

Normal Conducting Steady-State Toroidal  
Magnet Systems for Ignited Tokamaks

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ABSTRACT

Normal conducting steady-state toroidal magnet systems are investigated, emphasis being placed on applications to large ignited next generation tokamaks.

The study is based on water-cooled tape wound D-coils. The data for the TF magnet systems are calculated in a consistent manner with a computer program including the plasma, shield, ohmic heating coil system and geometric requirements for blanket modules, beam ducts etc. An optimisation procedure is used to find those TF coil systems which minimize cost-relevant quantities.

The main results are that normal conducting TF coil systems of ignited next-generation tokamaks (following JET, TFTR etc.) can be operated in a stationary mode and fed from the grid. Cost for electricity is a relatively small portion of the investment cost even in the case of long integral burn times ( $>10^7$  s).

## 1. INTRODUCTION

Within future tokamak development a certain fixed point is given by systems of the DEMO-INTOR type in that they require an ignited, relatively long burning plasma, superconducting toroidal field coils and a breeding blanket or blanket modules. Each of these goals requires a big step beyond the present status. Coupling all these options in a single experiment following JET or TFTR would constitute a major risk to achieving these aims.

Apart from a trivial time sequence procedure, decoupling of the risks could be achieved by making use of normal conducting TF coil systems for ignited plasmas and developing superconducting TF coils and ignited plasmas in separate devices. Such a procedure has already been proposed in "European Long-term Planning", in which emphasis is placed on time schedule requirements.<sup>(1)</sup>

In this study tape-wound, water-cooled D-coils with low mechanical stresses are considered with a view to application in ignited tokamaks. Such coils have been successfully manufactured for the JET device with specifications close to those of next-generation tokamaks. Apart from the greater experience in manufacturing and operating large, normal conducting TF magnets further advantages over superconducting magnets are:

- simpler mechanical structure of the winding and simpler cooling
- smaller radiation shielding
- immunity to rapid magnetic field variations (disruptions)
- easier (radial and lateral) support of the winding
- easier dismantling (no cryostats)
- simpler detection and discharge systems.

The main drawback of normal conducting TF coils is the high resistive losses. Furthermore, since next-generation tokamaks should operate in the long-pulsed mode, steady-state operation, and hence steady-state cooling, is required. There is an obvious trade-off between resistive losses and cooling requirements on the one hand and stored magnetic energy and coil thickness on the other. This paper is to clarify quantitatively whether next-generation ignited tokamaks with resistive steady-state TF coils with acceptable size and resistive losses can be built. For this purpose a model that allows for the self-consistent layout of such systems is developed. The solutions are optimized with respect to cost-relevant figures of merit and take into account all relevant technical and geometrical constraints.

Though this paper is devoted to next-generation tokamaks it is pointed out that the model also applies to current, non-ignited experiments as well as to hybrid and pure fusion reactors.

Other concepts for steady-state normal conducting TF coils were studied in Refs. (2, 3, 4), the underlying philosophy being different in part. In our study emphasis was placed on risk minimization, and consequently medium-field, low-stress coils, the fabrication of which is standard, were considered. For easier dismantling the analysis of Refs. (2, 3) is based on rectangular TF coils assembled from copper plates. In contradiction to this and our study, Ref. (4) deals with compact tokamaks with Bitter-type TF systems exposed to high mechanical stresses.

## 2. MODEL DESCRIPTION

### 2.1 Geometry

The analysis is based on tape-wound D-coil systems as shown in Fig. 1. The plasma is centred at  $R_0$  (plasma major radius). The elongated plasma cross-section is characterized by the minor radius  $a$  and the plasma elongation  $s$ . The plasma is surrounded by a scrape-off layer of thickness  $l$ , a vessel of thickness  $u$  and a shielding of thickness  $D$ .

The D-coil is characterized by the minimum and maximum distances of the centre line from the torus axis,  $R_1$  and  $R_2$  respectively, and by the coil thickness  $\Delta$ . The coils are tapered in the inner coil region in order to make optimum use of the limited area in this region. Each coil is subdivided into  $m$  pancakes, the typical case where  $m = 2$  being shown in Fig. 1. The toroidal field coils are supported by the central support cylinder of thickness  $t_1$  (see Fig. 2). The ohmic heating coil has a thickness  $\Delta t$  and a radius  $R_{OH}$  as shown in Fig. 2.

There are further important geometrical parameters, namely  $g$ ,  $\Delta R$  and  $d$ , of which  $g$  and  $\Delta R$  are visualized in Fig. 1. The parameter  $g$  characterizes the free space available for the installation of, for instance, a poloidal divertor, while  $\Delta R$  characterizes the space available for the installation of, for instance, blanket

modules. The parameter  $d$  is the free space between two coils in the equatorial plane and gives the space for beam ducts etc. Depending on the specific system under consideration, lower limits  $g^m$ ,  $\Delta R^m$  and  $d^m$  exist for the quantities  $g$ ,  $\Delta R$  and  $d$ .

## 2.2 Basic Equations

The following set of equations forms the central part of the model. They can obviously be evaluated in the given order, once the first two groups of parameters of Table I are given. The relations are only commented on if necessary. All units are MKSA units if not otherwise stated.

### 2.2.1 Geometric Relations

$$a^* = a \sqrt{s} \quad (1)$$

(radius of an equivalent circular plasma)

$$R_o = a A \quad (2)$$

(major radius)

$$V = 2 \pi^2 a^{*2} R_o \quad (3)$$

(plasma volume)



$$R_1 = R_0 - a - l - u - D - \Delta/2 \quad (4)$$

(see Fig. 1)

$$h_0 = s a + l + u + D + g^m \quad (5)$$

(see Fig. 1)

$h_0$  is the minimum value of  $h$ , such that  $g \geq g^m$ .

$$R_{20} = R_0 + a + l + u + D + \Delta/2 + \Delta R^m \quad (6)$$

$R_{20}$  is the minimum value of  $R_2$  such that  $\Delta R \geq \Delta R^m$ .

$$R'_{20} = R_1 + \frac{(d^m + 2D)N}{2\pi} + \Delta \quad (7)$$

$R'_{20}$  is the minimum value of  $R_2$  such that  $d \geq d^m$ .

$$R''_{20} = R_1 ((1.257 \times 10^2 + 29.91 (h_0 + \Delta/2)/R_1)^{0.5} - 10.10) \quad (8)$$

$R''_{20}$  is the minimum value of  $R_2$  such that  $g \geq g^m$ .

