

TE_{01} mode. A complete theory exists of the radiation properties of the Grill in realistic situations /33/.

As the C-point (77) is at the very edge of the plasma for the frequencies under consideration (Fig. 3), efficient tunneling can be easily obtained from a rf-antenna which is able to create an electric field essentially parallel to $\vec{B}_o$ and to concentrate most of the rf-power in the accessible part of the $n_{\parallel}$-spectrum according to (78) (the inaccessible part of the rf-field is trapped near the vacuum wall and eventually absorbed, resulting at best in low grade heating). If the accessibility condition (78) is violated there exists an interval $T_1 < x < T_2$ in which the two roots k_x^2 of the cold-plasma dispersion relation are complex conjugate and the waves evanescent: T_1 and T_2 are T-points, Fig. 9.

In the UHR-frequency domain it is impossible in practice to slow down waves along $\vec{B}_o$, as the required launching structures would have to be placed at a distance of the order of the millimeter wave length from the hot plasma. Thus the injected waves have real direction angles. The ray paths are traced by using the geometrical-optics equations /34/

$$
\begin{aligned}
d\vec{r}/ds &= (d\vec{r}/dt)/|\vec{v}_g| = -\,\mathrm{sgn}(\partial D/\partial\omega)(\partial D/\partial\vec{k})/|\partial D/\partial\vec{k}| \\
d\vec{k}/ds &= (d\vec{k}/dt)/|\vec{v}_g| = \mathrm{sgn}(\partial D/\partial\omega)(\partial D/\vec{r})/|\partial\vec{D}/\partial\vec{k}|
\end{aligned}
\tag{79}
$$

where $D(\omega(\vec{k},\vec{r}),\vec{k},\vec{r}) = 0$ is the local dispersion relation (53), s the arc length along the ray, and $\vec{v}_g$ the group velocity. Equations (79) are integrated numerically for given initial (at $s = s_o$) conditions $\vec{r}_o$, $\vec{k}_o$. Propagation purely perpendicular to the magnetic surfaces is most easily visualized by transposing to the poloidal cross section of the plasma torus the Clemmow-Mullaly-Allis (CMA) diagram giving for a plasma slab the C- and R-points of the X- and O-waves in the $\{(\omega_{pe}/\omega)^2, \omega_{ce}/\omega\}$-plane (Fig. 10). We obtain Fig. 11. The X-wave, thanks to the 1/R-dependence of B_o, is accessible only from the inside of a vertical cylinder of radius R_ω where $\omega_{ce}(R_\omega) = \omega$; it is deflected toward regions of weaker B_o-amplitude up to the UHR surface - actually a T-surface if thermal effects are retainded - where it mode-converts to an electron Bernstein mode which is eventually cyclotron damped /35/. The O-wave penetrates up to the $\omega_{pe} = \omega$ surface, being reflected away from regions of relatively-high density. If $k \neq 0$, the O- and X-waves are no longer uncoupled: it exists a $k_{\parallel}$ interval (Fig. 12 and Ref. /36/), for which the O-wave energy is transmitted slightly beyond the C-poing ($\omega_{pe}^2 = \omega^2$) and is almost entirely converted to the X-wave at the cold-plasma T-point (it is further transferred to an electron Bernstein mode, etc....).

With regard to Fig. 12 notice that when $|k_{\parallel}|$ is larger than the value for which the T-point coincides with the C-point, the branch which exhibits the UH-resonance is the ordinary wave in our labeling.

5. A ponderomotive effect in LH current drive

The measured value of the steady state current driven by LH waves in an initially Maxwellian plasma of sufficiently low density /37/ is orders of magnitude higher than expected given the nominal $n_{\parallel}$ spectrum of the couplers /11/

$$I_{/11/} \simeq 8.1\, n a^2 T^{-1/2} e^{9-w_M^2}\left(n_{\parallel m}^{-2} - n_{\parallel M}^{-2}\right). \quad (80)$$

Here I is in MA, n is in units of 10^{20} electrons per m^3, a - the plasma minor radius - is in m, T - the electron temperature - in keV, $w_M^{2} = 255,9/n_{\parallel M}^2 T$; and subscript m (M) denotes the lower (upper) limit of the $n_{\parallel}$ spectrum, taken for simplicity to be rectangular. Eq (80) reproduces the experimental values if $w_M \simeq 3$, i.e. $n_{\parallel M} \simeq 5.3\, T^{-1/2}$; however, the $n_{\parallel M}$ value deducible from the nominal spectra in /37/ is pretty much one half of that value, and this gives $\exp(9-w_M^2) = \exp(-27) \simeq 1.88 \cdot 10^{-12}$. In what follows we tentatively suggest that the required doubling of $n_{\parallel M}$ and concomitant broadening and upshifting of the $n_{\parallel}$ spectrum is produced by the quasi static ponderomotive force caused on the electrons by the LH wave. If ω is sufficiently in excess of ω_{LH} (Eq.(68)) to avoid linear mode conversion to the warm plasma wave for any $n_{\parallel}$ within the plasma then $\vec{E}$ is essentially electrostatic, the temperature can be neglected, and

$$E_\perp^2 / E_\parallel^2 = n_\perp^2 / n_\parallel^2 = -\varepsilon_3/\varepsilon_1$$
$$\simeq (m_i/m_e)\, \omega_{LH}^2 / (\omega^2 - \omega_{LH}^2) \,. \tag{81}$$

In this case the ponderomotive force is the gradient of the momentum independent ponderomotive potential /38/

$$(e\phi)_{PM} \simeq \sum_s (e_s^2/m_s)\left(E_\parallel^2/\omega^2 + E_\perp^2/(\omega^2-\omega_{LH}^2)\right)/4$$
$$\simeq e^2 E_\parallel^2 / 4 m_e \omega^2 \simeq \mathcal{E}/n \,, \tag{82}$$

where $\mathcal{E}$ is the time average reactive energy density including both electric field energy and particle kinetic energy /14/

$$\mathcal{E} \simeq \omega_{pe}^2 E_\parallel^2 / 16\pi (\omega^2 - \omega_{LH}^2) \,. \tag{83}$$

In the presence of ponderomotive potential (82) a quasi-neutral plasma is in Boltzmann equilibrium

$$n_s = n(r)\, e^{-\mathcal{E}/n(r)(T_e+T_i)}$$

or

$$\delta n/n \simeq -\mathcal{E}/n(T_e+T_i) \,. \tag{84}$$

Now the group velocity trajectories of the LH waves (Eqs.(81)) are independent of wave number: e.g.

