Plasma-External Circuit Interaction in the Burn Control with Vertical Field

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IPP 1/208 May 1982

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Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

IPP 1/208 F.Casci, E.Minardi Plasma-External Circuit Interaction in the Burn Control with Vertical Field (in English)

Abstract

The plasma-external circuit interaction is discussed with a simplified model in which the controlling vertical field is created by a single circuit. The interaction is characterized by two dimensionless parameters, a coupling parameter $\xi = D\nu/\gamma L_v$ and the parameter of the linear feedback $\tilde{U} = \bar{U}\nu\tau_o/R\gamma L_v$. Here $\bar{U}$ is the characteristic voltage involved in the feedback process; D depends linearly on the mutual inductance and its R-derivative (R is the major radius); $\nu \equiv B_v/I_v$ (B_v is the vertical field created by the current I_v); L_v is the inductance of the external circuit, γ involves the index of vertical field and τ_o is the confinement time. The circuit parameters ξ and $\tilde{U}$ together with the plasma parameter $A \equiv (p\partial\tilde{B}_v/\partial p)/\ /R\gamma$ (where $\tilde{B}_v$ is the Shavranov equilibrium field) characterize completely the stability of the system and the excursion Δ_∞/R. The excursion is parametrized in terms of ξ, A and the dimensionless feedback voltage $\tilde{U}$.

I. INTRODUCTION

The compression-decompression method for the burn control, recently discussed in the contexts of the Zephyr and INTOR designs [1] is based on the relationship between the plasma pressure and the shift of the plasma column produced by a variation of the external vertical field. In the frame of a zero-dimensional picture the time behaviour of the pressure perturbation $p_1(t)$ is related to the shift $\Delta(t)$ by the equation [2] :

$$\dot{p}_1 - \frac{p_1}{2\tau_o} + \frac{p_m}{2\tau_o}\frac{\Delta}{R_o} + \frac{10}{3}\, p_m \frac{\dot{\Delta}}{R_o} = 0 \qquad (1)$$

Here p_m is the pressure at marginal ignition, τ_o is the confinement time and R_o is the major radius; the subscript 0 refers to the initial time; an Alcator scaling was assumed for τ.

The shift Δ, as can be derived from the linearized Shavranov equilibrium equation, has the following form:

$$\Delta = \frac{1}{\gamma_v}\left[B_{1v} - \frac{\partial \tilde{B}_v}{\partial p_o}(p - p_o) - \frac{\partial \tilde{B}_v}{\partial I_p}(I_p - I_{po})\right] \qquad (2)$$

Here $p(t) = p_1(t) + p_m$, B_{1v} is the change of the vertical field which produces the shift and $\tilde{B}_v$ is given by the following relation:

$$\tilde{B}_v = \frac{\mu_o I_p}{4\pi R}\left(\ln\frac{8R}{a} + \frac{l_i}{2} - \frac{3}{2}\right) + \frac{4\pi p a^2}{I_p R} \qquad (3)$$

where I_p is the plasma current, a is the plasma radius, l_i is the internal inductance and $p = p_i = p_e$ is the pressure of one of the two (ion and electron) species; finally γ_v is defined as follows:

$$\gamma_v \equiv \left.\frac{\partial \tilde{B}_v}{\partial R}\right|_{a^2R=\text{const}} - \frac{\partial B_{ov}}{\partial R} - \frac{\partial B_{1v}}{\partial R} \qquad (4)$$

where $B_{ov}(R)$ is the unperturbed external vertical field; the constraint a^2R = const following from the toroidal flux conservation is taken into account in the R-derivative.

