The Role of Current-Driven Drift Mode on Various Scaling Laws

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ABSTRACT

The anomalous thermal heat conduction and particle diffusion caused by current driven drift modes (CD-mode) 4),5) can affect the energy balance of an ohmically heated plasma appreciably. Transport coefficients scaling as $\frac{1}{n}$ are derived using simple assumptions for the fluctuation level. Energy balance equations with the parameters of the W VII-A stellarator with ohmic heating are evaluated in order to explain the temperature scaling $T_e = T_e(\bar{n})$, $\bar{n}$ = density. At high density the anomalous transport coefficient increases with density but the increase is not large enough to explain the rapid deterioration of confinement at high density as found in W VII-A. In the low density regime the CD-mode may explain the observed scaling. The ions are assumed to behave neoclassically through the analysis.

I. INTRODUCTION

Theoretically, the effect of micro-instabilities on the macroscopic behavior of plasmas is generally calculated by estimating the transport coefficients, mainly diffusion and thermal conduction rates. In energy balance analyses, a concept of drift wave heating has been firstly remarked by Coppi[2] and has been discussed in several articles[2),3]. Krall and Mcbride[3] have recently formulated the turbulent contribution into a set of the transport coefficients (particle convection, thermal conduction) in slab geometry for both electrons and ions and have examined the contribution from trapped electron mode as well as trapped ion modes. In this paper we also make a complete set of energy balances both for electrons and ions, taking into account the anomalous heat exchange between electrons and ions owing to drift waves. The presence of microscopic instabilities is considered to cause both anomalous particle and heat fluxes as well as an anomalous heat exchange between electrons and ions. We here examine a drift wave contribution especially Current-Driven (CD) mode which is excited by electrons in time independent energy balance equations. In this case, the anomalous heat transfer from electrons to ions is expected. On the other hand, in the case where ion-branch of drift waves (driving mechanisms are ion inverse Landau damping) dominates, the anomalous heat transfer will occur from ions to electrons. Including the drift wave contributions, both loss and heating, we obtain scaling laws between various plasma parameters, for instance, the scaling laws between the central density and the central temperatures of ions and electrons for assumed profiles.

In energy balance analyses we have to consider effects of the thermal conductivity, particle convection, radiation, the charge exchange losses, ohmic input together with the anomalous heating and heat exchange owing to micro instabilities. In this article, we put our emphasis on the drift wave's contribution, both losses and heating. In order to figure out its role, we try to reduce the contributions from radiation, and charge exchange losses i) by simply replacing a plasma radius by an effective radius, and ii) by multiplying the energy balance equations with the electron temperature as a weight by which can enhance the efficiency with the plasma.

In this calculation, we assume that electron thermal conduction and particle convection as well as ion particle convection are anomalous and are caused by CD mode[4), 5)], while ion thermal conduction is at most neoclassical loss[6)].

Our calculations are compared to experimental results for tokamak-like discharges in the W VII-A machine[7)] by estimating the scalings between the values of the central electron and ion temperatures, Teo and Tio, and the mean plasma density $\overline{n}$ for given profiles. We find that the role of drift wave fluctuations is remarkable and that it can even explain the experimental results apart from some ambiguities which come from an uncertainty of the ion temperature profile. Concerning the ion energy balance, the ion temperature profiles and its central value, several authors[8), 9), 10)] have been discussing the radial ion energy balance equation taking the ion thermal conduction (neoclassical), the particle convection, the charge exchange loss as well as the Coulomb collisions with electrons into account. In their treatment, the particle convection term is equated to the ionization loss by using the continuity condition[8)] which needs a measurement of the radial neutral atom profile. Furthermore, the electron energy balance is not discussed and assumed to be given. However, we here we include the anomalous transport coefficients in order to find out the inclusive effect of drift waves in energy balances and obtain a set of equations. In comparing our calculations to the experimental results, we introduce an effective plasma radius in two senses; the first is that tokamak-like discharges mostly have a limitation of the central current density owing to MHD activities, so-called saw teeth oscillations, and also have constant profiles of temperature and density in the vicinity of the plasma centre where we expect the thermal conductivity to be very large. This can be interpreted that the plasma radius is effectively reduced. Secondly, the radiation outside the plasma periphery cools the temperature by narrowing the plasma effective radius. By these considerations, we introduce the effective radius into analyses in order to find out the gross properties of the plasma parameter.

A change of ion profile owing to the anomalous energy transfer from electrons and the convection loss, both have different radial distributions, is also discussed.

The structure of this article is as follows:

In section II we formulate the basic energy balance equations, taking the inclusive effect of the CD drift mode, both in collisionless and collisional regimes, into consideration. In section II, we theoretically solve them and derive the gross confinement times, temperature scalings in terms of averaged plasma density. The other effects on them, such as effective radius, impurity density, are also discussed. We also give a short argument about the ion temperature profile. Then, we compare our calculation thus derived with the experimental results of W VII-A[7), 11)]. In section IV some discussions are presented.

II. BASIC EQUATIONS

In this section, we formulate the energy balance equations for both electrons and ions, taking into account anomalous cross field particle fluxes of electrons and ions and anomalous heat flux of electrons. Ion heat flux is assumed to be at most neo-classical[6)]. Contrary to the case of ref. 3), we analyze them in the stationary state. In order to figure out the contributions of the low-frequency electro-static mode, especially the current-driven (CD) drift mode both in collisionless and collisional regimes[4),5)],to energy balance equations, we try to exclude the other contributions from our analyses. We assume that parallel resistivity, η , is given by the classical value.

In the presence of fluctuations, the time independent energy balance equation in the cylindrical coordinate (r, θ, z) is given as[5)]

$$\frac{1}{r}\frac{d}{dr}(Q_r r) + 2P_{rr}\frac{1}{r}\frac{d}{dr}\left(-\frac{\Gamma_r}{n}\right) = 2q v_{d\theta} B\Gamma_r + 2q\langle \tilde{\Gamma}\cdot\tilde{E}\rangle - P_{cx} \quad (1)$$

where Γ_r and Q_r are particle and heat fluxes respectively and are defined by[12)],

and $$\Gamma_r \equiv \sum_k \frac{\langle \tilde{n}_k \cdot \tilde{E}_{k,\theta}\rangle}{B}$$

and

$$Q_r \equiv \sum_k \left[\sum_i Q_{rii} + \Gamma_r P_{ii}\right] \quad , i = r, \theta, z$$

$$\equiv \sum_k \frac{1}{B}\left[\langle (2\tilde{P}_\perp + \tilde{P}_\parallel)\cdot\tilde{E}_\theta\rangle - \frac{P_\perp}{n}\langle \tilde{n}\cdot\tilde{E}_\theta\rangle\right]$$

where B is a strong main magnetic field in the Z direction, n, q are density and charge, $\tilde{P}_\parallel = \int m(v_z - u)^2 \tilde{f}\, d^3v$, $\tilde{P}_\perp = \int m(v_r^2 + (v_\theta - v_{d\theta})^2)\tilde{f}\, d^3v$

$\tilde{f}$ is the deviation from the equilibrium distribution function, U is the longitudinal electron velocity which is related to the plasma current $j = -enu$, $v_{d\theta}$ is the diamagnetic velocity in the presence of the density gradient ($\chi = \frac{d}{dr}\ln n$) and the temperature gradient ($\chi_T = \frac{d}{dr}\ln T$) and is defined by

$$v_{d\theta} \equiv (\chi + \chi_T)\frac{T}{qB} .$$

The quantities without tilde stand for equilibrium value which is defined in a similar way above[12], and Pcx stands for a charge exchange loss. In Qr indicated above, there are two terms which come from the conductive part and from the convection part. In eq. (1), the second term on the left-hand side and the first term on the right-hand side correspond to the work done by the Hall electric field and the slowly diffusing plasma. The term $\langle \tilde{\Gamma} \cdot \tilde{E} \rangle$ includes the heat exchange between electrons and ions through fluctuations as well as radiation.

