Boundary Conditions for Tokamak

Transport Equations from

Scrape-off Layer Modelling

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Abstract

A simple model for the scrape-off zone of a tokamak plasma, which can be
treated analytically, is considered. It involves a description of space
dependent parallel and perpendicular transport as well as ionization of
neutrals. The information contained in this model can be fully transformed
into boundary conditions for the bulk plasma. In the case of light particles
and hydrogen, the boundary conditions relate the densities, the particle
fluxes and the ionization rate at the boundary linearly to each other. In
the case of heavy impurities densities and fluxes are related to the flux
of neutrals coming from the wall. The model allows for the description of
impurity shielding and a balance between inward and outward impurity
diffusion at the boundary. The shielding efficiency of the scrape-off
zone is found to be strongly determined by the hydrogen boundary con-
dition. The model is to be used for the treatment of ratiation layers in
a ZEPHYR-like ignition experiment.

Introduction

In tokamak transport codes usually the values of densities and temperatures or the fluxes of heat and particles are prescribed at the boundary. This may be justified partly for the description of hydrogen but is inadequate in the case of impurity transport. In that case the particle flux at the boundary is frequently identified with the neutral flux of impurities from the wall. This does not admit any outflow of impurities and thus, unrealistically, leads to a continuously increasing impurity content in the bulk plasma, independently of the transport law for the impurities. For a consistent treatment the bulk plasma and the scrape-off layer have to be treated simultaneously. Particles then enter the system purely by volume sources.

In this paper we consider a simple, analytically tractable scrape-off layer model which, however, contains all relevant physical effects, such as space dependent parallel and perpendicular transport and ionization of neutrals. It can be completely incorporated into the plasma description in terms of boundary conditions for the bulk plasma particle transport equations. In these boundary conditions only central plasma boundary values appear. The model thus can be implemented into current tokamak transport codes without major modifications.

A consistent description of the plasma boundary is of special importance for the analysis of problems such as impurity accumulation or the built up of radiating layers as have been studied recently for the ZEPHYR ignition experiment /3/.

Basic Equation and Assumptions

We treat the scrape-off zone as a slab of thickness d according to Fig. 1. The indexes s and c are used to denote scrape-off and central plasma quantities respectively.

Fig. 1

plasma
boundary

slab

↓

wall

r   x = 0   x   x = d

r = a
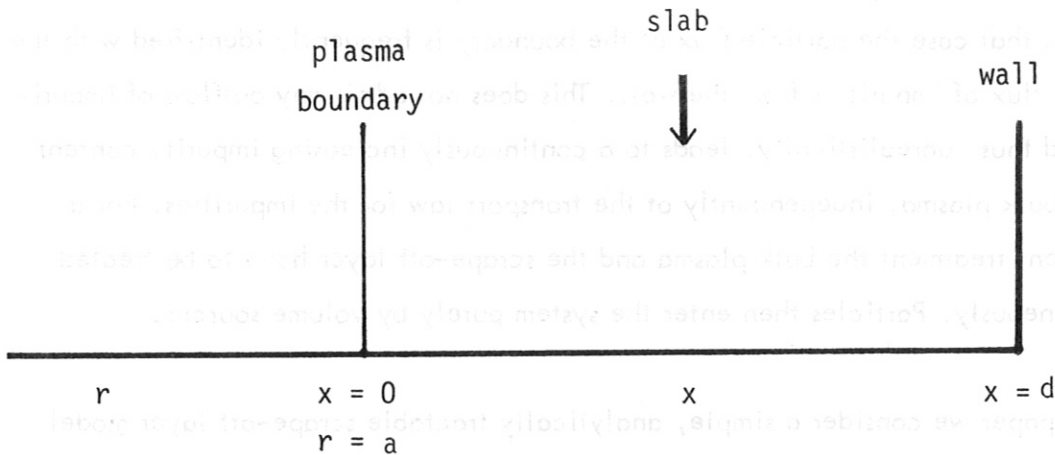
The following equation is used to describe parallel and perpendicular particle transport in the scrape-off zone /1/:

$$\frac{\partial n_s^{\sigma,m}}{\partial t} + \frac{\partial}{\partial x} \Gamma_s^{\sigma,m} = - \frac{n_s^{\sigma,m}}{\tau_{\parallel}^{\sigma,m}} + S_s^{\sigma,m} \,. \tag{1}$$

