Separatrix Location Using Magnetic Measurements in ASDEX Tokamak

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Abstract

A technique for determining the location of the separatrix in the magnetic flux distribution produced by a tokamak plasma with a poloidal divertor is described and its accuracy assessed. The method assumes the plasma to be in equilibrium and calculates a current density distribution from a least squares fit to magnetic flux and field measurements outside the plasma. Subject to some reasonable restrictions on the form of the current density profile, a reliable indication of the current distribution within the plasma may be obtained if, in addition, the poloidal beta of the plasma is known.

Introduction

The operation of a poloidal divertor tokamak such as ASDEX /1/ requires precise information about the location of the separatrix, both to ensure that a suitable magnetic configuration for operation of the divertor is achieved as well as to aid in interpretation of experimental investigations of plasma behaviour in the scrape-off layer. Because the separatrix is a purely geometric property of the magnetic field distribution, its location cannot be determined directly and must therefore be deduced from calculations of the magnetic fields produced by currents in the plasma and external conductors.

In order to determine the magnetic field structure over the entire cross-section of the discharge chamber, one would require detailed measurements of the current density distribution within the plasma. As experimental techniques for obtaining such information with a sufficient degree of accuracy do not currently exist, some assumptions about the nature of the current distribution must be made in order to constrain the range of magnetic field configurations thereby obtained to a sufficient degree that the available measurements can be used to obtain an accurate indication of the separatrix location.

The basic assumption used in the technique to be described below is that the plasma is in a state of equilibrium which thereby implies that the current density may be expressed as a function of the poloidal flux only. By assuming a certain functional form for this dependence, it has been found that the parameters of this function may be adjusted to fit the resulting magnetic field distribution to experimental magnetic measurements and thus obtain the separatrix location. This report presents a description of the computational technique used to fit the current density function to the measurement data, together with an assessment of the expected accuracy of the separatrix determination. A discussion of the practical applications of this method is also given as well as some remarks on the theoretical aspects of the calculation procedure.

Description of Computational Algorithm

The magnetic field configuration of a toroidal plasma current which is assumed to be in equilibrium may be described in terms of a poloidal flux function Ψ, defined by expressing the total magnetic field in the form

$$\underset{\sim}{B} = f \, \underset{\sim}{\nabla}\phi + \underset{\sim}{\nabla}\phi \times \underset{\sim}{\nabla}\Psi$$

where $f = RB_\phi$ is the flux function of poloidal currents /2/ and R, z, ϕ are cylindrical co-ordinates. It is assumed that all quantities are independent of the toroidal angle ϕ, so that the problem may be considered in two-dimensions. From the equation for plasma equilibrium, $\underset{\sim}{J} \times \underset{\sim}{B} = \underset{\sim}{\nabla} P$ and the above representation of the magnetic field, one may obtain the Grad-Shafranov equation /2/ in the form

$$\Delta^* \Psi = R^2 \underset{\sim}{\nabla} \left(\frac{1}{R^2} \underset{\sim}{\nabla} \Psi\right) = -\mu_o R^2 \frac{\partial P}{\partial \Psi} - f \frac{\partial f}{\partial \Psi} \qquad (1)$$

where the total pressure P, and f are functions of Ψ. Using the fact that the toroidal current density J_ϕ may be expressed as $\mu_o J_\phi = \frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \Psi}{\partial R}\right) + \frac{1}{R}\frac{\partial^2 \Psi}{\partial z^2}$, the equation may also be written as

$$\Delta \Psi = -\mu_o R J_\phi (\Psi) \qquad (2)$$

By assuming some functional form for $J_\phi(\Psi)$ which is compatible with the description involving $P(\Psi)$ and $f(\Psi)$ given in equation (1), an equilibrium magnetic configuration may thus be determined by solving equation (2) subject to appropriate boundary conditions.

In order to obtain an equilibrium solution consistent with the magnetic field produced by current carrying conductors outside the plasma region, the total poloidal flux Ψ_t must be calculated from

$$\Psi_t = \Psi_{pl} + \Psi_{ext}$$

where Ψ_{ext} is the flux function of currents in external conductors and Ψ_{pl} is determined from the plasma current as a solution of the equation

$$\Delta^* \Psi_{pl} = - \mu_o R J_\phi (\Psi_t)$$

This equation has been solved iteratively using a direct Poisson solver technique in the infinite domain together with a Green's function calculation to determine the Ψ values on the boundary of a finite domain /2,3/ using ficticious mirror currents in an unbounded domain. It has been assumed in the present calculations that the plasma is symmetric about the R axis so that the solution is computed only in the upper half plane.

In evaluating the current density function $J_\phi(\Psi)$, the location of the plasma boundary must be taken into account since J_ϕ must vanish outside the plasma region. For a plasma without a poloidal divertor, this boundary is generally defined by a flux surface which intersects a material limiter. With a divertor configuration, however, the boundary is determined by the separatrix whose location is calculated from the value at the X-point (saddle point) of the flux distribution.