If we limit ourselves to the case where there are thermal fluctuations, this $\langle \tilde{\Gamma} \cdot \tilde{E} \rangle$ gives three contributions, i) the classical heat exchange between electrons and ions owing to Coulomb collisions, ii) ohmic input to electrons, that is ηj^2, and iii) the radiation caused by high-frequency thermal fluctuations, P_{rad}. Taking into account the low-frequency fluctuations, we obtain the term $\langle \tilde{\Gamma} \cdot \tilde{E} \rangle$ for both ions and electrons as follows:

$$\langle \tilde{\Gamma} \cdot \tilde{E} \rangle = \begin{cases} 2\Gamma_{re} \varkappa T_e + n\nu_{ei}(T_e - T_i) & \text{(2-A)} \\ \\ -2\Gamma_{re} \varkappa T_e - n\nu_{ei}(T_e - T_i) + \eta j^2 - P_{rad} & \text{(2-B)} \end{cases}$$

In order to obtain eqs. (2-A) and (2-B), we use relations $z\Gamma_{ri} = \Gamma_{re}$ together with $n_e = Z n_i = n$ since the ambipolarity across the magnetic field line and the quasi-neutrality are strictly conserved for low frequency fluctuations[13]. From now on, we use subscripts i and e for ions and electrons, Z is a charge of an ion, the classical equipartition frequency is given by

$$\nu_{ei} = 4(2\pi)^{1/2} e^4 [Z] \frac{n^2 \sqrt{m_e}}{m_i} T_e^{-3/2}$$

where $[Z] = \sum_j n_j m_j Z_j^2 / n_e m_i$, and the parallel conductivity

$$\eta = 1.03 \times 10^{-2} Z \ln\Lambda \, T_e^{-3/2} \ (\text{Ohm cm})$$

Substituting eqs. (2-A) and (2-B) into eq. (1), we obtain energy balance equations for electrons and ions in the presence of low-frequency fluctuations. Combining both equations, we obtain the total energy balance equation, that is

$$\frac{1}{r}\frac{d}{dr}\{r(Q_{ri}+Q_{re})\}+2(P_{ri}+P_{re})\frac{1}{r}\frac{d}{dr}(r\frac{\Gamma_{re}}{n}) \tag{3}$$

$$=2\Gamma_{re}\{\varkappa(T_e+T_i)+\varkappa_{T_i}T_i+\varkappa_{Te}T_e\}+\eta j^2-P_{rad}-P_{cx}$$

For ions, we obtain

$$\frac{1}{r}\frac{d}{dr}(rQ_{ri})+2P_{ri}\frac{1}{r}\frac{d}{dr}(r\frac{\Gamma_{re}}{n}) \tag{4}$$

$$=2\Gamma_{re}\{\varkappa(T_e+T_i)+\varkappa_{T_i}T_i\}+n\nu_{ei}(T_e-T_i)-P_{cx}$$

These equations (3) and (4) are a starting base of our analyses. In order to solve these equations, we should know the particle and heat fluxes. Here we take the current-driven (CD) drift mode in both collisionless and collisional regimes into consideration as a main source of anomalous particles fluxes and electron heat flux and examine its contribution to the energy balances of elctrons and ions.

The electron heat and part fluxes owing to CD mode are given by matrix form as

$$\begin{bmatrix}\vec{\Gamma}\\ \vec{Q}\end{bmatrix}=-\begin{bmatrix}D_{11}, & D_{12}\\ D_{21}, & D_{22}\end{bmatrix}\begin{bmatrix}\nabla n\\ \nabla T\end{bmatrix} \tag{5}$$

where

$$D_{11}=[\frac{u}{v_e}+\varepsilon F_1+\varepsilon\sqrt{\frac{\nu}{\omega_{xm}}}F_2]D\ ;\ D_{12}=-\varepsilon\frac{F_3}{2}\frac{n}{T}D$$

$$D_{21}=[2\frac{u}{v_e}+2\varepsilon F_1+3\varepsilon\sqrt{\frac{\nu}{\omega_{xm}}}F_2]TD$$

$$D_{22}=[3\varepsilon F_3+2\varepsilon\sqrt{\frac{\nu}{\omega_{xm}}}F_4]nD$$

and $$D = \frac{\chi \nu}{2} \frac{T}{eB} F_0$$

where $$\omega_{\chi m} = \frac{\chi T_e}{eB} g_i^{-1} \; ; \; \nu = \nu_{eff} = Z_{eff} \frac{\omega_{pe}^4 \ln\Lambda}{3n(\sqrt{2\pi} v_e)^3}$$

The coefficients F_i are a slowly varying function of a shear parameter χL_s where L_s is the shear length. For the typical averaged value of $L_s = 10$, $F_o = 1.27$, $F_1 = F_2 = 0.1$, $F_3 = 1.1$, $F_4 = 1.2$. The smallness parameter ε is in the order of $(\chi L_s \frac{m_2}{m_i})^{1/2}$ and v_e is the electron thermal speed $\sqrt{T_e/m_e}$.

In the collisionless regime (lower density region), the terms which are proportional to $\frac{\omega}{v_e}$ dominate in eq. (5). On the other hand, in the collisional regime $\sqrt{\frac{\nu}{\omega_{\chi m}}}$ terms play main roles. In deriving this matrix formula, we assume the quasi-linear saturation level by choosing K_r to be an inverse of the mode localization width obtained in non-local analyzes[4], that is $|\frac{e\phi}{T_e}|^2 = f \frac{\chi^2/2}{K_\theta^2 + K_r^2}$, where we will use f to be an amplitude parameter which is to be determined empirically (~ 1). We summed up the modes provided each mode can be treated independently. For ion fluxes, we use the relation $\Gamma_{re} = Z\Gamma_{ri}$ and we assume that the ion heat flux is given by the neoclassical value since a recent analysis has shown that it remains small even in the presence of drift mode[6].