Here $n_s^{\sigma,m}$ is the density of particle species σ with charge state m. $\Gamma_s^{\sigma,m}$ is the corresponding perpendicular particle flux. $\tau_{\parallel}^{\sigma,m}$ characterizes parallel transport. $S_s^{\sigma,m}$ is the source of the respective type of particles (sum of ionization and recombination terms). Here only particle transport is considered, but the extension to energy transport is obvious.

The scrape-off zone quantities are linked to the bulk plasma quantities by the boundary conditions

$$\Gamma_c^{6,m}(a) = \Gamma_s^{6,m}(o) \tag{2}$$

and

$$n_c^{6,m}(a) = n_s^{6,m}(o) . \tag{3}$$

Equation (1) will be simplified by the following assumptions:

$$\Gamma_s^{6,m} = - D_\perp^{6,m} \frac{\partial n_s^{6,m}}{\partial x} , \tag{4}$$

$$d^2/D_\perp^{6,m} = \tau_\perp^{6,m} \gg \tau_\parallel^{6,m} , \tag{5}$$

$$D_\perp^{6,m} \text{ and } \tau_\parallel^{6,m} \text{ do not depend on } n_s^{6,m} \text{ and } x , \tag{6}$$

$$\frac{\partial}{\partial t} \equiv 0 \tag{7}$$

and

$$\frac{\partial T_s}{\partial x} = 0 . \tag{8}$$

The simple "ansatz" according to eq. (4) essentially expresses the poor knowledge of diffusion in the scrape-off zone. We consider free streaming and Bohm diffusion as typical candidates for parallel and perpendicular transport mechanisms respectively. In that case eqs. (5) and (6) are fulfilled. Typically $\tau_\parallel \ll \tau_p$ holds, where τ_p is the bulk plasma particle confinement time. Hence the approximation expressed by eq. (7) is adequate. The assumption of flat temperature profiles (eq. (8)) is made to make the problem analyticalle tractable.

Equation (1) now takes the form

$$\Delta^{6,m}\, n_s^{6,m\,\prime\prime} - n_s^{6,m} + \tau_{\shortparallel}^{6,m}\, S_s^{6,m} = 0, \tag{9}$$

with $\Delta^{6,m} = (D_\perp^{6,m}\, \tau_{\shortparallel}^{6,m})^{1/2} = d\,(\tau_{\shortparallel}^{6,m}/\tau_\perp^{6,m})^{1/2}$ and $n_s' = dn_s/dx$.

The homogeneous part has the general solution

$$n_s^{6,m} \sim e^{\pm\, x/\Delta^{6,m}}. \tag{10}$$

Note that $\Delta^{6,m}/d < 1$ holds because of eq. (5).

We now assume n_s^6 to be the total density of one species: $n_s^6 = \sum_m n_s^{6,m}$.
$S_s^6 = \sum_m S_s^{6,m}$ is then given by

$$S_s^6 = n^e\, n^{6,0} \langle 6 v \rangle_{ion}^6, \tag{11}$$

where $\langle 6 v \rangle_{ion}^6$ is the ionization rate for particle species 6. n^e and $n^{6,0}$ are the electron and neutral density respectively. In addition we assume that $\Delta^{6,m}$ and $\tau_{\shortparallel}^{6,m}$ do not depend on m. This is valid at least for the special scrape-off zone transport model to be considered. With these assumptions the equation for n_s^6 results from eq. (9) by omitting the index m. In what follows the index 6 mostly will be omitted for convenience. We consider two limiting cases with respect to the ratio λ/Δ, where $\lambda \approx (\langle 6 v \rangle_{ion}\, n^e/V_o)^{-1}$ is the penetration length for a neutral with velocity V_o:

Case a) $\lambda/\Delta \gg 1$. This condition typically prevails for hydrogen and light impurities. In that case we have

$$\frac{1}{S_s}\frac{\partial S_s}{\partial x} \ll \frac{1}{\Delta}. \tag{12}$$

Case b) $\lambda/\Delta \ll 1$. This case may prevail for heavy impurities. Then, if wall sputtering by charge exchange neutrals is assumed to dominate the impurity production, the

source function S is localized in the scrape-off zone, owing to the de-
crease of n^e (and T). This is an appropriate assumption for a divertor boun-
dary. In case of a material limiter sputtering at the limiter may play a role.
It can easily be included into the analysis.

The space dependence of the source function S is schematically visualized in Fig. 2
for these two cases.

Fig. 2

S case a)   S case b)

x = 0   x = d*   x = d
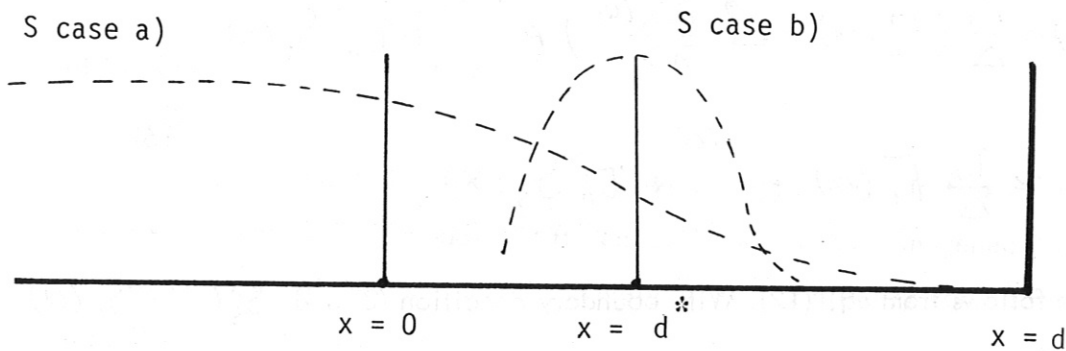
Boundary Conditions for Light Impurities (Case A)

From eqs. (6) and (12) one finds that

$$\overline{n}_s = S_s \, \tau_{\shortparallel}$$

(13)

is an approximate special solution of the inhomogeneous eq. (9). With eqs. (10) and (13) the general solution of eq. (9) fulfilling the boundary condition

$$\lim_{d \to \infty} n_s(d) \longrightarrow o \tag{14}$$

is given by

$$n_s(x) = \tilde{n}\, e^{-x/\Delta} + S_s(x)\, \tau_{\shortparallel} . \tag{15}$$

By applying the boundary condition (2) and using eq. (4) we can express $\tilde{n}$ in terms of $\Gamma_c(a)$:

$$n_s(x) = \frac{\tau_{\shortparallel}}{\Delta}\left(\Gamma_c(o) + \Delta^2 \frac{\partial S_s(o)}{\partial x}\right) e^{-x/\Delta} + \tau_{\shortparallel}\, S_s(x)$$

$$\simeq \frac{\tau_{\shortparallel}}{\Delta}\Gamma_c(o)\, e^{-x/\Delta} + \tau_{\shortparallel}\, S_s(x) . \tag{16}$$

The last step follows from eq. (12). With boundary condition (3) and $S_s(o) = S_c(a)$ we finally get

$$\frac{\Delta}{\tau_{\shortparallel}}\, n_c(a) = \Gamma_c(a) + \Delta\, S_c(o) . \tag{17}$$

The relation (17) is the desired boundary condition to be applied to the particle transport equations describing the central plasma region. It will be demonstrated later that the coefficients $\tau_{\shortparallel}$ and Δ can be related to bulk plasma boundary values of densities and temperatures. Equation (17) applies to light ions, especially to hydrogen. It admits a simple physical interpretation. When interpreted as the global particle balance equation of the layer $o \le x \le \Delta$ of thickness Δ, it states that the flux out of this layer by parallel streaming is balanced by the influx across the plasma boundary and by the source of particles through ionization.