It is the aim of the present note to complement the treatment based on the equations above by adding the circuit equation for the current creating the vertical field. We suppose that this field is created with a single circuit of negligible resistivity, self-inductance L_v and mutual inductance M_{pv}. For the definition of these quatities in terms of the geometry and of the index of the vertical field see [3] and [4]. The point to be noted is that, although M_{pv} can be relatively small, namely such that

$$M^2_{pv} \ll L_p L_v \qquad (5)$$

(where L_p is the plasma inductance) the R derivative of M_{pv} (subject to the constraint a^2R = const) is by no means negligible in the expression (12) below. This implies that the shift of the plasma column is accompanied by a change in the mutual inductance which effects significantly the coupling between the plasma and the external circuit

II. THE CIRCUIT EQUATIONS

The equations for the plasma current I_p and the current I_v in the vertical circuit are the following:

$$\begin{aligned} &\frac{d}{dt}(L_p I_p) + \frac{d}{dt}(M_{pv} I_v) + R_p I_p = U_p \\ &L_v \frac{dI_v}{dt} + \frac{d}{dt}(M_{pv} I_p) + R_v I_v = U \end{aligned} \qquad (6)$$

where the R-dependence of $L_p(R)$ and $M_{pv}(R)$ will be considered up to first order

$$\begin{aligned} &R = R_o + \Delta \\ &L_p(R) = L_p(R_o) + L_p'(R_o)\Delta \\ &M_{pv}(R) = M_{pv}(R_o) + M_{pv}'(R_o)\Delta \end{aligned} \qquad (7)$$

Neglecting resistive effects by assuming $U_p = R_p I_p$, the first equation (6) is immediately integrated to give

$$I_p(t) = \frac{C - M_{pv}(R)I_v(t)}{L_p(R)} \tag{8}$$

where

$$C = L_1(R_o)I_p(0) + M_{pv}(R_o)I_v(0) \tag{9}$$

We now linearize the Eqs (6) taking as zero order $I_{po} \equiv I_p(0)$ and $I_{vo} = U_o/R_v$:

$$\begin{aligned} I_p(t) &= I_{po} + I_{p1}(t) \\ I_v(t) &= I_{vo} + I_{v1}(t) \\ U(t) &= U_o + U_1(t) \end{aligned} \tag{10}$$

Using (8) and (9) one obtains from the second equation (6), neglecting the resistivity (assuming superconductivity), the following equation for $I_{v1}(t)$:

$$\dot{\Delta}D + \dot{I}_{v1}F = U_1 \tag{11}$$

where

$$D \equiv - \left[\frac{I_{po}}{L_p} (M'_{pv}L_p - M_{pv}L'_p) + \frac{M_{pv}M'_{pv}}{L_p} I_{vo} \right]_{R = Ro} \tag{12}$$

$$F \equiv \left(L_v - \frac{M^2_{pv}}{L_p} \right)_{R = Ro} \tag{13}$$

In order to derive a set of equations for $I_{v1}(t)$ and $p_1(t)$ we must express Δ_1 given by (2), in terms of these two variables. We note that from (8) one obtains at first order

$$I_{p1}(t) = H(R_o)I_{v1}(t) + G(R_o)\Delta(t) \tag{14}$$

$$H(R_o) = - \frac{M_{pv}(R_o)}{L_p(R_o)}, \quad G(R_o) = - \frac{1}{L_p(R_o)}(M'_{pv}(R_o)I_{vo} + L'_p(R_o)I_{po}) \tag{15}$$

On the other hand B_{1v} in equation (2) can be related to I_v by the linear relationship

$$B_{1v} = \nu_v I_{v1} \tag{16}$$

where ν_v is a coefficient depending on the geometry (see [4]). From (14), (16) and (2) one then derives the following expression for Δ

$$\Delta = \frac{1}{\gamma}\left[\nu I_{v1} - \frac{\partial \tilde{B}_v}{\partial p_o}(p_1 + p_m - p_o)\right] \tag{17}$$

where

$$\gamma \equiv \gamma_v \left(1 + \frac{1}{\gamma_v} \frac{\partial \tilde{B}_v}{\partial I_p} G\right)$$

$$\nu \equiv \nu_v \left(1 - \frac{1}{\nu_v} \frac{\partial \tilde{B}_v}{\partial I_p} H\right) \tag{18}$$