III. ENERGY BALANCES

A) Total and Ion Energy Balance

In this section we first analyze the global plasma parameter dependences concerning the heat content within the plasma column. For assumed profiles of plasma density, $n = n_0 \exp[-r^2/2L_n^2]$ and the temperatures $T_j = T_{oj} \exp[-\frac{r^2}{2L_{T_j}^2}]$ we solve the basic equations obtained in the previous section. In order to emphasize the inside of the plasma, we multiply the electron equation with $\frac{r}{a} nT_e/n_oT_{eo} \equiv \frac{r}{a} \exp[-\frac{1+\eta_e}{2L_n^2} r^2]$ ($\eta_e \equiv \frac{\chi_{Te}}{\chi} = \frac{L_n^2}{L_{Te}^2}$ is the ratio of the temperature gradient to the density gradient for electrons and η_i the equivalent for ions) and integrate eqs. (3) and (4) over the plasma radius up to r = a. Here we take simply $a = \sqrt{2}\, L_n$ (e - folding length). Instead of integrating over the plasma column, one can solve eqs. (3) and (4) directly; however, for the sake of comparing to experimental results, we take this procedure. Performing the partial integration of eq. (3), we obtain

$$\frac{n(a)T_e(a)}{n_o T_{eo}}\left[Q_{re}(a)+Q_{ri}(a)+2\left\{T_e(a)+\frac{T_i(a)}{Z}\right\}\Gamma_{re}(a)\right]$$
$$+\int_0^a \frac{nT_e}{n_o T_{eo}}(1+\eta_e)\left[Q_{re}+Q_{ri}+2\Gamma_{re}(T_e+\alpha_o T_i)\right]\frac{2r}{a}dr \qquad (6)$$
$$=\int_0^a \frac{nT_e}{n_o T_{eo}}\left\{\eta j^2 - P_{rad} - P_{cx}\right\}\frac{r}{a}dr$$

where

$$\alpha_o = 1+(1-Z)\frac{1+\eta_i}{(1+\eta_e)Z}$$

$$\frac{n(a)T_e(a)}{n_o T_{eo}}\left[Q_{ri}(a)+\frac{2}{Z}\Gamma_{re}(a)T_i(a)\right]$$
$$+\int_0^a \frac{nT_e}{n_o T_{eo}}\left[Q_{ri}(1+\eta_e)+2\Gamma_{re}\left\{(1+\eta_e)\alpha_o T_i - T_e\right\}2r\frac{dr}{a}\right. \qquad (7)$$
$$=\int_0^a \frac{nT_e}{n_o T_{eo}}\left[n\nu_{ei}(T_e-T_i)-P_{cx}\right]\frac{rdr}{a}$$

where the first terms in eqs.(6) and (7) are neglected since the electron pressure is very small at the plasma edge. From now on we neglect such edge contribution which may appear in the other terms. We will also neglect the terms of radiation and charge exchange losses in order to find out how much drift modes contribute to the energy balance in the plasma. In addition, if the radiation source and charge exchange loss are concentrated to the outside region of the plasma boundary, then this assumption may have some validity in analyses. In eqs. (6) and (7) the terms which are proportional to Γ_{re} and Q_{re} come from the drift wave contribution. You find that the drift mode causes both <u>heating</u> and <u>loss</u>, those rates depend on the radial location and profiles of the plasma parameters, for example n, T_e, T_i, Z

Taking into account the radial dependence of the particle flux Γ_{re}, heat fluxes Q_{re} and Q_{ri} where we take Q_{ri} to be neo-classical in the plateau regime, and assuming that the current profile is determined by the electron temperature profile, i.e. $j = j_0 (T_e/T_{e0})^{3/2}$ and that the conductivity η, is classical, we finally obtain the total energy balance equation in the integrated form as

$$\beta_0 Q_{ri0} + T_{e0}(\alpha_3 \Gamma_{cu}^{a} + \alpha_4 \Gamma_{ce}^{a}) + \alpha_0 T_{i0}(\alpha_1 \Gamma_{cu}^{a} + \alpha_2 \Gamma_{ce}^{a}) = \eta_0 j_0^2 \frac{a^2}{(2+5\eta_e)} \tag{8}$$

where

$$Q_{ri0} = 0.585\left(\frac{a}{R}\right) \frac{\sqrt{A}\, n_0 T_{i0}^{5/2}}{Z^2 B^2 a \chi}$$

$$\Gamma_{cu}^{a} = 15.1 f j_0 \frac{\sqrt{T_{e0}}}{B} \; ; \; \Gamma_{ce}^{a} = 0.759 f \sqrt{n_0^3 Z q_0 \frac{R}{B}}$$

$$\eta_0 j_0^2 = 4.91 \cdot 10^{-2} Z j_0^2 T_{e0}^{-3/2} \tag{8'}$$

where A is a mass number, and $\alpha_0 \sim \alpha_4, \beta_0$ are form factors (including Z, here we take Z to be homogeneous in the radial direction) given by

$$\alpha_0 = 1 + (1-Z)(1+\eta_i)/(1+\eta_e)Z \; , \; \alpha_1 = 2(1+\eta_e)\left(2/1+3\eta_e+\eta_i\right)^3$$

$$\alpha_2 = 4\sqrt{\frac{3\pi}{\eta_e}}\left(\frac{5}{2} + \frac{9}{4}\eta_e + \eta_i\right)/\left(\frac{5}{2} + \eta_e + \eta_i\right)^{7/2} ; \; \alpha_3 = \frac{32(1+\eta_e)}{(1+4\eta_e)^3}$$

$$\alpha_4 = \sqrt{\frac{3\pi}{\eta_e}}(1+\eta_e)\left(5 + 2\frac{F_4 \eta_e}{F_2}\right)\left(5 + \frac{13\eta_e}{2}\right)/\left(\frac{5}{2} + 2\eta_e\right)^{7/2}$$

$$\beta_0 = \eta_i (1+\eta_e)/2\left(1 + \frac{\eta_e}{2} + \frac{5}{4}\eta_i\right)^2$$

Plasma parameters are measured as follows: a (10 cm), R (m), B(T), T_{io}, T_{eo} (0.1 keV), $n_o(10^{13}/cm^3)$ and j_o(1 MA/m^2). The safety factor at the plasma periphery q_a is given by 2.5 B/R I_o. The rotational transform ι is made by both ohmic current and external magnetic field.

For the ion energy balance equation, we obtain

$$\beta_o Q_{rio} + \alpha_o T_{io}(\alpha_1 \Gamma_{cu}^a + \alpha_2 \Gamma_{ce}^a) - T_{eo}(\alpha_3' \Gamma_{cu}^a + \alpha_4' \Gamma_{ce}^a)$$
$$= (n\nu_{ei})_o \frac{a^2}{6+\eta_e}(T_{eo} - \alpha_5 T_{io}) \tag{9}$$

where α_3', α_4', and α_5' are also form factors as $\alpha_3' = (2/1+4\eta_e)^3$; $\alpha_4' = 2\sqrt{3\pi/\eta_e}\,(\frac{5}{2}+\frac{13}{4}\eta_e)/(\frac{5}{2}+2\eta_e)^{7/2}$ and $\alpha_5 = (3+\eta_e/2)/(3+\eta_i+\frac{\eta_e}{2})$. The term of the heat transfer owing to Coulomb collisions is given by the right-hand side of eq. (9) where $(n\nu_{ei})_o \equiv 3.85 \cdot 10^{-2}(Z/A)\, n_o^2 T_{eo}^{-3/2}$
The units used here are the same as for eq. (8). In deriving eqs. (8) and (9) we neglected the small correction terms in the fluxes formula, eq. (5). The terms which have Γ_{cu}^a come from the $\frac{u}{v_e}$ terms in eq. (5), may dominate in the collisionless regime, and Γ_{ce}^a terms are from $\sqrt{\frac{\nu}{\omega_{*m}}}$ terms (collisional regime). From eq. (8) one can see that the neoclassical loss (Q_{rio} term) in the total energy is small if the ion temperature gradient is not extremely large and that the drift wave loss balances to the ohmic input.

