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New Scaling Laws for Trapped-Particle Anomalous
Transport in Tokamaks

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Abstract:

An improved formula for anomalous transport caused by the dissipative trapped-ion instability (without shear effect) is derived numerically and by two independent analytical methods. The new shearless result is also used to derive the anomalous diffusion with shear effect by a general method already published.

Earlier numerical calculations by Saison et al. [1] of the anomalous diffusion caused by the dissipative trapped-ion instability in a toroidal plasma <u>without shear</u> gave Bohm-like diffusion. A diffusion formula that <u>includes shear</u> effects was derived from this shearless result [2] . In the meantime, more refined calculations [4, 5] have produced more accurate results for the cases without and with shear effect. A brief account of these new results is given in this letter. As in the earlier work, the starting point is set by the Kadomtsev-Pogutse 2D trapped-fluid equations [3] , viz.

$$\tilde{n}_t^j + \nu_j \tilde{n}^j + \nu_o \varrho_y + A\left(\tilde{n}_x^i \tilde{n}_y^e - \tilde{n}_y^i \tilde{n}_x^e\right) = 0, \qquad (1)$$

with $j = i,e$. Here $n^j(x,y,t)$ are the trapped-particle density perturbations, the subscripts x,y,t designate derivatives, $\varrho = \tilde{n}^i - \tilde{n}^e$, ν_j are the effective collision frequencies of the trapped particles, $A = cT/2eBN^P(1-\delta_o)$, $N^P(x)$ is the particle equilibrium density of one species, $\delta_o = n^o/N^P$ is the relative number of magnetically trapped particles (in equilibrium), $v_o = A\delta_o N^P/r_n$, with $r_n = \left|N^P/N_x^P\right|$. Eqs. (1) are now refined by applying, in addition, a Gaussian spectral cut-off operator to the time increments of the trapped-ion and trapped-electron density perturbations. By this device, a regularization of the equations at short wavelengths is effected. The quasineutrality condition is used to determine the electric potential $\Phi(x,y,t)$ from ϱ. From Φ one can determine the spatial distribution of $\underline{\underline{E}} \times \underline{\underline{B}}$ drift velocities, and the velocity field and the density perturbations yield

a y-averaged trapped-particle flux in the radial x-direction and
the corresponding ambipolar, anomalous diffusion coefficient.

The initial-value problem was numerically solved for broad-
spectrum initial perturbations in the rectangle $0 \leq x \leq a$, $0 \leq y \leq b$,
with $b = \pi a$, a = minor plasma radius. The usual boundary conditions
[1, 2] were employed, viz. periodicity in y with period b and zero
perturbations for $x = 0$ and $x = a$. The number of grid points and
the width of the Gaussian cut-off in the y-direction were adjusted
according to the values of the dominant mode number. In this way,
numerical errors were kept comparatively small in all calculations.
The results are summarized in Table 1. The anomalous diffusion coeffi-
cient is shown to scale as (cgs-units)

$$D \approx 3.5 \times 10^{-2} \delta_o a^2 \nu_i \; . \tag{2}$$

As in the earlier calculations [1], a dominant mode number m_y is
visible that obeys

$$m_{dom} \approx 4 m_{marg} \; , \tag{3}$$

where m_{marg} is the marginally stable mode number. As an illustration,
Fig. 1 shows late-time m_y Fourier transforms for an $m_{marg} = 1$
equilibrium, and Fig. 2 a typical wave structure $n_i(x,y)$, n_i = trapped-
ion density. One recognizes a periodic structure in the y-direction
with $m_y = 4$ and $m_y = 5$ in the outer and inner plasma regions,
respectively, and V-like curves of equal phase.

The new shearless result, eq. (2), is confirmed by two independent methods. One method is a combination of a similarity analysis [1, 2, 4] and the numerical result [1, 5] stating that the solutions have approximately m periods in the angular y-direction, with $m \approx 4\,m_{marg}$, m_{marg} = marginally stable mode number. The scaling of eq. (2) is recovered from this derivation. The second method consists in considering exact, special solutions of the Kadomtsev-Pogutse fluid equations, eq. (1), that satisfy the ansatz

$$\tilde{n}^{i}(x, y, t) = n_{1}^{i}(x) + n_{2}^{i}(x)\, \cos\left[ky - \omega t - \varphi^{i}(x)\right] \qquad (4)$$

for the trapped-particle density perturbations. The K_y cut-off can be taken into account in this analysis. The resulting anomalous diffusion formula reads

$$D \approx \frac{1}{12}\, \delta_{o}\, \nu_{i}\, a^{2} , \qquad (5)$$

which is similar to eq. (2), but numerically larger by a factor of ≈ 2. The K_y cut-off does not enter eq. (5). The solutions n^{i} of eq. (4) are exact because the nonlinear terms of eq. (1) do not produce second harmonics. It should be noted that D of eq. (5) is independent of r_{n}, b, and B.

