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ANOMALOUS RESISTIVITY AND VISCOSITY DUE TO

SMALL - SCALE MAGNETIC TURBULENCE

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6/222 April 1983

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

ANOMALOUS RESISTIVITY AND VISCOSITY DUE TO SMALL - SCALE MAGNETIC TURBULENCE

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Abstract

The effective resistivity η_a and viscosity μ_a through which small-scale MHD turbulence affects large - scale magnetic structures in a low-β plasma are derived. While μ_a only depends on the turbulent magnetic field and is always positive, η_a becomes negative when the magnetic turbulence energy density exceeds the kinetic one. This generalizes a previous result in the theory of 2D MHD turbulence. The origin of a recently published result giving a positive definite resistivity is pointed out.

I INTRODUCTION AND BASIC EQUATIONS

In a recent letter [1] it was pointed out that in a low-β plasma such as in a tokamak small-scale magnetic turbulence acts as a negative effective resistivity on large-scale magnetic field perturbations. This leads to amplification of the large-scale fields and is a very likely mechanism for explaining the explosive phase of the major disruption in tokamaks. This new paper now treats in greater detail the derivation of the turbulent diffusion coefficients, the resistivity η_a and the viscosity μ_a.

For a low-β plasma in a strong, externally generated magnetic field which is assumed to be in the z direction, the MHD equations are well approximated by the reduced equations [2] for the vector potential ψ of the magnetic field $\vec{B}_\perp = \hat{z} \times \nabla\psi$, $B_\perp << B_z$, produced by currents flowing in the plasma (essentially along B_z) and the stream function ϕ of the incompressible perpendicular plasma flow $\vec{v}_\perp = \hat{z} \times \nabla\phi$ (the parallel flow $v_\parallel$ decouples):

$$\frac{\partial\psi}{\partial t} - \vec{B}\cdot\nabla\phi = \eta j, \tag{1}$$

$$\frac{\partial u}{\partial t} + \vec{v}_\perp\cdot\nabla u = \vec{B}\cdot\nabla j + \mu \nabla_\perp^2 u , \tag{2}$$

with $j = \nabla_\perp^2\psi$, $u = \nabla_\perp^2\phi$.

The magnetic field $\vec{B}_\perp$ or ψ is now divided into three parts

$$\psi = \psi_o + \psi_\ell + \psi_s \quad . \tag{3}$$

Here ψ_o describes the average field, e.g. the mean poloidal field in a tokamak, ψ_ℓ the large-scale magnetic perturbations corresponding to tearing modes with low poloidal mode numbers m in a tokamak, and ψ_s the small-scale, i.e. high m-number, perturbations. To make this distinction of different spatial (and time) scales unambiguous, we assume a mode spectrum as illustrated in Fig. 1, with two well-separated populations and with the k_s part sufficiently broad, $\Delta k_s \sim k_s$, to allow strong turbulent mixing. Usually turbulence spectra are not of this type; rather they decrease monotonically. In this case the processes discussed here describe only a certain aspect of the full dynamics of the turbulence. This point will be returned to later, when possible applications are discussed. Averaging ψ over the small scales k_s gives $< \psi > = \psi_o + \psi_\ell$, averaging over large scales $<< \psi >> = \psi_o$. The scale separation should be valid in all spatial directions which precludes ψ_ℓ having localized structures like those of linear tearing modes around the resonant surface. This, however, is no real restriction since the main contributions to ψ_ℓ are from finite-amplitude perturbations where the radial mode-structures are broadened over dimensions corresponding to the magnetic island widths in the case of tearing modes. It is also assumed that possible spatial variations of ψ_o are weak over typical x_ℓ scales. Hence in tokamaks the island widths should be small compared with the plasma radius, which defines the radial scale length of the average poloidal field. Similarly, we divide the stream function

$$\phi = \phi_\ell + \phi_s \qquad (4)$$

since ϕ_o is usually negligible.

