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NEGATIVE ANOMALOUS RESISTIVITY - A MECHANISM OF THE MAJOR DISRUPTION IN TOKAMAKS

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Abstract:

A mechanism is given to explain the explosive phase of the major disruption in tokamak-like plasmas. It is based on the phenomenum, that small-scale magnetic turbulence acts on large scale magnetic fields as a negative magnetic diffusivity D. By means of a model equation it is found that negative D causes very rapid growth of tearing modes with a pronounced threshold behavior. Comparison with exact numerical simulations shows remarkably good agreement.

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

Numerical simulations of major disruptions in tokamaks[1) 2)] reveal the presence of intense small-scale MHD turbulence during the onset of the explosive phase of the disruption characterized by the rapid growth of low m-number modes, notably the (m,n) = (2,1) mode, stochastization of the central part of the plasma and rapid widening of the current profile. It has been argued[1)] that these small-scale oscillations cause anomalous resistivity which accelerates tearing mode growth and may thus explain the fast time scale of the disruption. In this letter we outline a somewhat different mechanism giving rise to sharp onset and truely explosive growth.

The starting point is a recent result in the theory of two-dimensional MHD turbulence[3)]. It has been shown that the effect of small-scale MHD turbulence on large-scale magnetic fields can be written in terms of a magnetic diffusivity which is negative if the magnetic energy of the turbulence exceeds the kinetic energy. We first show that this result is also valid for three-dimensional MHD processes in a low-β plasma. We then investigate the effect of a negative resistivity on tearing modes at small and finite amplitudes, using a model equation, and finally compare the results with numerical simulations of major disruptions.

We restrict ourselves to the low-β reduced MHD equations[4)] for the vector potential ψ of the poloidal magnetic field, $\vec{B} = (\hat{z} \times \nabla\psi,\ B_z)$ with $B_z \gg |\nabla\psi|$, and the stream function ϕ of the perpendicular flow, $v = (\hat{z} \times \nabla\phi,\ 0)$:

$$\frac{\partial\psi}{\partial t} - \vec{B}\cdot\nabla\phi = \eta\,\nabla^2\psi \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\nabla^2\phi = \vec{B}\cdot\nabla\nabla^2\psi + \mu\nabla^2\nabla^2\phi \tag{2}$$

We consider a cylindrical plasma column and divide ψ and ϕ into an average part, a large-scale pertubation corresponding to low-m-number tearing modes and a small-scale part corresponding to high-m Alfven modes

$$\psi = \psi_o + \psi_\ell + \psi_s \tag{3}$$

$$\phi = \phi_\ell + \phi_s \tag{4}$$

From (1), (2) we obtain the equations for tearing modes (linear in the amplitudes ψ_ℓ, ϕ_ℓ)

$$\frac{\partial\psi_\ell}{\partial t} - \vec{B}_o\cdot\nabla\phi_\ell = \langle\vec{B}_s\cdot\nabla\phi_s\rangle_\ell + \eta\,\nabla^2\psi_\ell \tag{5}$$

$$\frac{\partial\nabla^2\phi_\ell}{\partial t} - \vec{B}_o\cdot\nabla j_\ell - \vec{B}_\ell\cdot\nabla j_o = \langle\vec{B}_s\cdot\nabla j_s\rangle_\ell + \mu\,\nabla^2\nabla^2\phi_\ell \tag{6}$$

where $j_\ell = \nabla^2\psi_\ell$ and the brackets $\langle\ \rangle$ mean averaging over small scales. The term $\langle\vec{v}_s\cdot\nabla\nabla^2\phi_s\rangle$ from eq. (2) is found to be negligible and is hence omitted. In the presence of an average magnetic field small-scale MHD perturbations are essentially Alfven waves. From eqs. (1), (2) we find the approximate equations

$$\frac{\partial\psi_s}{\partial t} = (\vec{B}_o + \vec{B}_\ell)\cdot\nabla\phi_s + \vec{B}_s\cdot\nabla\phi_\ell \tag{7}$$

$$\frac{\partial\phi_s}{\partial t} = (\vec{B}_o + \vec{B}_\ell)\cdot\nabla\psi_s \tag{8}$$