Boundary Condition for Heavy Impurities (Case B)

To keep the discussion as simple as possible we use the following model source function in case b):

$$S_s = S_0 \, e^{-|x - d^*|/\varsigma} \tag{18}$$

ς characterizes the degree of localization of S_s around d^*. Insertion of S_s into eq. (9) immediately yields the special solution

$$\overline{\overline{n}}_s = \frac{\tau_{\shortparallel} S_0}{1 - (\Delta/\varsigma)^2} \, e^{-|x - d^*|/\varsigma} \tag{19}$$

of the inhomogeneous equation.

For this solution the derivative dn_s/dx and hence the particle flux Γ_s is discontinuous at $x = d^*$. By adding different solutions of the homogeneous equation in the regime $x < d$ and $x > d$ respectively, the discontinuity can be removed. The solution

$$\overline{n}_s = \frac{\tau_{\shortparallel} S_0}{1 - \Delta^2/\varsigma^2} \left(e^{-|x - d^*|/\varsigma} - \frac{\Delta}{\varsigma} \, e^{-|x - d^*|/\Delta} \right) \tag{19}$$

has the required property. It was arbitrarily chosen so that $dn_s/dx \big|_{x = d^*} = 0$ holds. The general solution fulfilling the boundary condition (14) now reads

$$n_s = \tilde{n} \, e^{-x/\Delta} - \frac{\tau_{\shortparallel} S_0}{1 - \Delta^2/\varsigma^2} \left(e^{-|x - d^*|/\varsigma} - \frac{\Delta}{\varsigma} \, e^{-|x - d^*|/\Delta} \right) \tag{21}$$

Expressing $\tilde{n}$ in terms of Γ_c (a) by boundary condition (2) and applying boundary condition (3) we get in complete analogy with case a):

$$n_c(a) = \frac{\tau_{\shortparallel}}{\Delta} \, \Gamma_c(a) + \frac{\tau_{\shortparallel} S_0}{1 - \Delta^2/\varsigma^2} \left(e^{-d^*/\varsigma} \left(1 + \frac{\Delta}{\varsigma} \right) - 2 \frac{\Delta}{\varsigma} \, e^{-d^*/\Delta} \right) \tag{22}$$

In the limit $\wp \ll \Delta$ (localized source) eq. (22) simplifies to

$$n_c(a) = \frac{\tau_{\shortparallel}}{\Delta} \Gamma_c(a) + 2 \tau_{\shortparallel} S_o \frac{\wp}{\Delta} e^{-d^*/\Delta} \qquad (23)$$

Noting that

$$\Gamma_o = \int_{-\infty}^{\infty} S \, dx \simeq \int_{o}^{d} S_s \, dx \simeq 2 \wp S_o \qquad (24)$$

holds, where Γ_o is the neutral flux of the respective species coming from the wall, one finally gets

$$\frac{\Delta}{\tau_{\shortparallel}} n_c(a) = \Gamma_c(a) + \Gamma_o e^{-d^*/\Delta} \qquad (25)$$

This relation is similar to eq. (17) except that the source $S_c(a)$ of particles due to ionization at the boundary, which plays no role in this case, is replaced by the flux $\Gamma_o e^{-d^*/\Delta}$ of particles which reach the boundary by inward diffusion from d^* where they are born. The reduction by the shielding factor $e^{-d^*/\Delta}$ results from the competition between parallel and perpendicular transport. The factor $(1 - e^{-d^*/\Delta})$ can be interpreted as the shielding efficiency of the scrape-off zone.