Finally the combination of (1), (11) and (17) results in the following system of coupled equations:

$$\dot{p}_1\left(1 - \frac{10}{3}A\right) - \frac{p_1}{2\tau_o}(1 + A) + \frac{10}{3}\frac{p_m \nu}{\gamma R_o}\dot{I}_{v1} + \frac{p_m \nu}{2\tau_o R_o \gamma} I_{v1} = \frac{A}{2\tau_o}(p_m - p_o) \tag{19a}$$

$$- \dot{p}_1 \frac{DAR_o}{p_m} + \dot{I}_{v1} F(1 + \xi) = U_1 \tag{19b}$$

where

$$A \equiv \frac{p_m}{R_o \gamma} \frac{\partial \tilde{B}_v}{\partial p_o} \tag{20}$$

$$\xi \equiv \frac{D\nu}{\gamma F} \tag{21}$$

The parameter ξ, which depends on the mutual inductance and its R-derivative (see (12) and (13)), characterizes the effects of the plasma-external circuit interaction.

III. LINEAR FEEDBACK

We now assume that $U_1(t)$ is controlled linearly by $\dot{p}_1$, putting

$$U_1(t) = \bar{U}\, \tau_o \frac{\dot{p}_1}{p_m} \tag{22}$$

where $\bar{U}$ is the characteristic voltage expressing the gain of the feedback system.

It follows from this assumption that $B_1(t)$ is controlled linearly by $p_1(t)$ (compare with equations (16) and (19b)). This scheme is equivalent to the so called "pseudostatic" stabilization discussed earlier by Borrass [5].

The stability of the system of equations (19), under the assumption (22), is easily discussed looking for solutions of the form $\exp\lambda t$. One obtains the following condition for stability:

$$2\tau_o\lambda = \frac{1 + \xi + A - \tilde{U}}{1 + \xi - \frac{10}{3}(A - \tilde{U})} < 0 \tag{23}$$

where

$$\tilde{U} = \frac{\tau_o\, v\, \bar{U}}{R_o \gamma F} \tag{24}$$

The parameter $\tilde{U}$ contains the effect of the feedback. One can distinguish two cases:

1) $\xi > -1$

The system is stable for

$$A - \tilde{U} < -(1 + \xi) < 0 \tag{25}$$

or, alternatively, for

$$A - \tilde{U} > \frac{3}{10}(1 + \xi) > 0 \tag{26}$$

2) $\xi < -1$

The system is stable for

$$A - \tilde{U} > -(1 + \xi) > 0 \tag{27}$$

or, alternatively, for

$$A - \tilde{U} < \frac{3}{10}(1 + \xi) < 0 \tag{28}$$

Stability can always be achieved with a proper choice of the feedback parameter $\tilde{U}$. However this choice depends, more strictly than in the case of negligible M_{pv}, on the geometry of the system and on the index of the vertical field. This also holds for the asymptotic value ($t \to +\infty$) of the excursion, which is given by the expression

$$\frac{\Delta_\infty}{R_o} = \frac{p_{10}}{p_m} \frac{A - \tilde{U}}{1 + \xi + A - \tilde{U}} \tag{29}$$

where P_{10} is the initial deviation from marginal ignition condition. One can easily see that in the parameter domain for stability there is the following limitiation for Δ_∞:

$$\frac{\Delta_\infty}{R_o} > \frac{p_{10}}{p_m} \frac{3}{13} \tag{30}$$

The lower limit corresponds to a strictly adiabatic behaviour [1]. Within this limitation one can play with the parameters A, ξ and $\tilde{U}$ in order to keep the excursion as low as possible. Examples of the dependence of the stability and the excursion on the feedback parameter are given in the figures.