See also eq. (10).

$$R_2 = \max \{R_{20}, R'_{20}, R''_{20}\} \quad (9)$$

$R_2$  is the minimum value such that the constraints  $g \geq g^m$ ,  $\Delta R \geq \Delta R^m$  and  $d \geq d^m$  are met.

$$h = R_1 (3.344 \times 10^{-2} (R_2/R_1)^2 + 6.757 \times 10^{-1} R_2/R_1 - 0.7891) - \Delta/2 \quad (10)$$

This is a polynomial fit of an analytic expression given in Ref. (10). The same relation was used in eq. (8).

$$g = h - s a - l - u - D \quad (11)$$

(see Fig. 1).

$$d = \frac{2\pi (R_2 - R_1 - \Delta)}{N} - 2 D \quad (12)$$

(coil distance in the equatorial plane)

$$\Delta R = R_2 - R_{20} + \Delta R^m \quad (13)$$

(see Fig. 1)

Equations (11), (12), (13) give the actual values of  $g$ ,  $d$  and  $\Delta R$ .

$$R = R_0 + a \quad (14)$$

$$\epsilon [\%] = 5.4 \times 10^2 \frac{R R_1 (R_2/R_1 - 1)^{1.5} (R_2 - R_1)}{N^4 ((R_2 - R)^2 + (\pi R_2/2N)^2)^{1.5}}$$

$$\frac{\pi^3 + 4\pi N^2 (1 - R/R_2) R/R_2}{\pi^2 + 4 N^2 (1 - R/R_2)^2} \quad (15)$$

(field ripple at  $R^{(5)}$ )

### 2.2.2 Plasma-Related Quantities

$$\bar{\beta}_t = 0.12 s/A \quad (16)$$

$\bar{\beta}_t$  is the volume averaged toroidal beta. This scaling of  $\bar{\beta}_t$  is widely accepted for not too strong elongation ( $s \lesssim 2.0$ ) and not too low aspect ratios ( $A \gtrsim 2.5$ ).

$$B_t^0 = \left( \frac{(1 - C_s) C}{\bar{\beta}_t a^2} \right)^{1/4} \quad (17)$$

Equation (17) is the ignition condition for a plasma which is dominated by anomalous electron heat conduction of the ALCATOR-type. ALCATOR-type losses typically dominate neoclassical ion heat conduction, except for rather high densities. Bremsstrahlung losses can typically be neglected, provided that the working point of the system is not located too far into the ignition regime. In this approximation the ignition condition simplifies to  $Q_\alpha - Q_{\text{loss}} \geq 0$ , where  $Q_\alpha$  is the  $\alpha$ -heating term and  $Q_{\text{loss}}$  the transport loss term in the energy balance. With  $Q_\alpha \sim \bar{\beta}_t^2 B_t^0{}^4 f(T)$  and  $Q_{\text{loss}} \sim T/a^2$  it then follows that ignition requires that  $\bar{\beta}_t^2 B_t^0{}^4 a^2 \geq C$ , where  $C$  can be taken as constant, if one assumes that the working temperature is close to the minimum of  $T/f(T)$ . The constant  $C$  measures the safety of ignition, a value of  $C \approx 1.8$  corresponding to marginal ignition. Other transport models can easily be implemented. Confinement to ALCATOR scaling was made in this study to allow comparison with systems such as JET and INTOR, which have been studied in detail on the basis of this model.

Driven systems can be treated by taking  $C_s > 0$ . Considering a system with the same working temperature as an ignited one with margin  $C$ , but driven by auxiliary heating with power density  $Q_h$ , one has  $(Q_\alpha + Q_h)/Q_{\text{loss}} = C$ . Setting  $Q_h/Q_{\text{loss}} = C_s$  it then follows that  $\bar{\beta}_t^2 B_t^0{}^4 a^2 = (1 - C_s) C$ , which is equivalent to eq. (17).

$$p_f \text{ [MW m}^{-3}\text{]} = 2.0 \bar{\beta}_t^2 B_t^0{}^4 \quad (18)$$

Using this approximation for the fusion power density  $p_f$ , it is assumed that  $T$  lies in a range where  $\langle \sigma v \rangle \sim T^2$  is valid for the reaction rate. This is typically compatible with the assumption about  $T$  in the derivation of eq. (17). The factor of proportionality in eq. (18) depends on both the  $T$  and  $n$  profiles. The value of 2.0 can be achieved by moderately peaked  $T$  and nearly flat  $n$  profiles. Even higher values could be obtained if the  $n$ -profiles were more peaked in large tokamaks than predicted by current transport codes.

$$P_f \text{ [MW]} = V p_f \quad (19)$$

(fusion power)

$$P_\alpha \text{ [MW]} = 0.2 P_f \quad (20)$$

( $\alpha$ -power)

$$N_n \text{ [MW m}^{-2}\text{]} = 0.4 P_f \frac{a^*{}^2}{a^* (1+s)/(2s^{1/2})+1} \quad (21)$$

(mean neutron wall load)

$$P_h \text{ [MW]} = \frac{C_s P_\alpha}{(1-C_s) \eta_h} \quad (22)$$

(heating power in a driven system)

$$W_{int} \text{ [MW y m}^{-2}\text{]} = \frac{N_n \tau_{tot}}{3.154 \times 10^7} \quad (23)$$

(integral wall load)

$$\tau_{iso} \text{ [s]} = \frac{1.13 \times 10^5}{N_n} e^{15.35 D} \quad (24)$$

$\tau_{iso}$  is the lifetime of the insulation, which determines the lifetime of normal conducting coils. A maximum allowable dose rate of  $5 \times 10^9$  rad and the equivalence  $1 \text{ rad} = 10^{13} \text{ n/m}^2$  was used. (6, 7)

$$T_c \text{ [g]} = 1.779 \times 10^{-6} P_f \tau_{tot} \quad (25)$$

(total tritium consumption)

