Compression Laws in the Presence of Transport and α- Heating in a Tokamak Plasma

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In this note formulas are given which describe the behaviour of the plasma quantities under compression in the presence of transport and α - heating. Instantaneous energy deposition and the approximation $\langle \sigma v \rangle \sim T^2$ are assumed. The results are expressed in terms of the parameters $\varepsilon \equiv (T-T_m)/T_m$ and $\sigma \equiv \tau_c / 2\tau_t$, where T is the plasma temperature, T_m is the marginal temperature for ignition, τ_c the compression time and τ_t the relevant energy confinement time. It is shown that up to first order in ε and σ a compression in which the plasma volume is changing by ΔV, remains purely adiabatic provided that $\sigma \left| 1 - (1 - \Delta V/V)^{5/3} \right| \ll 1$, $\sigma\varepsilon \ll 1$, which are weaker than the usual $\sigma \ll 1$.

The deviations from adiabaticity at higher orders are easily calculated. The formulas are applied to the calculation of the relevant quantities in the compressions - decompressions cycles around the ignition point which allow the temperature control of an ignited plasma. Explicit formulas are given for the temperature excursions, the duration of each side of the cycle and the electric energy and power directly generated in the external coils of the controlling vertical field. The application of these results to the Zephyr experiment and to INTOR is discussed.

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1. Transport Equations Including Compression

The transport equations including compression are easily derived from the energy conservation per unit volume and from the particle conservation

$$\delta q = \frac{dU}{V} + p\frac{dV}{V} , \quad nV = const \tag{1}$$

Here $U = (3/2) nVKT$ and $p = nKT$; q is the heat production per unit volume.
Let us indicate with $\Sigma_\ell S_\ell$ the sum of all positive and negative heat sources per unit time and volume. Then applying (1) to each species j one obtains

$$\frac{3}{2}\frac{d}{dt}\left(n_j K T_j\right) = \Sigma_\ell S_\ell^j - \frac{5}{2} n_j K T_j \frac{1}{V}\frac{dV}{dt} \qquad (2)$$

We consider a situation in which the only important heat sources are the α- heating S_α^j, the transport S_{tr}^j and the ion-electron transfer $S_\Delta^i = -S_\Delta^e$. The ions, with density $n_i = n_D + n_T$, are assumed to be at the same temperature T_i. The density of the α- particles is neglected and the α- energy deposition is assumed to be instantaneous. The expressions for S_α^j and S_{tr}^j are as follows

$$S_\alpha^j = \frac{1}{4} W_\alpha K n_i^2 \langle\sigma v\rangle f_{\alpha j} \quad , \quad S_{tr}^j = -\frac{n_j K T_j}{\tau_{Ej}} \qquad (3)$$

where W_α = 3500 kev, K = 1.6 x 10^{-16} Joule/ kev, $f_{\alpha j}$ is the fraction of the α- energy going to the j particles ($f_{\alpha e} + f_{\alpha i} = 1$), τ_{Ej} is the energy confinement time for the species j and the following approximation is taken for $\langle\sigma v\rangle$:

$$\langle\sigma v\rangle = \left(\frac{\langle\sigma v\rangle}{T_i^2}\right)_{T_{io}} \cdot T_i^2 = 10^{-24} T_i^2 \; (\mathrm{Kev}, m^3, s) \qquad (4)$$

This approximation is very good for T_i between 8 and 20 kev /1/.

Summing up the two equations (2) for the ions and the electrons, noting that $n_i = n_e \equiv n$ and assuming $T_i \approx T_e \equiv T$, one obtains from Eq. (2):

$$\frac{dp}{dt} = \alpha p^2 - p\left(\frac{1}{2\tau_t} + \frac{5}{3}\frac{1}{V}\frac{dV}{dt}\right) \quad \left(\frac{1}{\tau_t} \equiv \frac{1}{\tau_{Ei}} + \frac{1}{\tau_{Ee}}\right) \qquad (5)$$

Here $p = nKT$, where n and T are space averaged quantities (we take the density average of T) and

$$\alpha = \frac{\delta}{12}\frac{W_\alpha}{K} 10^{-24} \qquad (6)$$

The quantity δ is a coefficient which includes the effect of the space averaging on the α - heating and also takes into account the possible non perfect confinement of the α- particles.

