MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

WHAT IS WRONG WITH PETSCHEK'S MODEL OF MAGNETIC RECONNECTION ?

D. Biskamp

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Abstract:

It is shown that Petschek - type models of fast magnetic reconnection are not valid in the limit of small resistivity owing to the properties of the diffusion region. Since the structure of this region does not permit a high outflow speed, the reconnection rate is strongly dependent on the resistivity and, in general, quite small.

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

One of the most fundamental concepts in the theory of fast magnetic reconnection is the model introduced by Petschek[1]. (A detailed review of Petschek's theory and its later refinements is given in Ref. 2.) Since its appearance this model has been generally accepted and used to explain various types of explosive magnetic events, i.e. processes where large amounts of magnetic energy are released in a short period, notably in solar flares. Recently, however, increasing evidence which casts some doubt on the validity of Petschek's theory and related approaches has emerged[3) - 7)]. In this letter I have therefore re-examined the problem of fast magnetic reconnection across an x-type neutral point and considered in detail the structure of the diffusion region. One finds that it is precisely in the assumptions about this small but crucial region around the x-point where Petschek's and similar theories are in error, which leads to a failure of the whole concept.

Petschek's model in its simplest form is a stationary two-dimensional magnetohydrodynamic (MHD) configuration, as shown schematically in Fig. 1. It is characterized by two pairs of slow shocks standing back to back which sharply bend and accelerate the inward plasma flow. Finite resistivity plays an important role only in the small region around the x-point, the so called diffusion region. The ratio of the plasma velocity immediately upstream of the diffusion region to the upstream Alfvén speed $u/v_A = M$ is called the reconnection rate, since it indicates how fast the oppositely oriented magnetic flux tubes transported toward the x-point are reconnected. This local definition of M is useful for our discussion. (Conventionally the reconnection rate is defined by the asymptotic quasi-homogeneous magnetic field and flow velocity; the difference in definition does not, however, lead to largely different values of M.) Since in most applications of interest the resistivity is effectively very small, the most welcome feature of Petschek's and related theories is that they allow large reconnection rates, $M_{max} \lesssim 1$, essentially independent of the resistivity.

Petschek's theory is based on the two-dimensional incompressible MHD equations. In this case the plasma density can be assumed homogeneous without essential loss of generality, $\rho = \rho_o$. Either no magnetic field or a strong uniform one (which guarantees incompressibility) is assumed in the third direction. The dynamics are then described by two scalar functions, a magnetic flux function $\psi(x,y)$, $\vec{B} = \hat{z} \times \nabla\psi$, and a stream function $\phi(x,y)$, $\vec{v} = \hat{z} \times \nabla\phi$. The current density j and vorticity ω are in the z-direction, $j = \nabla^2\psi$, and $\omega = \nabla^2\phi$. Normalizing to the units B_o = typical magnetic field, L = typical macro-length scale and $v_A = B_o/\sqrt{\rho_o}$ the dynamic equations are

$$\frac{\partial\psi}{\partial t} + \vec{v}\cdot\nabla\psi = \eta\, j \tag{1}$$

$$\frac{\partial\omega}{\partial t} + \vec{v}\cdot\nabla\omega = \vec{B}\cdot\nabla j + \mu\nabla^2\omega \tag{2}$$

Here η and μ are now the inverse magnetic and kinetic Reynolds numbers. One usually assumes $\eta \gg \mu$, so that the viscosity term is negligible. The basic question in the theory of magnetic reconnection is, how fast can reconnection be at large magnetic Reynolds numbers, i.e. for $\eta \ll 1$ (no problem arises if η is sufficiently large).

Since eqs. (1), (2) are well suited to numerical treatment on present-day computers, several numerical studies of spontaneous as well as driven reconnection have been performed in recent years (reconnection which occurs in a closed system owing to, for instance, the free magnetic energy of an MHD unstable configuration is called spontaneous in contrast to reconnection in an open system enforced by the boundary condition, which is called driven .) In Refs. 3 and 4 the evolution of the island coalescence instability was investigated, in Ref. 5 results on the m = 1 resistive kink instability are given, while in Refs. 6 and 7 various types of driven reconnection processes have been analysed. In all cases the behavior for sufficiently small resistivity is quite different from a Petschek-type process. Instead, a long quasi one-dimensional diffusion layer is generated, and the reconnection rates are quite small, roughly $M \sim \eta^{1/2}$. Thus the question arises, what is wrong with Petschek's model.