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Appendix

NUMERICAL EXAMPLE

As an example let us apply our results to the geometry of INTOR.

The values of the parameters used in the numerical calculations are the following:

Plasma parameters

R_o = 5.2 m
a = 1.64 m (equivalent minor radius taking into account an elongation of 1.6)
I_p = 6.4 MA
β_p = 2.7
τ_o = 1.5 s

Parameters of the Vertical circuit

R = 6.75 m
r_v = 5.77 m (equivalent distance plasma-coil as defined in /4/)

$$n_v = - \frac{R_o}{B_{ov}} \frac{B_{ov}'}{R_o} = -1.$$

The values of the self- and mutual inductances and their space derivatives are calculated as follows:

L_p = 8.7 10^{-6} henry
L_p' = 2.4 10^{-6} henry/m
L_v = 15.7 10^{-6} henry
M_{pv} = 3.3 10^{-6} henry
M_{pv}' = 2.2 10^{-6} henry/m.

Calculated values of other characteristic parameters

γ = - 0.08 T/m
ν = 0.045 10^{-6} henry/m^2
ξ = 1.06
I_{ov} = 12.2 MA.

Since $\xi > -1$, the stability ranges are those described by the conditions (25) and (26). As shown in Fig. 1 the shift associated with stability corresponds to the following ranges of $\tilde{U}$:

$$\tilde{U} < -1.4242 \quad \text{and} \quad \tilde{U} > 1.2642 \qquad (31)$$

Assuming a value of the feedback parameter according to (31), ($\tilde{U} = -2$), one obtains:

- characteristic feedback voltage: $\bar{U} = 195$ Volt
- average circulating power related to the feedback: 13 MW (assuming 1 % deviation from marginal ignition).

It is worth pointing out that, in view of the flatness of the left branch of Fig. 1, corresponding to the stable region, it is useless to choose a value for the feedback parameter too close to the lower limit given by (31).