Having thus outlined the method used to calculate the magnetic field from the current density distribution function, the question of how to determine this function $J_\phi(\Psi)$ from magnetic measurements outside the plasma must now be considered. It is important to choose the functional form for $J_\phi(\Psi)$ and the number of measurements to be used in such a way as to allow sufficient generality in the range of current distributions which can be described but, at the same time, ensure that a reasonable degree of uniqueness in the solution can be expected. As a guide to developing a technique which meets these requirements, we consider the expression for the magnetic flux of a toroidal plasma in equilibrium derived by Shafranov /4/ as a first order approximation to an expansion in the inverse aspect ratio

$$\Psi = -\mu_o R_p I_p (\ell n \frac{8R_p}{r} - 2) +$$

$$+ \frac{\mu_o I_p}{2} (\ell n \frac{r}{a} + (\Lambda + \frac{1}{2})(1 - \frac{a^2}{r^2})) \, r \cos \Theta$$

where r is the poloidal radius, Θ the poloidal angle and a the minor radius of the plasma. From this formula it can be seen that at a given poloidal radius, the magnetic field outside the plasma is most strongly dependent on the plasma current I_p, its major radius R_p, and the coefficient of asymmetry of the current distribution $\Lambda = \beta_p + \frac{\ell i}{2} - 1$ (β_p is the poloidal β of the plasma and ℓ_i its internal inductance per unit length /4/). On the basis of this observation, which has also been found to be true of numerically determined equlibria, we have elected to describe the plasma in terms of 3 measurements. They are the total plasma current I_p, the difference $\delta\Psi$, between flux measurement loop located on opposite sides of the plasma and the magnetic field difference δB, between magnetic probes located as shown in Fig.1. The measurements were chosen in this manner because the $\delta\Psi$ signal is characteristic of the radial position R_p, while δB provides information about the current distribution factor, Λ

Following from the assumption that the current distribution is described in terms of the parameter set I_p, R_p and Λ, it would be indicated that the function $J_\phi(\Psi)$ should be defined in terms of 3 free parameters whose values are determined to fit the measurement data. A difficulty with this approach arises, however, because it is possible to have different internal current distributions with different β_p and ℓ_i values but the same value for Λ. The external measurements cannot detect these differences, however, and this non-uniqueness in the solution produces severe difficulties in trying to find the parameters of the $J_\phi(\Psi)$. It was therefore found necessary to use a form of current distribution function which did not allow this degree of freedom. Based on these considerations, the following form of this function has been used

$$\mu_o J\,(\Psi_t) = (\beta\, R + \frac{(1-\beta)}{R}\, R_o^2)\,(\alpha_1\, \Psi_d + \alpha_2\, \Psi_d^2) \qquad (3)$$

where $\Psi_d = \Psi_t - \Psi_b$, Ψ_b is the value of the total flux on the plasma boundary and R_o is the major radius of the magnetic axis. The β value determines approximately (within 3 %) the β_p of the resulting equilibrium and α_1, α_2 are the free parameters to be matched to the measurement data.

The basic element of the algorithm for determining the separatrix location may thus be formulated in terms of a procedure to fit the parameters of the function defined by equation (3) to the experimental measurements. This calculation may be outlined as follows:

Given I_p, $\delta\Psi$, δB, Ψ_{ext} and an estimate for β, the computation begins with some starting guess Ψ_{pl}^o for Ψ_{pl}. This may be derived from initial values of the parameters α_1 and α_2 or, in the more general case, using a filament current approximation. Values of the parameters α_1 and α_2 which provide a best fit in the least-squares sense to the measurement information are then found using an iteration scheme consisting of the following 3 steps

1. Determine the Ψ-value at the plasma boundary from the saddle point in the distribution of

$$\Psi_t^n = \Psi_{pl}^n + \Psi_{ext}$$

and from that the plasma boundary and the radius of the magnetic axis, R_o^n.

2. Solve the 2 linear systems

$$\Delta^* \Psi_{pli}^{n+1} = -\mu_o R(\beta R + \frac{(1-\beta)(R_o^n)^2}{R})(\Psi_d^n)^i$$

$$\text{for} \quad i = 1,2$$

using the method described in /3/ which requires essentially only two fast Poisson solver steps for each system. The solution of the full linear system

$$\Delta^* \Psi_{pli}^{n+1} = -\mu_o R(\beta R + \frac{(1-\beta)(R_o^n)^2}{R})(\alpha_1 \Psi_d^n + \alpha_2 (\Psi_d^n)^2)$$

is then just $\Psi_{pl} = \alpha_1 \Psi_{pl1} + \alpha_2 \Psi_{pl2}$

3. Determine α_1^{n+1}, α_2^{n+1} to minimize

$$F(\alpha_1,\alpha_2) = w_1 (I_p - \int(\beta R + \frac{(1-\beta)(R_o^n)^2}{R})(\alpha_1 \Psi_d^n + \alpha_2 (\Psi_d^n)^2) dS)^2$$

$$+ w_2 (\delta\Psi - \Delta\Psi_t^{n+1})^2 + w_3 (\delta B - \Delta B_t^{n+1})^2$$

where w_1, w_2 and w_3 are error weights. The allowable ranges of values for α_1 and α_2 are constrained by the condition

$$\alpha_1 > - 2 \alpha_2 \text{ Max } |\Psi_d^n|$$

which guarantees that the current density distribution has a maximum value at the magnetic axis. The special form of current distribution function given by equation (3) makes the minimization of $F(\alpha_1, \alpha_2)$ particularly simple and the convergence of the algorithm easy to control.