Equation (2) can be modified to include an important shear effect, viz. the effective radial localization of modes due to strong Landau damping [2, 6] . Comparison with experiments should be done using the modified formula. The method given in

[2] yields the anomalous diffusion coefficient with shear effect, viz.

$$D_s \approx 3.5 \times 10^{-2} \, \delta_0 \, \nu_i \, (\Delta a)^2 , \tag{6}$$

where Δa is the distance between properly chosen mode-rational surfaces (where $K_{\shortparallel} = 0$). It is given by [2, 6]

$$\Delta a = \min \left\{ a, \, r_q, \, \frac{r_q}{m_{dom}} \right\} , \tag{7}$$

with $r_q = |q/q_x|, q(x)$ = safety factor. For low enough temperatures eqs. (6) and (7) give, for $b = 2\pi r$, r = local minor radius,

$$D_s \approx 2.2 \times 10^{-3} \, \delta_0 \left(\frac{r_q}{r} \right)^2 \frac{\vartheta_0^2}{\nu_e} , \tag{8}$$

which is small compared with $D_{KP} = \frac{1}{2} \delta_0 \vartheta_0^2 / \nu_e$. At sufficiently high temperatures, shear becomes ineffective because $m_{dom} = 1$. Equation (6) then applies with $\Delta a = \min \left\{ a, \, r_q \right\}$. In evaluating eqs. (6) and (7) it must be ensured that a number of existence conditions for the dominating mode of the instability are also satisfied [2, 4]. If not all conditions are satisfied, one must put $D_s = 0$, contrary to eq. (6).

The ambipolar, trapped-particle anomalous diffusion also causes a convective, anomalous energy flux. A zero-dimensional analysis permits one to evaluate equivalent energy confinement times τ_E from the above process as well as from the neoclassical ion and anomalous electron heat conductivities. The latter are discussed in

refs. $\begin{bmatrix} 7, 8, 9 \end{bmatrix}$. The combined numerical results of these three transport mechanisms, as evaluated for several toroidal plasma devices, will be published elsewhere.