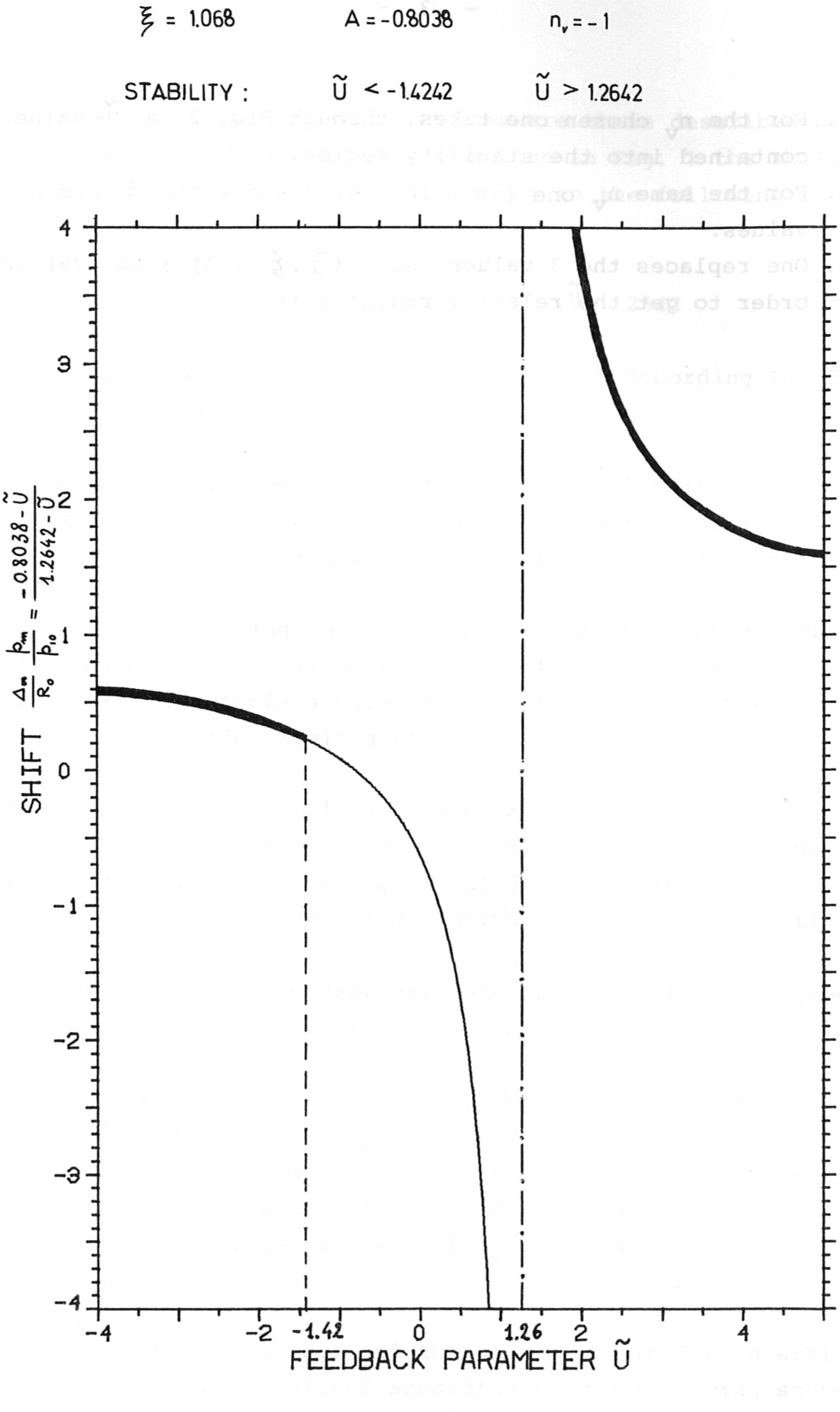
This in fact would imply a greater average circulating power in the feedback system whereas there is no sensible reduction in the radial shift (this is about 3 cm when one assumes a suitable $\tilde{U}$- value and 1 % deviation from marginal ignition).

Assuming n_v as independent variable, whereas all other geometrical parameters are fixed, one obtains:

- stable and unstable regions for the feedback parameter $\tilde{U}$ as function of the index n_v as in Fig. 2 where the stability regions, corresponding to the conditions (25) to (28), have been shaded
- the coupling parameter ξ as function of n_v (Fig. 3)
- the A parameter as function of n_v according to the definition (20) (Fig. 4).