$$(dx/dz)_{rays} = v_{gx}/v_{gz} = \pm n_\parallel / n_\perp \tag{85}$$

Thus the field radiated by a finite length LH antenna tends to concentrate around patterns of constructive interference, called resonant cones /39/. Since in the experiments referred to in /37/ the direct absorption of the injected spectrum is negligible, the waves which are transmitted to the plasma will travel around the torus a very large number of times, with essentially unchanged form. We postulate that these waves keep their space-coherence to a sufficiently high degree to be allowed to write that on a substantial fraction of the plasma volume $E^2(r)$ is schematically given by

$$E_{\parallel}^2 \simeq E_0^2\{1+\cos(n_{\parallel M} z - n_\perp r)\}/2 , \tag{86}$$

where for brevity reasons we have omitted the effect of cylindrical geometry. As a result

$$\begin{aligned} n_\perp = n_\perp(z,r) &\simeq n_{\parallel 0}\,\omega_{pe}/\omega \simeq n_{\parallel 0}\,(1+\delta n/2n)\langle\omega_{pe}\rangle/\omega \\ &\simeq n_{\parallel 0}\{1-(1+\cos(n_{\parallel 0} z - n_\perp r))\varepsilon_0/2nT\}\cdot \\ &\quad \cdot\langle\omega_{pe}\rangle/\omega , \end{aligned} \tag{87}$$

where $\varepsilon_0 \equiv \varepsilon_0(E_0^2)$. Now Jacobi identity (13) can be used to find that the following Fourier components are created within the plasma

$$E_{\parallel s}^2 = J_s^2(\lambda)\,E_{\parallel 0}^2/J_0^2(\lambda) \quad , \quad |s| = 0,1,2,\ldots \tag{88}$$

with

$$\begin{aligned} &\lambda \equiv k_{\perp 0}\, r\, \varepsilon_0/4n(T_e+T_i) , \\ &\vec{K}_s = (1+s)\vec{k}_0 \quad ; \quad w_s = w_0/(1+s) . \end{aligned} \tag{89}$$

Doubling $n_{\parallel 0}$ corresponds to s = +1, tripling to s = +2, and so on.

ε_o is related as follows to the absorbed power P_a, which we anticipate to be essentially due to one single s-Fourier component, and be given by Eq. (38)

$$P_a \simeq V\varepsilon_s \cdot 2\sqrt{\pi}\,\omega w_s^3 e^{-w_s^2} = V\varepsilon_o/\tau_{as} \tag{90}$$

$$\omega\tau_{as} \simeq e^{w_s^2}/2\sqrt{\pi}\, w_s^3 J_s^2(\lambda)\ ; \tag{91}$$

here $\varepsilon_s \equiv \varepsilon(E_s^2)$, V is essentially the plasma volume, and τ_{as} is the energy absorbtion time. Then Eq. (89) becomes

$$\lambda = \omega\tau_{as}\mu\ ;\quad \mu = k_{\perp 0}\, r\, P_a/2\omega V n T. \tag{92}$$

Requiring that, as in the experiments /37/, P_a be sensibly equal to the transmitted power

$$P_t \simeq \varepsilon_G A_G c\ , \tag{93}$$

where ε_G is the average energy density at the Grill mouth, and A_G the cross sectional area of the Grill, leads to

$$\varepsilon_o/\varepsilon_G \simeq (c\tau_{as}/2\pi R)\, A_G/\pi a^2 \tag{94}$$

The RHS of (94) can have a large value (see below) and plays the role of a stacking factor or of the quality factor of a resonant cavity.

Eqs. (91) and (92) now determine τ_{as} as a function of P_t. Anticipating that $\lambda_s \ll |s|$, we find

$$\lambda_s^{1+2s} = (2^s s!)^2 \mu \, e^{w_s^2} / 2\sqrt{\pi}\, w_s^3$$

$$\omega \tau_{as} = \left\{ \frac{2^{s-1} s! \, \mu^{-2s} e^{w_s^2}}{\sqrt{\pi}\, w_s^3} \right\}^{(1+2s)^{-1}} \tag{95}$$

with

$$\mu \simeq 0.75 \, \frac{10^{-6} \, n_{\parallel M} P_t}{f a R T \sqrt{n}}$$

where P_t is in MW, f - the Grill frequency - is in GHz, the other units being those of Eq. (80). With the parameters of /37/, $\mu \simeq 0 \, (10^{-6})$, $\lambda_s \lesssim 10^{-1}$. With $n_{\parallel 0} = 2.66$:

$$\tau_{a1} \simeq \left(10^{9} f \mu^{2/3}\right)^{-1} \ll \tau_{a\,|s|>1} ; \tag{96}$$

thus typically $\tau_{a_1} \simeq 10 \mu s$, which gives with Eq. (94)
$\varepsilon_0 / \varepsilon_G \simeq 10^3 A_G / \pi a^2$.

It is well known that the experiments have demonstrated maintaining of substantial current by LH waves alone, only when the plasma density is below some critical density, while at higher densities RF current can no longer be driven. The critical value could correspond to the point where the T-point for diverting energy from the primary wave to the warm plasma branch (Fig. 3 and Ref. /19/) appears in the core of the plasma. When this happens there is an abrupt drop of the absorption time with respect to value (96) which stops the penderomotive process responsible for the $n_{\parallel 0}$ upshift. If $w_s^2 \gg 3/2$, the relevant electrostatic equation for a warm single-ion-species plasma /40/ says that there will be no T-point as long as

$$n < n_{crit} = \frac{0.45\, f^2\, m_i/m_D}{1 + .15\, n_{\parallel s}\sqrt{T_i} - 1.18\, f^2 m_i/m_D B^2} \qquad (97)$$

where m_D is the deuteron mass, B is in Tesla, and the other units are as in Eqs. (80) and (95). However, other loss mechanisms, for instance purely collisional dissipation, could stop the penderomotive effect at even lower densities.

6. A Relativistic Effect for Arbitrarily Low Speed

Although the bulk electron temperature of fusion plasmas is at most in the 10 KeV range, relativistic effects cannot always be neglected in the electron motion: two such cases are the electron run-away process and electron cyclotron heating with X-waves propagating almost across $\vec{B}_o$. In the latter case, the reletivistic effect is there even if electron speed constantly remains arbitrarily low. Consider an em−field rotating around the direction of a uniform $\vec{B}_o$-field and constant along it, with frequency equal to the non relativistic electron gyrofrequency. The electron motion can be calculated exactly /41/: $v(t)$ is a purely periodic function of time, oscillating between 0 and $v_{Max} \propto (E/Bo)^{2/3}$, with period $\tau \propto 1/\omega(E/B_o)^{2/3}$. The period is essentially the time it takes the angle between $\vec{v}$ and $\vec{E}$ to become dephased by $\pi/2$. What matters here, is the action $\int_0^{\tau} E_{kin}\, dt$, rather than the kinetic energy E_{kin}.

