PLASMA SURFACE IMPEDANCE FOR COUPLING TO THE ION-CYCLOTRON AND ION-BERNSTEIN WAVES

Satish Puri

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Plasma Surface Impedance for Coupling to the Ion-Cyclotron and Ion-Bernstein Waves

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Abstract

Analytic expressions for the surface impedance for coupling radiofrequency energy into a slab-plasma at the ion-cyclotron range of frequencies are obtained. For the fast-wave coupling, the modification due to finite n_y, corresponding to azimuthal variation in cylindrical geometry, is included. For the slow-wave coupling to the ion-Bernstein modes, the plasma surface impedance exhibits critical dependence upon the electron density and ion temperature at the plasma edge.

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1. INTRODUCTION

For heating a toroidal plasma at the ion-cyclotron range of frequencies (ICRF), one may use either the quasi-compressional fast (f) wave [1-12] or the quasi-torsional slow (s) wave [13-21].

The important antenna design parameter ρ, the plasma surface impedance for these waves has hitherto been estimated using computational and semianalytical methods [6, 7, 10]. In this paper we derive ρ through purely analytical means.

The plasma model and the "local" dielectric tensor description outlined in Ref. 16 will be used. Since the edge region where coupling occurs is dominated by density variations, the relatively slow variation of the magnetic field will be ignored and $B_o = B_o(x=0)$ where the x-axis lies along the increasing plasma density with origin at the plasma edge. The plasma density will be assumed to vary linearly from $n_e(x=0) = 0$ to $n_e(x=x_p) = \tilde{n}_e$, where tilde denotes the maximum value.

All length are normalized by multiplication through $k_o = \omega/c$, the free space impedance η is normalized to unity, the differentiations denoted by a prime are with respect to $ik_o dx$ while the exponential variation $\exp i(n_y y + n_z z - \omega t)$ is suppressed, where $\underline{n} = \underline{k}/k_o$ is the refractive index.

2. FAST WAVE COUPLING

The Fourier analyzed Maxwell's equations may be written as

$$n_y E_z - n_z E_y = H_x \ , \tag{1}$$

$$n_z E_x - E_z' = H_y \ , \tag{2}$$

$$E_y' - n_y E_x = H_z \ , \tag{3}$$

$$-n_y H_z + n_z H_y = \epsilon_x E_x + \epsilon_y E_y \ , \tag{4}$$

$$n_z H_x - H_z' = \epsilon_y E_x - \epsilon_x E_y \ , \tag{5}$$

$$H_y' - n_y H_x = -\epsilon_z E_z \ . \tag{6}$$

In order to determine the plasma surface impedance

$$\rho_f = E_y / H_z \ , \tag{7}$$

we will first obtain a differential equation for E_y from the set of Maxwell's equations (1-6). It is possible to obtain a second degree equation in E_y provided we neglect E_z and E_z' in (1) and (2). This is reasonable in view of the fact that for fast wave coupling one deliberately chooses an antenna orientation which avoids finite E_z in order to suppress surface heating due to electron Landau damping. That this assumption is justified may also be confirmed, a posteriori.

Differentiating (3), we obtain

$$E_y'' - n_y E_x' - H_z' = 0, \tag{8}$$

which on eliminating H_z' using (5) and (1) gives,

$$E_y'' - \gamma_1 E_y + \epsilon_y E_x - n_y E_x' = 0, \tag{9}$$

where

$$\gamma_1 = (\epsilon_x - n_z^2) . \tag{10}$$

Combining (2) and (4) gives E_x and E_x' as

$$E_x = -(\epsilon_y/\gamma_1) E_y - (n_y/\gamma_1) H_z , \tag{11}$$

and

$$E_x' = [(\epsilon_x' \epsilon_y - \gamma_1 \epsilon_y') E_y - \gamma_1 \epsilon_y E_y' + n_y \epsilon_x' H_z - \gamma_1 n_y H_z'] / \gamma_1^2 \tag{12}$$