As for the ion energy balance, the second and third terms on the left-hand side show the drift wave contribution (both heating and loss). The drift waves can heat the ions, but is also causes the ion anomalous particle flux[13]. In general, the drift wave contributes to the heat loss (due to particle convection) in general discharges of the plasma. Let us briefly discuss about eq. (8) for the fixed other parameters n_o, T_{eo}, T_{io} we see that even the ratio of T_{io} to T_{eo} ($\equiv R_T$) changes appreciably the relation between T_{eo} and n_o does not change so much and one can find that T_{eo} is a decreasing function of n_o for both collisionless and collisional regimes.

Once one knows the relation between T_{eo} and n_o, one can find the T_{io} dependence on n_o or T_{eo} from eq. (9). If the ion thermal convection is small, eq. (9) shows that T_{io} is the increasing function of n_o for a relatively low density region and will saturate or decrease according to the increase of n_o. Before considering the absolute value or discussing eqs. (8) and (9) in more detail, let us discuss about the ion temperature profile change owing to the CD drift mode.

B) Ion Temperature Profile Change

Since the damping mechanism of the drift wave has been understood to be ion Landau damping (through the convection by the magnetic shear) and the free energy is extracted from electrons, there must be some duct of energy via the drift waves. This is the heating mechanism on ions. However, this drift wave also causes anomalous ion particle flux across the magnetic field so that we have to consider the competition between them.

However, these two effects have different radial distributions. Therefore, some part of the plasma column might be heated by waves. In the following, we show that the outer region of the plasma column can be heated while the inner part of the column is cooled by them. Thus, the ion temperature can exceed the electron temperature in the outer region of the plasma column. This possibility comes from the condition that the drift wave input is greater than the drift wave loss. This necessary condition is directly derived from eq. (4), comparing the second term of the left-hand side (loss) and the first term on the right-hand side (heating), obtaining

$$\frac{2}{Z}\left\{\kappa\frac{d}{dr}(\kappa r)+\frac{\kappa^3 r}{2}-\kappa^2\kappa_{Te}\,\nu-\frac{Z}{2}\kappa^2(\kappa_{T_i}+\kappa)r\right\}T_i(r)<\kappa^3 r\,T_e(r) \qquad (10)$$

For Gaussian profiles

$$\frac{2}{Z}\left\{\frac{2r^2}{L_n^4}+\frac{r^4}{L_n^6}\left(1-2\eta_e-\frac{Z}{2}(1+\eta_i)\right)\right\}<\frac{T_{eo}}{T_{io}}\frac{r^4}{L_n^6}\exp\left\{-\left(\frac{r^2}{L_{Te}^2}-\frac{r^2}{L_{Ti}^2}\right)\right\} \qquad (10')$$

From (10) and (10') we see that the radial dependences of input and loss are different and that the heat input is localized to the outer region of the plasma column. Therefore, one can expect the fact that the ion temperature in the outer region can exceed the electron temperature if this condition is satisfied.

C) Comparison with the Experiment

In order to figure out the drift wave contribution in the energy balances of both electrons and ions in experiments, our calculations are compared with the experimental measurements of n, T_e, and T_i, and the energy confinement times for tokamak-like operation in W VII-A[7), 11)]. The procedures of analyses are as follows:

i) First, we calculate the expected total and electron confinement times.

ii) Then, from the energy balance equations for ions and electrons, we derive the relationship between n_o, T_{eo} and T_{io} assuming the other parameters to be fixed. The Z dependence as well as the current density j_o dependence are included.

iii) Finally, using the scaling laws for n_o, T_{eo}, and T_{io}, the energy confinement times of electron and total are calculated and are compared with the experiment.

We choose typical discharges of W VII-A. On the one hand, the radiation from the plasma periphery dominates and, on the other hand, there are discharges which do not show the significant amount of radiations[17)]. We will concentrate on results from the latter discharges. Our analyses for the energy confinement time of τ_E shows, defining

$$\frac{\int (nT_e + n_i T_i) r dr}{\tau_E} \equiv \int \eta j^2 r dr \equiv \frac{1}{S}\int \eta j^2 \left(\frac{nT_e}{n_o T_{eo}}\right) r dr \qquad (11)$$

where S is the form factor given as $\frac{3}{2}\eta_e/(1+\frac{5}{2}\eta_e)$ from eq. (8) neglecting the Q_{rio} term. $(R_T \equiv T_{io}/T_{eo})$.

$$\frac{1}{\tau_E} = \frac{2(2+5\eta_e)(1+\eta_e)(1+\eta_i)}{3\eta_e[(1+\eta_i)+R_T(1+\eta_e)]} \frac{[(\alpha_3+\alpha_0\alpha_1 R_T)\Gamma^a_{cu}+(\alpha_4+\alpha_0\alpha_2 R_T)\Gamma^a_{ce}]}{n_o a^2} \tag{12}$$

For the electron confinement time, using eqs. (8) and (9) and replacing $nT_e + Zn_iT_i$ by nT_e in eq. (11), we obtain

$$\frac{1}{\tau_{Ee}} = \frac{1+(1+\eta_e)\frac{R_T}{1+\eta_i}}{\tau_E} - \frac{2(2+5\eta_e)(1+\eta_e)}{3\eta_e(6+\eta_e)} \frac{(n\nu_{ei})_o}{n_o}(1+\alpha_5 R_T) \tag{13}$$

Note that a stands for an effective plasma radius in which we can effectively include the radiation loss by narrowing the plasma radius, a , and a' will be introduced as an effective transport layer reduced by both MHD activities and the radiation[14].

From eqs. (12) and (13) together with the Γ^a_{cu} and Γ^a_{ce} in eq. (8'), you find that there is an optimum density though the electron and ion temperature dependences on the density are not yet included. The experimental condition of W VII-A covers both the collisionless and the collisional regimes, that is,on the low density side ($n \approx 10^{13}/cm^{-3}$) collisionless CD mode dominates, and at the high density side the collisional CD mode is dominating (R = 2m, B=3T, a $\leq$ 11 cm).