2. Integration of the Transport Equation

Eq. (5) is a Riccati equation which can be integrated exactly, giving

$$p(t) = p_0 \left(\frac{V_0}{V(t)}\right)^{5/3} \frac{\exp - \int_{t_0}^{t} dt / 2\tau_t(t)}{1 - \alpha p_0 \int_{t_0}^{t} dt' \left(\frac{V_0}{V(t')}\right)^{5/3} \exp - \int_{t_0}^{t'} dt'' / 2\tau_t(t'')} \tag{7}$$

where a possible time dependence of τ_t resulting from the compression, was taken into account.

In the absence of compression Eq. (7) takes the form

$$p(t) = p_0 \frac{\exp - (t - t_0)/2\tau_{t_0}}{1 - 2\alpha p_0 \tau_{t_0} + 2\alpha p_0 \tau_{t_0} \exp - (t - t_0)/2\tau_{t_0}} \tag{8}$$

One can see that p(t) is increasing or decreasing depending whether

$$p_0 \gtrless p_m \equiv \frac{1}{2\alpha \tau_{t_0}} \tag{9}$$

where p_m is the marginal value of p for ignition, associated with the temperature

$$T_m = p_m / n_0 K$$

In order to obtain from Eq. (7) an algebraic expression for $p(t_0 + \tau_c)$ for a given change of the plasma volume during a time τ_c, we assume the approximation $\tau_c / 2\tau_t(t) < 1$ and moreover we approximate $\tau_t(t)$ in the range $t_0 < t < t_0 + \tau_c$ with an intermediate constant value $\bar{\tau}_t$.

Thus, up to second order in $\sigma \equiv \tau_c / 2\bar{\tau}_t$, one obtains from Eq. (7)

$$p(t_0 + \tau_c) = p_0 \left(\frac{V_0}{V(t)}\right)^{5/3} \frac{1}{1 + (\sigma + \frac{1}{2}\sigma^2)\left[1 - (1+\xi_0)(1 - \frac{\Delta V}{\bar{V}})^{5/3}\right]} = p_0 \left(\frac{V_0}{V(t)}\right)^{5/3} \frac{1}{1 + \sigma\left[1 - (1 - \frac{\Delta V}{\bar{V}})^{5/3}\right]} + O(\sigma \xi_0) \tag{10}$$

where

$$\xi_0 = \frac{T_0 - T_m}{T_m} \tag{11}$$

and $\bar{V} = V_0 + \Delta V$ is some intermediate value of V(t) in the range $t_0 < t < t_0 + \tau_c$.

It is then seen that if

$$\sigma \left| 1 - (1 - \frac{\Delta V}{\bar{V}})^{5/3} \right| \ll 1 \ , \quad \sigma \xi_0 \ll 1 \tag{12}$$

the ideal adiabatic law $p/p_0 = (V_0/V)^{5/3}$ holds. The condition (12) can be understood physically in the following way. The change of p adiabatically related to a change of volume from V_0 to $\bar{V}$ is given by $p_0 - \bar{p}$, where $\bar{p}/p_0 = (V_0/\bar{V})^{5/3}$. Now the transport losses on $p_0 - \bar{p}$ in a time τ_c are given by $\sigma(p_0 - \bar{p})$ and these losses should be negligible in order for the ideal adiabaticity to hold. So one has the condition

$$\sigma|p_0 - \bar{p}| \ll p_0 \tag{13}$$

which is equivalent to the inequality (12). For a plasma near ignition the non adiabatic effects of transport and α- heating cancel each other at the lowest order and one is left with (12) as the only condition for adiabaticity instead of the usual $\sigma \ll 1$.