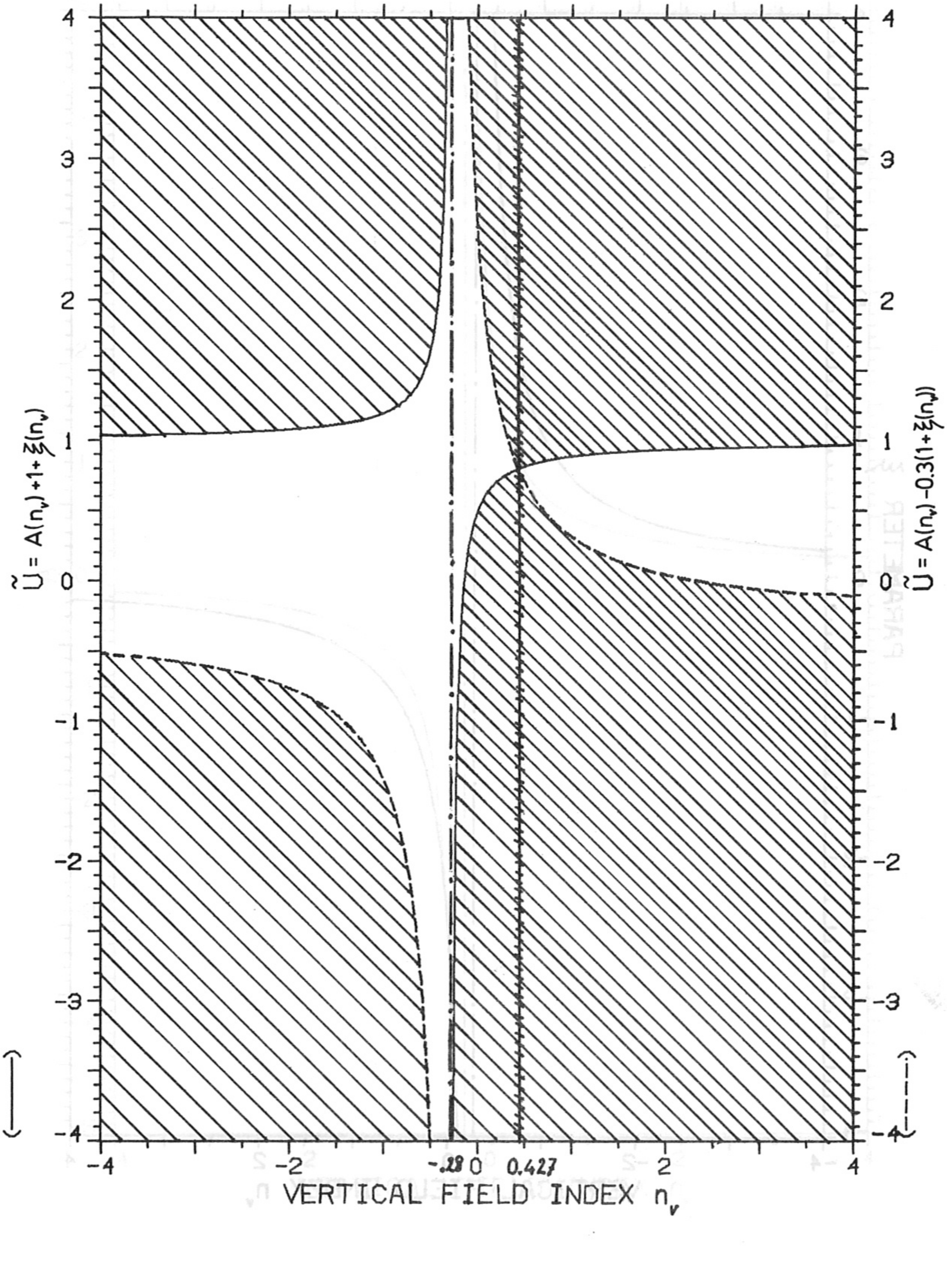
The relative radial shift of the plasma column for an arbitrary n_v can be calculated according to the following steps:

1. For the n_v chosen one takes, through Fig. 2, a $\tilde{U}$-value contained into the stability region.
2. For the same n_v one finds in Fig. 3 and 4 the ξ and A values.
3. One replaces the 3 values above ($\tilde{U}$, ξ, A) into (29) in order to get the relative radial shift.



ξ = 1.068
A = -0.8038
n_v = -1
STABILITY :
Ũ < -1.4242
Ũ > 1.2642
SHIFT Δ∞/R₀ p_m/p₁₀ = (-0.8038 - Ũ)/(1.2642 - Ũ)
FEEDBACK PARAMETER Ũ
4
3
2
1
0
-1
-2
-3
-4
-2
-1.42
1.26

FIG. 1



4
3
2
1
0
-1
-2
-3
-4
-4
-2
-.28
0
0.427
2
4
VERTICAL FIELD INDEX n_v
$\tilde{U} = A(n_v) + 1 + \xi(n_v)$
$\tilde{U} = A(n_v) - 0.3(1 + \xi(n_v))$