Algorithm Tests - Accuracy and Convergence

In order to be able to use the procedure described above for interpretation of experimental results, it is necessary to obtain some assessment of the range of current distrubtions that can be treated and the expected accuracy in the separatrix location. This has been done by performing an equilibrium calculation using different forms of the current density distribution $J_\phi(\Psi)$, to generate the required I_p, $\delta\Psi$ and δB information and then comparing the results of the inverse calculation with the initial equilibrium. These tests were made using the functional form for $J_\phi(\Psi)$ given by equation (3) with various values of α_1, α_2 and β as well as with a function of the form

$$\mu_o J_\phi(\Psi_t) = (\alpha_1 R + \frac{\alpha_2}{R})\ (\Psi_t - \Psi_b)^\gamma \qquad (4)$$

for which $\beta_p \approx \dfrac{1}{1 + \dfrac{\alpha_2}{\alpha_1 R_o^2}}$ and can be adjusted to produce profiles with a large range of ℓ_i values.

A significant result of these tests, which provided a good coverage of the range of current distributions that could be reasonably expected to occur in the experiment (excluding hollow profiles) was that convergence of the algorithm was obtained in all cases investigated. In addition, it was always observed that when the minimum of the function $F(\alpha_1,\alpha_2)$ was sufficiently close to zero, the separatrix location determined from the inverse calculation was in good agreement with that of the input equilibrium so that the solutions determined by this method were found to be unique.

As a particular example of the performance of this technique, which provides a good illustration of a number of aspects of the behaviour of the algorithm, we consider the equilibrium produced by using the current density function given by equation (4) with $\alpha_1 = \alpha_2 = 1$ and $\gamma = 1.5$. For the external conductor configurations and dimensions corresponding to the ASDEX experiment, the flux contours and separatrix location are shown in Fig.2. This current distribution was found to have a poloidal beta of $\beta_p = 0.72$ and internal inductance $\ell_i = 1.26$ so that $\Lambda + 1 = \beta_p + \frac{\ell i}{2} = 1.35$. Using the measurement information obtained from this equilibrium, the inverse calculation was made initially assuming a value for ß of 0.3. The flux contours and measurement information obtained from this calculation are given in Fig.3.

These results indicate that good agreement with the input data was obtained despite the large error in estimating the ß value. (The relative errors in the I_p and $\delta\Psi$ values were less than $1o^{-3}$ while the δB value, which is always the most difficult to satisfy, was found correct to within 3 % of the input value). The value of $\Lambda + 1$ for this current distribution was 1.36, in good agreement with that of the initial equilibrium while the location of the separatrix was found to coincide very well with that of the input data, there being no noticeable differences in the separatrix contours given in the flux plots. This result was found to be true of all cases calculated using ß values in the range 0.3 to 0.9, providing good justification for the assumption that the magnetic field distribution external to the plasma is essentially determined by the Λ parameter.

A limit to the range of ß values that can be used as input is found if the inverse calculation is made with $\beta = 1.0$. As indicated in the results of Fig.4, the agreement with the input data is relatively poor and there is a correspondingly large error in the location of the separatrix. The reason for this behaviour is that the current distributions which can be obtained with the current density function given by equation (3) have ℓ_i values which lie within the range 0.8 to 2.2. With a β value of 1.0, the inverse calculation would require a current distribution with ℓ_i = 0.7 in order to have $\Lambda + 1 = 1.35$ which is not possible with the given form of $J_\phi(\Psi)$. Because the range of ℓ_i values that can be produced with this particular form of $J_\phi(\Psi)$ are well representive of current density profiles of general experimental interest, there would appear at present to be no justification for using a more general form for $J_\phi(\Psi)$ in the inverse calculation such as that given by equation (4), as long as the limitation on the range of ℓ_i values is taken into account when an estimate for β is made.

In the situation where the estimate for β is reasonably close to the actual β_p of the experimental data, not only the separatrix location is well determined but also the internal current distribution can be accurately reproduced. This aspect of the inverse calculation is illustrated by the results given in Fig.5 where a value of β = 0.75 was used. An ℓ_i value of 1.24 was obtained in good agreement with the ℓ_i value of 1.26 corresponding to the input data, while both the current density profiles and flux contours were also quite well reproduced. (The root-mean-square relative errors in the current density and flux distributions compared to the initial equilibrium being 2.3×10^{-2} and 3.5×10^{-3} respectively). This result is a particularly good indicator of the ability of the algorithm to determine a current distribution corresponding to experimental data because the form of $J_\phi(\Psi)$ used in the initial equilibrium computation cannot be analytically represented by the function used in the inverse calculation.

The extent of similarity between these two equilibria provides reason to conclude that, to a good degree of approximation, the current density distribution within a plasma in equilibrium may be well characterized essentially by the parameters β_p and ℓ_i. Because the present work was primarily concerned with development of a technique for location of the separatrix, however, the extent to which this observation is true in general has not been investigated in detail for a large range of current density functions or values of the ratio of divertor to plasma current. Subject to the restriction on the class of current distributions which can be described by the functional form used in this technique, it is nevertheless possible to anticipate that a good indication of the current distribution within the plasma can be obtained if an accurate experimental measurement of β_p is available in addition to the external magnetic measurements.