From eqs. (1) and (2) one obtains the equations for the large-scale perturbations ($\vec{B}_o = (\hat{z} \times \nabla\psi_o, B_z)$) :

$$\frac{\partial\psi_\ell}{\partial t} - \vec{B}_o\cdot\nabla\phi_\ell - \vec{B}_\ell\cdot\nabla\phi_\ell = <\vec{B}_s\cdot\nabla\phi_s> + \eta\nabla_\perp^2\psi_\ell \ , \qquad (5)$$

$$\frac{\partial u_\ell}{\partial t} - \vec{B}_o\cdot\nabla j_\ell - \vec{B}_\ell\cdot\nabla j_o + \vec{v}_\ell\cdot\nabla u_\ell - \vec{B}_\ell\cdot\nabla j_\ell \qquad (6)$$

$$= - <\vec{v}_s\cdot\nabla u_s> + <\vec{B}_s\cdot\nabla j_s> + \mu\nabla_\perp^2 u_\ell$$

and the small-scale perturbations

$$\frac{\partial\psi_s}{\partial t} - \vec{B}_o\cdot\nabla\phi_s - \vec{B}_\ell\cdot\nabla\phi_s - \vec{B}_s\cdot\nabla\phi_\ell = \vec{B}_s\cdot\nabla\phi_s + \eta\nabla_\perp^2\psi_s \ , \qquad (7)$$

$$\frac{\partial u_s}{\partial t} - \vec{B}_o\cdot\nabla j_s - \vec{B}_\ell\cdot\nabla j_s - \vec{B}_s\cdot\nabla j_\ell + \vec{v}_\ell\cdot\nabla u_s + \vec{v}_s\cdot\nabla u_\ell \qquad (8)$$

$$= - \vec{v}_s\cdot\nabla u_s + \vec{B}_s\cdot\nabla j_s + \mu\nabla_\perp^2 u_s \ .$$

The average terms $< \vec{B}_s\cdot\nabla\phi_s >$ in eq. (5) and $- <\vec{v}_s\cdot\nabla u_s> + <\vec{B}_s\cdot\nabla j_s>$ in eq. (6) give rise to anomalous resistivity and viscosity, respectively. To evaluate these terms, we consider an expansion of ψ_s, ϕ_s in the amplitudes of the large-scale perturbations ψ_ℓ, ϕ_ℓ

$$\psi_s = \psi_s^{(0)}(x_s) + \psi_s^{(1)}(x_s,x_\ell) + \ldots \qquad (9)$$

and similarly for ϕ_s. Here $\psi_s^{(0)}$, $\psi_s^{(1)}$, $\phi_s^{(0)}$, $\phi_s^{(1)}$ are periodic functions of x_s and $\psi_s^{(1)}/\psi_s^{(0)} \sim B_\ell/B_o$.

Unlike the conventional analysis in MHD turbulence theory, the present treatment does not introduce an overall Fourier decomposition of fields which requires overall statistical homogeneity. Instead, it is only for the small-scale motions that a state of local homogeneous turbulence is assumed, while the large-scale fields $\vec{B}_\ell$, $\vec{v}_\ell$ are deterministic and inhomogeneous. Since the small-scale field $\vec{B}_s$ is in general small compared with the large-scale field $\vec{B}_\ell$, the nonlinear terms on the r.h.s of eqs. (7) and (8) are small. Hence, to lowest order, these equations seem to reduce to

$$\frac{\partial \psi_s}{\partial t} - \vec{B}_o \cdot \nabla \phi_s = 0 \qquad (10)$$

$$\frac{\partial u_s}{\partial t} - \vec{B}_o \cdot \nabla j_s = 0 \qquad (11)$$

which describe free Alfvén waves propagating along $\vec{B}_o$ with equal kinetic and magnetic energies $< v_s^2 > = < B_s^2 >$. This, however, is an unrealistically strong restriction. Though the small-scale modes are Alfven-like with kinetic and magnetic mode energies of the same order, their ratio is usually not exactly unity. The ratio depends on the way in which these modes are excited. In a realistic, monotonically decaying MHD turbulence spectrum the small-scale modes are continuously stirred by interaction with the adjacent part of the spectrum. If the latter has distinctly disparate kinetic and magnetic mode energies, the smaller scales, too, will not reach complete equipartition. Since only interaction with the large-scale modes is explicitly included in the equations for ψ_s and u_s, we may mimic the effect of neighbouring parts of the k_s spectrum by a stirring force f_s, g_s which should be independent of the large-scale fields. Hence, instead of eqs. (10) and (11), the lowest-order equations become