### 2.2.3 Toroidal Field System

$$B_m = \frac{B_t^0 R_o}{R_1 + \Delta/2} \quad (26)$$

(magnetic field at  $R_1 + \Delta/2$ )

$$K = \frac{1}{2} \ln\left(\frac{R_2}{R_1}\right) \quad (27)$$

(coil parameter (8))

$$\sigma = \frac{K}{2\mu_o} \frac{B_m^2 (R_1 + \Delta/2)^2}{R_1 F \Delta} \quad (28)$$

$\sigma$  is the mechanical tensile stress for D-coils. The study anticipates that the TF coils are wound with conductors fabricated from microalloyed copper (oxygen-free

copper with 0.1 % silver). The yield limit of this conductor used for the JET toroidal coil system (9) is in the range of 200 - 250 MPa.

$$\sigma_D = \frac{B_m^2 (R_1 + \Delta/2)^2}{2\mu_0 R_1^2} \quad (29)$$

(pressure on the support cylinder due to the centripetal force)

$$\rho = 2 \times 10^{-8} \quad (30)$$

(specific resistivity of copper at 80° C)

$$P_{\text{ohm}} [\text{MW}] = \frac{4\pi^2 10^{-6}}{\mu_0^2} \frac{\rho K e^K (R_1 + \Delta/2)}{F \Delta} B_m^2 ((R_1 + \Delta/2) I_1(K) + R_1 I_0(K)) \quad (31)$$

$P_{\text{ohm}}$  denotes the resistive losses, and  $I_0$  and  $I_1$  are the modified Bessel functions of zero and first order respectively.

$$V_{\text{Cu}} = 4 \pi^2 \Delta K e^K R_1 F (R_1 I_1(K) + (R_1 + \Delta/2) I_0(K)) \quad (32)$$

(volume of copper)

$$\rho_{\text{Cu}} = 8.9 \times 10^3 \quad (33)$$

(density of copper)

$$M_{Cu} = \rho_{Cu} V_{Cu} \quad (34)$$

(mass of copper)

$$E_m [MJ] = \frac{2\pi^2 10^{-6}}{\mu_o} K e^K B_m^2 R_1 (R_1 + \Delta/2)^2$$

$$(K I_o(K) + (K - 1) I_1(K)) \quad (35)$$

(stored magnetic energy)

$$j = \frac{B_m (R_1 + \Delta/2)}{\mu_o \Delta F R_1} \quad (36)$$

(peak current density in TF coils)

Equations (31) and (32) can easily be derived once the average length of a conductor per turn is known (see eq. (38)). Equation (35) is taken from Ref. (10).

Equation (36) follows from Ampere's law.

#### 2.2.4 Cooling

$$P_o [MW] = \frac{P_{ohm}}{m N} \quad (37)$$

(ohmic losses per pancake)

$$l_o = 2 \pi R_1 K e^K (I_o(K) + I_1(K)) \quad (38)$$

$l_o$  is the average length of a conductor per turn (10).

$$L = n_L l_o \quad (39)$$

(length of a conductor per pancake)

$$Q_L = 2.389 \times 10^{-1} \frac{P_o}{n_s \Delta T} \quad (40)$$

(water flow per cooling channel)

$$Q = Q_L m n_s N \quad (41)$$

(total water flow)

$$d = \left( \frac{8 (F_I - F) \Delta R_1}{m n_L N n_c} \right)^{1/2} \quad (42)$$

(hydraulic diameter of cooling channel)

$$\Delta P = 2.44 \times 10^{-4} \frac{Q_L^2 L n_c}{n_s d^5} \quad (43)$$

(pressure drop per cooling channel)

$$v = \frac{4 Q_L}{\pi d^2} \quad (44)$$

(velocity of water)

$$T_b [C^\circ] = T_o + \Delta T / 2 \quad (45)$$

(mean temperature of water)

$$\dot{q} [W m^{-2}] = \frac{10^6 P_o}{\pi d L n_c} \quad (46)$$

(heat flux)

$$\eta [W m^{-2} C^\circ^{-1}] = 1.42 \times 10^3 (1 + 1.5 \times 10^{-2} T_b)$$

$$\frac{v^{0.8}}{d^{0.2}} \quad (47)$$

(heat transfer coefficient)

$$\Delta T_w [C^\circ] = \dot{q} / \eta \quad (48)$$

(temperature difference between conductor surface and coolant)



$$T_{\max} [C^{\circ}] = T_o + \Delta T + \Delta T_w \quad (49)$$

(maximum coil temperature)

$$L_{\text{pump}} [\text{MW}] = 7.854 \times 10^{-2} \Delta P d^2 v m N n_s \quad (50)$$

(total pumping power)

The above treatment of the cooling problem is based on Ref. (11).

### 2.2.5 Core Constraint

To take into account the core constraint, the coupling of the OH coil to the plasma ring has to be considered. Figure 2 shows the OH coil arrangement and the notations used.

$$t_1 = 0.1 \quad (51)$$

$t_1$  is the thickness of the central support cylinder. A fixed value is used throughout.

$$R_{\text{OH}}^{\circ} = R_1 - \Delta/2 - t_1 \quad (52)$$

(maximum OH coil radius; see Fig. 2)

$$\Delta t = 0.58 R_{\text{OH}}^{\circ} \quad (53)$$

$\Delta t$  is the thickness of the OH coil. The factor of 0.58 is explained below.

$$R_{OH} = R_{OH}^O - 0.5 \Delta t \quad (54)$$

(OH coil radius)

$$I_p = \frac{2\pi B_t^O a^{*2}}{\mu_O q(a)R_O} \frac{1 + s^2}{2s} \quad (55)$$

(plasma current in case of an elongated plasma (9))

$$l_i = 1 \quad (56)$$

(internal plasma inductance per unit length in units of  $\mu_O R_O/2$ )

$$L_p = \mu_O R_O (\ln(8R_O/a^*) - 2 + l_i/2) \quad (57)$$

(plasma inductance)

$$f = 2 \quad (58)$$

(enhancement factor for poloidal flux; see note below)

$$B_{OH} = \frac{f I_p L_p}{2 \pi R_{OH}^2} \quad (59)$$

$B_{OH}$  is the field at the OH coil. Equation (59) is the approximate transformer equation for the OH coil - plasma ring system. Resistive losses are estimated by taking  $f > 1$ . Equation (59) is pessimistic in that contributions by the vertical field are ignored.