Measurements show that η_e is close to unity, while η_i has a very small value, i.e., the ion temperature distribution is broader,and in the outer region in the plasma column, the ion temperature may exceed the electron temperature! We present a possibility of drift wave heating on ions in subsection (B). It may explain the profile of the ion temperature qualitatively. However, as indicated in eq. (10), the heating is very sensitive to the ratio, $T_{io}/T_{eo} \approx R_T$, as well as the profiles themselves. We do not enter this problem in detail. For the ion temperature profile measured in the experiment, we see that the neoclassical loss is small in the total energy equation and is neglected.

From eq. (8) we obtain (taking $\eta_e \approx 1$ and $\eta_i = 0.1$) the relation

$$T_{eo}^{5/2}\left[0.87 J_o\left(1+0.49R_T\left(\frac{3-Z}{Z}\right)\right)T_{eo}^{1/2}+2\sqrt{Z\gamma_o}\, n_o^{3/2}\right]\frac{f}{B} = 38 Z a^2 J_o^2 \tag{14}$$

where n_o is interpreted by $\bar{n} = \int ndl$ (line integrated density which has the relation $n_o a\sqrt{\pi} = \bar{n}$ for Gaussian profile, and $\bar{n}$ is measured in ($10^{14}/cm^2$), T_{eo} and T_{io} are in (0.1 keV), j_o is in (1 MA/m^2) and B is in (3T).

For the ion energy balance we get from eq. (9')

$$n_o^{3/2} T_{eo} \sqrt{Z} \left(0.52 R_T \frac{3-Z}{Z} - 0.13\right) + T_{eo}^{3/2} \left\{\frac{3-Z}{Z}, 1.35 R_T - 0.32\right\} j_o$$

$$= 1.76\, Z \frac{n_o}{\sqrt{T_{eo}}} (1 - 1.4 R_T) \frac{a^2 B}{f} \qquad (15)$$

combining eqs. (13) and (14), we obtain for j_o = 2, B = 3

$$R_T = \frac{0.055(\bar{n} a T_{eo})^{3/2} + 0.64 T_{eo}^2 + 0.652\, Z \bar{n}^{-2} \frac{a^2}{f}}{\left\{0.22 (\bar{n} a T_{eo})^{3/2} + 2.7 T_{eo}^2 \left\{\frac{3-Z}{Z} + 0.78\, Z \bar{n}^{-2} \frac{a^2}{f}\right\}\right.} \qquad (15')$$

where we also neglect the neoclassical thermal conduction term in eq. (15).

In order to compare with the experimental results, let us discuss about the effective plasma radius in the experiment. Although the plasma radius of W VII-A is about 10 cm, in the vicinity of the plasma column,MHD-saw-teeth oscillations dominate, setting a limitation of the central current density (which may correspond to the q=1 restriction). This MHD activity causes a non-equilibrium state around the plasma axis ($r < r_0$) where we expect very large transports, and makes the effective radius be much shorter. Including the radiation effect, we use "a" as the effective radius, and "a'" as an effective transport layer which is reduced by both radiation and MHD activities.As shown in Fig. 1, we have at least two factors of ambiguities concerning the radius. By now, we used f as an amplitude parameter which will be used to include these two ambiguities for fitting with the measurements.

In Fig. 1 we show the relation between r_0, a' and a. For the convenience of interpretation, we use

$$n = \begin{cases} n_0 & (r < r_0 \\ n_0 \exp\{-(r-r_0)^2/a'^2\} & ; r > r_0 \end{cases}$$

which gives the effective plasma radius "a" to be

$$a^2 = a'^2 + \left(\frac{e}{e-1}\right) r_0^2$$

from the total number conservation.

In Fig. 2, we compare our calculations derived in eqs. (14) and (15') with the experimental measurements for $j_0 = 2$ MA/m^2, B = 3 T, and $n = n_0 / 1+(r/b)^4$ (b = 6.5 cm). The density distribution roughly corresponds to the case, $r_0 = 3.2$ cm, a' = 4 cm, that is a = 5.7 cm. In order to obtain the value of an amplitude parameter, f, we use one data (T_{eo} = 350 eV, $\bar{n} = 2.6$ (10^{14}/cm^2) and get f $\approx$ 1. Two lines of T_e vs $\bar{n}$ in Fig. 2 show the dependence on the different Z values, the upper line indicates the case for Z = 1.4 and the lower one corresponds to the case for Z = 1.0. Experimental measurements for H_2 gas are also plotted with subscripts which indicate the value of Z. These theoretical lines are satisfactorily obtained even if we neglect the terms which are proportional to R_T in eq. (14).

Although the relation between T_{eo} and $\bar{n}$ is insensitive to the ion temperature itself (when it is not extremely high), the ratio of ion and electron temperatures (R_T) or T_{io} itself is very sensitive to the ion temperature profile, as indicated in eqs. (9) and (15). So that we show the expected ion temperature derived from our calculation by the dashed region in Fig. 2. The lower boundary of the region corresponds to the $\eta_i = 0$ case, while the upper line shows the case of $\eta_i = 1$ being Z = 1, also experimental data corresponding to the same data as used for the T_{eo}-$\bar{n}$ curve. One can see that agreements are fairly good.

We now return to the energy balance equations. The density dependence on T_{eo} is derived from the total energy balance equation in which we finally know that the drift wave loss balances to the ohmic input within the plasma column. As an example, we reunite eq. (12) for the case $\eta_e = 1$, $\eta_i = 0$, and $Z = 1$ ($\alpha_0 = 1$, $\alpha_1 = 0.5$, $\alpha_2 = 0.73$, $\alpha_3 = 0.5$/and $\alpha_4 = 10.$) yielding

$$\frac{1}{\tau_E} = \frac{1.8\,\bar{f}(a,a')}{(1+2R_T)\bar{n}aB}\Big[15\,j_0\sqrt{T_{eo}}\,(0.51+0.5R_T)$$

$$+\,0.32\sqrt{R}\left(\frac{\bar{n}}{a}\right)^{3/2}(10.6+0.73R_T)\Big]\,(ms)^{-1}$$

Substituting eqs. (14) and (15) into eq. (16), we finally obtain the energy confinement vs. line integrated density $\bar{n}$. The calculations are shown in Fig. 3 for the parameters listed before and $Z = 1$, $a = 5.7$ where $\bar{f}$ (a, a') is also the fitting parameter. For the same value of $\bar{f}$, we compare the expected τ_E for the case $\eta_i = 1$ and also shown in Fig. 3. The results show the tendency that the confinement time τ_E will decrease with the increment of the density, which can be seen in fairly good agreement with the experiment. As we noted, the discrepancy may come from the radiation and charge exchange losses. The rough estimation of some parts of radiation loss contribution to energy loss is incorporated in terms of the effective plasma radius, a. If we consider the case where the radiation loss is dominated in the outside of the plasma column, the plasma edge may be cooled by it and the effective radius may become smaller. This contribution appears for instance in eq.(14), showing that the total energy input by the ohmic heating rate is to be reduced to compensate the radiation loss. As to the dependence of the energy confinement times on the effective radius as well as the effective transport layer, one can see from eqs. (12) and (14) that the optimum density for the confinement time will shift to the lower density side and the absolute value of τ_E may decrease appreciably. For the comparison, the dependence of τ_E on the effective radius is shown in Fig. 4 for the cases of a = 5.7 cm and a = 10 cm. We can see the drastic change of τ_E on the effective radius a. However, this calculation does not give the steep decrease of τ_E nor a density limitation even for the case $\bar{n} \rightarrow \infty$. Therefore, the satisfactory comprehension of the experiments needs to estimate the other losses, such as the radiation and charge exchange loss. In particular, we dropped a radiation part from eq. (12) and only retained the effect from the radius. The radiation loss may be much more noticeable in the high density region. Therefore, in order to obtain the reasonable agreement with experiments for the density limitation, the arguments from the drift wave contribution are not satisfactory.