From (3) and (11) one may solve for E_x and H_z,

$$E_x = -(\epsilon_y/\gamma_2)\,E_y - (n_y/\gamma_2)\,E_y' \tag{13}$$

and

$$H_z = (n_y \epsilon_y/\gamma_2)\cdot E_y + (\gamma_1/\gamma_2)\,E_y' \tag{14}$$

where,

$$\gamma_2 = \epsilon_x - n_y^2 - n_z^2 \,. \tag{15}$$

From (12) and (14) using (5), (1) and (11), then gives

$$E_x' = \left[\left(-\gamma_1\gamma_2 n_y - n_y\epsilon_y^2 + \epsilon_x'\epsilon_y - \gamma_2\epsilon_y'\right)E_y + \left(n_y\epsilon_x' - \gamma_1\epsilon_y\right)E_y'\right]/\gamma_1\gamma_2 \,. \tag{16}$$

Substituting for E_x and E_x' from (13) and (16) in (9), finally gives the desired differential equation

$$E_y'' - \left[n_y^2\,\epsilon_x'/(\gamma_1\gamma_2)\right]E_y' - \left[\alpha(\epsilon_R - n_z^2) - n_y^2 + n_y\epsilon_y'(n_y^2 + n_z^2 - 1)/\gamma_1\gamma_2\right]E_y = 0 \tag{17}$$

where,

$$\alpha = (\epsilon_L - n_z^2)/(\epsilon_x - n_z^2)\,, \tag{18}$$

$$\epsilon_L = \epsilon_x - i\,\epsilon_y\,, \tag{19}$$

and

$$\epsilon_R = \epsilon_x + i\epsilon_y \quad . \tag{20}$$

In obtaining (17) we used the relation ,

$$\epsilon_x' \epsilon_y - (\epsilon_x - n_y^2 - n_z^2)\epsilon_y' = (n_y^2 + n_z^2 - 1)\epsilon_y' . \tag{21}$$

Writing ,

$$E_y = (\gamma_2/\gamma_1)^{1/2} \; \mathcal{E}_y \, , \tag{22}$$

gives from (17),

$$\mathcal{E}_y'' - \left[\alpha(\epsilon_R - n_z^2) - n_y^2 + n_y^2 \delta\right] \mathcal{E}_y = 0 , \tag{23}$$

where

$$\delta(x) = \left[(\epsilon_x - n_y^2/4 - n_z^2)\epsilon_x'^2 + \gamma_1\gamma_2(n_y^2 + n_z^2 - 1)\epsilon_y'/n_y\right] / \gamma_1^2 \gamma_2^2 . \tag{24}$$

For $n_y = 0$, (17) or (23) reduce to an Airy equation if we treat the slowly varying parameter $\alpha \sim O(1)$ as a constant.

For finite n_y, the presence of $\delta(x)$ in (23) creates no difficulties if the gradients are slow enough so that

$$\delta(x) << 1 \quad . \tag{25}$$

The dominant contribution to $\delta(x)$ in (24) comes from the first term. To satisfy (25) one requires that

$$\left(\frac{x}{x_p}\right) >> \left(\frac{\omega_{ci}}{\tilde{\omega}_{pi}}\,\frac{1}{x_p}\right)^{2/3} . \tag{26}$$

For a reactor sized machine with $B_o = 50$ kG at the outer plasma radius, $\tilde{n}_e = 10^{15}$ cm^{-3}, $x_p = 0.1$, the RHS of (26) $\lesssim O\,(0.1)$ and (25) will be satisfied over the region of interest surrounding the magnetosonic cutoff. Nevertheless for smaller machines, specially when operating at reduced densities, the condition (25) may be only marginally met.

Accordingly we treat the following two cases separately,

(i) weak density gradients so that $\delta << 1$, and

(ii) steeper density gradients where $\delta \lesssim O(1)$.

For still steeper gradients such that $\delta > O(1)$, the analytic approach becomes extremely cumbersome.