Application of the Procedure to Experimental Results

With the information about currents in external conductors as well as the I_p, $\delta\Psi$ and δB data determined from the experiment, the computational program requires only an estimate for β_p to be supplied in order to perform the separatrix location determination. When no experimental measurement of this quantity is available, it is generally possible to assume a value in the range 0.1 to 0.6 for an ohmically heated tokamak or, in the case where auxiliarly heating is used, estimate β_p from the heating power based on theoretical predictions or experimental experience. As discussed previously, the β_p value is not important to the determination of the separatrix location provided that the allowable range of ℓ_i values is kept in mind.

In the calculation of the flux Ψ_{ext}, produced by currents in conductors external to the plasma, it has been assumed that only the driven currents in the vertical field and divertor field coils have a significant effect on the magnetic field distribution. The influence of the ohmic heating windings has not been considered because the magnetic fields produced by these coils are small over most of the discharge region. Additional conductors for passive stabilization of plasma motions are also included in the ASDEX design and these are located near the entrance to the divertor chambers (Fig.6). In order to assess the influence of induced currents in these conductors on the location of the separatrix, a calculation of the magnetic field configuration produced with the maximum expected current flowing in these conductors has been made. The results are given in Fig.6 for a plasma current of 2oo kA, indicate that even with the relatively large current of 1o kA in the passive conductors, the separatrix location is relatively little effected, most importantly in the region of the divertor throat, because of the strongly localized nature of the field produced by the divertor coil triplet.

The influence of fields produced by induced currents on the operation of the inverse calculation has not been examined in detail because initial interest in divertor discharges will concentrate on relatively steady conditions. Should experiments indicate larger induced currents than currently predicted or interest develop in unsteady conditions where induced currents are significant, their influence could also be included in the present calculations by using the facilities for measuring these currents which presently exist in ASDEX.

Conclusion

The technique described in this report has proved to be capable of locating the separatrix in a magnetic field distribution produced by the plasma current in a tokamak with poloidal divertor using equilibrium calculations of the current density distribution within the plasma. Measurement of three quantities; the plasma current, the poloidal flux change and poloidal field change across the plasma, together with currents in external conductors, have been found to provide sufficient information to accurately determine the separatrix location. If, in addition, the poloidal beta of the plasma is determined from an independent measurement, this may be used to obtain a good indication of the current density distribution within the plasma.

Acknowledgements

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References

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/2/ Lackner,K., Comput. Phys. Comm. 12 (1976) 33.

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Figure Captions

Figure 1:

Schematic sectional view of ASDEX tokamak indicating location of flux measurement loops (Ψ_1 and Ψ_2) and poloidal magnetic field measurement coils (B_1 and B_2). Using these measurements, the quantities $\delta\Psi = \Psi_1 - \Psi_2$ and $\delta B = B_1 - B_2$ are calculated as input for the separatrix location program.

Figure 2:

Poloidal flux contours for a plasma equilibrium calculated using $J_\phi(\Psi)$ given by equation (4) and $\alpha_1 = \alpha_2 = 1$, $\gamma = 1.5$. The measurement data corresponding to this current distribution are $\delta\Psi = 0.13289$ and $\delta B = -5.1658 \cdot 10^{-3}$.

Figure 3:

Results of the inverse calculation with $\beta = 0.3$. $\delta\Psi = .13276$ and $\delta B = -5.0149 \cdot 10^{-3}$.

Figure 4:

Poloidal flux distribution calculated from measurement data with $\beta = 1.0$. Best fit to measurement data was $\delta\Psi = 0.07387$ and $\delta B = -1.1825 \cdot 10^{-2}$.

Figure 5:

Results of inverse calculation using $\beta = 0.75$. $\delta\Psi = .13298$ and $\delta B = -5.267 \cdot 10^{-3}$.

Figure 6:

Comparison of separatrix location with 1o kA induced current in passive conductors A and B (dotted line) with separatrix without induced currents (full line).