IV. DISCUSSIONS

In order to reduce the charge-exchange loss (P_{cx}) and the radiation loss (P_{rad}) in the energy balance equation <u>within</u> the plasma, we multiplied the electron pressure distribution as a weight of integration. If the radiation from the inner side of the plasma column contributes appreciably to the energy balance, it must be included in the analyses for comparison with the experiments. However, in the first step, we devote our calculations to finding out how much the drift wave fluctuations can contribute to the energy balance so that the other effects are neglected. Furthermore, as to the charge-exchange loss, even its rate increases with the density, the neutral density may decrease with the increment of the total density[8] and with the increment of the effective plasma radius[10]. It may be, however, possible that its loss still plays an important role in the energy balances, even today's interest is to approach the high density plasma with a small aspect ratio.

The radiation loss in the energy balances may be partially included in our analyses by considering the narrowing of the effective plasma radius a, which is easily interpreted either as a shrinkage of the plasma confinement layer or as a steepening of the density gradient (or both).

In section III(C) we neglected the neoclassical part of the ion heat loss because in the experiment η_i is very small and its contribution is considered to be small. However, in the case of η_i being large, the neoclassical loss again may become important in Ti. In addition, if η_i becomes large as to exceed a certain value ($\eta_i \approx 1.5$), the shear damping on the electron exciting drift mode cannot be expected[4] so that also ion particle loss may be much enhanced; furthermore, an ion exciting drift mode may appear playing an important role on the loss mechanisms. This ion branch of the drift modes might cause anomalous heating on the electrons (if T_i is sufficiently high) as well as anomalous losses, as analogous to our present analyses. Therefore, in the case where η_i is large, one must take the contribution from the ion drift mode into account in reflecting on the energy balance problems.

Although we include the various kinds of drift wave contributions, there remain other open questions to complete the energy balance analyses. We use the assumption that the parallel conductivity remains classical (the so-called Spitzer resistivity and Z_{eff}) while there remains a question whether the drift wave may change it appreciably or not.

In section III(B), we mentioned the ion temperature profile. Even the drift wave can heat the ion it cannot explain satisfactorily since the total heat transfer from electrons to ions is expected to be small. So one has to consider other anomalous heating mechanisms on ions, for example, the anomalous viscosity and heating owing to the ion cyclotron drift modes[11].

Finally, we should mention that these results depend on the choice of the saturation amplitude. The other choices must be examined and compared with each other to obtain the reasonable scalings. Other than the choice of the saturation level, our calculations are not uniquely derived from the results obtained in refs. 4) and 5) because we treated the integrated values of the plasma parameters with assumed profiles and did not solve their expected profiles directly.

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REFERENCES

1) B. Coppi, Phys. Rev. Lett. 25, 851 (1970).

2) N.T. Gladd and W. Horton,Jr., Phys. Fluids 16, 879 (1973).

3) N.A. Krall and J.B. McBride, Nucl. Fusion 17, 713 (1977).

4) S. Inoue, K. Itoh and S. Yoshikawa, Nucl. Fusion 18, 755 (1978).

5) S. Inoue, K. Itoh, T. Tange, K. Nishikawa and S. Yoshikawa, 7th Int. Conf. on Plasma Physics and Contr. Nuclear Fusion Research (Innsbruck 1978) CN-37/W-3, and Internal Report of I.P.P. (Nagoya University) IPPJ-3/8.

6) . Shigeta, T. Tange and K. Nishikawa, private communication, and also in Internal Report of Institute for Fusion Theory (Hiroshima University) HIFT-6.

7) Proc.8th Europ.Conf.on Contr.Fus.and Plasma Physics, II,73, Prague 1977

8) L.A. Berry, J.F. Clarke, and J.J. Hogan, Phys. Rev. Lett. 32, 363 (1974).

9) P.E. Stott. Plasma Physics, 18, 251 (1976).

10) M. Brusati, S.L. Davis, J.C. Hosea, J.P. Strachan, and S. Suckewer, Nucl. Fusion, 18, 1205 (1978)

11) 6th Int.Conf.on Plasma Physics and Contr.Nucl.Fusion Research, II,81, Berchtesgaden 1976 (IAEA-CN-35/D2).

12) T. Tange, S. Inoue, K. Itoh, and K. Nishikawa, Internal Report of Institute for Fusion Theory (Hiroshima University) HIFT-2.

13) S.Inoue, K. Itoh, T. Tange, and T. Tuda, Internal Report of I.P.P. (Nagoya) IPPJ- , to be published.

FIGURE CAPTIONS

Fig. 1 The schematic drawing of the density profiles. The region $r < r_0$ suffers from MHD activity and is considered to be flat. And the region denoted by a' is the effective transport layer.

Fig. 2 Theoretically expected temperatures (T_{eo} and T_{io}) dependence on $\int ndl$ (line integrated density). The upper line of T_{eo} corresponds to the case Z = 1.4 and the lower one is for Z = 1.0. The ion temperature is shown by the dashed region. The boundaries are given by the cases for $\eta_i = 0$ (lower) and $\eta_i = 1$ (upper).
The marks ● and ⊗ are experimental values of T_{eo} and T_{io} respectively. The subscript shows the value of Z.

Fig. 3 The expected total energy confinement time τ_E as a function of $\int ndl$ for the $\eta_i = 0$ case and the $\eta_i = 1$ case are shown together with the data denoted by ● .

Fig. 4 The dependence of τ_E on the effective plasma radius is shown.