2.1 Coupling to Weak Density Gradients

The two simplifying assumptions which would reduce (23) to an Airy equation are $\delta << 1$ and $\alpha \sim O(1)$, a constant. Expressing ϵ_R as

$$\epsilon_R = 1 + x(\partial\epsilon_R/\partial x) , \tag{27}$$

one obtains from (23)

$$\frac{\partial^2 \mathcal{E}_y}{\partial x^2} - \alpha\left(n_y^2/\alpha + n_z^2 - 1 - x\frac{\partial\epsilon_R}{\partial x}\right)\mathcal{E}_y = 0 . \tag{28}$$

At the magnetosonic cutoff x_c, the RHS of (28) vanishes, giving

$$x_c = (n_y^2/\alpha + n_z^2 - 1)(\partial\epsilon_R/\partial x)^{-1} . \tag{29}$$

If we let

$$\xi = x_c - x , \tag{30}$$

equation (28) reduces to

$$\frac{\partial^2 \mathcal{E}_y}{\partial \xi^2} - p^2 \xi \mathcal{E}_y = 0 , \tag{31}$$

where,

$$p^2 = (n_y^2 + \alpha n_z^2 - \alpha)/x_c . \tag{32}$$

The solution of the standard Airy equation (31) is given by

$$\mathcal{E}_y = Ai\,(p^{2/3}\xi\, e^{2\pi i/3}) \tag{33}$$

where we assume that there is no returning wave from the plasma core. From (22) and (33) we get the electric field as

$$E_y = (\gamma_2/\gamma_1)^{1/2}\, Ai\,(p^{2/3}\xi\, e^{2\pi i/3})\,. \tag{34}$$

Differentiating (34) gives

$$E_y' = (\gamma_2/\gamma_1)^{1/2}\, p^{2/3}\, e^{\pi i/6}\, \frac{\partial A_i}{\partial x}(p^{2/3}\xi\, e^{2\pi i/3}) \tag{35}$$

Using (34) and (35) in (14) we obtain H_z (note that $\epsilon_y = 0$ at the surface) and finally the plasma surface impedance,

$$\rho_f = (\gamma_2/\gamma_1)_{x=0}\, e^{-\pi i/6}\, p^{-2/3}\, Ai(\psi)\,[\partial Ai(\psi)/\partial x]^{-1}, \tag{36}$$

where,

$$\psi = e^{2\pi i/3}\, p^{2/3}\, x_c\,. \tag{37}$$

For the frequently encountered case $p^{2/3}\xi \ll 1$, the Airy functions may be approximated by the first terms in the series expansions, so that (36) reduces to

$$\rho_f = (\gamma_2/\gamma_1)_{x=0}\, 3^{-1/3} \left[\Gamma(1/3)/\Gamma(2/3)\right] e^{-\pi i/6} (x_c/\alpha)^{1/3} (n_y^2/\alpha + n_z^2 - 1)^{-1/3} \tag{38}$$

In order to obtain an estimate of x_c, let us write ϵ_R as

$$\epsilon_R = 1 + \frac{\omega_{pe}^2(x)}{\omega\omega_{ce}}\left[1 - \frac{f_D}{1+\omega/\omega_{cD}} - \frac{f_T}{1+\omega/\omega_{cT}}\right]$$

$$= 1 + \frac{\beta}{2}\frac{m_D}{m_e}\frac{\omega_{pe}^2(x)}{\omega_{ce}^2}, \tag{39}$$

where,

$$\beta = \frac{2\omega_{cD}}{\omega}\left[1 - \frac{f_D}{1+\omega/\omega_{cD}} - \frac{f_T}{1+\omega/\omega_{cT}}\right] \sim O(1), \tag{40}$$

where we assume an equal mixture of deuterium and tritium ions. At the magnetosonic cutoff

$$\epsilon_R = n_y^2/\alpha + n_z^2 = 1 + \frac{\beta}{2}\frac{m_D}{m_e}\frac{\omega_{pe}^2(x_c)}{\omega_{ce}^2} \tag{41}$$