The term $<\vec{B}_s \cdot \nabla \phi_s>_\ell$ in eq. (5) gives rise to the anomalous resistivity and is therefore the most important small-scale contribution. To lowest order this term is linear in the tearing mode amplitude. It is evaluated iteratively by inserting the solution of eqs. (7), (8) into either factor of $<\vec{B}_s \cdot \nabla \phi_s>$

$$<\vec{B}_s \cdot \nabla \phi_s>_\ell = \nabla_\ell \cdot <\vec{B}_s \phi_s>$$

$$= \nabla_\ell \cdot (<\vec{B}_s^{(1)} \phi_s^{(2)}> - <\vec{v}_s^{(1)} \psi_s^{(2)}>) \tag{9}$$

where

$$\psi_s^{(2)} = \int^t (\vec{B}_\ell \cdot \nabla \phi_s + \nabla \phi_\ell \cdot \vec{B}_s) \, dt'$$

$$\simeq \vec{B}_\ell \cdot \int^t \nabla \phi_s \, dt' + \nabla \phi_\ell \cdot \int^t \vec{B}_s \, dt' \tag{10}$$

$$\phi_s^{(2)} = \int^t \vec{B}_\ell \cdot \nabla \psi_s \, dt' \simeq \vec{B}_\ell \cdot \int^t \nabla \psi_s \, dt' \tag{11}$$

The time integral is taken along the (zeroth order) characteristic of eqs. (7), (8). Assuming the absence of cross-correlations, $<\psi_s \phi_s> = 0$, as usually done in MHD turbulence theory, only the first term in $\psi_s^{(2)}$ in eq. (10) contributes in eq. (9). Thus we find

$$<\vec{B}_s \cdot \nabla \phi_s>_\ell = \nabla_\ell \cdot (<\vec{v}_s \int^t \vec{v}_s \, dt'> - <\vec{B}_s \int^t \vec{B}_s \, dt'>) \cdot \nabla \psi_\ell \tag{12}$$

Assuming an isotropic (in r,θ) Alfven spectrum and $\tau_V \simeq \tau_B = \tau$, where the Lagrangian correlation times τ_V and τ_B are defined by

$$\tau_V = \int_{-\infty}^{t} < \vec{v}_s(t) \cdot \vec{v}_s(t') > dt' \Big/ < v_s^2 >$$

$$\tau_B = \int_{-\infty}^{t} < \vec{B}_s(t) \cdot \vec{B}_s(t') > dt' \Big/ < B_s^2 >$$

the r.h.s. of eq. (5) takes the form of a diffusion term $(D + \eta)\,\nabla^2 \psi_\ell$ with

$$D = \frac{\tau}{2} \left(< v_s^2 > - < B_s^2 > \right) \tag{13}$$

The anomalous magnetic diffusivity is thus negative if the magnetic energy density of the small-scale turbulence exceeds the kinetic one. This generalizes Pouquet's result[3) and is in contrast to a recently published study[5) where the magnetic contribution is erroneously concluded to yield a positive resistivity.

The anomalous viscosity contained in $< \vec{B}_s \cdot \nabla j_s >$ in eq. (6) is evaluated in a similar way. Here only the second term in eq. (10) contributes giving rise to a positive coefficient

$$\mu_a = \frac{\tau}{2} < B_s^2 > \tag{14}$$

which agrees with Pouquet's result, as can be seen after partial integration of the corresponding expressions in Ref. 3, assuming that large and small scales are well separated.