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FIGURE CAPTIONS

1. Phase-space trajectories in a coordinate system moving with a low-frequency wave with $B_{1\parallel} = -\, b\, B_o \cos(kz - \omega t)$

2. Schematic plot of the heating rate γ_H versus the collision frequency ν_{coll} in a wave-particle resonance case (upper curve) and in gyrorelaxation (lower curve).

3. Schematic plot of k_x^2 versus $x = (\omega_{pe}/\omega)^2$ in the LHR - frequency range for ordinary (O) and extraordinary (X) waves with a sufficiently large $(k_z c/\omega)^2$ value (see condition (77) below). When the plasma temperature tends to zero the broken line which is a warm plasma asymptote approaches the vertical $\omega = \omega_{LHR}$.

4. Profiles of the square of the radial wave-number k(x) when there is a R-point together with two C-points (cases (a) and (b)), with one C-point (case c) and with no C-points (case d).

5. Geometrical-optics plot of the C- and R- curves of the extraordinary wave, Eq.(61), in the I^2HR frequency domain on the Tokamak minor cross-section. The Tokamak center is to the left of the figure. The wave is evanescent in the shaded regions.

6. Schematic plot of $(k_\perp v_A/\omega)^2$ versus $(k_{//} v_A/\omega)^2$ in a single ion species plasma for $\omega > \omega_{ci}$ (lower curve) and $\omega < \omega_{ci}$ (upper curves).

7. The quasi-toroidal and cylindrical co-ordinates.

8. Schematic rf-coil arrangement for torsional TTMP $(|m| = |n| = 1)$ (after /27/).

9. Schematic plot of k_x^2 versus $x = (\omega_{pe}/\omega)^2$ for a cold plasma in the LHR-frequency range when the accessibility condition (77) is not fulfilled : $1 < N_{//}^2 < (1 - \omega^2/\omega_{ci}\omega_{ce})^{-1}$ (for the accessible case see Fig. 3).

10. The Clemmow-Mullaly-Allis (CMA) diagram for exact perpendicular wave propagation in the UHR-frequency domain. The O-wave is evanescent at the right of $(\omega_{pe}/\omega)^2 = 1$. The X-wave is evanescent in the shaded regions.

11. Geometrical-optics plot of the C- and R- curves for O- and X-waves at normal incidence in the UHR-frequency domain on a Tokamak minor cross-section. The Tokamak center is to the left of the figure. The O-wave is evanescent inside the $(\omega_{pe}/\omega)^2 = 1$ curve. The X-wave is evanescent in the shaded region.

12. Square of the perpendicular index of refraction, $N_\perp^2$ as a function of plasma density $(\omega_{pe}/\omega_{ce})^2$ for various values of $N_{\shortparallel}^2$ at a fixed value of $\omega_{ce}/\omega = 2/3$. $N_{\shortparallel}^2 = 0$ (....), 0.16 (-..-), 0.25 (-.-.-), 0.4 (——), 0.66 (-----) (slightly modified after /36/).