$$\frac{\partial \psi_s^{(o)}}{\partial t} - \vec{B}_o \cdot \nabla \phi_s^{(o)} = f_s \tag{10a}$$

$$\frac{\partial u_s^{(o)}}{\partial t} - \vec{B}_o \cdot \nabla j_s^{(o)} = g_s \ . \tag{11a}$$

It is emphasized that the dynamics of $\psi_s^{(o)}$, $u_s^{(o)}$ will not be used in the following derivation; it only enters implicitly through the correlation times.

II TURBULENT DIFFUSION COEFFICIENTS

Let us now turn to the evaluation of the anomalous diffusion terms in eqs. (5) and (6), considering first $<\vec{B}_s\cdot\nabla\phi_s>$. One has

$$<\vec{B}_s\cdot\nabla\phi_s> = <(\nabla_\ell+\nabla_s)\cdot\vec{B}_s\phi_s> = \nabla_\ell\cdot<\vec{B}_s\phi_s> , \qquad (12)$$

so that in zeroth order this term vanishes, $\nabla_\ell\cdot<\vec{B}_s^{(0)}\phi_s^{(0)}> = 0$. In the next order one obtains

$$<\vec{B}_s\phi_s>^{(1)} = <\vec{B}_s^{(0)}\phi_s^{(1)}> + <\vec{B}_s^{(1)}\phi_s^{(0)}>$$

$$\cong <\vec{B}_s^{(0)}\phi_s^{(1)}> - <\vec{v}_s^{(0)}\psi_s^{(1)}> \quad . \qquad (13)$$

The first-order quantities obey the equations

$$\frac{\partial\psi_s^{(1)}}{\partial t} - \vec{B}_o\cdot\nabla\phi_s^{(1)} = \vec{B}_\ell\cdot\nabla\phi_s^{(0)} - \vec{v}_\ell\cdot\nabla\psi_s^{(0)} \qquad (14)$$

$$\frac{\partial u_s^{(1)}}{\partial t} - \vec{B}_o\cdot\nabla j_s^{(1)} = \vec{B}_\ell\cdot\nabla j_s^{(0)} - \vec{v}_\ell\cdot\nabla u_s^{(0)} \quad . \qquad (15)$$

Since in the computation of $<\vec{B}_s\phi_s>^{(1)}$ only the lowest-order contributions are needed (in contrast to the anomalous viscosity terms in eq. (6) where, as will be seen, cancellations occur), one may approximate $u_s^{(1)} = \nabla^2\phi_s^{(1)} \cong \nabla_s{}^2\phi_s^{(1)}$ and $j_s^{(1)} \cong \nabla_s{}^2\psi_s^{(1)}$ in eq. (15) and extract the Laplacian $\nabla_s{}^2$:

$$\frac{\partial\phi_s^{(1)}}{\partial t} - \vec{B}_o\cdot\nabla\psi_s^{(1)} = \vec{B}_\ell\cdot\nabla\psi_s^{(o)} - \vec{v}_\ell\cdot\nabla\phi_s^{(o)} . \tag{15a}$$

Equations (14) and (15a) are easily solved:

$$\psi_s^{(1)} = \frac{1}{2}\int^t dt'\left[A_+(t') + A_-(t') + B_+(t') - B_-(t')\right] \tag{16}$$

$$\phi_s^{(1)} = \frac{1}{2}\int^t dt'\left[A_+(t') - A_-(t') + B_+(t') + B_-(t')\right] . \tag{17}$$

