Static or Feedback Stabilization of the Burn in a Tokamak

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Abstract

The control of the burn in an ignited Tokamak using a space and time dependent external vertical magnetic field is discussed. It is shown that a static field, suitably shaped in space, is able to stabilize the burn for a certain range of the plasma parameters of physical interest. An oscillating magnetic field with constant frequency and amplitude fixed by the initial plasma parameters stabilizes the burn in all situations.

1. INTRODUCTION

One of the main problems which are met in the Tokamak operation in the ignited regime is represented by the thermal runaway associated with the α -heating exceeding the energy losses. Several methods were recently proposed in order to overcome this difficulty based either on an active control of the auxiliary heating or of a passive control of the thermal conductivity, as for instance by means of a magnetic ripple or also using compression-decompression cycles /1, 2/. It seems, however, to have escaped the general attention that a static vertical magnetic field B_0 with a suitable index $n = -(R/B)\partial B/\partial R$ is quite sufficient for stabilizing the burn in a significant range of plasma parameters. This possibility was also independently indicated by Wilhelm.

In the present report the theory of this effect will be developed starting from the space averaged transport equation. The conditions for the existence of an asymptotic state of equilibrium, which implies the stabilization of the burn are determined taking into account the non-linearities which eventually arise in situations when the initial unstable state of the ignited plasma is far from the marginal point of ignition. One can show that stabilization can be achieved in a certain range of values of the plasma parameters and of the index of the vertical field.

This range decreases with decreasing poloidal β and with the increasing power of the temperature dependence of the confinement time. It is then advisable to find a stabilization method which is not subject to these restrictions. We shall show that, at least in cases not too far from the point of ignition, a small modulation of the vertical field with a time period of the order of the confinement time, is sufficient to stabilize the burn in all situations provided that the amplitude of the modulation is prescribed in terms of the plasma parameters of the initial ignited state. This feedback stabilization has the advantage to allow a direct conversion of a fraction of the fusion power, because it can be assimilated to a succession of small compression - decompression cycles /2/.

2. BASIC EQUATIONS

We consider the equation of energy conservation including α- heating, transport and the external work p dV. The density of α- particles is neglected and the α- energy deposition is assumed to be instantaneous. Defining $n = n_i = n_e$ and assuming $T_e = T_i = T$

the global energy balance takes the form (see e.g. Ref. /1/)

$$\frac{d}{dt}\bar{p} = \alpha \bar{p}^2 - \bar{p}\left(\frac{1}{2\tau(t)} + \frac{5}{3}\frac{1}{V}\frac{dV}{dt}\right) \tag{1}$$

where $\bar{p} = \overline{nT}$ and the bar denotes a volume average except for temperatures where $\bar{T} = \overline{nT}/\bar{n}$. The bar will be omitted from now on for convenience. The α - production has been taken to be proportional to p^2, which is a good approximation between 7 and 20 keV; α is a coefficient which includes the effect of space averaging on the α - heating and which also takes into account the possible imperfect confinement of non thermalized α particles.

The quantity $\tau(t)$ is defined by the expression

$$\frac{1}{\tau} = \frac{1}{\tau_e} + \frac{1}{\tau_i} \tag{2}$$

where τ_e and τ_i are the confinement times for electrons and ions respectively. In the present Tokamaks $\tau_i \gg \tau_e$ so that $\tau \approx \tau_e$. The scaling $\tau \sim n a^2 T^m$ is considered throughout. Assuming particle conservation (this can be realized for instance, by a rail limiter of appropriate shape and complete recycling) one has $V \sim n^{-1}$. Thus one has

$$\tau = \tau_o \left(\frac{V_o}{V}\right)^{1/2} \left(\frac{T}{T_o}\right)^m \tag{3}$$

The change in the plasma volume is accompanied by a shift of the plasma column from the original position R_o to $R_o + \Delta$. It is convenient to express the basic equation (1) in terms of the unknowns p and Δ, using the expressions

$$\frac{1}{V}\frac{dV}{dt} = \frac{2}{R_o\left(1+\frac{\Delta}{R_o}\right)}\frac{d\Delta}{dt} \qquad \tau(t) = \tau_o\left(\frac{p}{p_o}\right)^m\left(1+\frac{\Delta}{R_o}\right)^{2m-1} \tag{4}$$