Estimation of $\tau_{\shortparallel}$ and Δ

To get some estimation of the coefficientes $\tau_{\shortparallel}$ and Δ, appearing in eqs. (17) and (25), we now specify the parallel and perpendicular transport mechanisms in the scrape-off zone.

For parallel transport we assume free streaming of the particles along the field lines. This means /1/

$$\tau_{\shortparallel} = \frac{4 L}{\langle v \rangle} \qquad (26)$$

where L is the average distance a particle has to fly until it reaches the limiter
or the divertor plates and $\langle v \rangle = \sqrt{8/\pi} \sqrt{T/m}$. For a toroidal limiter one has
for instance

$$L \simeq \pi \, q(a) \, R \, . \tag{27}$$

In practical units we get

$$\tau_{\shortparallel} = 8.10 \cdot 10^{-8} \, L \, \mu^{1/2}/T^{1/2} \tag{28}$$

($\tau_{\shortparallel}$ in s, L in cm, T in keV, $\mu = m/m_p$, m_p the proton mass).

For perpendicular transport we assume Bohm diffusion:

$$D_{\perp} = \alpha \, c \, T/16 \, e \, B \, , \tag{29}$$

where α is some arbitrary enhancement factor. In practical units we obtain for
$\tau_{\perp} = d^2/D_{\perp}$

$$\tau_{\perp} = 1.60 \cdot 10^{-7} \, d^2 \, B/\alpha \, T \tag{30}$$

($\tau_{\perp}$ in s, d in cm, B in kG, T in keV).

The following ZEPHYR-like data are taken to calculate an example: d = 9 cm, B = 90 kG,
L = 1270 cm, T = 0.1 keV, α = 1.

They yield for hydrogen (D-T-mixture)

$$\tau_{\shortparallel}^{H} = 5.14 \cdot 10^{-4} \text{ s,}$$
$$\tau_{\perp}^{H} = 1.17 \cdot 10^{-2} \text{ s,}$$
$$\Delta^{H} = 1.87 \text{ cm}$$

and for iron

$$\tau_{\parallel}^{Fe} = 2.43 \cdot 10^{-3} s,$$
$$\tau_{\perp}^{Fe} = 1.17 \cdot 10^{-2} s,$$
$$\Delta^{Fe} = 4.10 \text{ cm.}$$

Estimation of the shielding Efficiency for Heavy Impurities

To estimate the shielding factor $e^{-d^*/\Delta}$ in eq. (25) we compute the source function S_s for a heavy impurity such as iron.

The probability of an atom coming from the wall with velocity V_o to reach point x in the scrape-off layer without ionization is, in the case of normal incidence, given by

$$P = e^{-\beta} \tag{31}$$

with

$$\beta = \frac{1}{V_o} \int_x^{ol} \langle \sigma v \rangle_{ion} \, n^e \, dx . \tag{32}$$

For not too low temperatures $\langle \sigma v \rangle_{ion} \simeq$ const /2/. Since the degree of ionization of impurities is small at the relatively low boundary temperatures, we can furthermore assume $n_s^e \simeq n_s^H$, where n_s^H is the hydrogen density. The second term in eq. (15) is usually small so that we may use finally the approximate solution $n_s^e \simeq n_s^H (0)$ $e^{-x/\Delta^H} = n_c^H (a) \, e^{-x/\Delta^H}$ to evaluate ß:

$$\beta = \frac{\langle \sigma v \rangle_{ion} \, n_c^H(a)}{V_o} \int_x^{ol} e^{-x/\Delta^H} \simeq \frac{\Delta^H}{\lambda_a} e^{-x/\Delta^H} \tag{33}$$

where $\lambda_a^{-1} = \langle \sigma v \rangle_{ion} \, n_c^H(a)/V_o$. In deriving eq. (33) $\lambda_a \ll \Delta < ol$ was used.