Let us now proceed to investigate the effect of a negative resistivity on tearing modes. We start by observing that in a diffusion equation a negative diffusion coefficient leads to exponential growth with the smallest possible spatial scales dominating, which can be seen directly after Fourier transformation

$$\frac{\partial \psi_k}{\partial t} = k^2 |D| \; \psi_k \tag{15}$$

In the context of negative resistivity due to small-scale magnetic turbulence eq. (15) applies only for $k < k_s$, where k_s is the average turbulent wave number. To account for this effect, we replace D by D(k) in eq. (15) with $D \to 0$ for $k > k_s$, choosing the ansatz $D(k) = D \exp\{-k^2/k_s^2\}$. Transformation of the diffusion term to configuration space and insertion into eq. (5) yields

$$\frac{\partial \psi_\ell}{\partial t} - \vec{B}_o \cdot \nabla \phi_\ell = \eta \nabla^2 \psi_\ell + D\, f(r) \frac{k_s}{\sqrt{\pi}} \int_o^1 dr' \exp\{-k_s^2 (r-r')^2\} \; \nabla^2 \psi_\ell \tag{16}$$

Here D is given by eq. (13) and f(r) is a smooth shape function with $f(r_s) = 1$, r_s = resonant radius of tearing mode considered. Treating the anomalous viscosity in a similar way, the equation of motion (6) becomes

$$\frac{\partial \nabla^2 \phi_\ell}{\partial t} - \vec{B}_o \cdot \nabla j_\ell - \vec{B}_\ell \cdot \nabla j_o = \mu \nabla^2 \nabla^2 \phi_\ell \tag{17}$$

$$+ \mu_a \, f(r) \frac{k_s}{\sqrt{\pi}} \int_o^1 dr' \exp\{-k_s^2 (r-r')^2\} \; \nabla^2 \nabla^2 \phi_\ell$$

with μ_a given by (14). It should be mentioned, that our results will be rather insensitive to the form of the viscosity term, so that the r.h.s. of eq. (17) could as well be written in terms of a constant viscosity, $\mu_{eff}\nabla^2\nabla^2\phi_\ell$.

We consider the stability of the (2,1) tearing mode and choose a tokamak configuration characterized by the following safety factor profile (similar as in Ref. 2) :

$$q(r) = 1.15 \left(1 + \left(\frac{r}{0.5}\right)^8 \right)^{1/4}$$

The magnetic field is normalized such that $B_\theta(a) = 1$, the plasma radius a is taken to be unity, and as usual the diffusion coefficients η,.. are expressed in terms of $a\,v_{A\theta}(a)$. Equations (16) and (17) are solved numerically by advancing ψ_ℓ , ϕ_ℓ in time, which yields the most unstable (least stable) mode. Figure 1 gives as an example growth rates γ_{21} as a function of k_s for $D = -5 \times 10^{-4}$ and both $\eta = 5 \times 10^{-5}$ and 10^{-4}. The prominent feature is the threshold behavior at a critical value k_c. While for $k_s < k_c$ the negative resistivity has mainly a stabilizing effect, it is strongly destabilizing for $k_s > k_c$. The stability properties are rather complex in the neighborhood of the critical point, $0.9 \lesssim k_s/k_c \lesssim 1$, where in particular a real part of the frequency appears. Varying D, we find $k_c^{-1} \propto |D|^{2/5}$ which is thus related to the resistive layer width δ, $k_c\delta \simeq 1$, i.e. the region where the (positive or negative) resistivity is important. If k_s^{-1} becomes smaller than δ, the negative diffusion process indicated in (15) is activated, which drives the tearing mode strongly unstable. For $k_s \simeq k_c$ the global mode structure resembles that of the ordinary tearing instability, while for $k > 1.1\,k_c$ the mode becomes localized around r_s and γ approaches the value $k_s^2\,|D|$ predicted by (15).

Figure 1 shows that for a larger value of η the mode is somewhat more unstable or less stable for subcritical k_s, and that the value of k_c is somewhat increased. This tendency is quite plausible, since the positive resistivity contribution partially cancels the dominant negative one. It is also interesting to study the influence of negative D on stable tearing modes (with $\Delta' < 0$ in the usual notation), such as the (4,2) mode. In this case, too, explosive growth occurs for $k_s > k_c$, where k_c increases with mode number m. But in contrast to the (2,1) mode (with $\Delta' > 0$) the (4,2) mode is already slightly destabilized for $k_s < k_c$.