Taking this into account, a value of  $f = 2$  seems sufficient to compensate for the resistive losses during start-up and a burn of approximately one  $L_p/R_p$ -time length.

$$j_{OH} = \frac{B_{OH}}{\mu_0 \Delta t} \quad (60)$$

(OH coil current density)

$$\sigma_{OH} = \frac{B_{OH}^2 R_{OH}}{2 \mu_0 \Delta t} \quad (61)$$

(OH coil tensile stress)

Both quantities  $j_{OH}$  and  $\sigma_{OH}$  are subject to upper limits. One can choose  $\Delta t$  such that for fixed  $R_{OH}^0$  either  $j_{OH}$  or  $\sigma_{OH}$  has a minimum value.  $j_{OH}$  takes its minimum for  $\Delta t = 2/3 R_{OH}^0$ , while  $\sigma_{OH}$  is minimal if  $\Delta t = 0.5 R_{OH}^0$ . Since both minima are relatively weak, the mean value  $\Delta t = 0.58 R_{OH}^0$  was taken in eq. (53).

#### 2.2.6 Figure of Merit

A rough cost estimation is used as a figure of merit for the optimization procedure to be described below.

$$\phi_1 \text{ [DM]} = C_{e1} \frac{\tau_{tot} (P_{ohm} + P_h)}{3.6} \quad (62)$$

(cost of electricity; see eqs. (22) and (31))

$$\phi_2 \text{ [DM]} = C_{\text{magn}} E_m \quad (63)$$

(investment cost)

$$\phi_3 \text{ [DM]} = C_T T_c \quad (64)$$

(cost of tritium; see eq. (25))

$$\phi \text{ [DM]} = \phi_1 + \phi_2 + \phi_3 \quad (65)$$

$\phi_1$  gives the cost of the consumed electric energy.  
 $\phi_3$  is the cost of the burned tritium. Both the cost of installation and the cost for the tritium inventory can be roughly included by taking enhanced values for  $C_{el}$  and  $C_T$ . Equation (63) estimates the investment costs by the stored magnetic energy.

### 2.3 Constraints and Optimization

In practice, several of the quantities resulting from the equations of the preceding section underly constraints, which are of a geometrical nature or stem from materials or technical limitations. The following constraints are involved in our model:

$$g \geq g^m \quad (66)$$

$$\Delta R \geq \Delta R^m \quad (67)$$

$$d \geq d^m \quad (68)$$

Here  $g^m$ ,  $\Delta R^m$  and  $d^m$  have the meaning outlined above.

$$\epsilon \leq \epsilon^m \quad (69)$$

$$\sigma \leq \sigma^m \quad (70)$$

$\epsilon^m$  is the maximum field ripple, while  $\sigma^m$  gives the maximum tensile stress in the TF coils.

The core constraint is taken into account by the conditions

$$\sigma_{OH} \leq \sigma_{OH}^m \quad (71)$$

and

$$j_{OH} \leq j_{OH}^m \quad (72)$$

where  $\sigma_{OH}^m$  is the maximum tensile stress in the OH coils, and  $j_{OH}^m$  the maximum OH coil current density.

The total burn time should not exceed the insulation lifetime, so that

$$\tau_{\text{iso}} \geq \tau_{\text{tot}} \quad (73)$$

must hold. This condition essentially determines the shielding thickness if  $\tau_{\text{tot}}$  is prescribed.

In certain cases (material tests, blanket test modules) the wall load  $N_n$  and the integral wall load  $W_{\text{int}}$  are limited to below:

$$N_n \geq N_n^m, \quad (74)$$

$$W_{\text{int}} \geq W_{\text{int}}^m. \quad (75)$$

It is pointed out that all the quantities  $g, \Delta R, d, \varepsilon, \sigma, \sigma_{\text{OH}}, j_{\text{OH}}, \tau_{\text{iso}}, N_n, W_{\text{int}}$  result, when evaluating the set of equations given in the preceding section.

In general, the constraints do not determine a unique solution of the basic equations. This means that generally for a wide range of input parameters  $a, A, s, \dots$  the basic equations can be solved while simultaneously meeting the constraints according to eqs. (66) to (75). A unique solution is found by determining that combination of input parameters that minimizes the figure of merit  $\emptyset$  as given by eq. (65).

The optimization procedure runs as follows. The input parameters of the second group in Table I are kept fixed throughout. They are either not subject to manipulations (such as  $q(a)$ ) and/or only have a weak influence on relevant quantities ( $l, T_0, \Delta T$  etc.). Intervals  $[a_1, a_2], [A_1, A_2], \dots, [C_1, C_2]$  are given for the input parameters of the first group. The six-dimensional space spanned by these intervals is covered with a six-dimensional, equidistant grid. For each grid point the computer runs through eqs. (1) to (65) of Sec. 2.2. Whenever the resulting  $\emptyset$  value is larger than the stored one or if any of the constraints is violated, the results for this grid point are ignored and the computation is repeated for the next point. In the opposite case the results for this grid point, including  $\emptyset$  and the values for  $a, A, \dots, C$ , are stored.

After the computation has been performed for all grid points, the stored one is that yielding a solution with minimum  $\emptyset$ . Around this point a new grid of smaller size is defined and the whole procedure is repeated. This procedure is iterated to the desired accuracy.

Owing to its discrete nature the coil number  $N$  requires a distinct treatment. At first the optimum solutions are determined as described above,  $N$  being kept fixed. After repetition for all  $N$  with  $N_1 \leq N \leq N_2$ , where  $N_1, N_2$  are input parameters, the one that yields a minimum value for  $\emptyset$  is selected and taken as final solution.

The number of grid points, the number of subsequent grids and the reduction factor for the grid size can be prescribed as input parameters.

The number of grid points, and hence the computation time, strongly increases with the grid dimension. In order not to exceed the number of six variables,  $R_2$  was chosen according to eq. (9). This choice affords a trivial optimization since any reasonable figure of merit is minimized by choosing  $R_2$  in this way.