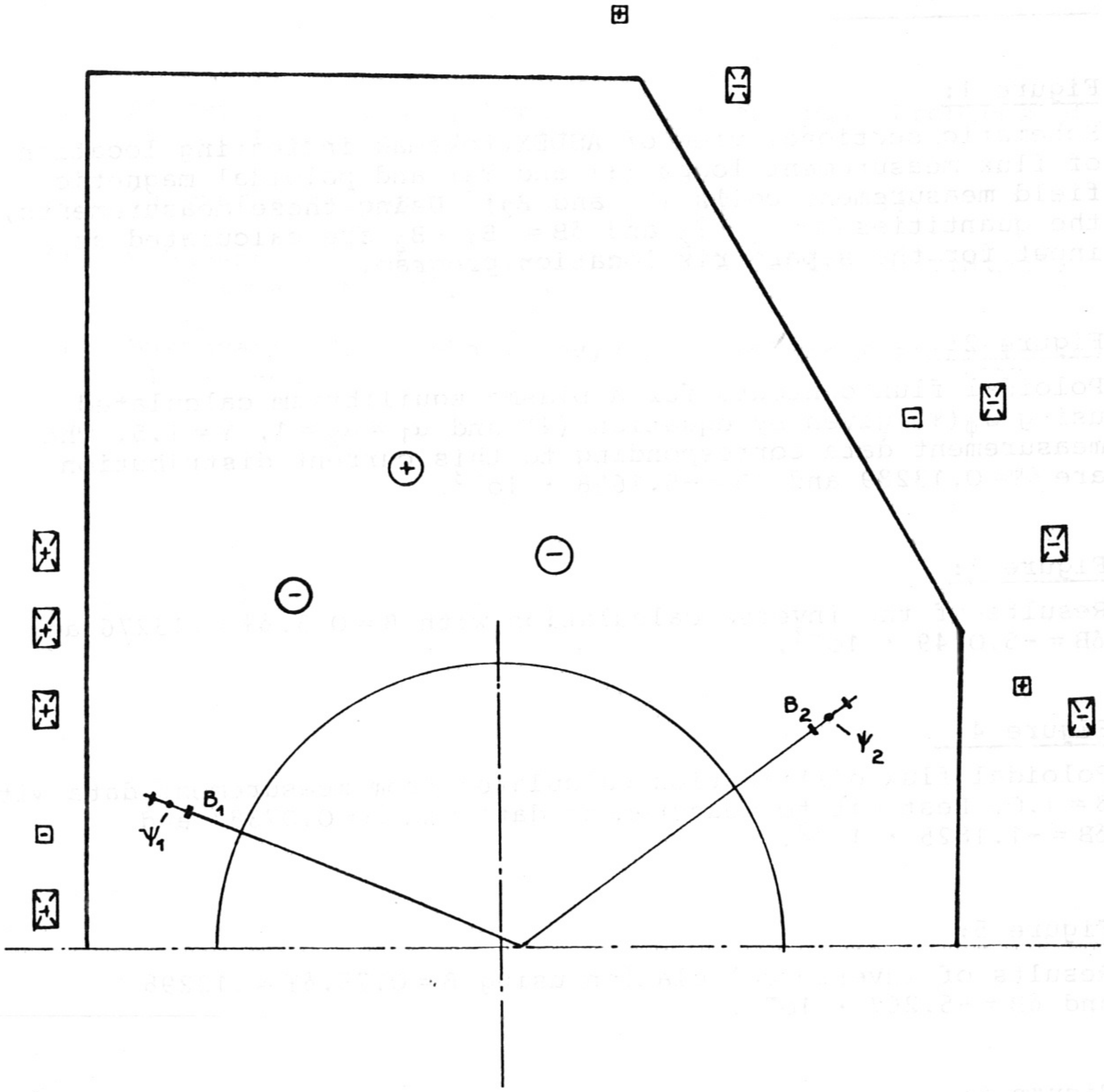

Figure 1

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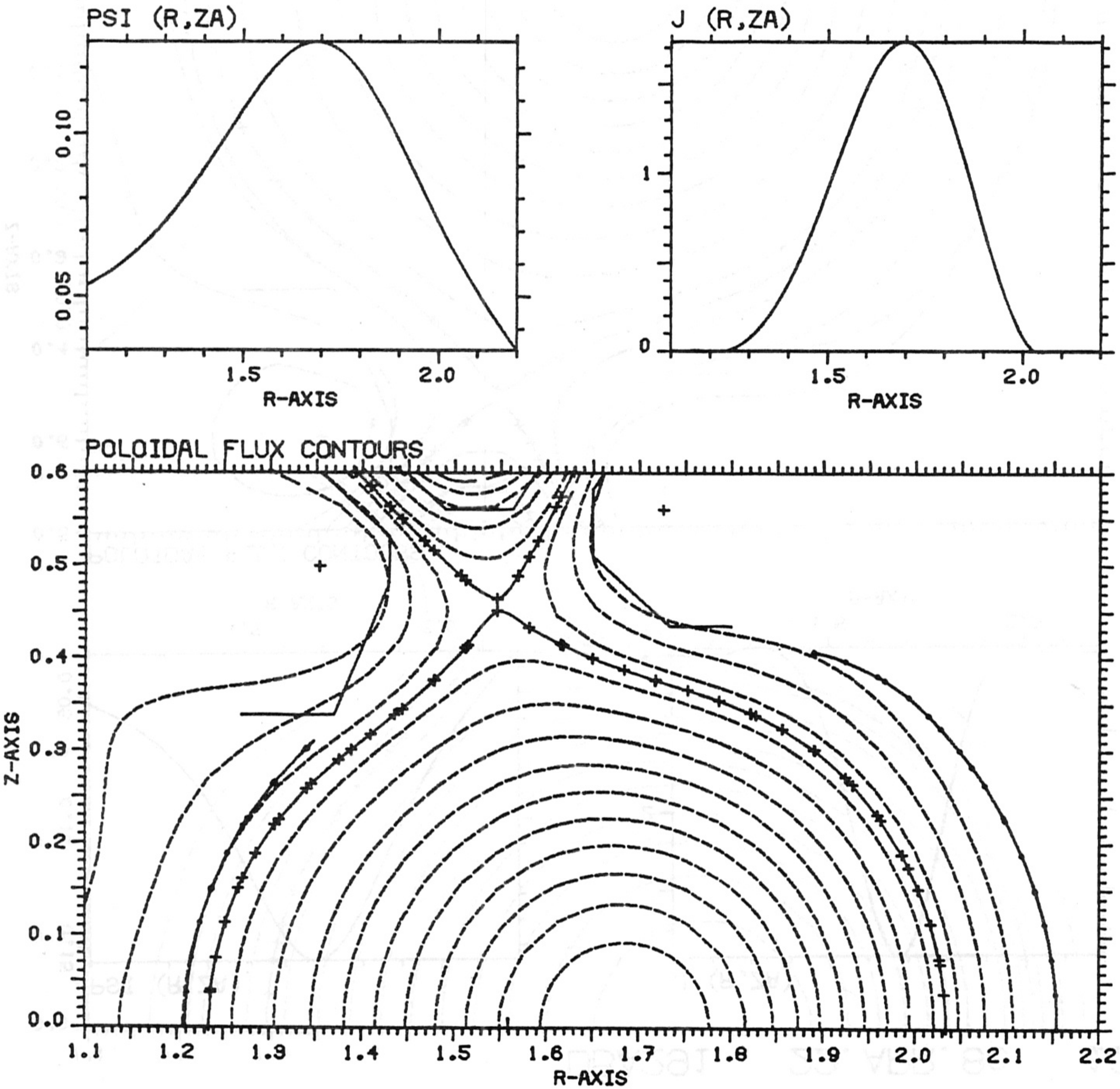