Also, since

$$\omega_{pe}^2(x_c) = (x_c/x_p)\,\tilde{\omega}_{pe}^2, \tag{42}$$

we obtain from (41) and (42)

$$x_c = \frac{2}{\beta}\frac{m_e}{m_D}(n_y^2/\alpha + n_z^2 - 1)\frac{\omega_{ce}^2}{\tilde{\omega}_{pe}^2}\, x_p, \tag{43}$$

which on substitution in (38) gives

$$\rho_f = 3^{-1/3} \frac{\Gamma(1/3)}{\Gamma(2/3)} e^{-\pi i/6} \frac{1-n_y^2-n_z^2}{1-n_z^2} \left(\frac{\omega_{ce}}{\tilde{\omega}_{pe}}\right)^{2/3} \left(\frac{2x_p}{\alpha\beta} \frac{m_e}{m_D}\right)^{1/3} . \tag{44}$$

Note that the approximations α, $\beta \sim O(1)$ are uncritical in the light of one-third power dependence.

By far the most surprising element of (44) is the lack of criticality that ρ_f displays with respect to the refractive indices n_y and n_z. This would enable uniform fast wave coupling at ICRF over a wide (n_y, n_z) spectrum range.

This result also provides added justification for neglecting the contribution $n_y^2\ \delta$ in (23); because this term acts in much the same manner as n_y^2 and n_z^2 and one may reasonably well infer that ρ_f is not overly sensitive to this term either.

At this stage one may determine E_z from (6) after some straightforward algebraic steps and verify that our initial neglect of E_z and E_z' in (1) and (2) was indeed justified.

2.2 Coupling to Steeper Density Gradients

In this section we treat the case when

$$\delta \lesssim O(1) \quad . \tag{45}$$

A rigorously exact treatment is rather involved. We, therefore, would limit the analysis so as to provide a useful check on computational results.

From (24), one may approximate

$$\delta(x) = \frac{x_p}{\tilde{\epsilon}_x \, x^3} + \frac{\omega}{\omega_c} \, \frac{n_y^2 + n_z^2 - 1}{n_y} \, \frac{x_p}{\tilde{\epsilon}_x \, x^2} \tag{46}$$

and

$$\frac{\partial \delta(x)}{\partial x} = \frac{-3 x_p}{\tilde{\epsilon}_x \, x^4} - \frac{\omega}{\omega_c} \, \frac{n_y^2 + n_z^2 - 1}{n_y} \, \frac{2 x_p}{\tilde{\epsilon}_x \, x^3} \quad . \tag{47}$$

Expanding $\delta(x)$ near the critical magnetosonic cut-off region x_c, gives

$$\delta(x) = \delta(x_c) + (x - x_c)[\partial \delta(x_c)/\partial x] \quad . \tag{48}$$

where x_c in this case is determined from (23), (4), (42) and (46) as the root of the algebraic equation,

$$\frac{\alpha\beta}{2x_p}\,\frac{m_D}{m_e}\,\frac{\tilde{\omega}_{pe}^2}{\omega_{ce}^2}\,x_c^4 + (\alpha - n_y^2 - \alpha\, n_z^2)\,x_c^3 + \frac{\omega}{\omega_c}\,\frac{n_y x_p}{\tilde{\epsilon}_x}(n_y^2 + n_z^2 - 1)x_c + n_y^2\,\frac{x_p}{\tilde{\epsilon}_x} = 0\,. \tag{49}$$

This reduces (23) to the form

$$\frac{\partial^2 \mathcal{E}_y}{\partial \xi^2} - q^2\,\xi\,\mathcal{E}_y = 0\,, \tag{50}$$

where,

$$q^2 = \left[n_y^2\left\{1 + \delta(x_c) + x_c\,\partial\delta(x_c)/\partial x\right\} + \alpha n_z^2 - \alpha\right]/x_c \tag{51}$$

and x_c is determined from (49) while δ and $\partial\delta/\partial x$ are given by (46) and (47) respectively. The remaining steps leading to the expression for ρ_f are identical to the treatment in Sec. 2.1.

Observe that in the approximate treatment of this section, we have endeavored to describe the correction term $\delta(x)$ accurately in magnitude and to the lowest order in slope near the critical region surrounding the magnetosonic cut-off. The deviations from the correct value away from the critical region are unlikely to introduce significant error specially in-view-of the relative insensitivity of ρ_f to the refractive index at large as was found to be the case in the preceeding section.