If the input intervals  $[a_1 a_2]$ ,  $[A_1 A_2]$ , ...,  $[C_1 C_2]$ ,  $[N_1 N_2]$  are appropriately chosen, the resulting values for  $a$ ,  $A$ ,  $\dots$ ,  $C$ ,  $N$  should not coincide with either of the boundaries. This, however, may be impossible if the range of some variable is limited by physical or technical constraints. The elongation  $s$ , for instance, generally tends to take the maximum value  $s_2$ . The upper boundary  $s_2$  of the input interval thus practically constitutes an additional constraint. A similar situation prevails for the ignition factor  $C$ , which generally takes its minimum value  $C_1$ , except when  $N_n^m$  is sufficiently high. In such cases the respective variable can be explicitly prescribed. The grid dimension, and hence the computation time, is then considerably reduced.



The optimization procedure described yields sufficient accuracy with modest computation time. A typical run with maximum grid dimension yielding an accuracy of a few p.p.th. requires less than 60 s on a CRAY I. A significant advantage of the method lies in the simplicity with which constraints can be taken into account.

### 3. RESULTS

The following three systems are discussed in greater detail and taken as reference points for subsequent parameter studies:

- I A pure physics experiment to study ignited, long burning D-T plasmas.
- II An ignited, long burning tokamak which has a neutron wall load and integral neutron flux that allows material tests.
- III A system with the same specifications as II but with additional space for the implementation of blanket test modules.

The complete set of input parameters for these three systems is given in Table II. In all three cases  $f = 2$  was set in eq. (59), allowing burn times of up to roughly one plasma  $L_p/R_p$ -time.

For system I the required integral burn time would follow from the envisaged experimental programme and hence is explicitly given. For systems II and III  $\tau_{tot}$  is self-consistently determined from  $W_{int}$  and  $N_n$ .

The complete set of output data is listed in Table III. In all three cases  $s$  and  $C$  take the boundary values,  $s_2$  and  $C_1$  respectively. With respect to the core constraint eq. (72) is typically more restrictive than eq. (71). As expected,  $D$  always adjusts so that  $\tau_{\text{iso}} = \tau_{\text{tot}} \cdot \sigma$  is always close to  $\sigma^m$ . In cases I and II the coil number  $N$  is mainly determined by the conditions  $d \geq d^m$  and  $\epsilon \leq \epsilon^m$ , resulting in a value  $\Delta R \geq 0$ . In system III the condition  $\Delta R \geq \Delta R^m$  is more restrictive, and solutions exist with different  $N$  but identical  $\emptyset$  that satisfy both eq. (68) and eq. (69). Among these the solution with minimum  $N$  was selected, yielding maximum  $d$ .

Several important conclusions can be drawn from Table III:

- Stationary cooling can be achieved without difficulties in all three cases. For typical values of  $F$ ,  $F_I$ ,  $n_L$ ,  $n_S$ ,  $T_O$  and  $\Delta T$  all critical parameters such as  $T_{\text{max}}$ ,  $\Delta P$  are found to be on the safe side. Furthermore, the flow velocity is sufficiently low to avoid cavitation.
- The resistive losses do not exceed 500 MW. The systems could thus be fed from the grid in principle.

- For all three systems both the cost of electricity and the cost of tritium are at least one order of magnitude smaller than the investment costs. Hence, though the estimation of the investment costs is very rough, all three systems would probably be dominated by the investment costs.
  
- There is a relatively big jump in stored magnetic energy between systems I and II. On the other hand, there is only a modest increase in stored energy when going from system II to III. Systems of type I (pure physics experiment) or type III (physics, materials testing, blanket test modules) are natural candidates for next-generation tokamaks with normal conducting TF coils.

The solutions of the model are largely determined by the constraints. In Fig. 3 the sensitivity of  $\phi$  to variations of the most relevant constraints and the integral burn time  $\tau_{\text{tot}}$  is visualized. System I was taken as reference system. The strong dependence on  $s$  is striking. It explains the general tendency of the systems to take the maximum possible value for  $s$ . Considering the dependence on  $\tau_{\text{tot}}$  and  $d^m$ , one recognizes that there is a continuous transition from system I to those of type II or III.

In our examples, since the cost of electricity does not significantly contribute to  $\phi$ ,  $\Delta$  reduces until  $\sigma$  reaches its limit value. There may be additional constraints, such as political ones, that make it desirable to reduce the resistive losses by increasing the coil thickness, despite the resulting increase in stored energy. Figure 4 shows how  $\phi$  and  $P_{\text{ohm}}$  vary if  $\Delta$  deviates from its optimum value. System I was taken as reference case. With increasing  $\Delta$  the relative gain in  $P_{\text{ohm}}$  equals the relative loss in  $\phi$ . A drastic reduction of the resistive losses by increasing the coil thickness would thus probably lead to an unacceptably high stored energy.

In devices the main purpose of which is neutron production it might be advantageous to operate in the non-ignited mode and drive the system by additional heating. This can be crudely simulated by taking  $C_s > 0$ . Figure 5 shows the variation of  $\phi$  with  $C_s$ . System III was taken as reference system. The discontinuities in Fig. 5 are related to jumps in the coil number  $N$ . Additional heating only has a weak effect on  $\phi$ , the reason being that  $C$  increases with increasing  $C_s$  so that  $C_{\text{eff}} = C(1 - C_s)$  remains nearly constant. The increase of  $C$  above its lower limit is required to keep  $B_t^0$  and hence  $N_n$  sufficiently high to meet the condition  $N_n \geq N_n^m$ .

#### 4. CONCLUSION

Water-cooled, steady-state resistive TF coil systems have been studied for application in next-generation ignited tokamaks. D-shaped, tape-wound coils were selected to minimize risk. Tried fabrication techniques are available for such coils, and a TF system with specifications close to those of next-generation tokamaks has been successfully manufactured for the JET device.

The feasibility of such systems was studied within the framework of a self-consistent layout model. Besides describing the main system components (plasma, TF coils, OH system, cooling), this model takes into account all relevant technical and geometrical constraints. The resulting data sets describe systems which, in addition, are optimized with respect to cost relevant quantities.

For a wide range of specifications (pure physics experiment, materials testing and blanket testing facilities) our investigation yields the following main results:

- The TF coils can be operated in steady-state without difficulties (low pressure drop in the coolant channel, low flow velocities).
- The resistive losses are sufficiently low to feed the TF coil system from the grid.