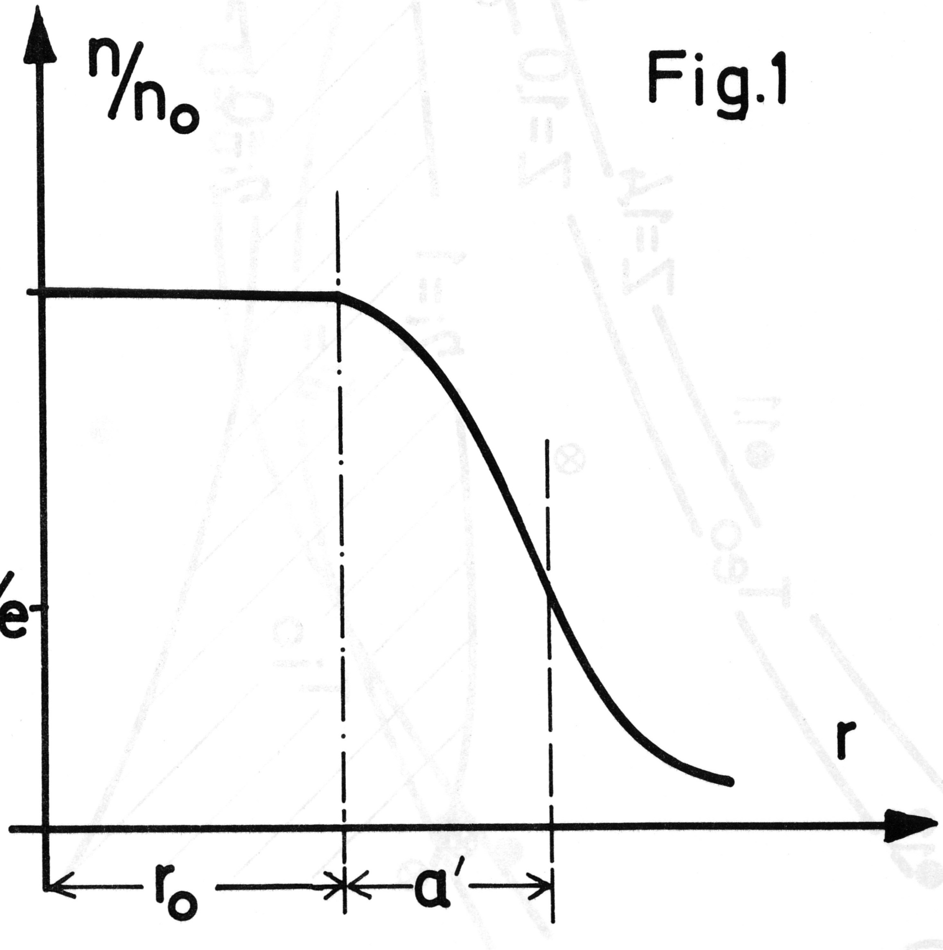


Fig.1

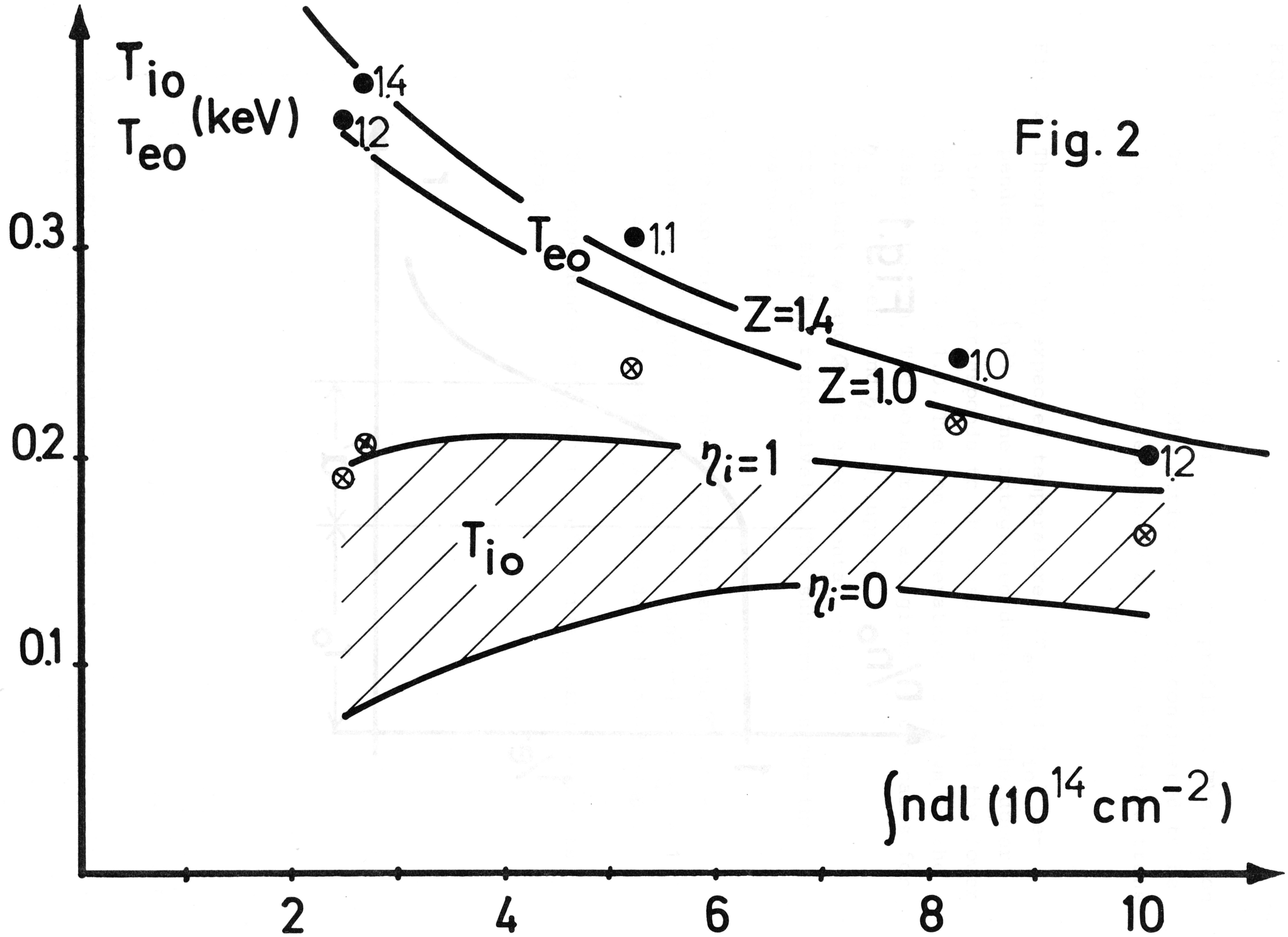


Fig. 2
T_{io} T_{eo} (keV)
0.3
0.2
0.1
T_{eo}
T_{io}
Z=1.4
Z=1.0
$\eta_i=1$
$\eta_i=0$
1.4
1.2
1.1
1.0
1.2
2
4
6
8
10
$\int ndl$ ($10^{14}cm^{-2}$)