The shift Δ is determined by the well known Shavranov equation for the equilibrium

$$B_1(R_o+\Delta, t) + B_o(R_o+\Delta) = B_V(R_o+\Delta, p_o+\Delta p) \tag{5}$$

where

$$B_v = \frac{IR}{c}\left\{\left(\ln\frac{8R}{a} + \frac{l_i}{2} - \frac{3}{2}\right)/R^2 + \frac{4\pi p c^2 a^2}{I^2 R^2}\right\} \tag{6}$$

with $R = R_o + \Delta$, $p = p_o + \Delta p$; $B_1(R,t)$ is an external perturbation of the original external vertical field $B_o(R)$ and the zero order quantities R_o, p_o satisfy the equation

$$B_o(R_o) = B_v(R_o, p_o) \tag{7}$$

Eq. (5) defines implicitly a function $\Delta = \Delta(p, R_o, t)$. In defining this function through (5) one must take into account the IR is constant in time. This follows from the conservation of the poloidal flux. The toroidal flux conservation gives $a^2 = a_o^2 R/R_o$.

Using (4), the basic equation (1) takes the general form:

$$\dot{p}\left(1+\frac{10}{3}p\frac{1}{R_o(1+\frac{\Delta}{R_o})}\frac{\partial\Delta}{\partial p}\right) = \alpha p^2 - p^{1-m}\left(1+\frac{\Delta}{R_o}\right)^{1-2m}\frac{1}{2\tau_o}p_o^m - \frac{10}{3}p\frac{1}{R_o(1+\frac{\Delta}{R_o})}\frac{\partial\Delta}{\partial t} \tag{8}$$

The theorem on implicit functions applied to $\Delta(p)$ as defined by (5) allows to express $\partial\Delta/\partial p$ as follows

$$\frac{\partial\Delta}{\partial p} = -\frac{1}{\gamma}\frac{\partial B_v}{\partial p} \tag{9}$$

where

$$\gamma = \left(\frac{\partial B_v}{\partial R}\right)_{IR=\mathrm{const}} - \frac{\partial B_o}{\partial R} - \frac{\partial B_1}{\partial R} \tag{10}$$

$$\frac{\partial B_v}{\partial p} = \frac{4\pi c a_o^2 R}{IR\, R_o} = \frac{\beta_\theta I}{p c R} = \frac{4\pi c a_o^2}{IR}\left(1+\frac{\Delta}{R_o}\right) \tag{11}$$

From (8) one has that every stationary asymptotic state p_∞ must be a solution of the equation

$$\alpha\, p_{\infty} - \left(\frac{p_0}{p_{\infty}}\right)^{m} \left(1 + \frac{\Delta(p_{\infty})}{R_0}\right)^{1-2m} \frac{1}{2\tau_0} = 0 \tag{12}$$

The question to be discussed is then the following: given an asymptotic state p_{∞}, defined by (12), under what conditions the equation (8) admits a solution which tends asymptotically to p_{∞} ?

3. STATIC CASE ($B_1 = 0$)

a) Alcator Scaling ($m = 0$)

We consider first the case without time dependence in the external field and with the Alcator scaling $\tau \sim n a^2$. Putting in (8) $y = p - p_{\infty}$, $x = t^{-1}$ and linearizing with respect to y one arrives at the equation

$$-\frac{d}{dx} \lg y = \frac{p_{\infty}}{x^2}\left[\alpha + \left(\frac{\beta_\theta I}{pcR}\right)_{\infty} \frac{1}{2R_0\gamma_{\infty}\tau_0}\right]\left[1 - \frac{10}{3}\left(\frac{\beta_\theta I}{pcR}\right)_{\infty} \frac{p_{\infty}}{R_{\infty}\gamma_{\infty}}\right]^{-1} \tag{13}$$

which describes the behaviour of the pressure for $t \to \infty$ (or $x \to 0$) in the neighbourhood of the asymptotic state p_{∞}. Integrating (13) one sees that in order that $y \to 0$ for $x \to +0$ one must have

$$\left[1 + \left(\frac{\beta_\theta I}{pcR}\right)_{\infty} \frac{p_i}{R_0\gamma_{\infty}}\right] \cdot \left[1 - \frac{10}{3}\left(\frac{\beta_\theta I}{pcR}\right)_{\infty} \frac{p_{\infty}}{R_{\infty}\gamma_{\infty}}\right]^{-1} < 0 \tag{14}$$

where $p_i \equiv (2\alpha\tau_0)^{-1}$. It is convenient to express the condition (14) with respect to the quantity $R_0\gamma_{\infty}$ which is directly related to the plasma parameters by the relation

$$R_0\gamma_{\infty} = R_0\left\{\frac{I_0 R_0}{cR^3}\left[\frac{7}{2} + \beta_\theta - 2\lg\frac{8R}{a} - l_i\right] - \frac{\partial B_0}{\partial R}\right\}_{R=R_{\infty},\, p=p_{\infty}} \tag{15}$$