Here A and B are the right-hand sides of eqs. (14) and (15a) respectively, and $F_\pm(t') = F(\vec{x}_s \pm \vec{c}_A(t-t'),t')$, where in our units $\vec{c}_A = \vec{B}_o$. Substituting expressions (16) and (17) in eq. (13) yields a number of correlation functions, either pure correlations $< \vec{B}_s \int \vec{B}_{s\pm}(t')dt' >$ and $< \vec{v}_s \int \vec{v}_{s\pm}(t')\, dt' >$ or cross correlations such as $< \vec{v}_s \int \vec{B}_{s\pm}(t')\, dt' >$. Let us first evaluate $< \vec{B}_s\phi_s >_p$, the contribution from the pure correlations. Using the relation $\vec{B}_\ell\cdot\nabla\psi_s = \hat{z}\times\nabla\psi_\ell\cdot\nabla\psi_s = -\vec{B}_s\cdot\nabla\psi_\ell$, we obtain

$$\begin{aligned} < \vec{B}_s\phi_s >_p = &-\frac{1}{2}\Big[< \vec{B}_s \int^t dt'(\vec{B}_{s+}(t') + \vec{B}_{s-}(t')) > \\ &- < \vec{v}_s \int^t dt'(\vec{v}_{s+}(t') + \vec{v}_{s-}(t')) >\Big]\cdot\nabla\psi_\ell \\ &+ \frac{1}{2}\Big[< \vec{B}_s \int^t dt'(\vec{B}_{s+}(t') - \vec{B}_{s-}(t')) > \\ &- < \vec{v}_s \int^t dt'(\vec{v}_{s+}(t') - \vec{v}_{s-}(t')) >\Big]\cdot\nabla\phi_\ell , \end{aligned} \tag{18}$$

omitting the superscripts of $\vec{B}_s$ and $\vec{v}_s$. For isotropic turbulence both propagation directions are equally probable:

$$< \vec{B}_s \int \vec{B}_{s+}(t')dt > = < \vec{B}_s \int \vec{B}_{s-}(t')dt' > = \tau_B < B_s^2/2 > \vec{I},$$

τ_B being the magnetic correlation time; similarly,

$$< \vec{v}_s \int \vec{v}_{s+}(t')dt > = < \vec{v}_s \int \vec{v}_{s-}(t')dt' > = \tau_V < v_s^2/2 > \vec{I} .$$

Hence the terms $\propto \nabla\phi_\ell$ cancel, and eq. (18) becomes

$$< \vec{B}_s \phi_s >_p = (\tau_V < \frac{v_s^2}{2} > - \tau_B < \frac{B_s^2}{2} >)\nabla\psi_\ell . \qquad (19)$$

The cross correlations $< \vec{B}_s(t) \int \vec{v}_s(t')dt' >$ are now discussed. It is argued that these vanish or are at least small for reasonable assumptions about the small-scale turbulence. It is easy to show that for free Alfvén waves, eqs. (10) and (11), the equal time correlation tensor $< \vec{B}_s(t)\, \vec{v}_s(t) >$ is constant in time, i.e. if initially zero, it will vanish at any time. For this result to hold also in the more general case of driven Alfvén-like waves, eqs.(10a) and (11a), the stirring forces must satisfy certain statistical relations. But even if these are only approximately satisfied, the cross correlation tensor is expected to be small. In the numerical simulations referred to in Ref. 1, the cross correlations for the small scale modes are in fact found to be small, typically $<\vec{B}_s\cdot\vec{v}_s>/\sqrt{(<v_s^2><B_s^2>)} \sim 10^{-2}$. Hence the time integral $\int<\vec{B}_s(t)\vec{v}_s(t')>dt' \sim \tau<\vec{B}_s(t)\vec{v}_s(t)>$ is also small. With cross correlations neglected, the anomalous magnetic diffusion term thus becomes

$$< \vec{B}_s\cdot\nabla\phi_s > = \eta_a \nabla^2\psi_\ell$$

where

$$\eta_a = \frac{\tau}{2} (< v_s^2 > - < B_s^2 >) \tag{20}$$

with $\tau_B \cong \tau_V = \tau$. The anomalous resistivity η_a thus becomes negative if the magnetic energy of the small-scale turbulence exceeds the kinetic energy. This generalizes a result previously obtained for 2D MHD turbulence[3], though derived in a different way. In fact, the three-dimensional structure of the turbulence in our case of a low-β plasma only appears in the integrals along B_o, while the dynamics is essentially two-dimensional.