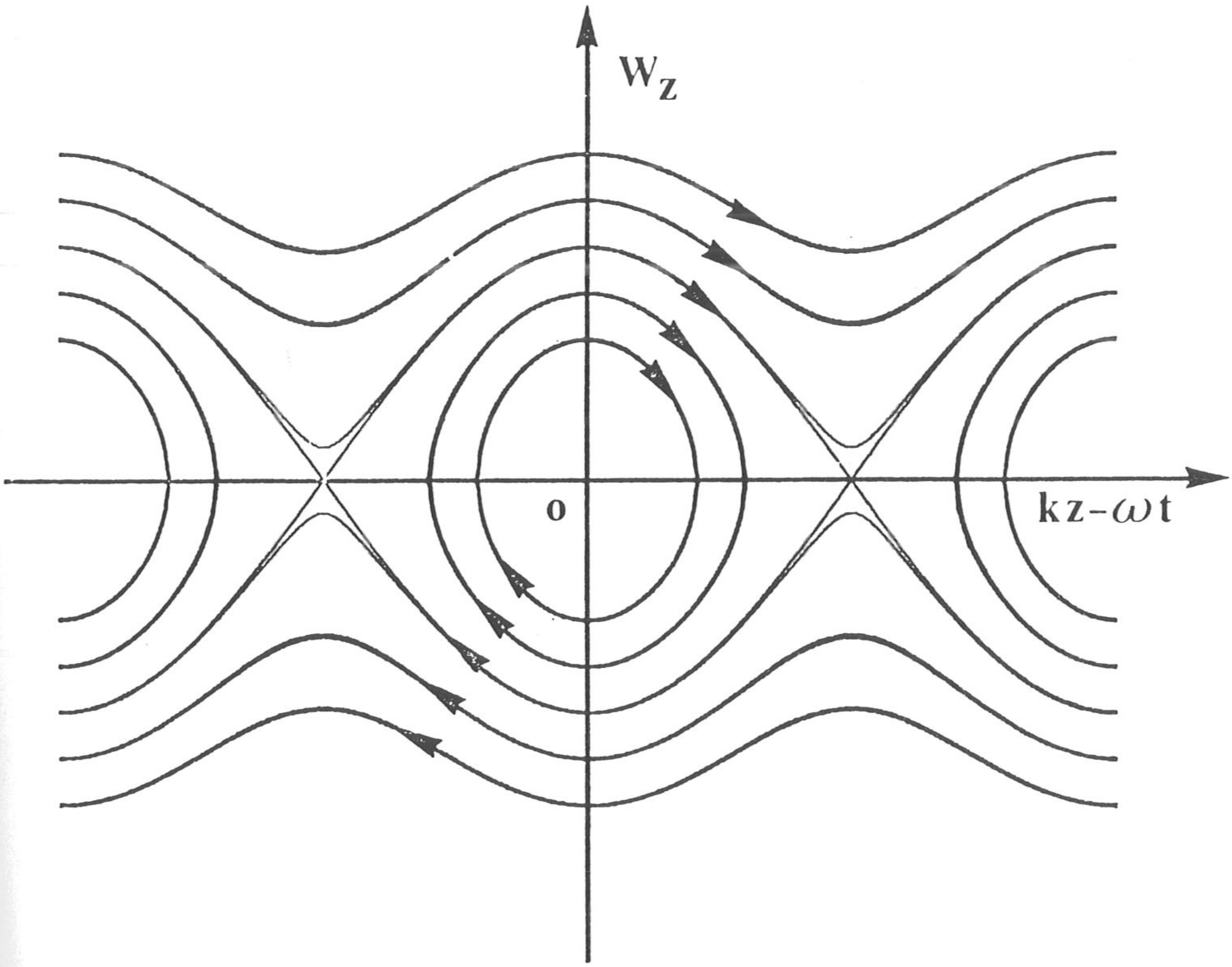

Fig. 1

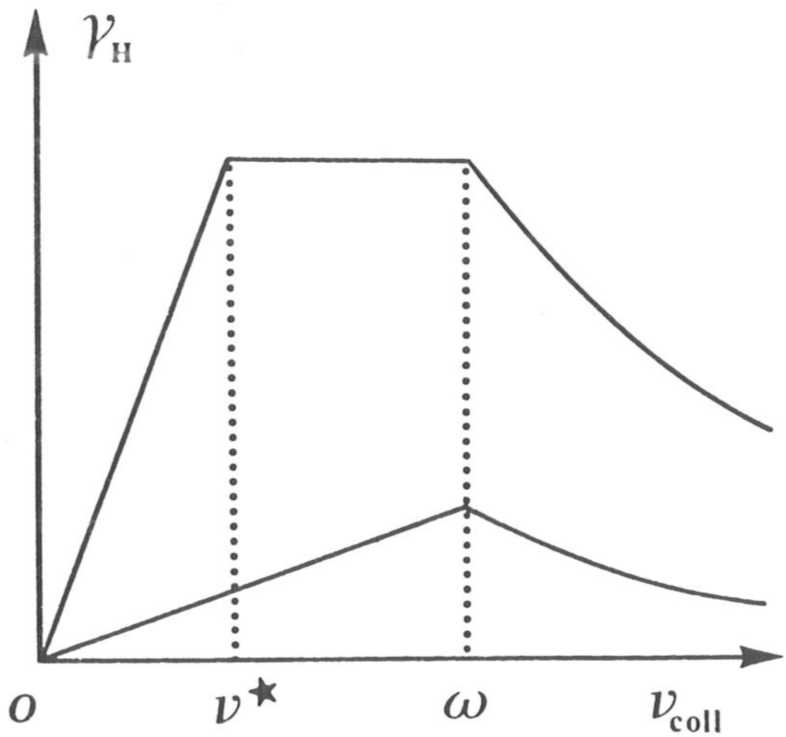

Fig. 2

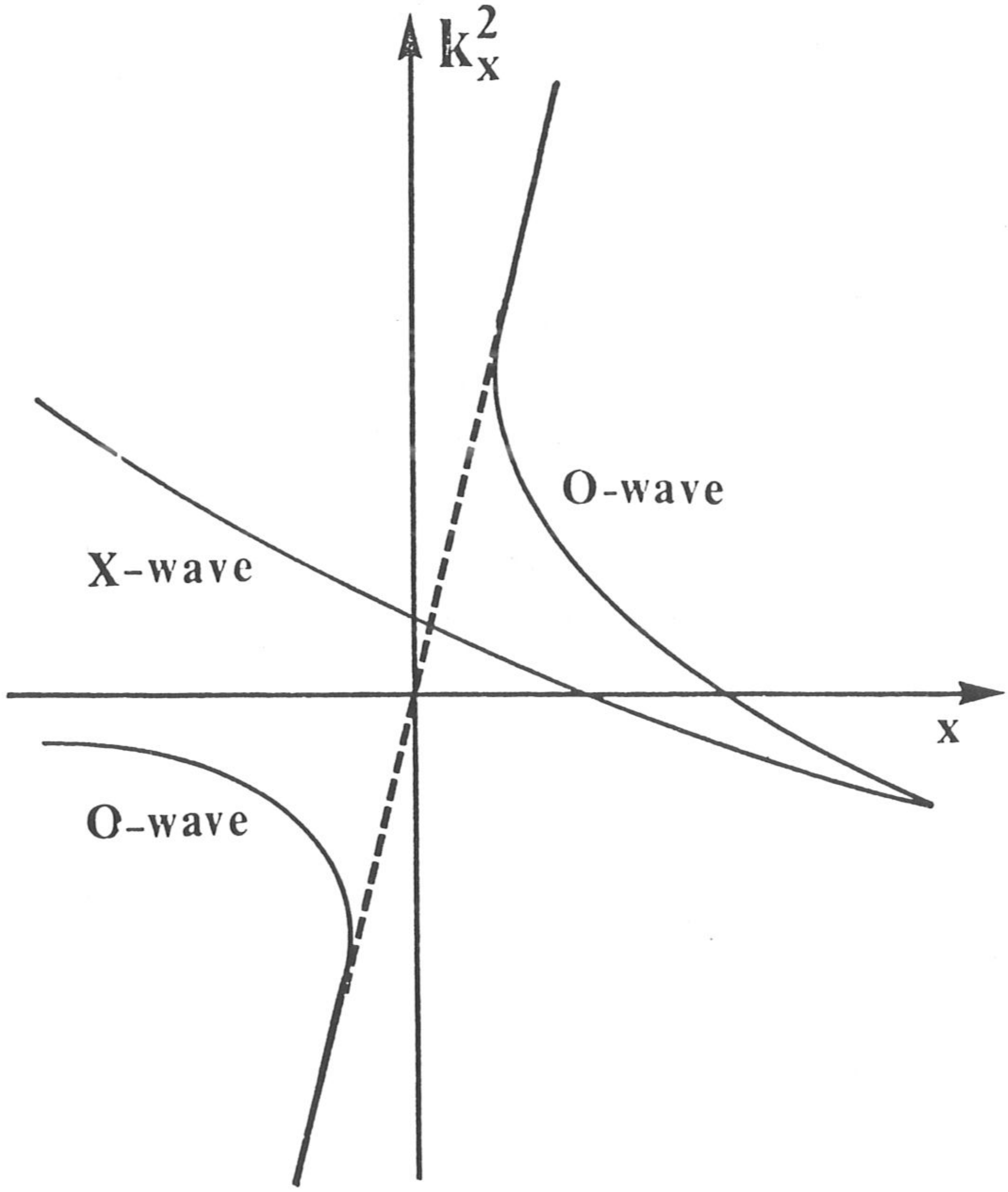