3. The Periodic Decompression - Compression Cycle

We will now apply our results to the regulating decompression-compression cycle represented in the $p-V$ plane by fig. (1). The interest of this cycle relies on the fact that not only it can be applied for the thermal regulation of an ignited plasma /2/ but also it allows in principle a partial conversion of the α-particle heating directly into electricity to be recovered in the circuit for the controlling vertical magnetic field. Indeed the electrical energy delivered by the power supply to the external coils in a cycle of duration τ, neglecting resistive losses, is given by

$$\int_t^{t+\tau} (U_0 I_0 + U_v I_v)\, dt = -\sum_j \oint p_j\, dV < 0 \tag{14}$$

so that one has an energy gain in the outer circuit (U and I are the applied voltage and current and the indices 0 and v refer to the ohmic and vertical circuits respectively) resulting from the fact that the plasma to be displaced has a smaller pressure during the compression than during the expansion phase.

Straightforward application of Eqs. (7) and (8) under the condition (12) gives the following expressions for p at the points B, C, D of the cycle

Transition A ⟶ B

$$p_B = p_A \left(\frac{V_A}{V_B}\right)^{5/3} \tag{15}$$

Transition B ⟶ C

$$p_C = p_A \left(\frac{V_A}{V_B}\right)^{5/3} \frac{\exp - \Delta t_B/\tau_B}{1 - \frac{p_A}{p_m}\left(\frac{V_A}{V_B}\right)^{5/3} \frac{\tau_B}{\tau_A}\left[1 - \exp - (\Delta t_B / 2\tau_B)\right]} \tag{16}$$

where Δt_B is the duration of the transition and τ_A, τ_B are the confinement times in A and B respectively.

Transition C $\longrightarrow$ D :

$$p_D = p_A \frac{\exp -(\Delta t_B / 2\tau_B)}{1 - \frac{p_A}{p_m}\left(\frac{V_A}{V_B}\right)^{5/3} \frac{\tau_B}{\tau_A}\left[1 - \exp -(\Delta t_B / 2\tau_B)\right]} \tag{17}$$

Transition D $\longrightarrow$ A :

The expression for p_A calculated applying (8) to this transition must be equal to the initial p_A, in order for the cycle to be closed. This gives the following condition for the duration Δt_A of the transition :

$$1 - \frac{p_A}{p_m}\left[\left(\frac{V_A}{V_B}\right)^{5/3} \frac{\tau_B}{\tau_A}\left(1 - \exp -(\Delta t_B/2\tau_B)\right) + \exp -(\Delta t_B/2\tau_B)\left(1 - \exp -(\Delta t_A/2\tau_A)\right)\right] = \tag{18}$$

$$= \exp -\left(\Delta t_A/2\tau_A + \Delta t_B/2\tau_B\right)$$

The curve of marginal ignition in the p-V plane is given by the relation

$$p_m(V) = p_m(V_A)\frac{\tau_A}{\tau(V)} = \begin{cases} p_m(V_A)\left(\dfrac{V}{V_A}\right)^{1/2} & \text{for the Alcator scaling} \\[2ex] p_m(V_A)\left(\dfrac{V}{V_A}\right)^{1/3} & \text{for the Neoclassical scaling} \end{cases} \tag{19}$$

Since the system is subigniting in B and igniting in D, from $p_B < p_m(V_B)$ and $p_D > p_m(V_A)$ one derives the following two conditions respectively

$$\frac{T_A - T_m}{T_m} < \frac{\tau_A}{\tau_B}\left(\frac{V_B}{V_A}\right)^{5/3} - 1 = C^{2\ell} - 1 \tag{20}$$

$$\frac{T_B - T_c}{T_B} < \frac{T_A - T_m}{T_A} \tag{21}$$

Here C is the compression factor $C^2 = V_B/V_A$; $\ell = 13/6$ and $\ell = 2$ respectively for the Alcator or the Neoclassical scaling; $T_B = T_A \, (V_A / V_B)^{2/3}$