The abrupt increase of γ to large values owing to negative D should be contrasted with the corresponding positive D case. Here γ is a smoothly growing function of k_s approaching the relatively small asymptotic value $\propto D^{3/5}$ for $k_s \to \infty$.

Since the explosive phase of the major disruption sets in at some finite amplitude of the (2,1) mode, it is important to extend the preceding results to nonlinear tearing modes. In this letter we only treat the simplest nonlinear model, the quasi-linear approximation, which, however, already incorporates the essential features of the nonlinear evolution. Hence we still neglect nonlinear terms in eqs. (16) and (17), but allow for the change of the average current distribution due to finite (2,1) amplitude. The quasi-linear equation also contains the anomalous (negative) resistivity term $D\nabla^2\psi_o$, but this is usually small compared with the quasi-linear diffusion term resulting from the low-m modes. We numerically follow the slow nonlinear growth due to a classical resistivity η and switch on the anomalous negative resistivity D, when the amplitude has reached a certain value. As in the linear case we find that the mode will either decay or grow rapidly (after a short decay phase), where the critical value k_c is smaller than in the linear case and is essentially related to the island width.

We shall now show that the mechanism just outlined may exlain the main features of the major disruption as observed in full-scale numerical simulations[1),2)]. In Ref. 2 it is found that during the period immediately preceding the hard phase of the disruption a steadily growing level of small-scale turbulence is generated, giving rise to growing values of k_s and D. The magnetic energy of the turbulence is consistently larger than the kinetic energy, hence D should be negative. It should also be noted that if the formation of the small-scale turbulence spectrum is affected by choosing too few modes in the simulation, the phase of rapid growth is suppressed. Since the energy spectra are monotonically falling off with increasing mode number, the low- and high-m parts are not strictly separated. It has, however, been pointed out in Ref. 2 that the character of the modes changes quite distinctly at $m \simeq 5\text{-}6$, being tearing-mode-like for smaller and Alfven-mode-like for larger m. Hence we define our high-m spectrum by $m > 6$. In a typical simulation run we have found by summing over these modes that $(\langle v_s^2\rangle - \langle B_s^2\rangle)/2 \simeq 5 \times 10^{-4}$ in the region of the $m = 2$ island at the time of onset of rapid growth. The Lagrangian correlation time is $\tau \simeq 0.5 - 1$ and hence $|D| \simeq 3\text{-}5 \times 10^{-4}$.

The anomalous viscosity is roughly $\mu_a \simeq 1.5\,|D|$, while the classical resistivity in this region is $\eta \simeq 5 \times 10^{-5}$. A measure of k_s can be obtained from the radial correlation function, $k_s \simeq 30$. Inserting these numbers, $D = -\,4 \times 10^{-4}$, $\mu_a = 6 \times 10^{-4}$, $k_s = 30$, we have solved the quasi-linear equations outlined above, smoothly switching on the anomalous resistivity and viscosity at the time when the $m = 2$ amplitude has reached about the onset value observed in the simulation run (at $t = 85$ in Fig. 2). The agreement is surprisingly good, which may be somewhat accidental in view of the qualitative character of the theory. We should like to point out, however, that the conditions are quite close to marginal stability. Even a slight decrease of k_s or $|D|$ will quench the growth, while a further increase would lead to an excessively rapid growth.

In summary, we have shown that the effect of a negative resistivity generated by small-scale magnetic turbulence may explain the main features of the explosive phase of the major disruption. The agreement between the simple quasi-linear model and the full-scale numerical simulation is remarkably good. Nevertheless we should like to emphasize that our model only deals with the interaction between low-m and high-m modes. The real dynamics of the disruption is certainly more complex, the interaction between the different low-m modes playing an important role.