The evaluation of the anomalous viscosity in eq. (6) is slightly more complicated. First the magnetic contribution $< \vec{B}_s \cdot \nabla j_s > = \nabla_\ell \cdot < \vec{B}_s \nabla^2 \psi_s >$ is treated. Proceeding as before, one finds

$$\begin{aligned} < \vec{B}_s \nabla^2 \psi_s >^{(1)} &= < \vec{B}_s^{(0)} \nabla^2 \psi_s^{(1)} > + < \vec{B}_s^{(1)} \nabla^2 \psi_s^{(0)} > \\ &= < \vec{B}_s^{(0)} (2\nabla_\ell \cdot \nabla_s + \nabla_\ell^2) \psi_s^{(1)} > \end{aligned} \tag{21}$$

since $\nabla^2 \psi^{(1)} = (\nabla_s^2 + 2\nabla_\ell \cdot \nabla_s + \nabla_\ell^2)\, \psi^{(1)}$ and

$< \vec{B}_s^{(0)} \cdot \nabla_s^2 \psi_s^{(1)} > = - < \vec{B}_s^{(1)} \nabla_s^2 \psi_s^{(0)} >$. When $\psi_s^{(1)}$, eq. (16), is substituted in eq. (21) and cross correlations are again neglected, the expression $< \vec{B}_s^{(0)} \nabla_s \psi_s^{(1)} >$ consists of terms $\propto < \vec{B}_s \int dt' \nabla_s \vec{B}_s(t') >$. Since the equal time correlation tensor

$< \vec{B}_s(t)\ \nabla_s \vec{B}_s(t) >$ vanishes, the time integral is small, too. Hence

$$< \vec{B}_s \nabla^2 \psi_s > = < \vec{B}_s^{(0)} \nabla_\ell^{\,2} \psi_s^{(1)} >$$

$$= \frac{1}{2} \int^t dt' \Big[< \vec{B}_s(t)(\vec{B}_{s+}(t') + \vec{B}_{s-}(t')) > \cdot \nabla\nabla^2 \phi_\ell$$

$$- < \vec{B}_s(t)(\vec{B}_{s+}(t') - \vec{B}_{s-}(t')) > \cdot \nabla\nabla^2 \psi_\ell \Big]$$

$$= \tau_B < \frac{B_s^{\,2}}{2} > \nabla\nabla^2 \phi_\ell \ . \qquad (22)$$

To compute the kinetic viscosity contribution in eq. (6), the exact equation (15) for $u_s^{(1)}$ instead of the approximate equation (15a) for $\phi_s^{(1)}$ must be used. As in eq. (21), we have

$$< \vec{v}_s \cdot \nabla\nabla^2 \phi_s > = \nabla_\ell \cdot < \vec{v}_s^{(0)} (2\nabla_\ell \cdot \nabla_s + \nabla_\ell^{\,2})\ \phi_s^{(1)} > \qquad (23)$$

The only non-vanishing contribution could come from the term $\vec{v}_\ell \cdot \nabla u_s^{(0)}$ in eq. (15):

$$\nabla^2 \phi_s^{(1)} = - \frac{1}{2} \nabla_s^{\,2} \int^t dt' \vec{v}_\ell \cdot \nabla \left[\phi_{s+}(t') + \phi_{s-}(t') \right] + \ldots \qquad (24)$$