Fig. 3

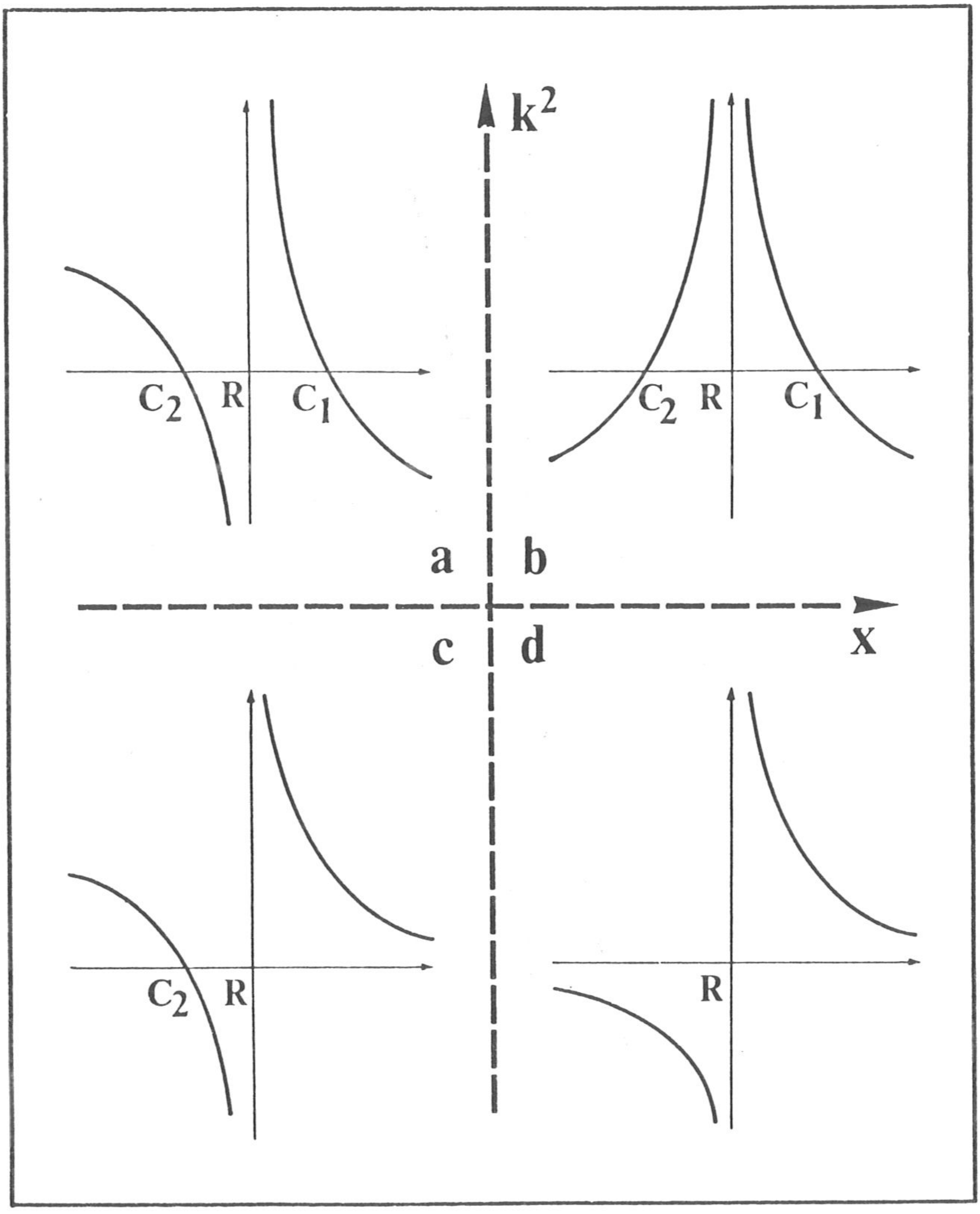

Fig. 4

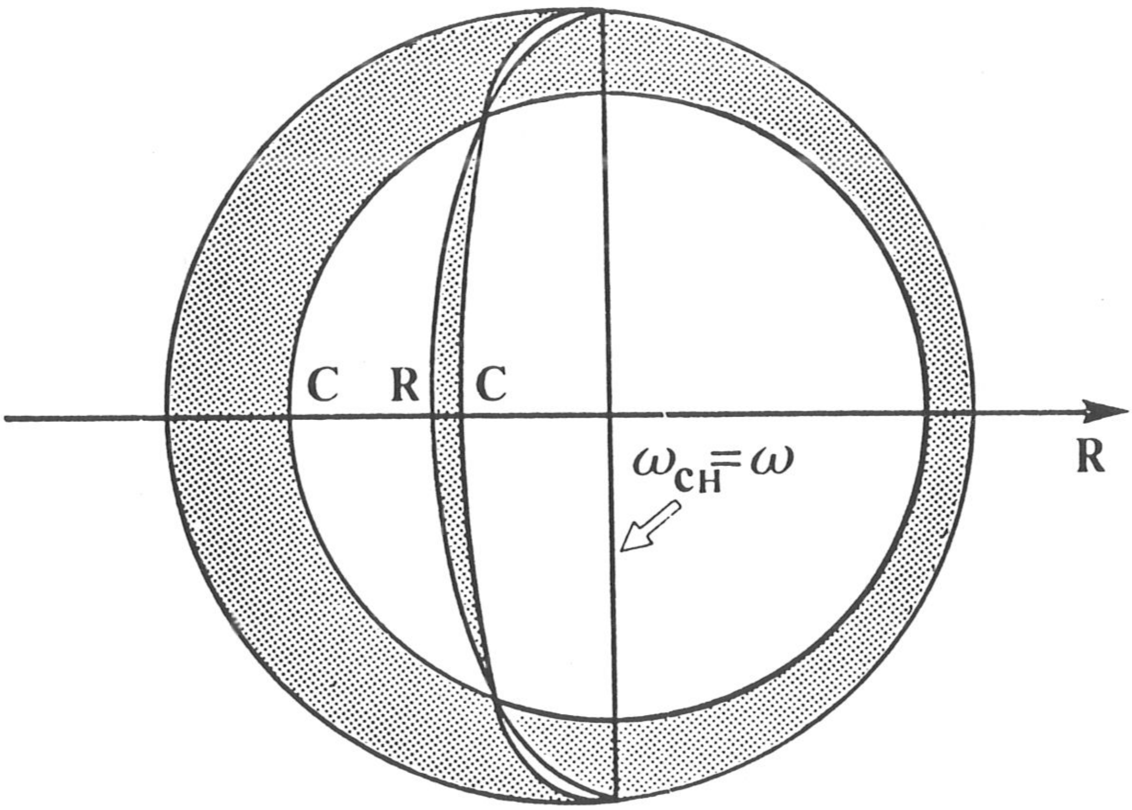

Fig. 5

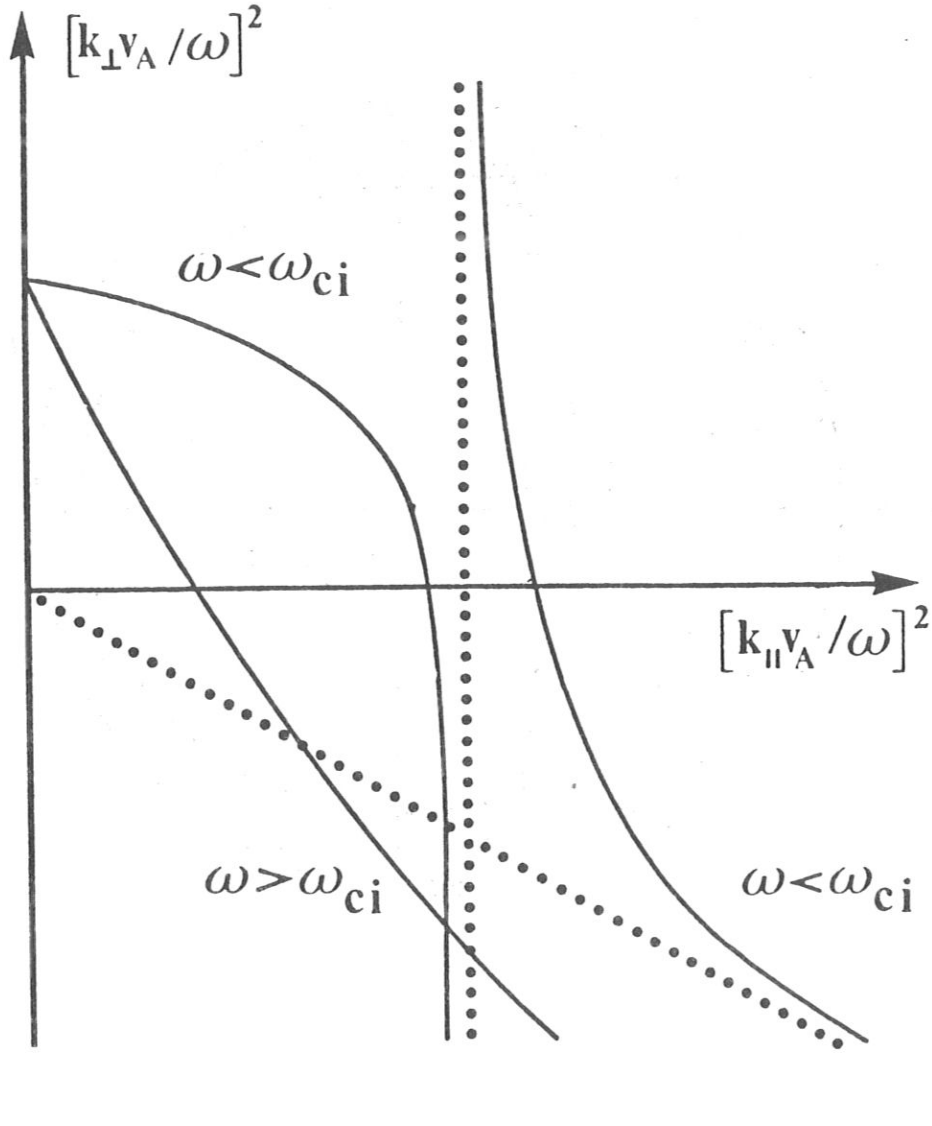
$[k_\perp v_A/\omega]^2$
$\omega<\omega_{ci}$
$[k_\parallel v_A/\omega]^2$
$\omega>\omega_{ci}$
$\omega<\omega_{ci}$