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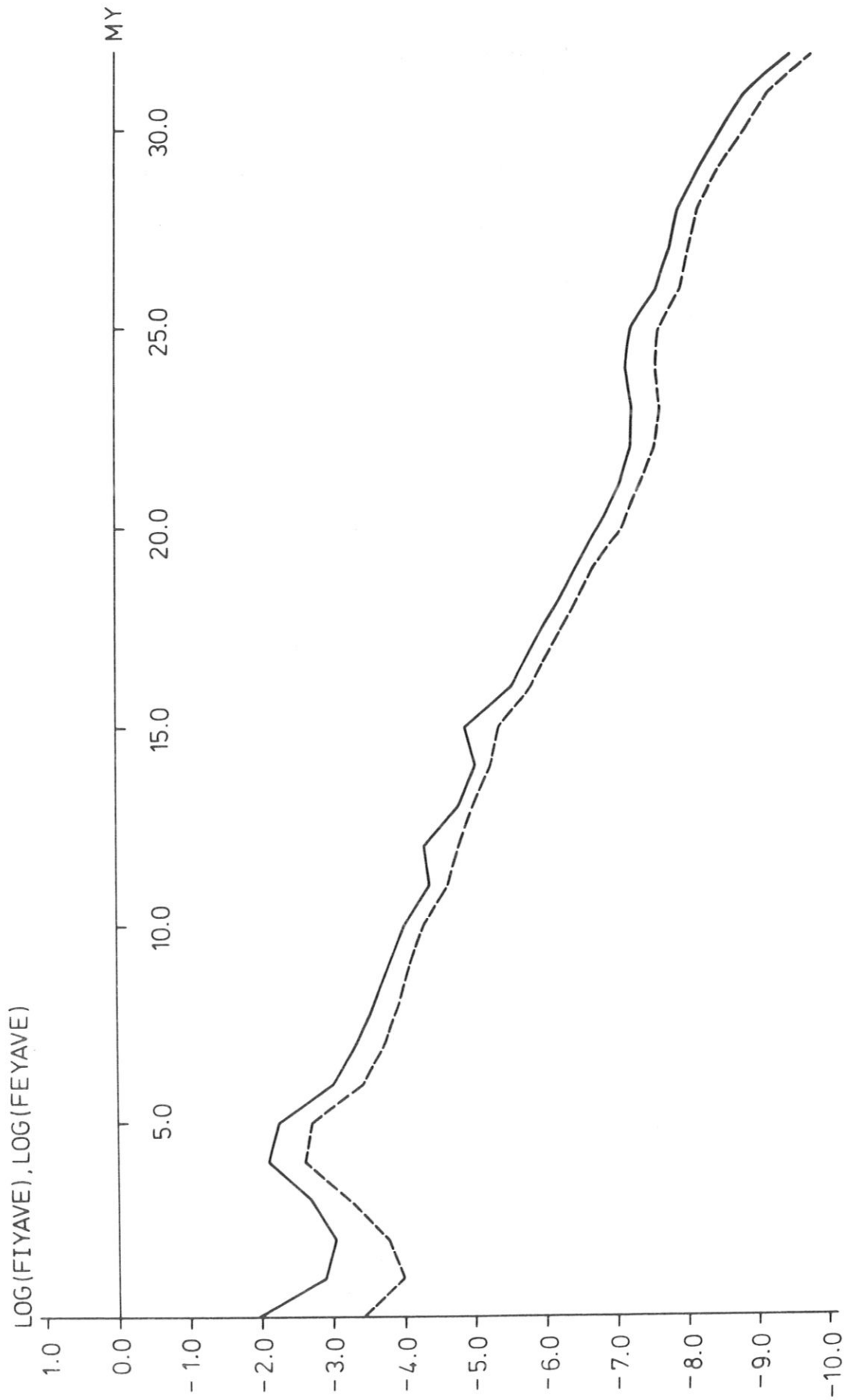Sherwood Meeting, Tucson, Arizona (1980).

m_{marg}	$D/(\delta_o \nu_i a^2)$	$D/(\delta_o a v_o)$	D/D_{KP}	$(e\Phi/T)_{max}$	m_{dom}	m_{cy}	m_{oy}	N_y
0.5	4.10×10^{-2}	4.43×10^{-3}	8.21×10^{-2}	1.43×10^{-1}	3	4	8	64
1.0	3.37×10^{-2}	7.27×10^{-3}	2.69×10^{-1}	1.42×10^{-1}	4(+ 5)	8	16	64
2.0	3.40×10^{-2}	1.47×10^{-2}	1.09	1.54×10^{-1}	8(+ 9)	16	32	128
3.0	4.19×10^{-2}	2.71×10^{-2}	3.02	1.73×10^{-1}	14(+15)	32	64	256
4.0	3.39×10^{-2}	2.93×10^{-2}	4.34	1.58×10^{-1}	16(+18)	32	64	256
5.0	3.02×10^{-2}	3.26×10^{-2}	6.04	1.61×10^{-1}	20(+...)	40	80	256
6.0	2.77×10^{-2}	3.59×10^{-2}	7.98	1.73×10^{-1}	22(+...)	48	96	256