It is useful to relate $\Delta T_B = T_B - T_C$ to the duration Δt_B of the transition B $\rightarrow$ C:

$$\Delta T_B = \frac{p_B - p_c}{n_B} = T_A \left(\frac{V_A}{V_B}\right)^{2/3} \left[1 - \frac{\exp - (\Delta t_B / 2\tau_B)}{1 - \frac{T_A}{T_{in}} \left(\frac{V_A}{V_B}\right)^{5/3} \frac{\tau_B}{\tau_A} \left[1 - \exp - (\Delta t_B / 2\tau_B) \right]} \right] \tag{22}$$

Then condition (21) imposes the following limitation on Δt_B:

$$\exp(\Delta t_B / 2\tau_B) < \left(1 - C^{-10/3}\right)\left(\frac{T_m}{T_A} - C^{-2\ell}\right) \tag{23}$$

The temperature excursion in A, $\Delta T_A = T_A - T_D$, is related to ΔT_B by the relation

$$\frac{\Delta T_A}{T_A} = \frac{\Delta T_B}{T_B} \tag{24}$$

Finally the area enclosed by the cycle is expressed as follows:

$$\oint p \, dV = \int_{V_A}^{V_B} p_B(V) \, dV + \int_{V_B}^{V_A} p_D(V) \, dV = \frac{3}{2} n_A T_A V_A \cdot \frac{T_A - T_B}{T_A} \cdot \frac{T_B - T_C}{T_B} \tag{25}$$

After multiplication by a factor 2 (in order to take into account the two particle species) the formula above gives the energy recovered by the external circuit under ideal conditions.

4. Case of Small Cycles: the Electric Power

We consider now the case of small decompressions- compressions and of small temperature excursions so that the following approximations can be introduced

$$\frac{V_B - V_A}{V_B} \ll 1 \qquad\qquad \frac{\Delta t_B}{2\tau_B} \ll 1 \tag{26}$$

The power of the electricity generation is defined by the expression

$$P = \frac{1}{\tau} \Sigma_j \oint p_j \, dV = \frac{2}{\tau} \oint p \, dV \tag{27}$$

where τ is the duration of the cycle, namely $\tau = \Delta t_A + \Delta t_B + 2\tau_c$.
Solving Eq. (18) after linearization, one obtains

$$\Delta t_A = \Delta t_B \left(\ell \frac{V_B - V_A}{V_B} \frac{T_m}{T_A - T_m} - 1 \right) \tag{28}$$

so that

$$\tau = \ell \frac{V_B - V_A}{V_B} \frac{T_m}{T_A - T_m} + 2\tau_c \tag{29}$$

Expressing $T_B - T_c$ with Eq. (22) and putting $\varepsilon_A = (T_A - T_m)/T_m$, the power P takes the form

$$P = \frac{p_A V_A}{\tau_B} \frac{T_B}{T_A} \frac{V_B - V_A}{V_B} \frac{\ell \frac{V_B - V_A}{V_B} - \varepsilon_A}{\ell \frac{V_B - V_A}{V_B} + \varepsilon_A \frac{2\tau_c}{\Delta t_B}} \tag{30}$$

This expression has a maximum as a function of ε_A . Assuming $2\tau_c/\Delta t_B < 1$, the maximum exists when

$$\varepsilon_A = \frac{T_A - T_m}{T_m} = \frac{\ell}{2} \frac{V_B - V_A}{V_B} \left(1 - \frac{\tau_c}{2\Delta t_B}\right) = \begin{cases} \frac{13}{12} \frac{V_B - V_A}{V_B} \left(1 - \frac{\tau_c}{2\Delta t_B}\right) & \text{(Alcator scaling)} \\ \frac{V_B - V_A}{V_B} \left(1 - \frac{\tau_c}{2\Delta t_B}\right) & \text{(Neoclassical scaling)} \end{cases} \tag{31}$$