3. SLOW WAVE COUPLING

Unlike the fast wave case where the wavelength remains large compared to the ion-Larmor radii, the slow-wave coupling is dominated by hot plasma effects, because the cold-plasma wave converts to ion-Bernstein like modes at a relatively low density ($n_e \sim 10^{11}$ cm^{-3}). Following the wave conversions, the WKB conditions are readily satisfied for the hot-plasma wave, which propagates to the plasma core without further reflection [16].

The distance between the plasma edge and the wave conversion region is, in practice, insignificantly thin or else is altogether absent, when a tenuous plasma extends up to the antenna [16-17]. In either case, one may assume, without introducing an appreciable error, that the antenna directly launches the propagating hot-plasma wave. Also, since the multicomponent aspect does not enter the coupling considerations in this section, we consider only a deuterium-electron plasma in this treatment.

For the reflectionless WKB propagation, one obtains for the backward Gross-Bernstein waves, the normalized surface impedance

$$\rho_s = E_z / H_y ,$$

$$= - n_x / \epsilon_z ,$$

$$= - (2\Lambda_i)^{1/2} / (r_{ci}\, \epsilon_z)$$

$$\simeq (2\Lambda_i)^{1/2} (c/v_{zi}) (\omega\omega_{ci}/\omega_{pe}^2) , \tag{52}$$

where,

$$\Lambda_i = (1/2)\, n_x^2\, r_{ci}^2 , \tag{53}$$

r_c is the gyroradius, v_z is the thermal speed and the dielectric tensor description corresponds to a hot, Maxwellian plasma. For the case $\Lambda_i \sim 3$, $\omega/\omega_{ci} \sim 1.3$, $n_e \sim 10^{11}$ cm^{-3}, $T_i \sim 100$ eV, $B_o \sim 50$ kG at the plasma edge, $\rho_s \sim O(1)$ implying a good match between the antenna and the plasma, thus confirming the numerical results of Ono et al.[19]. This optimistic picture, however, is subject to the mercurial temperament of the plasma near the edge where fluctuations in temperature and density could cause large fluctuations in the values of v_{zi}, ω_{pe} and $\Lambda_i^{1/2}$. Of these three parameters, $\Lambda_i^{1/2}$ may not be subject to gross variations (see Appendix A), whereas ω_{pe} and v_{zi} are much harder to pin down. Uncertainties of this nature, in fact, are a common feature of low wavelength plasma heating schemes for abvious reasons of critical dependence on plasma surface conditions.

The object of the above analysis has been to arrive at an approximate guideline for designing the coupling antennas. No attempt has been made, or is warranted, at refinement beyond the rough estimate because of uncertainties inherent in the experimental parameters.

One may readily verify that the perpendicular wavelength

$$\lambda_x = \sqrt{2}\,\pi\, r_{ci}\, \Lambda_i^{-1/2} , \tag{54}$$

is of the order of ion gyroradius and is much smaller than the typical gradient lengths. This, together with the slow variation of λ_x, justifies the use of local approximation for ϵ_x and for assuming the validity of WKB propagation in the derivation of (52).

4. DISCUSSION

The principal contribution of this paper is the derivation of plasma surface impedance both for the slow and fast wave coupling to ICRF in an analytical form.

For the fast wave coupling ρ_f is expressed in a simple form (36) and (44) for the case of weak density gradients. For stronger density gradients an estimate for ρ_f is obtained (sec. 2.2) to provide useful check on computational results.

For the slow wave coupling, the finite temperature effects are included and an approximate expression is derived to provide a guideline for coupling considerations. It is shown that the coupling would be dominated by uncertain surface conditions of electron density and ion temperature. Conditions at the sheath, no doubt will assume importance but further theoretical refinement would make little sense in view of the large uncertainties already in sport. Pointing out the futility of an accurate theoretical determination of ρ_s, in fact, may be considered as among the contributions of this paper.