A basic assumption in the traditional reconnection models of the Petschek type is that there exists a hierarchy of spatial scales. Embedded in a larger system whose size and shape depend on the particular global magnetic process under consideration, the Petschek configuration, Fig. 1, consists of a self-similar ideal hydromagnetic configuration, usually called the outer region, and the small region around the x-point, called the diffusion region, where the resistivity is important. A necessary condition for a global solution to exist is that the solutions in the sub-regions can be matched to each other. The main problem is caused by the diffusion region.

No rigorous solution has yet been given. In fact, most articles in the literature concerned with Petschek's theory only treat the outer region, simply ignoring the diffusion region. The few actual treatments of the latter, such as in Ref. 2, are based on several simplifying or even erroneous assumptions. The conventional picture of the diffusion region is that of a small Sweet-Parker[8)] layer connecting smoothly to the outer region. The plasma enters the diffusion layer at a relatively low speed $u = M v_A$, where it is uniformly accelerated up to the Alfven speed v_A, at which it continues to flow throughout the outer field-reversal region, the downstream region between the slow shocks as indicated in Fig. 1. Hence the reconnection rate is essentially determined by the angle formed by the shocks.

The numerical simulations, however, consistently show that the transition from the diffusion layer into the outer field-reversal region is not smooth. Figures 2 and 3 illustrate a typical simulation of driven reconnection. Plasma and magnetic field are injected at a constant rate at the boundaries $x = \pm 1$ and are leaving the system at $y = \pm 1$ (only a quadrant is actually computed in this particular case). After a short transient phase a stationary configuration is generated. Parameters, in particular η, are chosen in such a way, that the diffusion region has a finite length Δ, but is still smaller than the overall system size. The stereographic plot of the current density, Fig. 2, shows the diffusion region and the adjacent slow shock. The structure of the diffusion region is clearly more complex than anticipated. There is a distinct region of negative current density attached to the current layer which connects discontinuously (for $\mu \to 0$) to the current density of the outer field-reversal region. The negative current density strongly affects the plasma flow along the layer. While the plasma is accelerated in the region of positive current density up to the Alfvén speed as in the conventional picture of the diffusion region, it is strongly decelerated by the adjacent region of negative j, where the $\vec{j} \times \vec{B}$ force reverses sign, so that the downstream flow speed in the outer field-reversal region is again small compared with v_A. This behavior is clearly seen in the ϕ-plot in Fig. 3. It may be said that the negative j region acts like a plug to the flow within the diffusion region. Consequently the reconnection rate is small ($M \cong 0.03$ in this case), although the angle between the shocks is quite large.

A qualitative interpretation of the current behavior in the diffusion region is obtained by considering the requirements of Ampère's law for an x-point magnetic configuration. Since the current density at the neutral point strongly exceeds that within the shocks, the latter are unimportant for the magnetic field in the neighborhood of the diffusion layer. A filament of unidirectional current density, even if elongated, would, however, produce an O-point magnetic configuration. Only if the current layer is terminated at both ends by regions of reversed current density, the proper x-point field topology may be generated.