Table I. Anomalous trapped-ion diffusion coefficient D versus m_{marg}, the marginally stable m_y, for $\nu_i/\nu_e = 1.17 \times 10^{-2}$. Quantities: D_{KP} = Kadomtsev-Pogutse diffusion coefficient, Φ = electric potential, m_{dom} = dominant azimuthal mode number m_y, m_{cy} = half-width of numerical cut-off of m_y, m_{oy} = highest m_y present in the initial perturbation, N_y = number of grid intervals in the y-direction, m_{cx} = 16 = half-width of cut-off of m_x, N_x = 64 = number of grid intervals in the x-direction. The values of $(e\Phi/T)_{max}$ hold for $\delta_o = \frac{1}{2}$.
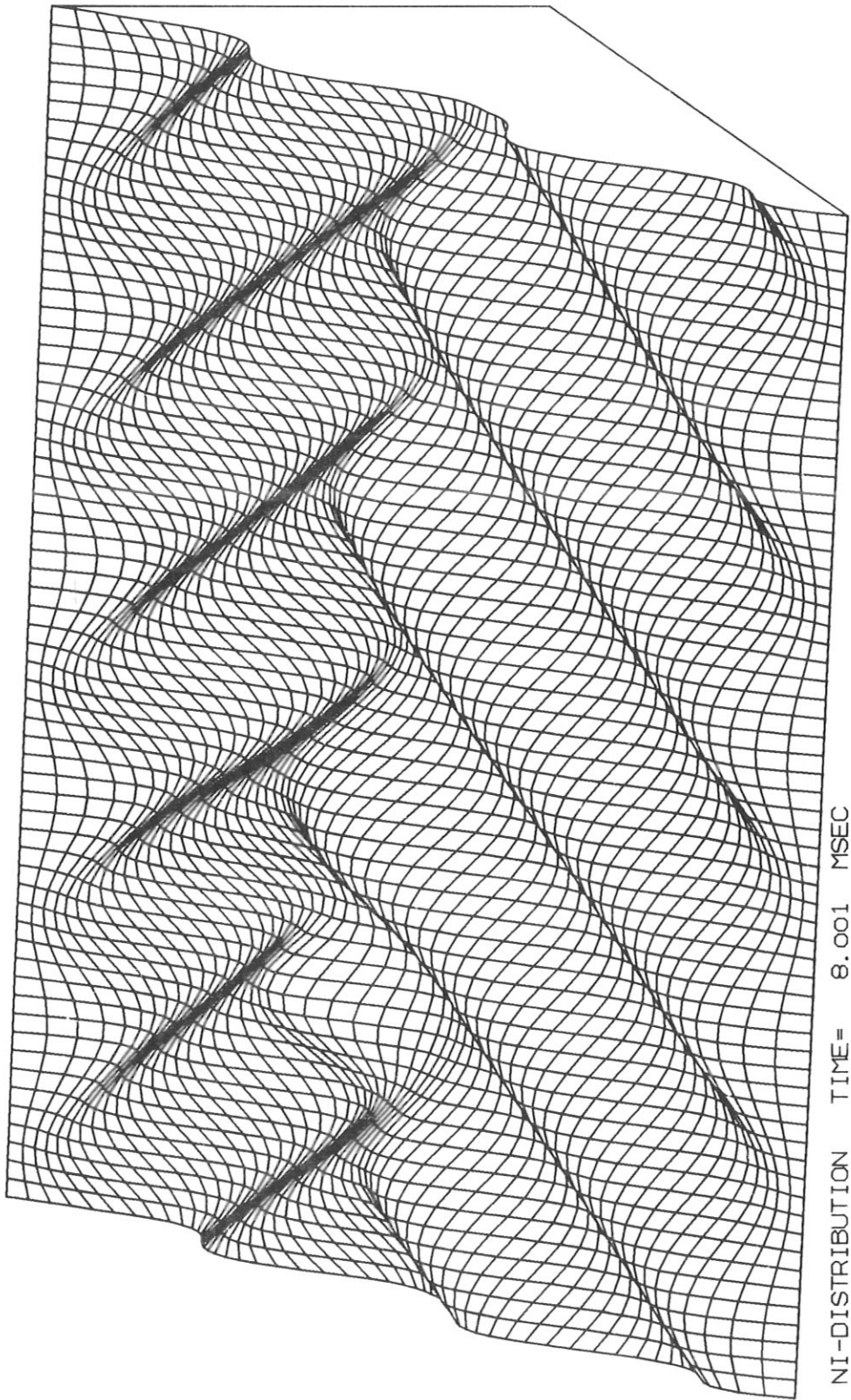
Figure Captions

Fig. 1   Fourier transforms in the y-direction of trapped-ion and

trapped-electron (dashed line) densities, root-square

averaged over x, for $m_{marg} = 1$ at late times.

Fig. 2   Trapped-ion density distribution in the x-y plane, for

$m_{marg} = 1$ at late times.

LOG(FIYAVE),LOG(FEYAVE)

TIME = 8.001 MSEC

NI-DISTRIBUTION   TIME=   8.0o1   MSEC