- The cost of electricity is relatively small compared with investment costs.
- The coils can be operated at low operational load (low mechanical stresses, tensile stresses below 80 MPa).

The main purpose of this paper is to assess the basic feasibility of normal conducting TF coil systems for long-pulsed, ignited tokamaks. When comparing such systems with, for instance, superconducting TF coil systems, complete lines of development towards a demonstration plant have ultimately to be compared from a cost-risk point of view. As a contribution to such an undertaking, similar calculations for superconducting TF coil systems are under preparation<sup>(12)</sup>.

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INPUT PARAMETERS

a	(m)	plasma minor radius	
A		aspect ratio	<u>GROUP I</u>
s		elongation	
D	(m)	shielding thickness	selfconsistently
$\Delta$	(m)	coil thickness	determined
C		ignition factor	
-----			
q(a)		plasma safety factor at boundary	
l	(m)	scrape-off layer thickness	
u	(m)	vessel thickness	
F		total filling factor	
$F_i$		isolation filling factor	
$n_L$		layers per pancake	
m		number of pancakes	<u>GROUP II</u>
$n_c$		number of cooling channels per layer	fixed
$n_s$		number of cooling channels per pancake	
$T_o$	(C <sup>o</sup> )	coolant inlet temperature	
$\Delta T$	(C <sup>o</sup> )	coolant temperature rise	
$t_{tot}$	(s)	integral burn time	
$C_{mag}$	(DM MJ <sup>-1</sup> )	specific cost of stored magnetic energy	
$C_{el}$	(DM kWh <sup>-1</sup> )	specific cost of electric energy	
$C_{trit}$	(DM g <sup>-1</sup> )	specific tritium cost	
-----			
$\epsilon^m$	(%)	maximum allowable field ripple	
$s^m$		maximum elongation	
$\sigma^m$	(MPa)	maximum tensile stress	
g <sup>m</sup>	(m)	minimum value for g (see Fig. 1)	
d <sup>m</sup>	(m)	minimum distance between two coils (equatorial plane)	<u>GROUP III</u>
$\sigma_{OH}^m$	(MPa)	maximum OH-trafo tensile stress	constraints
$j_{OH}^m$	(A m <sup>-2</sup> )	maximum OH-current density	
$\Delta R^m$	(m)	minimum value for $\Delta R$ (see Fig. 1)	
$N_n^m$	(NW m <sup>-2</sup> )	minimum neutron wall load	
$W_{int}$	(MW y m <sup>-2</sup> )	minimum integral wall load	
$\Delta P^m$	(at)	maximum pressure drop per channel	
$C^m$		minimum ignition factor	

Table I

Set of input parameters required for solution of the basic equations

		<u>I N P U T</u>			
SYSTEM		I	II	III	
<u>CONSTRAINTS</u>					
$\epsilon^m$	(%)	1	-	-	maximum allowable field ripple
$s^m$		1.5	-	-	maximum elongation
$\sigma^m$	(MPa)	80	-	-	maximum tensile stress
$g^m$	(m)	1.0	-	-	minimum value for g (see Fig. 1)
$d^m$	(m)	0.65	-	-	minimum distance between two coils (equatorial plane)
$\sigma_{OH}^m$	(MPa)	150	-	-	maximum OH-trafo tensile stress
$j_{OH}^m$	(A m <sup>-2</sup> )	$1.5 \cdot 10^7$	-	-	maximum OH-current density
$\Delta R^m$	(m)	0	-	1	minimum value for $\Delta R$ (see Fig. 1)
$N_n^m$	(MW m <sup>-2</sup> )	0	1	-	minimum neutron wall load
$W_{int}^m$	(MW y m <sup>-2</sup> )	0	1	-	minimum integral wall load
$\Delta p^m$	(at)	5	-	-	maximum pressure drop per channel
$C^m$		2.5	-	-	minimum ignition factor
<u>FIXED INPUT PARAMETERS</u>					
$q(a)$		2.5	-	-	plasma safety factor at boundary
$l$	(m)	0.09	-	-	scrape-off layer thickness
$u$	(m)	0.1	-	-	vessel thickness
$F$		0.7	-	-	total filling factor
$F_I$		0.8	-	-	isolation filling factor
$n_L$		16	-	-	layers per pancake
$m$		2	-	-	number of pancakes
$n_C$		2	-	-	number of cooling channels per layer
$n_S$		32	-	-	number of cooling channels per pancake
$T_0$	(C°)	20	-	-	coolant inlet temperature
$\Delta T$	(C°)	40	-	-	coolant temperature rise
$\tau_{tot}$	(s)	$2 \cdot 10^6$	+	+	integral burn time
$C_{mag}$	(DM MJ <sup>-1</sup> )	$1.5 \cdot 10^6$	-	-	specific cost of stored magnetic energy
$C_{el}$	(DM kWh <sup>-1</sup> )	0.15	-	-	specific cost of electric energy
$C_{trit}$	(DM g <sup>-1</sup> )	$7 \cdot 10^3$	-	-	specific tritium cost

Table II

Complete list of input parameters for reference systems I, II and III.  $C_{mag}$  was estimated from the approximate cost of  $2 \times 10^9$  DM for the extended JET version with  $E_m \approx 1.4$  GJ.