Fig. 6

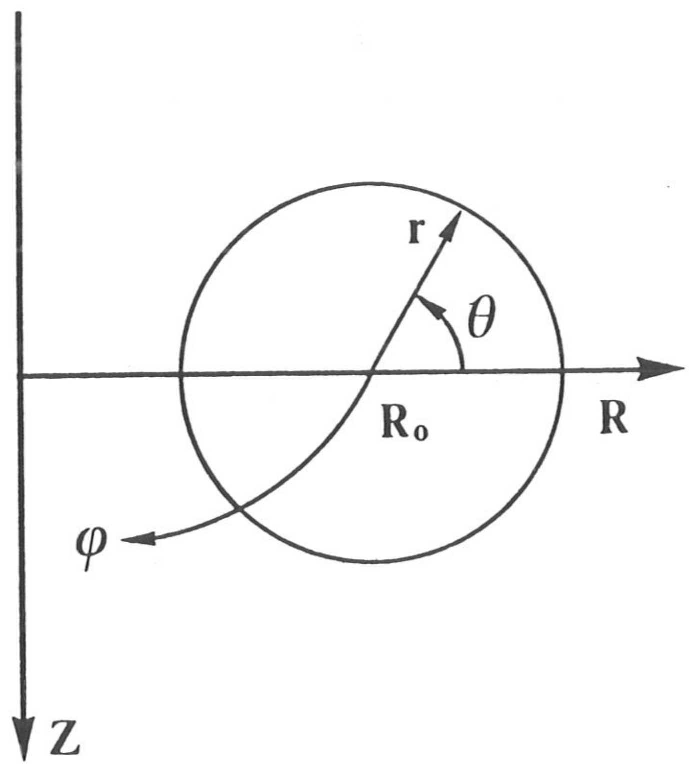

Fig. 7

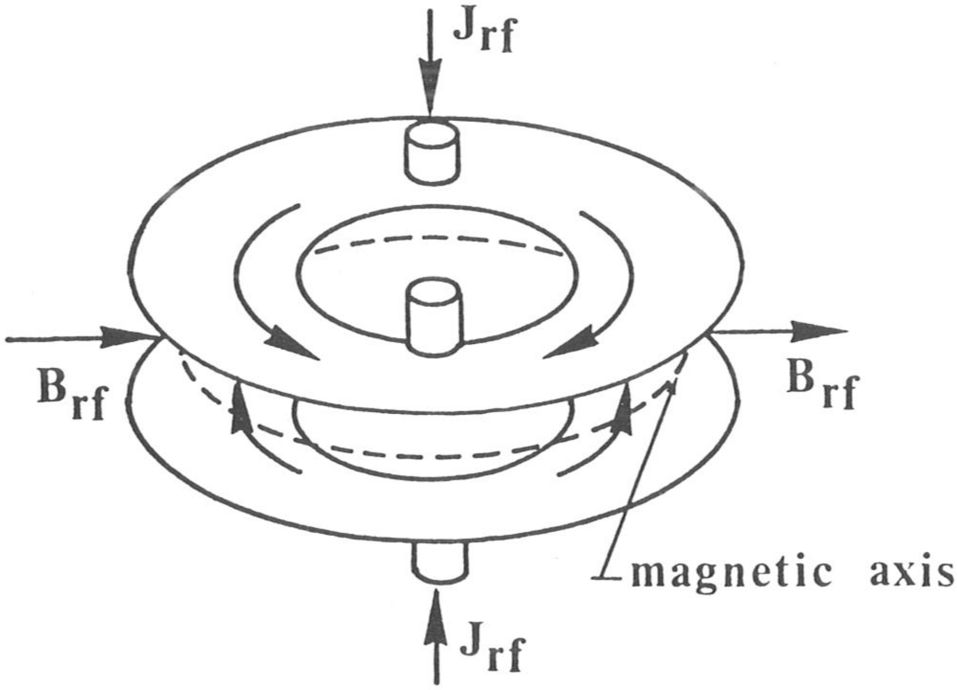

Fig. 8

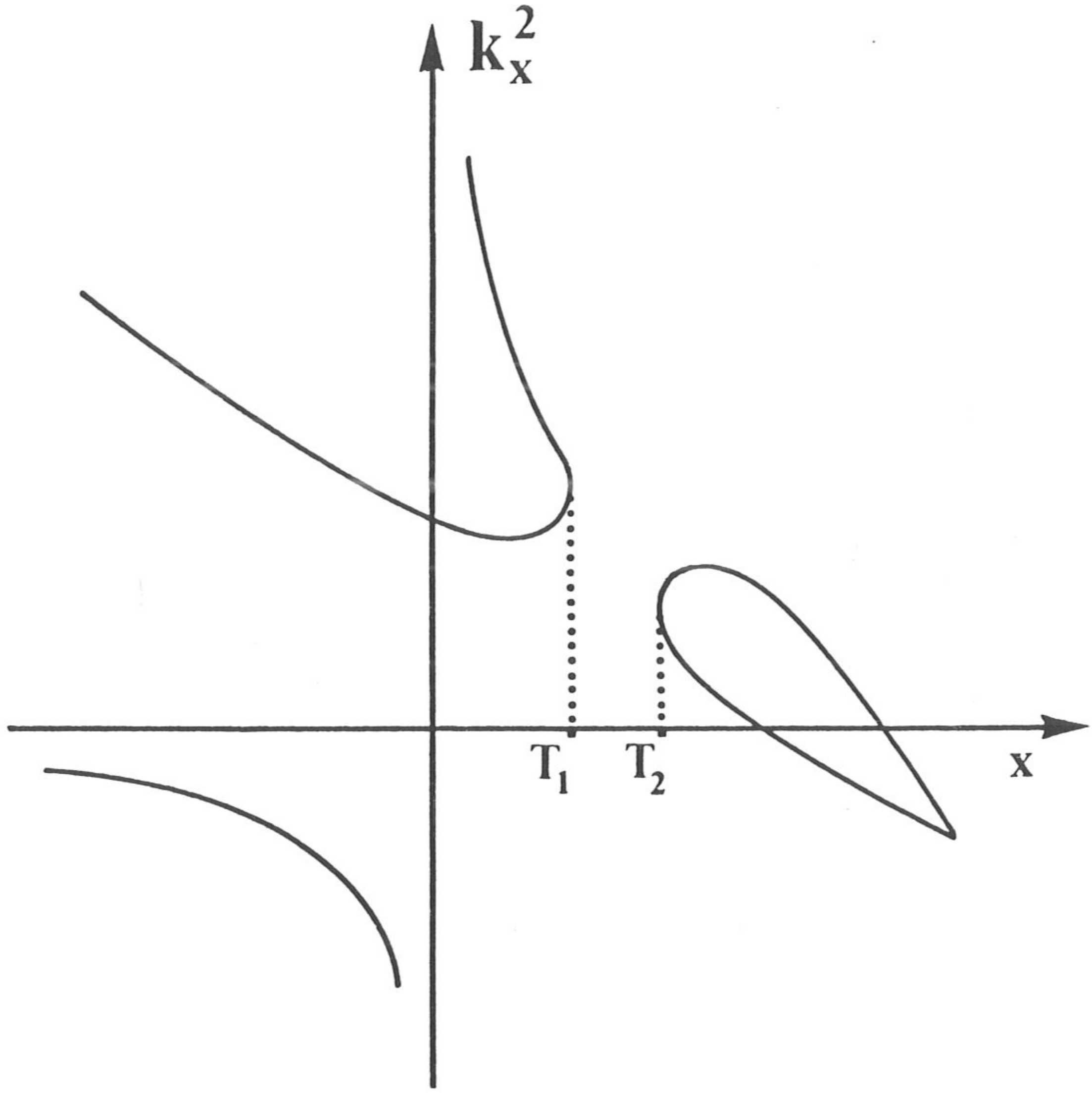

Fig. 9