APPENDIX A

To obtain an estimate of the lower limit on Λ_i, let us express ϵ_x for the warm plasma approximation ($\Lambda_i << 1$) as

$$\epsilon_x = \epsilon_x^c + \Lambda_i S_i \quad , \tag{A1}$$

where

$$S_i = \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} - \frac{\omega_{pi}^2}{\omega^2 - 4\,\omega_{ci}^2} \quad , \tag{A2}$$

and ϵ_x^c is the cold-plasma contribution. Combining (A1) with the dispersion relation (see e.g. Eq. 11 in Ref. 16),

$$A\,n_x^4 - B\,n_x^2 + C = 0 \quad , \tag{A3}$$

and (53), a quadratic equation for ϵ_x is obtained with the solution (for $n_x >> 1$, $|\epsilon_z| >> 1$),

$$\epsilon_x \simeq [u \pm (u^2 - 4v)^{1/2}]/2, \tag{A4}$$

where,

$$u = \epsilon_x^c + g S_i , \tag{A5}$$

$$v = g n_z^2 S_i , \tag{A6}$$

and,

$$\begin{aligned} g &= (1/2)\, \epsilon_z r_{ci}^2 , \\ &\simeq -(1/2)(v_{zi}/c)^2 (\omega_{pe}/\omega_{ci})^2 . \end{aligned} \tag{A7}$$

Near the plasma edge, $|g| << 1$. Also, excepting a thin or perhaps non-existent (but in either case insignificant) region near the plasma edge, $S_i \simeq - \epsilon_x^c >> 1$, so that for typical $n_z \sim O(1)$, $u >> v$ and (A4) simplifies to

$$\epsilon_x \simeq u < 0, \tag{A8}$$

and ,

$$\epsilon_x \simeq v/u \simeq n_z^2 (1 - g^{-1})^{-1} \ll 1, \tag{A9}$$

Of these two roots, the second corresponds to the warm-plasma wave being considered. Substituting the value of ϵ_x from (A8) in (A3) gives

$$\begin{aligned} n_x^2 &\simeq B/A \simeq -n_z^2 \epsilon_z / \epsilon_x , \\ &= \epsilon_z / g , \end{aligned} \tag{A10}$$

so that, from (A7) and (A10),

$$n_x \simeq \sqrt{2}\, r_{ci}^{-1} . \tag{A11}$$

This value of n_x, however, corresponds to $\Lambda_i = 1$, indicating the breakdown of the assumption leading to (A1). Including higher order terms in Λ_i indicates that the direction of the error implies that $\Lambda_i > 1$. Thus we may set the approximate lower limit on $\Lambda_i = 1$.

We now proceed to estimate an upper bound on Λ_i in (52). For $\Lambda_i >> 1$, one obtains for the electrostatic Gross-Bernstein wave using $\epsilon_x << 1$ (using Eq. 4 in Ref. 16),

$$\Lambda_i^{3/2} \simeq \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{\omega_{pi}}{\omega_{ci}}\right)^2 \frac{S_n}{1 + \omega_{pe}^2/\omega_{ce}^2} \qquad \text{(A12)}$$

where,

$$S_n \simeq \sum_{n=1}^{O(\Lambda_i)} \frac{n^2 \omega_{ci}^2}{\omega^2 - n^2 \omega_{ci}^2} \quad . \qquad \text{(A13)}$$

Assuming the edge conditions $n_e \sim 10^{11}$ cm^{-3}, $B_o \sim 50$ kG,

$$\Lambda_i^{3/2} \simeq S_n \quad . \qquad \text{(A14)}$$

Practical considerations dictate that there be no cyclotron resonance near the plasma edge in order to avoid surface heating. If we limit

$$\Delta\Omega = |(\omega - n\omega_{ci})/\omega_{ci}| \gtrsim 0.1 \,, \tag{A15}$$

and $S_n \lesssim 5n$ so that $\Lambda_i \lesssim 4\ n^{2/3}$. For heating up to the third harmonic the upper bound on $\Lambda_i \lesssim 8$. Thus the approximate range of $\Lambda_i^{1/2}$ in (52) is given by

$$1 \lesssim \Lambda_i^{1/2} \lesssim 3 \,. \tag{A16}$$

For rough estimates we take $\Lambda_i \sim 3$ to be a representative value.

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