Since $\nabla^2 \phi_s^{(1)} \cong (\nabla_s^{\,2} + 2\nabla_\ell \cdot \nabla_s) \phi_s^{(1)}$, it follows that

$$\phi_s^{(1)} \cong - \left(1 - \frac{2\nabla_\ell \cdot \nabla_s}{\nabla_s^{\,2}}\right) \vec{v}_\ell \cdot \nabla \int^t \phi_s \, dt' \ , \qquad (25)$$

where $1/\nabla_s^{\,2}$ is the inverse of the Laplacian. Substituting eq. (25) in eq. (23) yields

$$< \vec{v}_s \cdot \nabla \nabla^2 \phi_s > = \nabla_\ell \cdot < \vec{v}_s \left(4 \frac{\nabla_\ell \cdot \nabla_s \nabla_\ell \cdot \nabla_s}{\nabla_s^2} - \nabla_\ell^2 \right) \int^t dt' \vec{v}_s > \cdot \nabla \phi_\ell \qquad (26)$$

If isotropic turbulence is assumed and θ, the angle between $\nabla_s \phi_s$ and $\nabla_\ell \phi_\ell$ is introduced, this expression will vanish since it is proportional to $\int d\theta \sin^2\theta \, (4 \cos^2\theta - 1) = 0$. This can most easily be seen when Fourier transforming eq. (26) with respect to the small-scale coordinates x_s.

Finally, it should be mentioned that the term $\vec{v}_s \cdot \nabla u_\ell$ in eq. (8), which is a factor $O(k_\ell^2/k_s^2)$ smaller than $\vec{v}_\ell \cdot \nabla u_s$ and hence has been neglected in eq. (15), seems to make a contribution of the same order as the terms in eq. (26). Its total contribution, however, vanishes (to the order considered here). Because of the inherent smallness of the corresponding contribution to $\phi_s^{(1)}$, $\phi_{s\alpha}^{(1)}$, one has

$$< \vec{v}_{s\alpha}^{(1)} \nabla^2 \phi_s^{(0)} > + < \vec{v}_s^{(0)} \nabla^2 \phi_{s\alpha}^{(1)} > \cong$$

$$< \vec{v}_{s\alpha}^{(1)} \nabla_s^2 \phi_s^{(0)} > + < \vec{v}_s^{(0)} \nabla_s^2 \phi_{s\alpha}^{(1)} > = 0 \ .$$

The kinetic contribution to the anomalous viscosity thus vanishes and it is found from eq. (22) that

$$\mu_a = \frac{\tau}{2} < B_s^2 > , \qquad (27)$$

which also agrees with Pouquet's results [3)] for 2D turbulence if it is taken into account that because of the shape of the spectrum assumed, Fig. 1, no boundary contributions arise from partial integration of the spectral integrals.

III COMPARISON WITH PREVIOUS STUDIES

The problem of turbulent resistivity due to small-scale magnetic fluctuations has also been treated in a recent publication[4]. Though the spirit and methods of that paper are somewhat different to those involved here, emphasizing resonance phenomena and resonance broadening, the basic features of the derivation of the turbulent resistivity (Section VII in Ref. 4) are similar to our treatment. Surprisingly, the result is different from eq. (20), the magnetic contribution being positive instead of negative. The author has traced back this difference, which is <u>not</u> due to a simple sign error, and identified its origin. Instead of eqs. (1) and (2) for the potentials ψ and ϕ, Reference 4 uses equations for $\vec{B}$ and $\vec{v}$. While the $\vec{B}$ equation is exact,

$$\frac{\partial \vec{B}}{\partial t} = \nabla \cdot (\vec{B}\vec{v}) - \nabla \cdot (\vec{v}\vec{B}) \ , \tag{28}$$

and equivalent to eq. (1), the $\vec{v}$ equation is not,

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = \vec{B} \cdot \nabla \vec{B} \ , \tag{29}$$

the pressure force $- \nabla p^*$ being neglected. It is easy to see that these equations indeed give rise to a positive definite expression for η_a. Averaging eq. (28) over small scales gives (only the diffusion terms being written down)

$$\frac{\partial \vec{B}_\ell}{\partial t} = \nabla_\ell \cdot < \vec{B}_s \vec{v}_s > - \nabla_\ell \cdot < \vec{v}_s \vec{B}_s > \qquad (30)$$