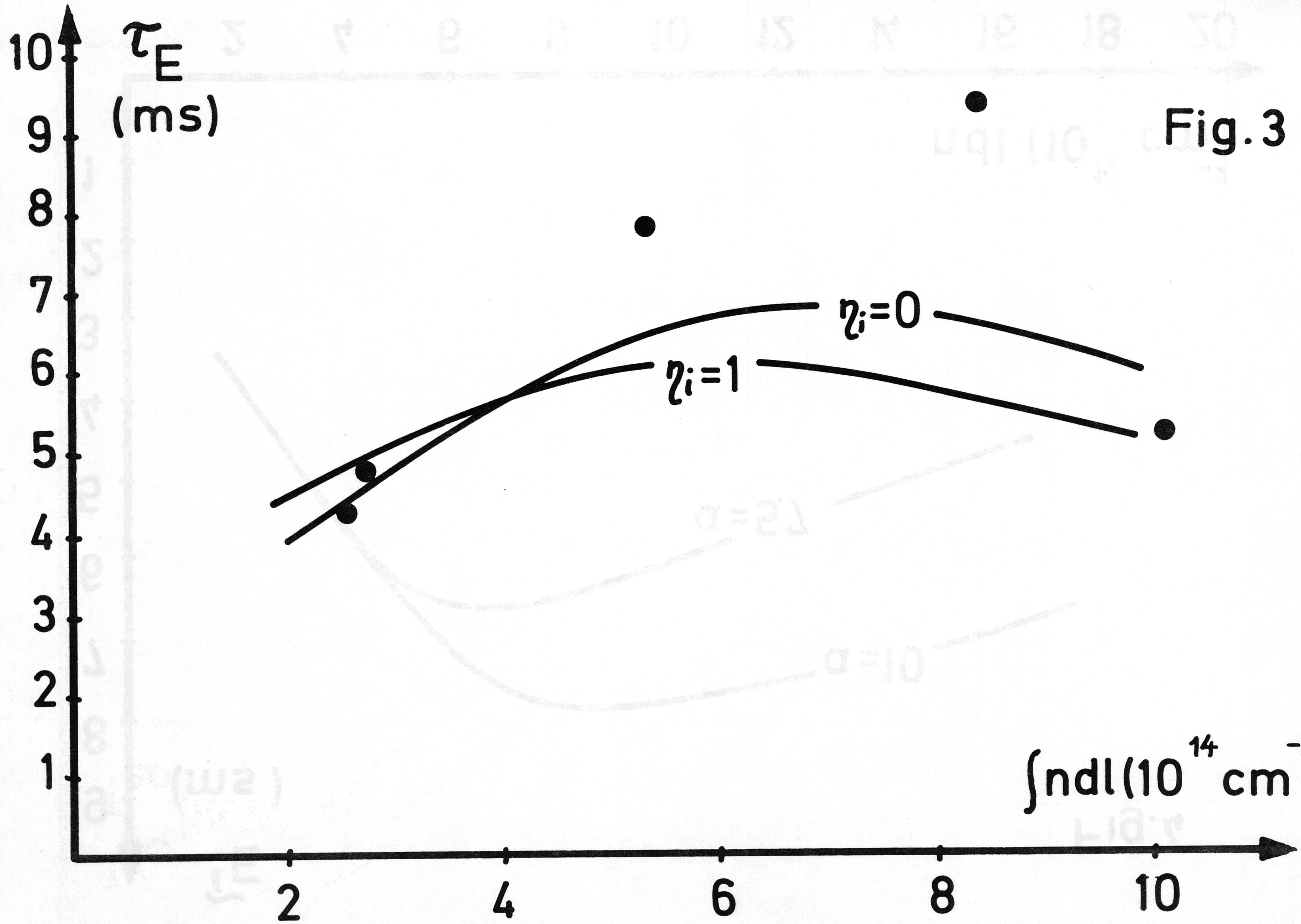
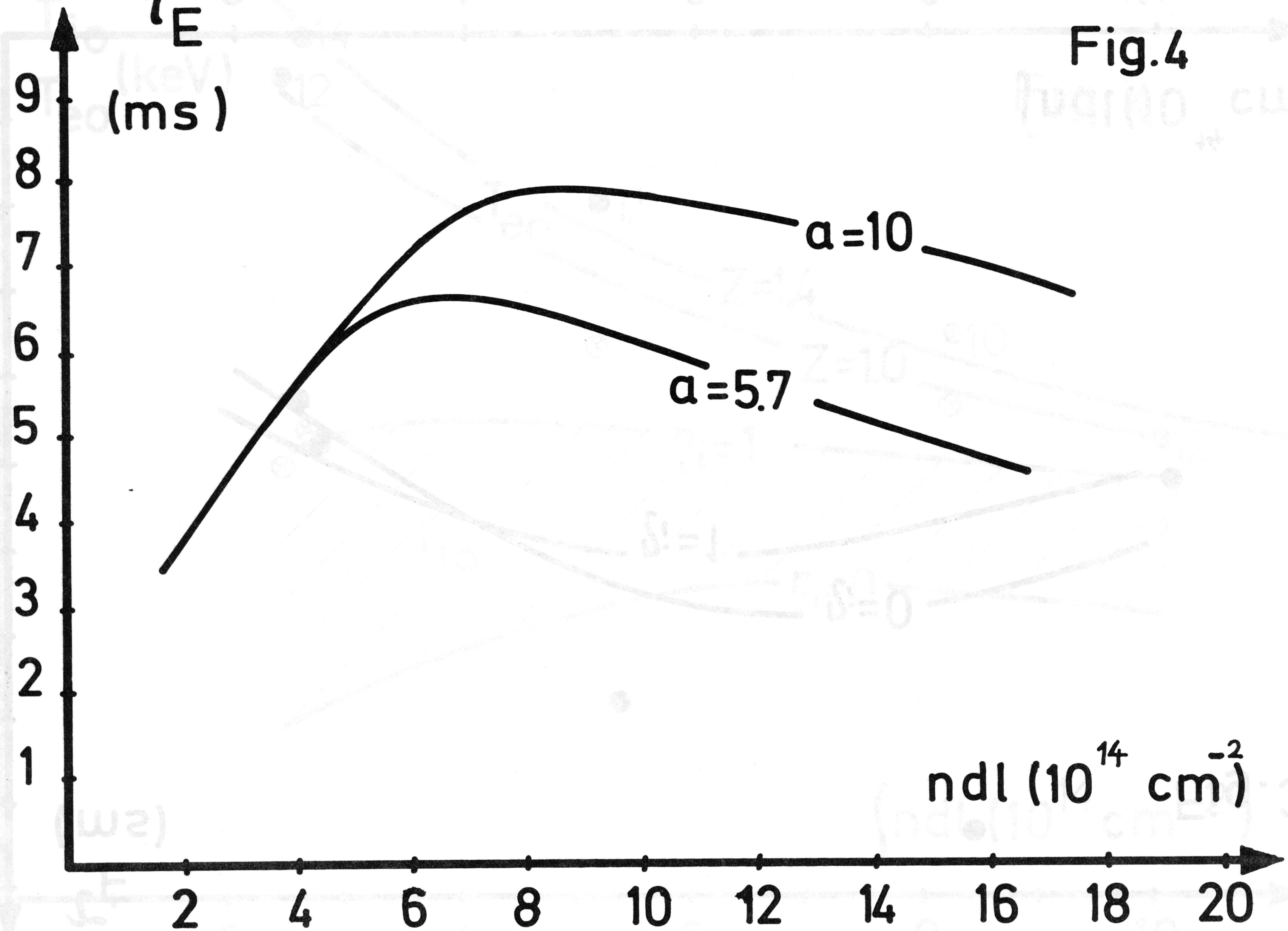


Fig. 3



$\tilde{\tau}_E$ (ms)
9
8
7
6
5
4
3
2
1
a=10
a=5.7
ndl (10^{14} cm^{-2})
2
4
6
8
10
12
14
16
18
20

Fig.4