It is possible to construct an analytic solution of (1), (2) in the diffusion region as a power series in y in terms of the zeroth-order current distribution $j_o(x)$, using only the symmetry properties $\psi(-x,y) = \psi(x,-y) = \psi(x,y)$; $\phi(-x,y) = \phi(x,-y) = -\phi(x,y)$

$$\psi = \psi_o(x) + y^2\,\psi_2(x) + \ldots \tag{3}$$

$$\phi = y\,\phi_1(x) + y^3\phi_3(x) + \ldots \tag{4}$$

It is found in the simulations that the current density in the diffusion region has a self-similar shape which fits surprisingly well the expression

$$j_o(x) = \frac{j_m}{\cosh^2 \frac{x}{\delta}} , \tag{5}$$

the well-known solution for a collisionless current sheet[8]. (It might be worthwhile to give some attention to this observation, which indicates that the profile (5) is of more general importance than anticipated.) With this assumption the lowest-order velocity becomes

$$v_x^{(1)} = -\frac{\eta}{\delta}\tanh\frac{x}{\delta} , \tag{6}$$

$$v_y^{(1)} = \frac{\eta y}{\delta^2}\,\frac{1}{\cosh^2\frac{x}{\delta}} . \tag{7}$$

Hence the upstream plasma velocity is $u = \eta/\delta$, the upstream Alfvén velocity $v_A = j_m \delta$, and the reconnection rate $M = \eta / j_m \delta^2$, which we assume to be small $M \ll 1$. The true expansion parameter in (3) and (4) is thus $M\, y/\delta$. The subsequent contributions ψ_2, ϕ_3 are readily computed. An interesting result is that at $x = 0$ ψ_2 and j_2 vanish (for $\mu = 0$), i.e. $B_x = O(y^3)$ along the current layer, which implies that the two branches of the separatrix do not intersect at a finite angle but merely osculate. It is interesting to note that this is not a consequence of the special current profile (5) but is generally true for $\mu = 0$, as can be seen from a power series expansion in x and y around the neutral point using only the symmetry properties of ϕ and ψ[10]. (It should be mentioned that previous calculations of the diffusion layer such as in Ref. 2 erroneously assume B_x to vary linearly with y). From a fourth order calculation for $x = 0$ one obtains $B_x / j_m \delta = \frac{4}{3} M^4 (y/\delta)^3$ and $j/j_m = 1 - \frac{4}{3} M^4 (y/\delta)^4$. Although the low order terms in (3) and (4) already reveal important features of the full solution, the power series expansion probably fails to converge for $M (y/\delta) \geqq 1$, i.e. when y reaches the length of the diffusion layer $\Delta \cong j_m \delta^3 / \eta$, which also corresponds to the point, where v_y reaches the Alfvén speed. Hence the complex discontinuous transition from the diffusion region to the ideal field reversal region cannot be described in this way.

I can therefore discuss only qualitatively what happens in a reconnecting system in the limit of small resistivity. To be definite consider the case of stationary driven reconnection. The important point is the η-scaling of Δ, the length of the diffusion layer. Decreasing η (for a fixed rate of plasma injection , $u_\infty B_\infty = \eta j_m$ = const) one expects reconnection to become more difficult, which leads to a piling up of magnetic flux in front of the layer and hence to an increase of the upstream field. Writing $B_o = j_m \delta \propto \eta^{-\nu}$, $\nu>0$, one has $\delta \propto \eta^{1-\nu}$ and $\Delta = j_m \delta^3/\eta \propto \delta^3/\eta^2 \propto \eta^{1-3\nu}$. Only for $\nu<\frac{1}{3}$ would the size of the diffusion layer shrink with η, as assumed in Petschek's theory. In the simulations, however, $\nu \gtrsim \frac{1}{2}$ and thus Δ increases and finally reaches the overall system size.

The diffusion layer which is a quasi one-dimensional current sheet, is nevertheless relatively stable with respect to tearing modes owing to the inhomogeneous plasma flow along the sheet. Instability sets in only, if the tearing mode growth rate exceeds the convective distortion rate $k\Delta v$, as has been discussed in Ref. 4, which occurs if η is sufficiently small. The nonlinear development of the tearing instability depends on the geometry of the overall configuration. If this is rather symmetric and the current layer has reached the maximum length allowed by the configuration, a single large magnetic island develops. It forms a new quasi stationary state, which strongly reduces the reconnection rate. This has been described in detail in Ref. 4. In the general unsymmetric case, however, the tearing instability has a less dramatic effect. The islands created are swept downstream along and out of the current layer before growing to large amplitudes. Their effect on the average reconnection rate seems to be rather small.