(in Ref. 4 the notation $\vec{B} = < \vec{B} > + \delta\vec{B}$ is used instead of $\vec{B}_\ell + \vec{B}_s$). Since we want to investigate the magnetic contribution, only $< \vec{B}_s^{(o)} \vec{v}_s^{(1)} >$ need be considered. From eq. (29) one obtains

$$\vec{v}_s^{(1)} = \int^t dt' (\vec{B}_\ell \cdot \nabla \vec{B}_s^{(o)} + \vec{B}_s^{(o)} \cdot \nabla \vec{B}_\ell) \quad . \qquad (31)$$

Inserting this into eq. (30), we find that there is only one non-vanishing term, the second term of eq. (31) inserted in the first term of eq. (30):

$$\frac{\partial \vec{B}_\ell}{\partial t} = \nabla_\ell \cdot < \vec{B}_s \int^t dt' \vec{B}_s > \cdot \nabla_\ell \vec{B}_\ell \qquad (32)$$

since $< \vec{B}_s \nabla_s \vec{B}_s > = 0$ and $\nabla_\ell \cdot \vec{B}_\ell = 0$. Hence starting from eq. (29), one indeed obtains a positive resistivity $\eta_a = \tau < B_s^2/2 >$. It should be noted that we did not use the fact that the plasma motion is incompressible. In fact, eq. (29) does not describe incompressible motions, since it is in general inconsistent with $\nabla \cdot \vec{v} = 0$. It is true that ∇p^* is small, but since the dominant first term in eq. (31) does not contribute to η_a, the pressure force nevertheless plays an important role, p^* being determined by Poissons equation $\nabla^2 p^* = \nabla \cdot (\vec{B} \cdot \nabla \vec{B} - \vec{v} \cdot \nabla \vec{v})$. For incompressible motions $v_s^{(1)}$ has to be computed from eq. (2):

$$\frac{d}{dt} \nabla^2 \phi_s^{(1)} \cong \frac{d}{dt} (\nabla_s^2 + 2\nabla_s \cdot \nabla_\ell) \phi_s^{(1)} = \vec{B}_\ell \cdot \nabla_s \nabla_s^2 \psi_s^{(o)}$$

Hence we have

$$\phi_s^{(1)} \cong \int^t dt' \left[\vec{B}_\ell \cdot \nabla_s \psi_s^{(0)} - \frac{2\nabla_\ell \cdot \nabla_s}{\nabla_s^2} \vec{B}_\ell \cdot \nabla \psi_s^{(0)} \right]$$

and

$$\vec{v}_s^{(1)} = \hat{z} \times (\nabla_s + \nabla_\ell)\, \phi_s^{(1)}$$

$$= \int^t dt' \left[\vec{B}_\ell \cdot \nabla \vec{B}_s - \vec{B}_s \cdot \nabla \vec{B}_\ell - \frac{2\nabla_\ell \cdot \nabla_s}{\nabla_s^2} \vec{B}_\ell \cdot \nabla_s \vec{B}_s \right] \qquad (33)$$

since $\hat{z} \times \nabla_\ell (\vec{B}_\ell \cdot \nabla \psi_s) = - \hat{z} \times \nabla_\ell (\vec{B}_s \cdot \nabla \psi_\ell) = - \vec{B}_s \cdot \nabla \vec{B}_\ell$. Inserting eq. (33) into eq. (30), we find that only the second term in eq. (33) gives a non-vanishing contribution, while the last term cancels when inserted into both terms of eq. (30). Hence we get

$$\frac{\partial \vec{B}_\ell}{\partial t} = - \nabla_\ell \cdot \langle \vec{B}_s \int^t dt' \vec{B}_s \rangle \cdot \nabla_\ell \vec{B}_\ell$$

and we again find a negative magnetic contribution to the anomalous resistivity $\eta_a = - \tau < B_s^2/2 >$ in agreement with our result in Sec. II.

I do not want to discuss whether the (compressible) equations (28), (29), considered as exact model equations, have any practical significance. However, I should like to emphasize that in the numerical simulations of the major disruption, which have been discussed in Ref. 1 and interpreted in terms of a negative resistivity, the exact incompressible equations (1), (2) are being used.