FIG.2

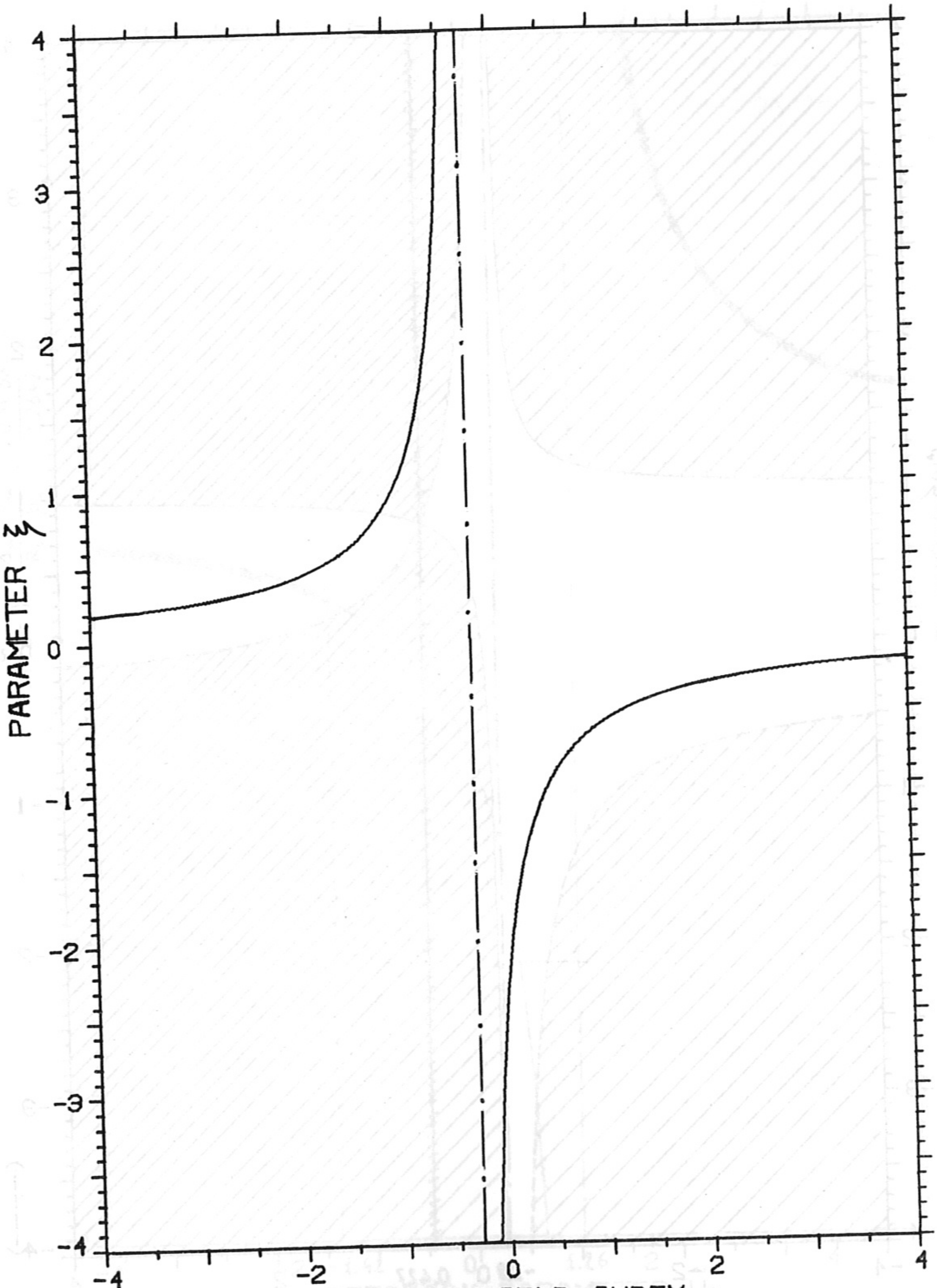