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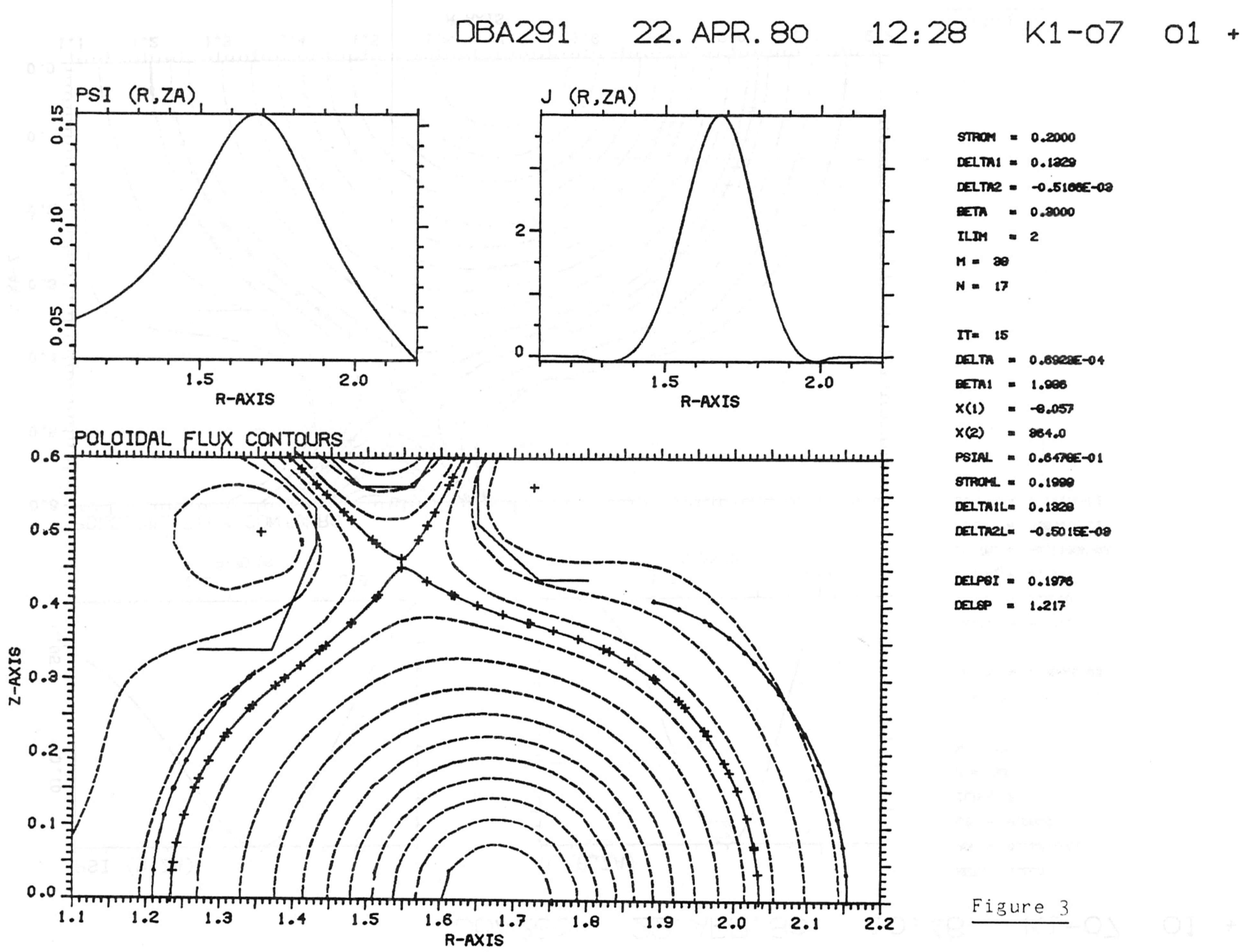
Figure 2



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R-AXIS
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Figure 3