In conclusion, it has been seen that Petschek's theory is not valid because of the hitherto unknown properties of the diffusion region. The analogy, often used, between a reconnecting magnetic configuration of the kind discussed here and two supersonic counterstreaming gas flows is not correct. While in the latter nothing particular happens at the stagnation point, the flow being completely determined by the system of shock waves, which arrange themselves automatically, it is only in the diffusion region in the magnetic system that the reconnection process occurs. If this process is too slow, the message is transmitted upstream by the fast magnetic mode (infinitely fast in the incompressible approximation), increasing the magnetic field and slowing down the flow. Only if the problem of the diffusion region is eliminated by artificially increasing the resistivity in this region (using an "anomalous" resistivity) is the Petschek configuration obtained[11).]

References

1) Petschek, H.E., in AAS/NASA Symp. on the Physics of Solar Flares, edited by W.N. Hess, p. 425, Washington, D.C. (1964)

2) Vasyliunas, V.M., Rev. Geophys. Space Phys. 13, 303 (1975)

3) Biskamp, D. and Welter, H., Phys. Rev. Lett. 44, 1069 (1980)

4) Biskamp, D., Phys. Lett. 87A, 357 (1982)

5) Biskamp, D., in Physics of Plasmas close to Thermonuclear Conditions, Vol. 1, p. 181, Brussels 1980

6) Brushlinskii, K.V., Zaborov, A.M., and Syrovatskii, S.I., Sov. J. Plasma Phys. 6, 165 (1980)

7) Biskamp, D., Physica Scripta, Vol. T2/2, 405, 1982

8) see Parker, E.N, Astrophys. J. Suppl. Ser. No. 77, 8 (1963)

9) Harris, E.G., U.S. Naval Research Laboratory Report NRL 4944 (1957)

10) Priest, E.R. and Cowley,S.W.H.,J. Plasma Phys. 14, 271 (1975)

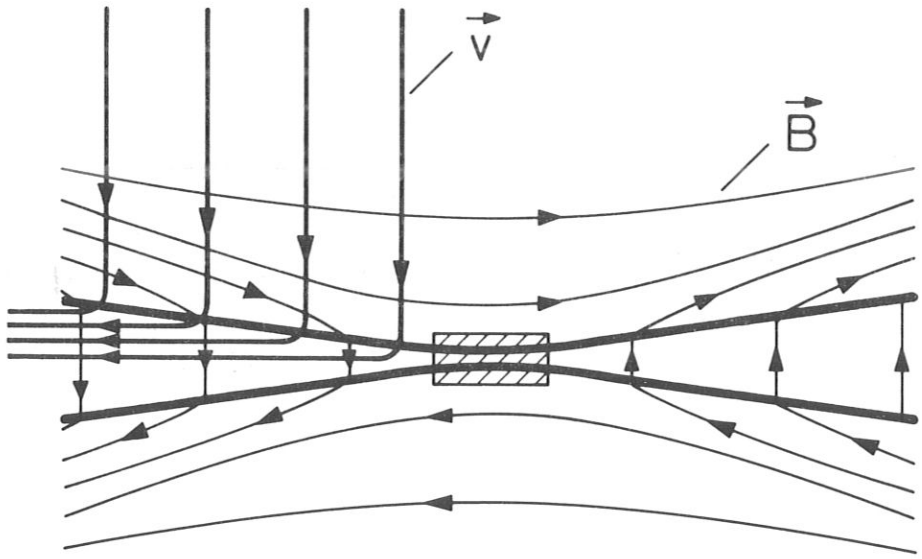
11) Sato, T., J. Geophys. Research 84, 7177 (1979)

Figure Captions

Fig. 1 Schematic drawing of Petschek's configuration

Fig. 2 Current density in a simulation of driven reconnection (only a quadrant is displayed)

Fig. 3 Flow pattern ϕ in the same simulation as in Fig. 2 (only half of the system is shown)