One has then

$$-\left(\frac{\beta_\theta I}{pcR}\right)_{\infty} p_i < R_0\gamma_{\infty} < \frac{10}{3}\left(\frac{\beta_\theta I}{cR^2}\right)_{\infty} R_0 \tag{16}$$

In the linear limit $p_\infty \sim p_i$, $R_o \sim R_\infty$ and the condition above can be written in the form

$$- \beta_v q_v < n + \frac{I}{cR}\left(\frac{7}{2} - 2\lg\frac{8R}{a} - l_i\right)\frac{1}{B_0} < \frac{7}{6}\beta_v q_v \tag{17}$$

where all quantities refer to the initial state and

$$n \equiv - \frac{R}{B_0}\frac{\partial B_0}{\partial R} \quad , \quad \beta_v \equiv \frac{16\pi p_0}{B_0^2} \quad , \quad q_v \equiv \frac{a_0^2 B_0 c}{2RI} \tag{18}$$

b) <u>Scaling $\tau \sim n a^2 T^m$</u>

The discussion above can be repeated without difficulty in the case $m \neq 0$ and the range of stabilization is now given by the inequality

$$\frac{(2m-1)\left(\frac{\beta_\theta I}{pcR}\right)_\infty \left(\frac{p_0}{p_\infty}\right)^m p_i}{\left(1+\frac{\Delta(B_\infty)}{R_0}\right)^{-2m}\left[1+\frac{p_i}{p_\infty}\left(\frac{p_0}{p_\infty}\right)^m\left(1+\frac{\Delta(p_\infty)}{R_0}\right)^m\right]^{1-2m}} < R_0\gamma_\infty < \frac{10}{3}\left(\frac{\beta_\theta I}{pcR}\right)_\infty p_\infty \frac{R_0}{R_\infty} \tag{19}$$

One sees that only the lower limit is affected by the temperature dependence, the range of stabilization becoming narrower with increasing m. In particular a situation with $\gamma_\infty < 0$ and $m > \frac{1}{2}$ cannot be stabilized.

4. FEEDBACK STABILIZATION ($B_1 = A \sin\omega t$)

We limit our considerations to the linear approximation and to the Alcator scaling. We make the "Ansatz"

$$B_1(R_0, t) = h \sin\omega t + (p - p_0)\frac{\partial B_v}{\partial p_0} \tag{20}$$

and look whether the amplitude h can be determined in such a way that a bounded oscillatory solution for p(t) and B (t) exists. The equation (8) for p takes the form

$$\dot{p} = \alpha p^2 - \frac{p}{2\tau_0}\left(1 + \frac{\Delta}{R_0}\right) - \frac{10}{3} p \frac{\dot{\Delta}}{R_0} \tag{21}$$

where the shift Δ, which is determined by the linearized equation (5), is given by the simple expression

$$\Delta = \frac{h}{\gamma} \sin \omega t \tag{22}$$

The solution p(t) is then the following:

$$p(t) = p_0 \frac{q_0(t) + O\left(\frac{\Delta^2}{R^2}\right)}{-p_0\alpha + K \exp t/2\tau_0 + O\left(\frac{\Delta^2}{R^2}\right)} \tag{23}$$

where

$$q_0(t) = 1 + \frac{h}{\gamma R_0}\left[\frac{1}{2\tau_0\omega}(\cos\omega t - 1) - \frac{10}{3}\sin\omega t\right]$$

$$q_1(t) = -2\tau_0 + \frac{h}{\gamma R_0}\frac{\tau_0}{1+4\tau_0^2\omega^2}\left[\frac{26}{3}\sin\omega t + \left(\frac{40}{3}\omega^2\tau_0^2 - 1\right)\frac{\cos\omega t}{\omega\tau_0} + \frac{1+4\tau_0^2\omega^2}{\omega\tau_0}\right]$$

$$K = 1 - 2p_0\tau_0\alpha + \frac{h}{\gamma R_0}\frac{52}{3}p_0\tau_0\alpha\frac{\tau_0\omega}{1+4\tau_0^2\omega^2} \tag{24}$$

The condition for a bounded oscillation is K = 0 or

$$\frac{h}{R_0\gamma} = \frac{3}{26}\,\frac{p_0 - p_i}{p_i}\,\frac{1+4\tau_0^2\omega^2}{\tau_0\omega} \tag{25}$$

This expression is minimum for $\tau_0\omega = 1/2$.