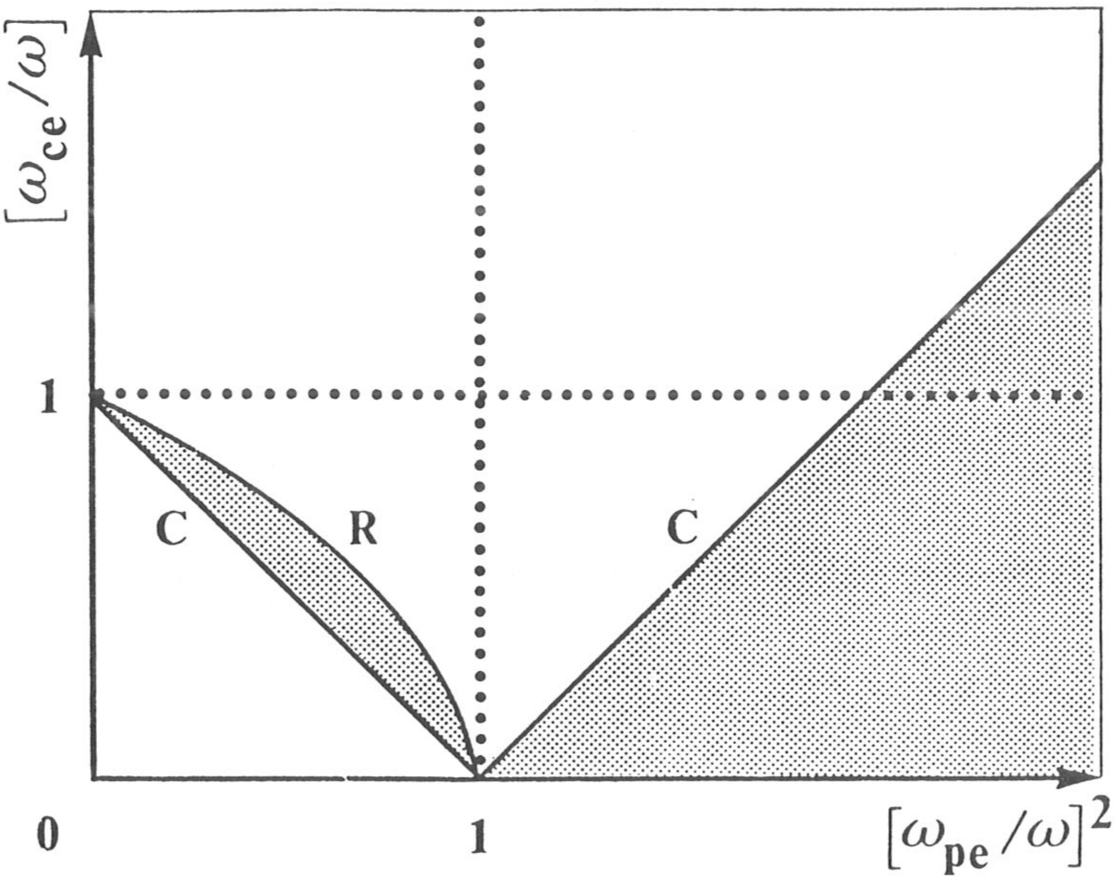

Fig. 10

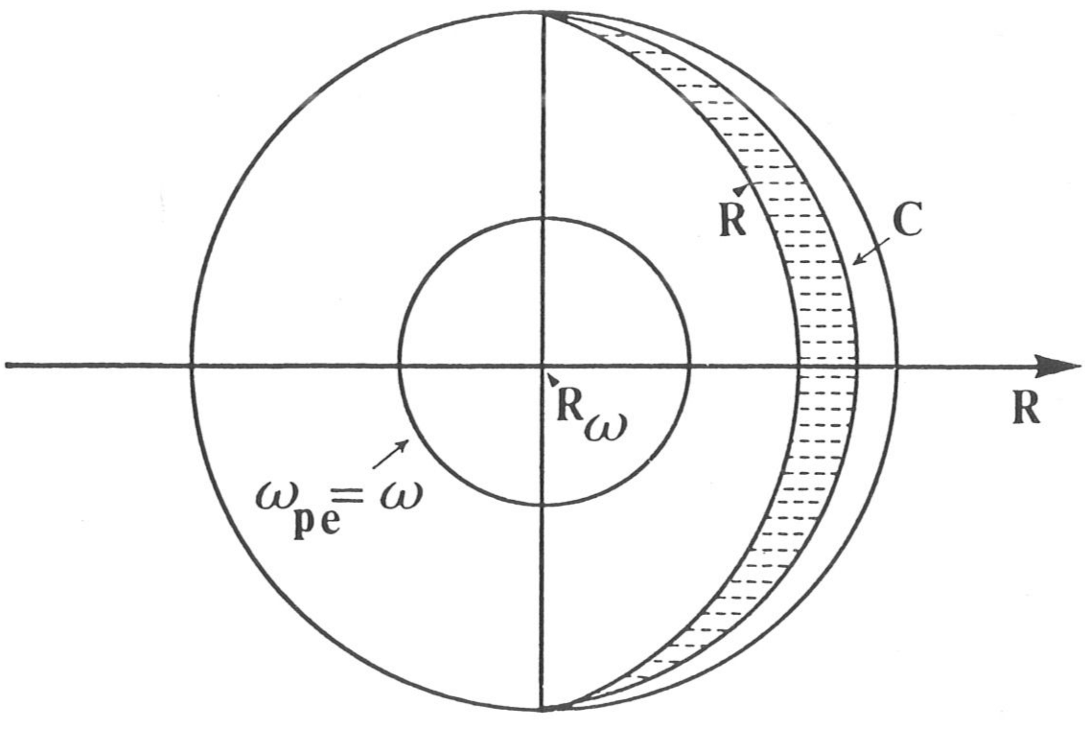

Fig. 11

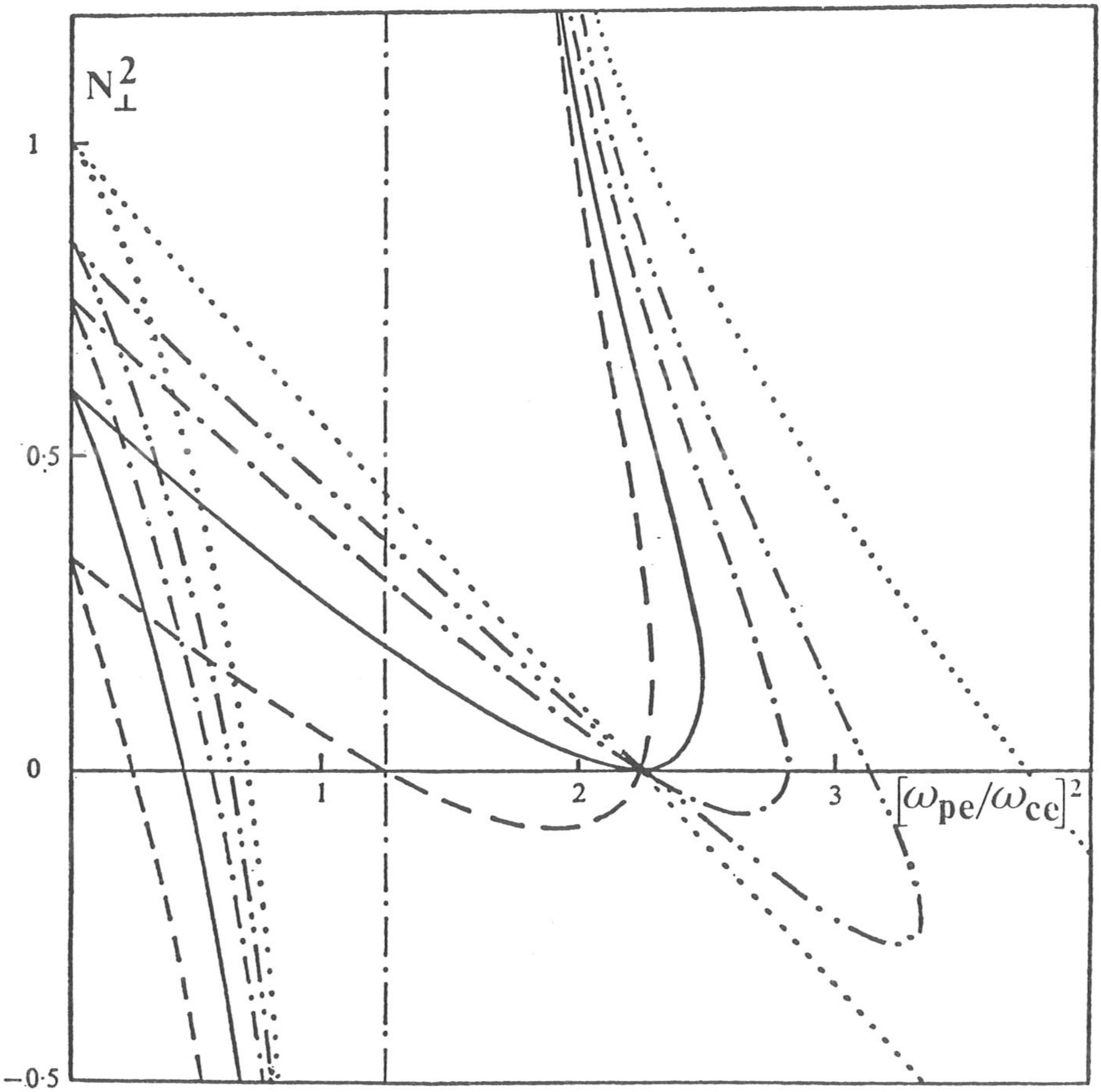

Fig. 12