DBA296 22. APR. 80 12:39 K1-07 01 +

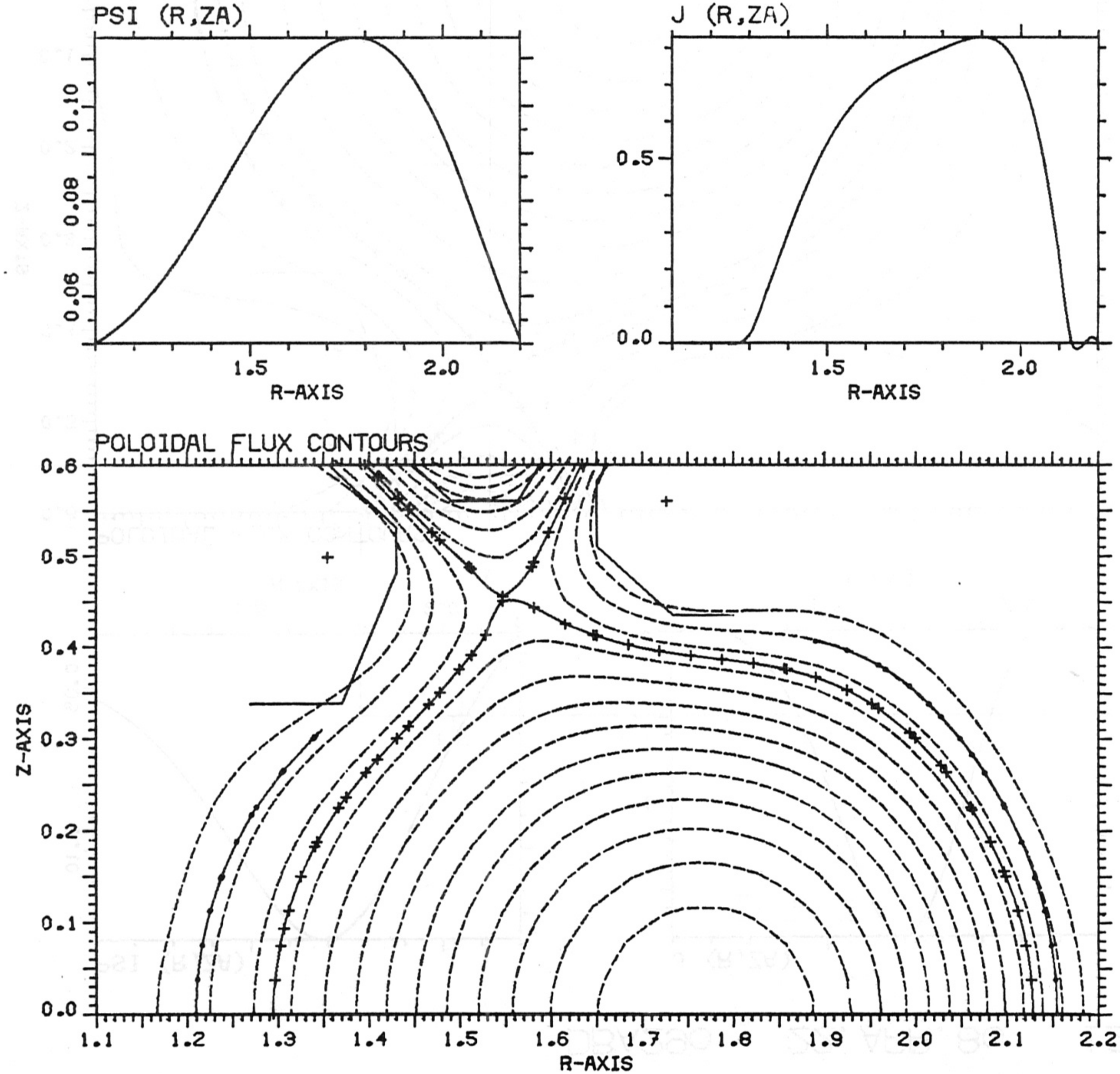

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Figure 4

DBA29o 22. APR. 8o 12:26 K1-o7 o1 +