Substitution of p(t) into (20) gives the following expression for the external feedback field

$$B_1 = \frac{p_0 - p_i}{p_i}\left(\frac{\beta_\theta I}{cR}\right)_0\left[-1 + A\sin(\omega t + \delta)\right] \tag{26}$$

where

$$A = \left(1 + a^2\right)^{1/2}, \quad \delta = \operatorname{arctg} a^{-1} \tag{27}$$

with

$$a = \frac{3}{26\tau_0\omega}\left[1 - \frac{40}{3}\tau_0^2\omega^2 + \frac{R_0^2\gamma c}{\beta_\theta I_0}\left(1 + 4\tau_0^2\omega^2\right)\right] \tag{28}$$

In practical situations h can never be chosen as to satisfy rigorously the condition (25). Let the real h differ of the amount Δh from the ideal h given by expression (25). Then the solution (23) for p(t) will still be approximately oscillatory and bounded during a time Δt satsifying the condition (see equations (23) and (24))

$$\frac{\Delta h}{\gamma R_0}\,\frac{26}{3}\,\frac{p_0}{p_i}\,\frac{\tau_0\omega}{1 + 4\tau_0^2\omega^2}\exp\frac{\Delta t}{2\tau_0} \ll 1 \tag{29}$$

so that the percentage error which is tolerable in order to stabilize the burn during the time Δt must satisfy the condition

$$\frac{\Delta h}{h} \ll \frac{\exp - \Delta t/2\tau_0}{(p_0 - p_i)/p_i} \tag{30}$$

The results above can be easily extended using linearization to the more general scaling $\tau \sim n a^2 T^m$.

5. RELATIONSHIP BETWEEN THE STATIC AND THE FEEDBACK STABILIZATION

The frequency ω of the feedback field is arbitrary and one is free to choose it in such a way that the amplitude A of the oscillating field is minimum. This is obtained by satisfying the condition a = 0 where a is given by (28). However, this is only possible when the plasma parameters and the index are such as to satisfy the relation

$$R\gamma = \frac{\beta_\vartheta I}{cR}\left(\frac{40\tau^2\omega^2}{3} - 1\right)\frac{1}{1+4\tau_0^2\omega^2} \tag{31}$$

It is easy to see that this relation can hold only when $R\gamma$ is comprised within the limits (16) for which the static stabilization is possible. In situations in which $R\gamma$ is outside these limits the amplitude of the feedback field connot be minimum.

6. CONCLUSION

We have shown that the burn in a Tokamak can be stabilized by the static vertical field in a given range of equilibrium parameters or by an oscillating feedback vertical field in all equilibrium situations. The latter method can be assimilated to a succession of compression-decompression cycles and then it has the advantage to allow direct energy conversion of a fraction of the fusion power /2/. If $P_\alpha = 3\alpha p_i^2 V$ is the total α - power heating at ignition, the fraction of fusion power directly recovered in the circuits of the feedback field is given by the expression

$$P \equiv \frac{\omega}{2\pi} 2\int_0^{2\pi/\omega} p(t)\frac{dV}{dt}dt = \frac{3}{13}\frac{p_i V}{\tau_0}\left(\frac{p_0 - p_i}{p_i}\right)^2\left(1+4\tau_0^2\omega^2\right) = \frac{2}{13}P_\alpha\left(\frac{p_0-p_i}{p_i}\right)^2\left(1+4\tau_0^2\omega^2\right) \tag{32}$$

The recovered power increases quadratically with ω . However, an upper bound to the frequencies which can be considered in Eq. (32) is imposed by the validity of the linear approximation.

/1/ K. Borrass, K. Lackner, E. Minardi, 9th Europ. Conf. on Controlled Fusion and Plasma Physics, Oxford 1979

/2/ E. Minardi, IPP 1/171, August 1979