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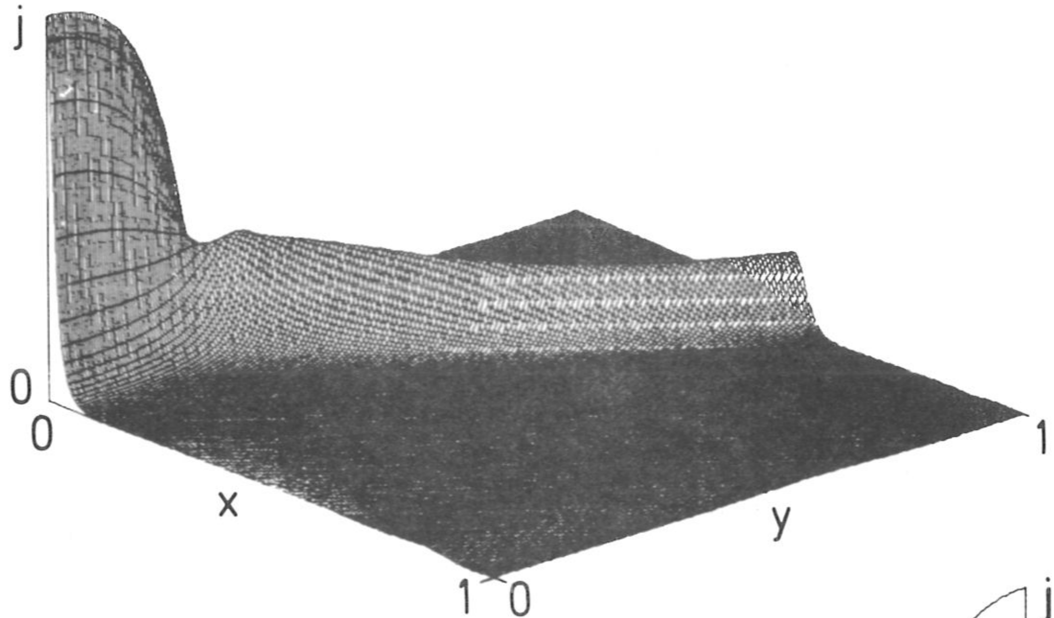
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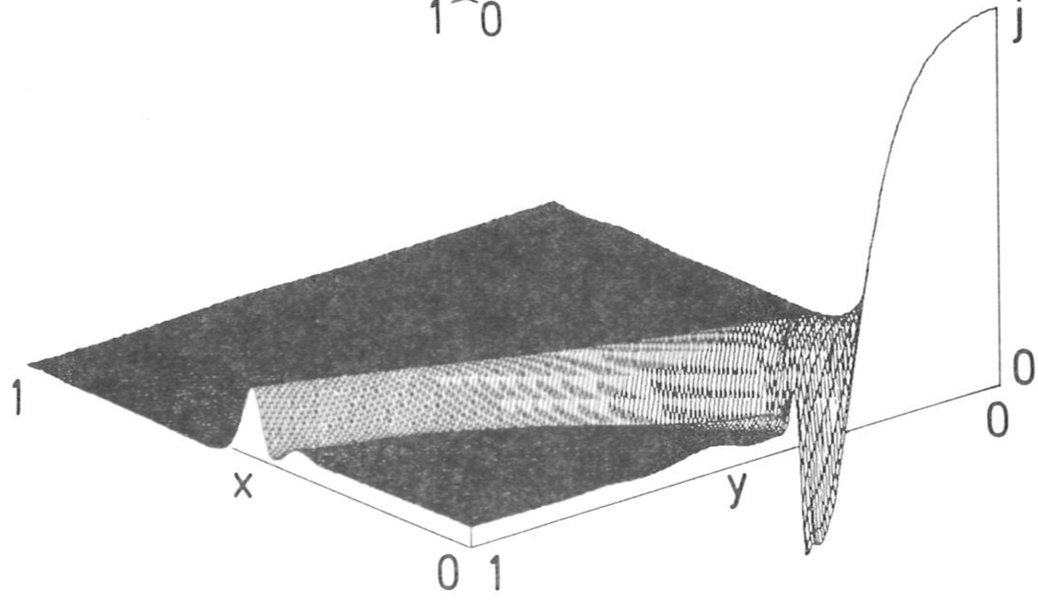