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References

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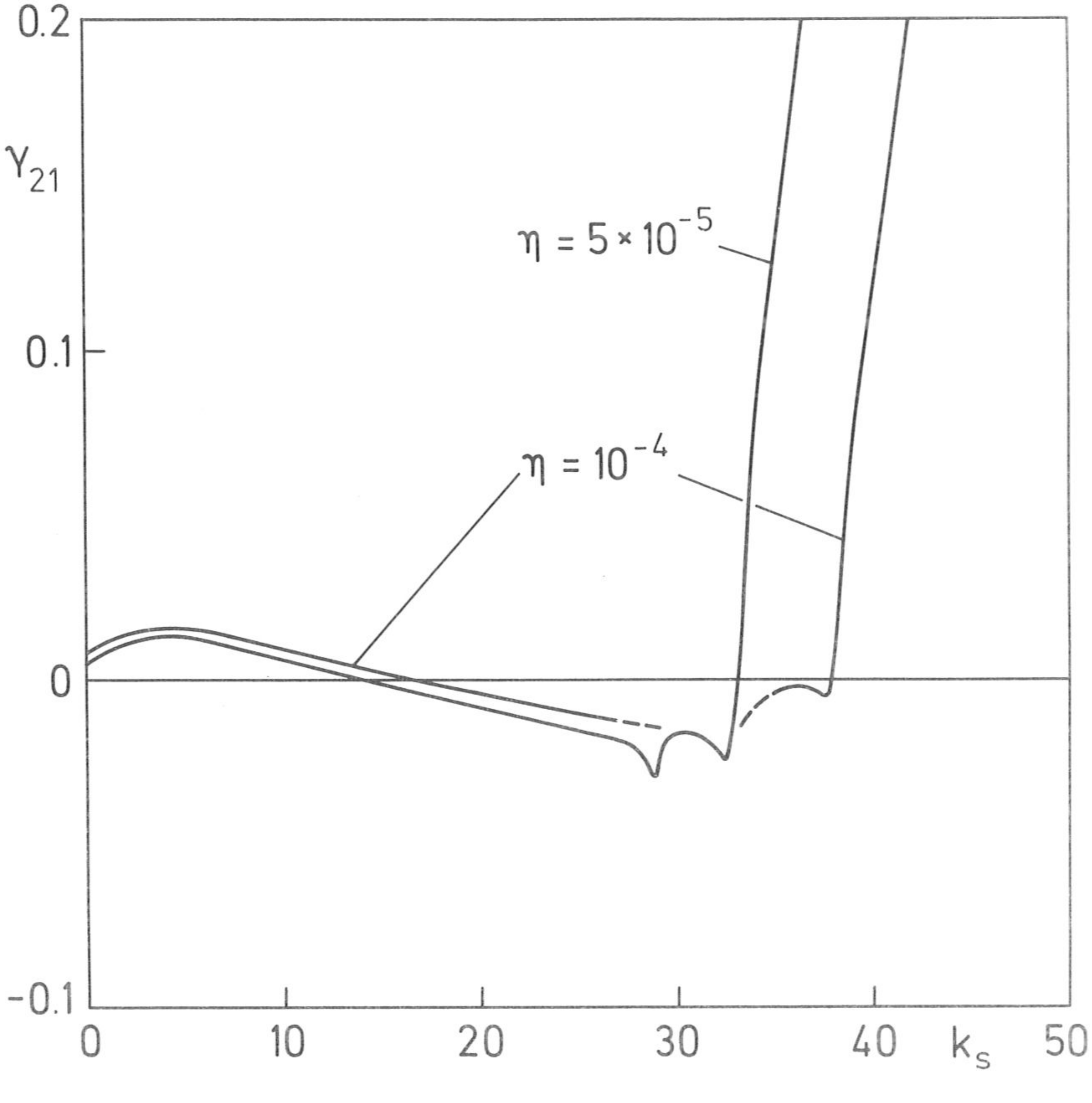
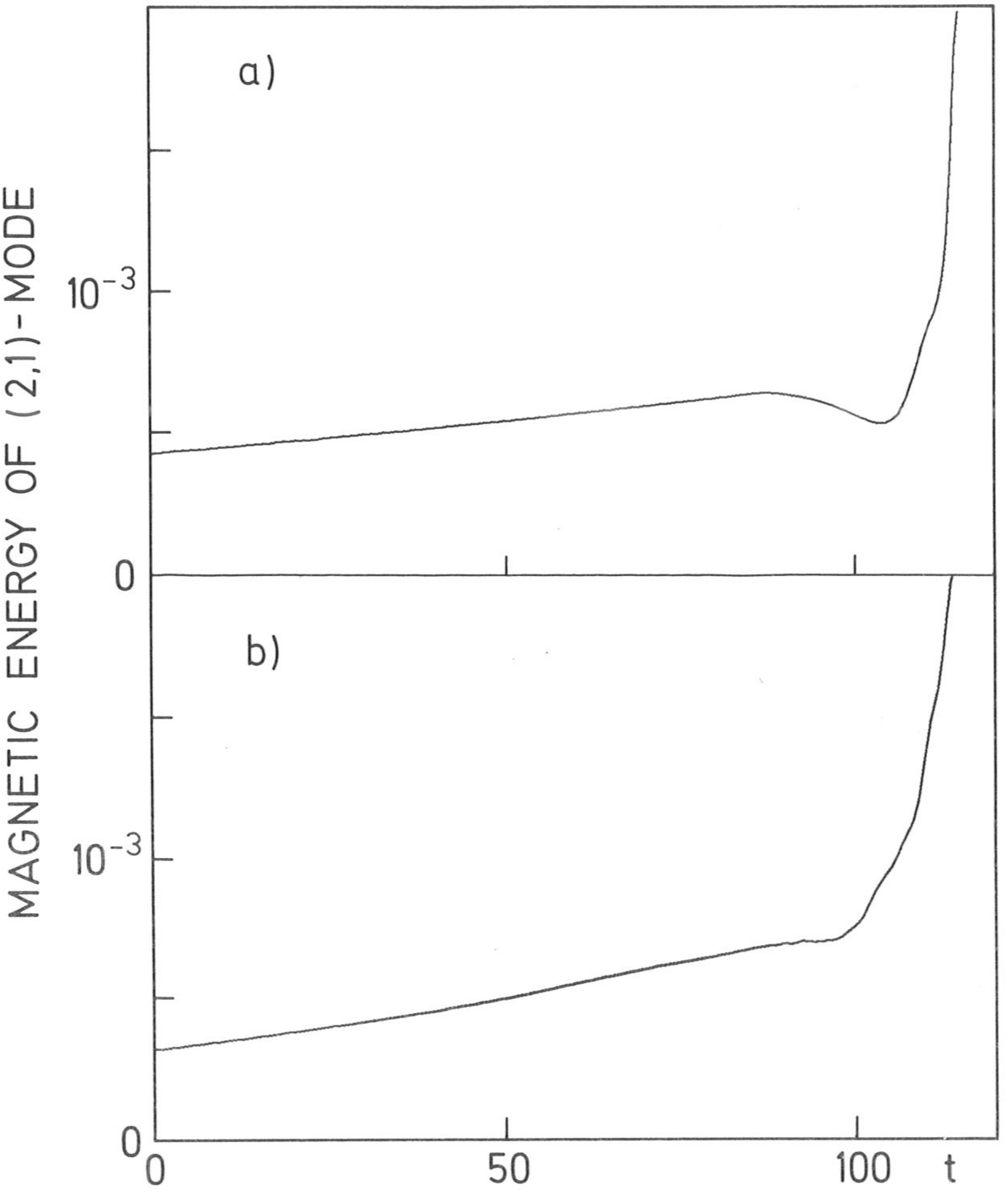
Figure Captions

Fig. 1 Linear growth rates γ_{21} due to negative anomalous resistivity

Fig. 2 Time evolution of the magnetic energy of the (2,1) mode

a) quasi-linear model; b) exact numerical simulation


0.2
0.1
0
-0.1
γ_{21}
$\eta = 5 \times 10^{-5}$
$\eta = 10^{-4}$
0
10
20
30
40
k_s
50



a)
b)
MAGNETIC ENERGY OF (2,1)-MODE
10^-3
10^-3
0
0
0
50
100
t