OUTPUT

SYSTEM	I	II	III	
<u>GEOMETRY</u>				
a (m)	1.56	1.43	1.53	plasma minor radius
A	2.50	2.91	2.78	aspect ratio
s	1.5	1.5	1.5	elongation
D (m)	0.19	0.37	0.37	radiation shield thickness
$\Delta$ (m)	0.50	0.69	0.65	coil thickness
$\Delta R$ (m)	0.70	0.88	1.0	see Fig. 1
$R_1$ (m)	1.71	1.83	1.84	see Fig. 1
$R_2$ (m)	6.78	7.37	7.65	see Fig. 1
d (m)	0.65	0.65	0.74	distance between two coils (equatorial plane)
g (m)	1.16	1.48	1.62	see Fig. 1
N	28	22	22	number of coils
<u>PLASMA</u>				
$\bar{\beta}_t$ (%)	7.20	6.18	6.46	average toroidal beta
$\epsilon$ (%)	0.99	1.00	0.97	field ripple
$B_t^0$ (T)	3.39	3.83	3.62	toroidal field at axis
$P_f$ (MW)	385	411	419	total fusion power
C	2.5	2.5	2.5	ignition factor
$N_n$ ( $MW m^{-2}$ )	0.98	1.07	1.00	neutron wall load
$W_{int}$ ( $MW y m^{-2}$ )	$6.22 \times 10^{-2}$	1.00	1.00	integral wall load
$\tau_{tot}$ (s)	$2 \times 10^6$	$2.95 \times 10^7$	$3.14 \times 10^7$	total burn time
<u>TORUS SYSTEM</u>				
$B_m$ (T)	6.74	7.32	7.10	maximum toroidal field
$\sigma$ (MPa)	80	80	80	tensile stress
$P_{OHM}$ (MW)	460	484	505	resistive coil losses
$E_m$ (GJ)	4.24	6.82	6.93	stored magnetic energy
$M_{Cu}$ (kg)	$8.14 \times 10^5$	$1.35 \times 10^6$	$1.35 \times 10^6$	mass of copper
j ( $A m^{-2}$ )	$1.76 \times 10^7$	$1.44 \times 10^7$	$1.46 \times 10^7$	peak current density
$\sigma_D$ (MPa)	23.8	30.1	27.8	pressure on the support cylinder
<u>OH-TRAFO</u>				
$\sigma_{OH}$ (MPa)	107	111	114	OH-trafo tensile stress
$J_{OH}$ ( $A m^{-2}$ )	$1.50 \times 10^7$	$1.50 \times 10^7$	$1.48 \times 10^7$	OH-trafo current density
<u>COOLING SYSTEM</u>				
$T_{max}$ ( $C^0$ )	70.4	72.9	72.0	maximum coil temperature
$d_c$ (m)	$1.95 \times 10^{-2}$	$2.67 \times 10^{-2}$	$2.61 \times 10^{-2}$	hydraulic diameter of cooling channel
$\Delta P$ (at)	4.40	1.81	2.35	pressure drop per channel
$L_{pump}$ (MW)	1.21	0.52	0.71	pumping power
Q ( $m^3 s^{-1}$ )	2.75	2.89	3.01	total coolant volume flow
v ( $m s^{-1}$ )	5.11	3.65	4.00	coolant flow velocity
<u>FIGURES OF MERIT</u>				
$\theta_1$ (DM)	$3.83 \times 10^7$	$5.95 \times 10^8$	$6.61 \times 10^8$	cost of electricity
$\theta_2$ (DM)	$6.36 \times 10^9$	$1.02 \times 10^{10}$	$1.04 \times 10^{10}$	investment cost
$\theta_3$ (DM)	$9.58 \times 10^6$	$1.51 \times 10^8$	$1.64 \times 10^8$	tritium cost
$\theta$ (DM)	$6.40 \times 10^9$	$1.10 \times 10^{10}$	$1.12 \times 10^{10}$	total cost