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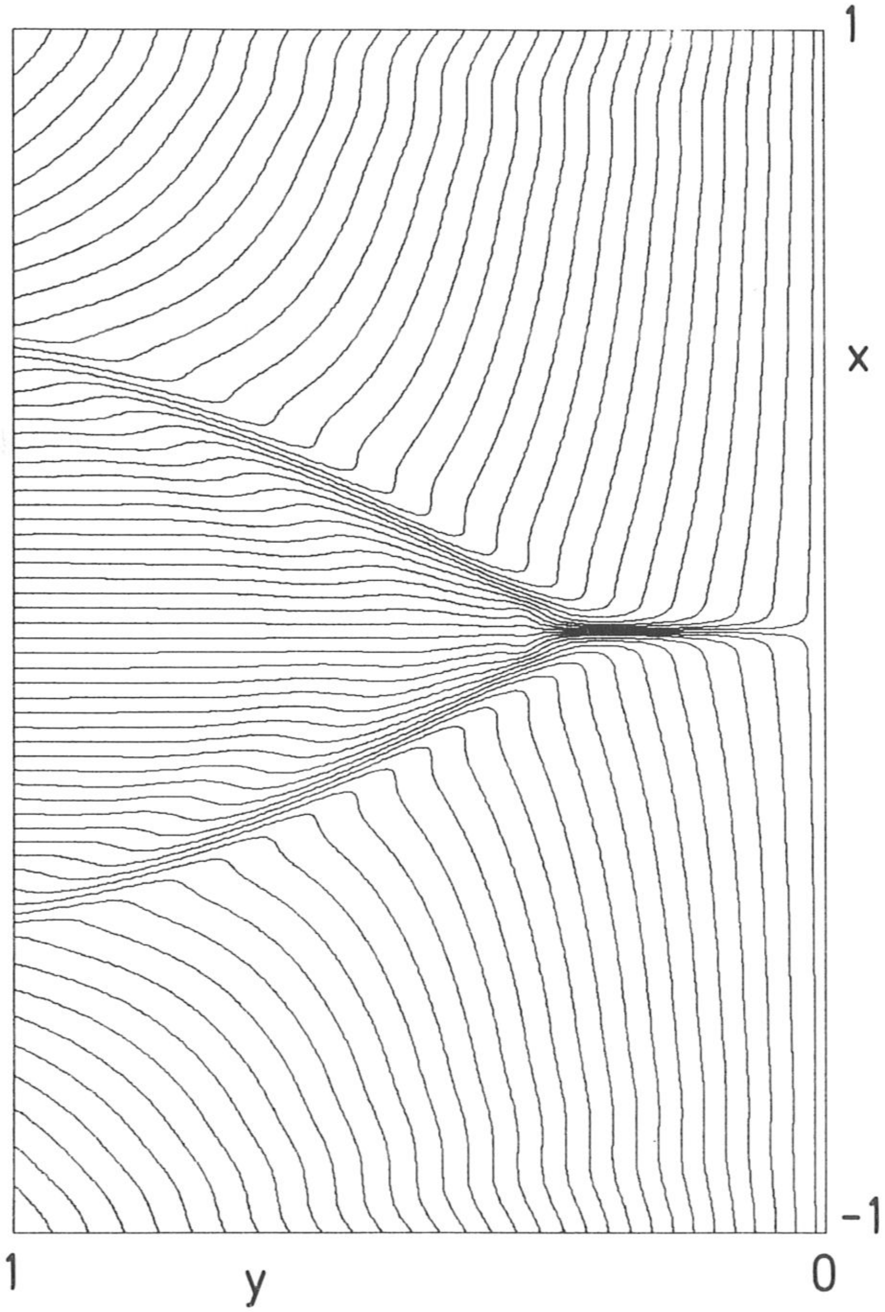

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