FIG. 3

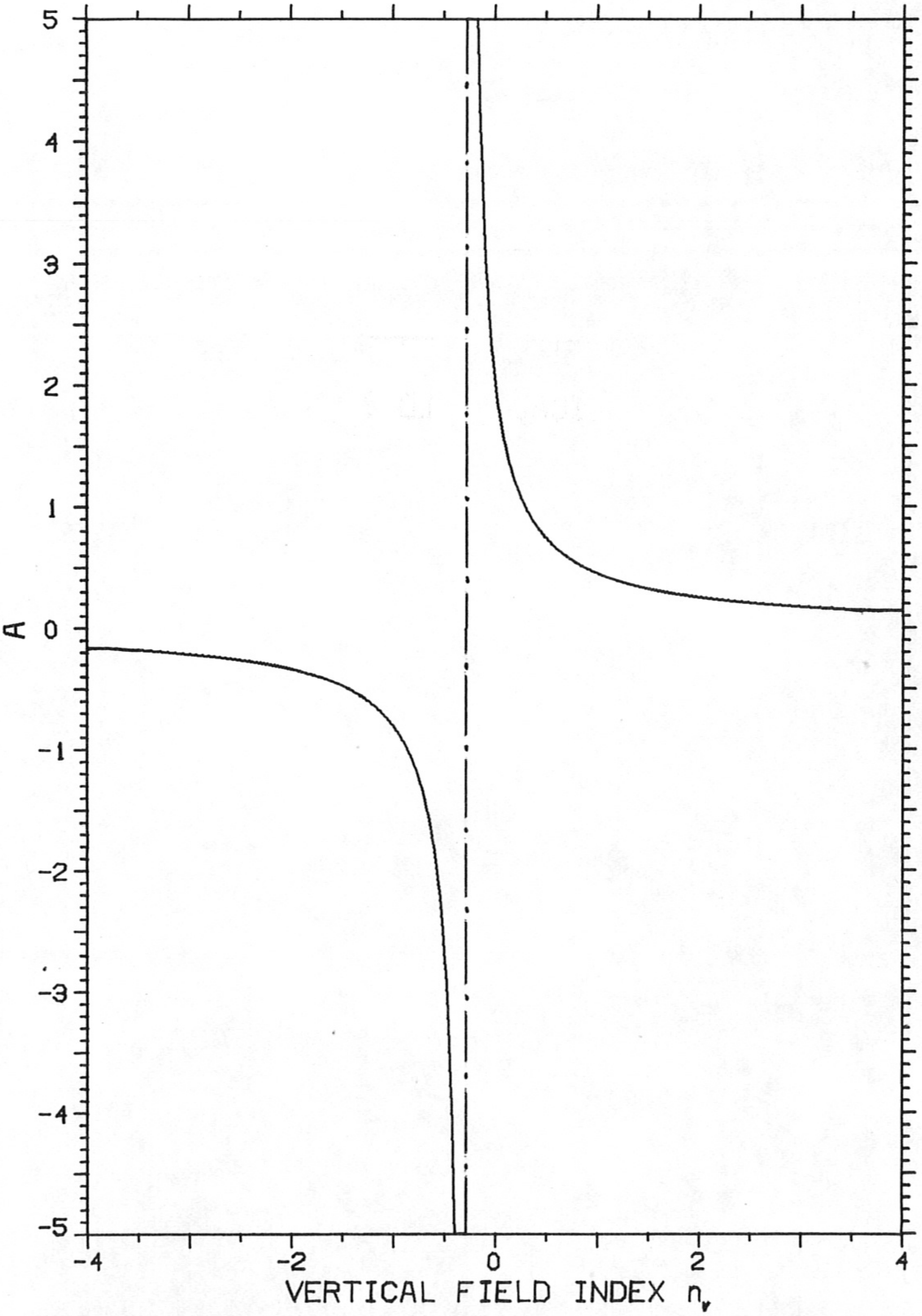


FIG. 4