Now we get for the source function

$$S_s = \Gamma_o \frac{dP}{dx} = \frac{\Gamma_o}{\lambda_{ol}} e^{-\left(\frac{\Delta^H}{\lambda_{ol}} e^{-\frac{x}{\Delta^H}} + \frac{x}{\Delta^H}\right)}.$$

(34)

S_s has indeed a maximum within the layer, if n_c^H (a) and hence λ_{ol}^{-1} is not too low, but unfortunately the localization is not very pronounced. For a crude estimation, however, we may set $d^* = x_m$, where x_m is the point where S_s takes its maximum. From eq. (34) one gets by elementary calculations

$$\frac{x_m}{\Delta^H} = \ell n \left(\frac{\Delta^H}{\lambda_{ol}}\right)$$

(35)

and hence

$$e^{-d^*/\Delta} = \left(\frac{\lambda_{ol}}{\Delta_H}\right)^{\frac{\Delta^H}{\Delta}}$$

(36)

for the shielding factor in eq. (25).

Now, taking iron as an example with $V_o = 3 \cdot 10^5$ cm s^{-1}, $\langle \sigma v \rangle_{ion} = 2 \cdot 10^{-7}$ s^{-1} /2/ and the transport model of the foregoing chapter yielding $\Delta^H / \Delta^{Fe} = (2.5/56)^{0.25} \simeq 0.46$ and $\Delta^H = 0.895$ $(\propto L \, T^{1/2} B^{-1})^{1/2}$, we obtain

$$e^{-\frac{d^{*Fe}}{\Delta^{Fe}}} = \left(\frac{1.68 \cdot 10^{12}}{n_c^H(ol)} \sqrt{\frac{B}{\propto L \sqrt{T}}}\right)^{0.46}.$$

(37)

With the above ZEPHYR-data one gets from eq. (37)

$$e^{-\frac{d^{*Fe}}{\Delta^{Fe}}} = \left(\frac{7.95 \cdot 10^{11}}{n_c^H(ol)}\right)^{0.46}.$$

(38)

Equation (37) emphasizes the importance of the correct boundary condition for hydrogen when computing the shielding efficiency of the boundary layer for heavy impurities. The influx of heavy impurities into the bulk plasma is, apart from the neutral flux Γ_o, mainly determined by the hydrogen boundary density. For typical values of n_c^H (a) the shielding factor is not much below unity.

Conclusion

A model for the scrape-off zone of a tokamak plasma is developed that describes space
dependent parallel and perpendicular diffusion as well as ionization of neutrals in the
scrape-off layer. Its content is transformed into boundary conditions for the bulk plasma
particle transport equations. Light impurities and hydrogen on the one hand and heavy im-
purities on the other hand are considered as two limiting cases. The boundary conditions
have for each species type the form of a linear relation between the bulk plasma flux
at the boundary, the density at the boundary and the source of particles by ionization
at the boundary (light particles, eq. (17) or the flux of neutrals coming from the wall
(heavy impurities, eq. (25)) respectively. The coefficients ($\tau_{\parallel}$, $\triangle$) appearing in
these linear relations can be fully expressed in terms of bulk plasma boundary densities
and temperatures, if the parallel and perpendicular particle transport laws are specified.
In this model heavy impurities enter the bulk plasma by diffusion from the scrape-off
zone. The model allows for the description of a balance between inward and outward
diffusion and thus the net flux of impurities at the boundary may vanish even in the case
of a continuous flux of impurity atoms from the wall (see eq. (25)). Shielding of the
scrape-off zone is also included. The shielding efficiency for heavy impurities is found
to be strongly determined by the bulk plasma hydrogen density at the boundary (eq. (37)).
This strengthens the importance of the boundary condition eq. (17) for light particles
like hydrogen. The advantage of the model lies in the fact that it contains nontrivial
information but nevertheless can easily be implemented into current tokamak transport
codes by simply changing the boundary conditions. The extension of this analysis to
heat transport as well as the inclusion of limiter sputtering is straightforeward.

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