Table III

Complete list of output data for reference systems I, II and III

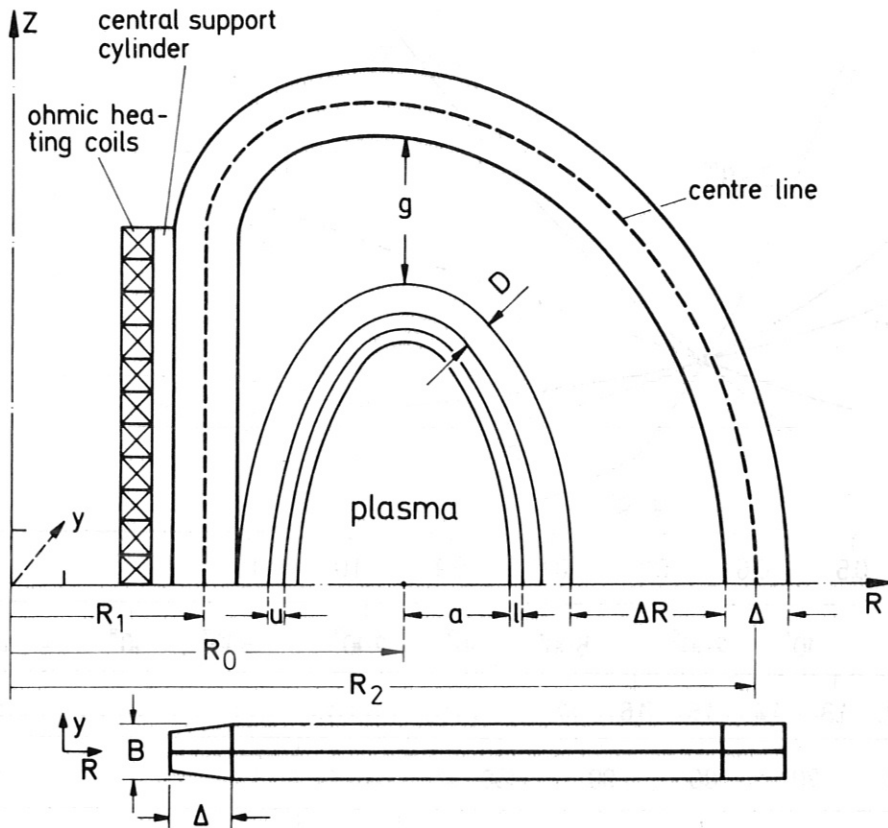


Fig. 1

Geometrical arrangement of main components

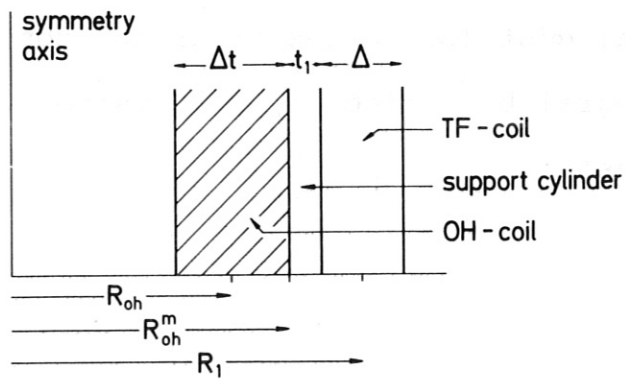


Fig. 2

Geometrical arrangement of the OH trafo and central support cylinder

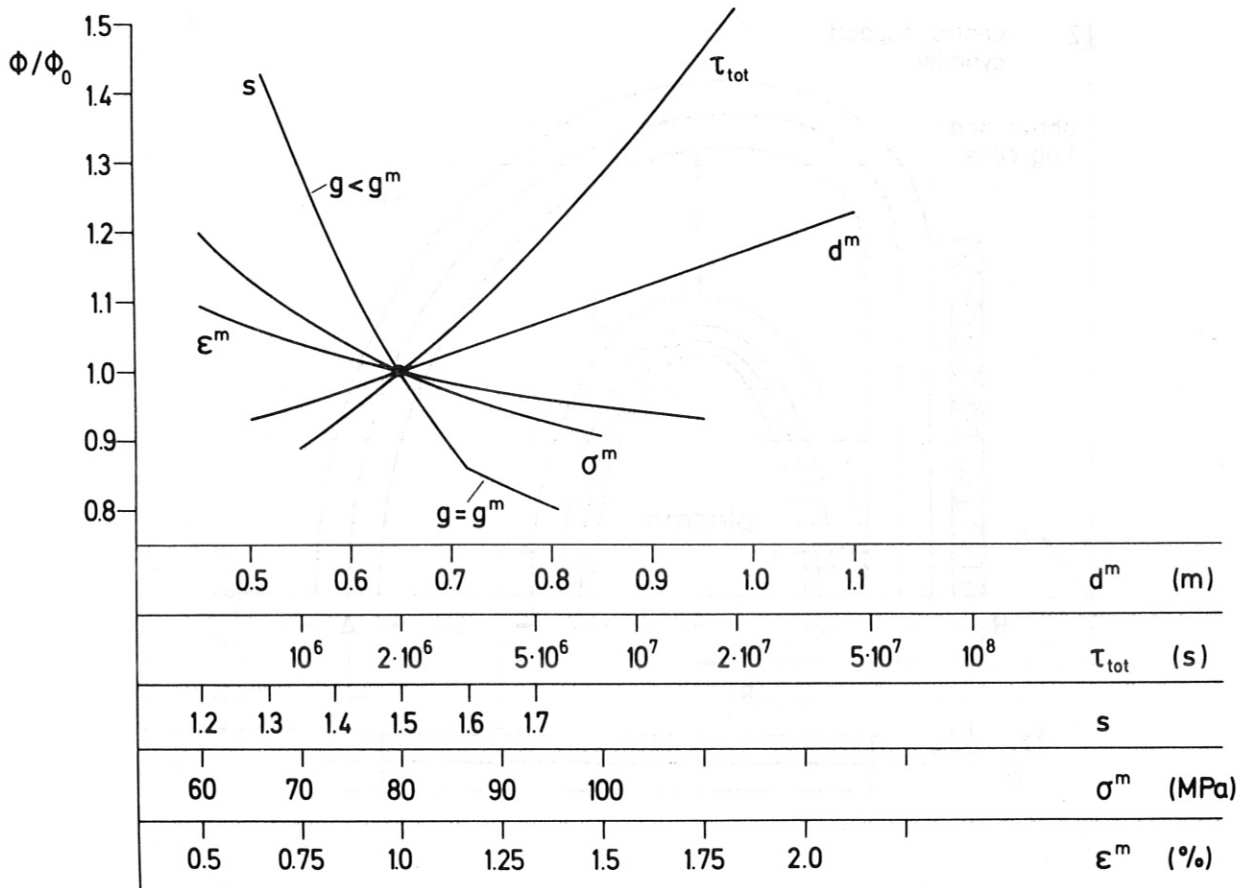


Fig. 3

Variation of the total cost ( $\Phi$ ) versus important constraints and the integral burn time ( $\tau_{tot}$ ). System I is taken as reference system

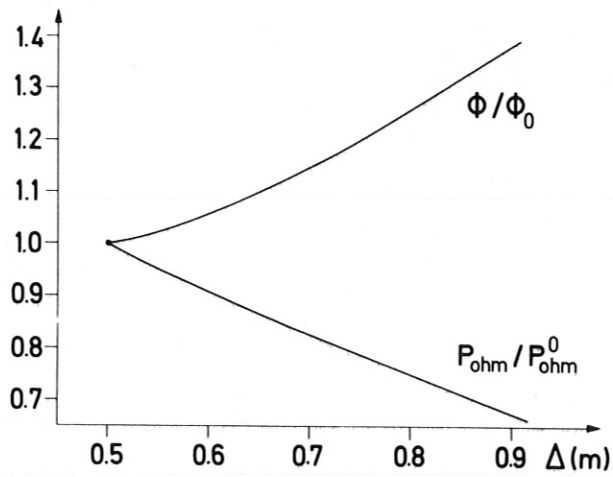


Fig. 4

Sensitivity of the total cost ( $\phi$ ) and resistive losses ( $P_{ohm}$ ) versus variations of the coil thickness ( $\Delta$ ). System I is taken as reference system

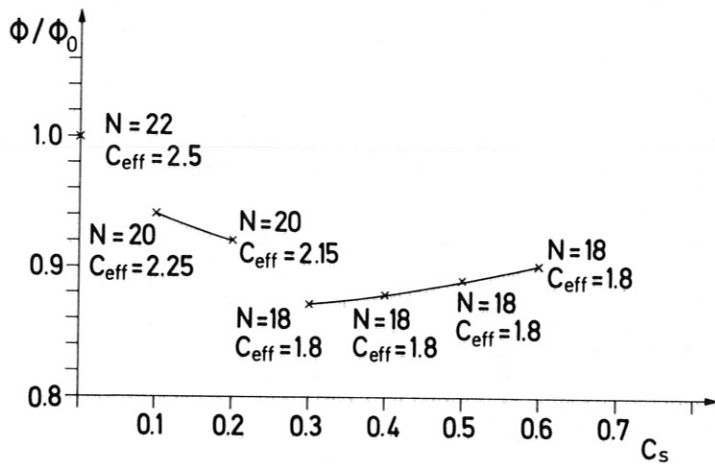


Fig. 5

Total cost variation in the case of a non-ignited, driven system.  $C_s$  measures the fraction of the plasma losses that is delivered by additional heating. System III is taken as reference system.