IV CONCLUSIONS

In conclusion, a few comments are made on the main assumptions entering the derivation of the diffusion coefficients (20) and (27). The basic assumption is that of two well-separated spatial scales x_s, x_ℓ, which seems to require a spectrum as indicated in Fig. 1. Since in practice MHD turbulence spectra usually decrease monotonically, our model system only describes part of the dynamics of the turbulence where the interaction with the intermediate scales is switched off. However, if the character of the modes changes rather abruptly in k-space, i.e. if there are two different types of modes, the requirements imposed on the separation of their spatial scales seem to be less strict. Such behaviour has been observed in numerical simulations of MHD turbulence generated during a major disruption in a tokamak-like plasma[5]. While modes with $m \leqslant 5$ are essentially tearing modes with $B_m^{\ 2} >> V_m^{\ 2}$, they become Alfvén-like for $m > 6$ with $B_m^{\ 2} \gtrsim V_m^{\ 2}$. Comparison of a model computation of the behaviour of the $(m,n) = (2,1)$ mode under the influence of η_a and μ_a with an exact numerical simulation shows good agreement[1], which indicates that at least in the major disruption the effect of the (negative) turbulent resistivity is the dominant process. It should also be stressed once more that the zero-order small-scale modes $\psi_s^{(o)}$, $\phi_s^{(o)}$ need not be free Alfvén waves since these modes are continuously re-excited owing to interaction with modes of the adjacent part of the spectrum in a cascade process. This is accounted for by a stirring force in eqs. (10a) and (11a), which because of the quasilocality of the cascade process is essentially independent of the long-wavelength part of the spectrum. The dynamics of $\psi_s^{(o)}$, $\phi_s^{(o)}$ only enter the diffusion coefficients through the correlation times. In addition, eqs. (20) and (27) may be considered to be "renormalized" in the sense that $< B_s^2 >$, τ_B, etc. are not computed with the dominant, somewhat fictitious zero-order quantities, but with the full actual small-scale fields $\vec{B}_s$, $\vec{v}_s$.

The concept of negative anomalous resistivity is, in the author's opinion, likely to play a role in various types of explosive magnetic phenomena other than disruptions in tokamaks, notably in solar flares.

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Figure Caption

Fig. 1 Illustration of the magnetic energy spectrum with clear separation of spatial scales k_ℓ, k_s.

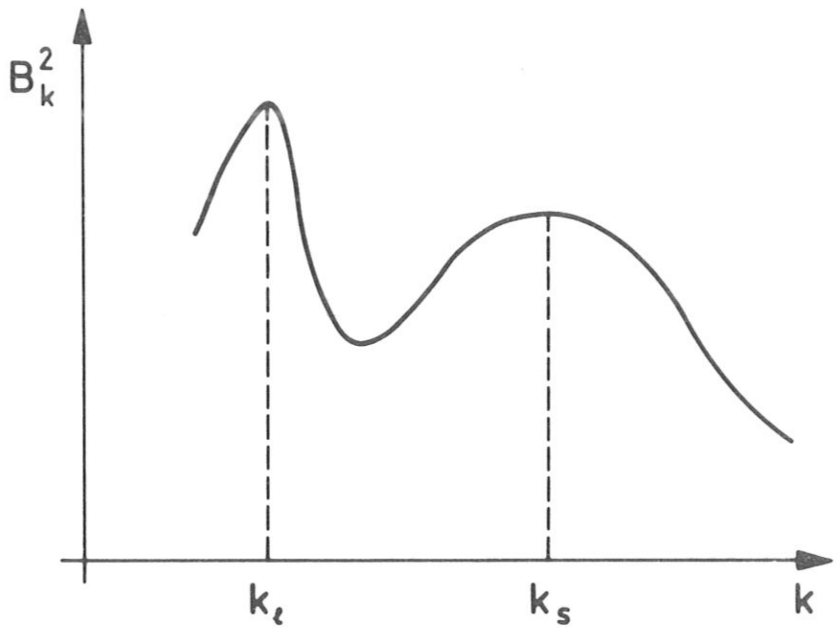

Fig. 1