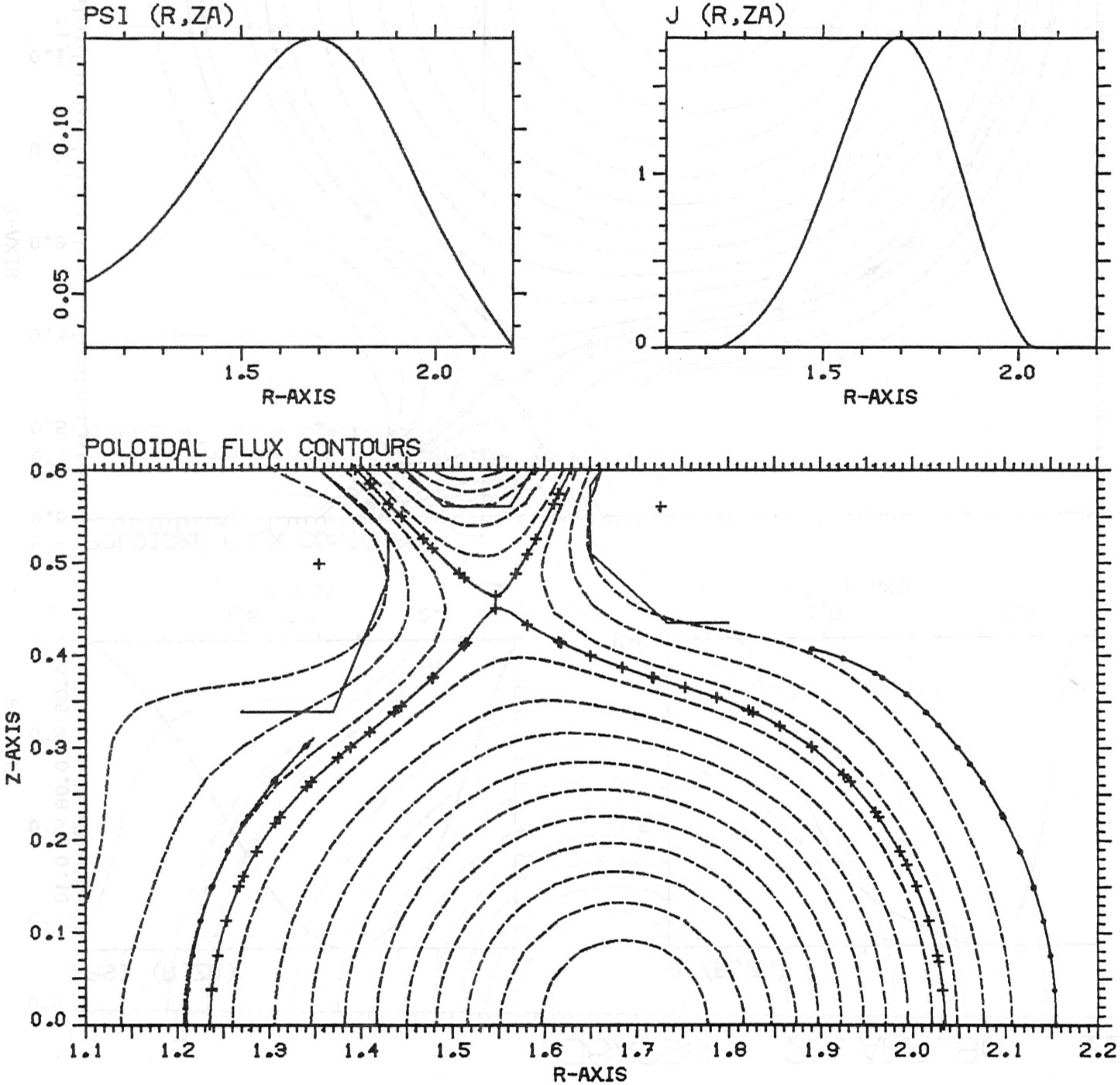

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Figure 5

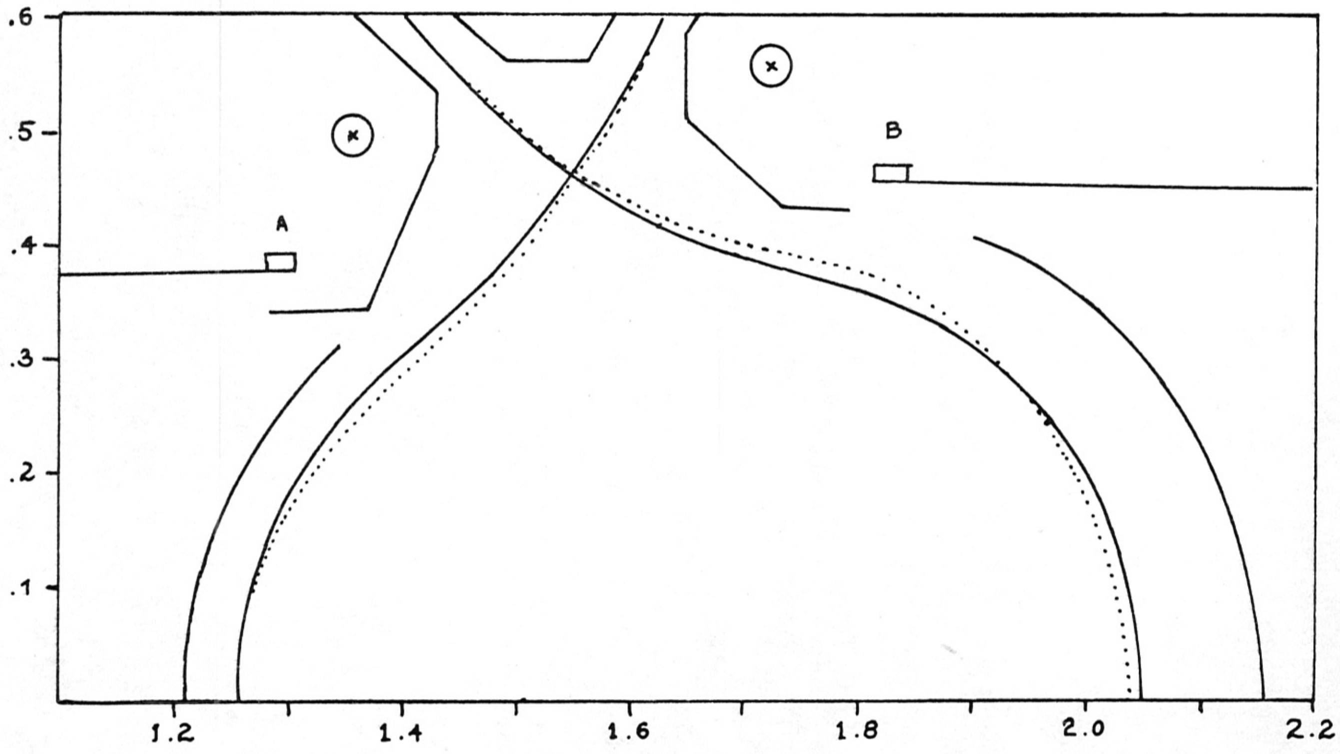

Figure 6