The maximum power which can be extracted from the cycle is then expressed as follows ($\ell \approx 2$)

$$P_{max} = \frac{1}{2} n_A V_A \frac{T_B}{\tau_B} \left(\frac{V_B - V_A}{V_B} \right)^2 \left(1 - \frac{\tau_c}{\Delta t_B} \right) \tag{32}$$

Finally we observe that under the optimization condition (31) the value of Δt_A is simply given by the expression

$$\Delta t_A = \Delta t_B + \tau_c \tag{33}$$

provided that $(V_B - V_A)/V_B \ll 1$.

5. Examples

a) The "Zündexperiment"

As an example, we give the predictions of the formulas above when the parameters of the Zündexperiment are introduced. We take C = 1.5, $V_B = 12.11\ m^3$, $n_A = 3.33 \times 10^{20}\ m^{-3}$ and $T_m = 10$ kev. Let us assume that in the ignition regime, before the application of the controlling cycle, the temperature reaches a value $T_A = 15$ kev. Then one has the following values:

State A $\qquad p_A = 8 \times 10^5\ \text{joule}/m^3 \qquad T_A = 15\ \text{kev}$

State B $\qquad p_B = 2.08 \times 10^5\ \text{joule}/m^3 \qquad T_B = 8.73\ \text{kev}$

In the transition B → C we take a temperature drop $\Delta T_B = 1.73$ kev. Solving Eq. (22) one finds that this occurs in a time $\Delta t_B = 0.57\ \tau_B$. The value of p in C can be directly calculated from ΔT_B, since, combining Eqs. (16) and (22), one obtains

$$p_C = p_B \left(1 - \frac{\Delta T_B}{T_A}\left(\frac{V_B}{V_A}\right)^{2/3}\right) \tag{34}$$

So, after the time Δt_B, the system is in the following state:

State C $\qquad p_C = 1.66 \times 10^5\ \text{joule}/m^3 \qquad T_C = 7\ \text{kev}$

and after a time $\Delta t_B + \tau_C$

State D $\qquad p_D = 6.4 \times 10^5\ \text{joule}/m^3 \qquad T_D = 12.02\ \text{kev}$

In order to reach the state A and close the cycle the temperature must increase of the amount $\Delta T_A = T_A - T_D = 2.98$ kev. This occurs in a time $\Delta t_A = 2.04\,\tau_B$.

The total duration of the cycle is then

$$\tau = 2.6\ \tau_B + 2\ \tau_C$$

Taking $\tau_B \simeq 0.250$ and $\tau_C \simeq 0.06$ one has $\tau = 0.77$. At this point we can immediately calculate the energy recovered by the external circuit under ideal conditions and the power of the electric generation, using (25) and (27). One obtains

$$2 \oint p\, dV = 1.07 \times 10^6\ \text{joule}, \qquad P = \frac{2 \oint p\, dV}{\tau} = 1.4 \times 10^6\ \text{watts}$$

b) INTOR

In the case of small compression-decompression cycles for burn control one can use (32) for the optimized power. Since we are in a situation very near to marginal ignition, P_{max} can be expressed directly in terms of the total α - power heating $P_\alpha \equiv$ $\equiv 3 \alpha p^2 V_A$:

$$P_{Max} = \frac{1}{3} P_\alpha \left(\frac{V_B - V_A}{V_B} \right)^2 \left(1 - \frac{\tau_c}{\Delta t_B} \right) \qquad (35)$$

Assuming a compression factor 1.05 for the burn control one has that $P_{max} \lesssim 10^{-2} P_\alpha \frac{1}{3}$.
It follows that P_{max} cannot exceed few 100 kW.

References

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/2/ Borrass, K., Lackner, K., and Minardi, E., To be presented at the Oxford Conference, September 1979

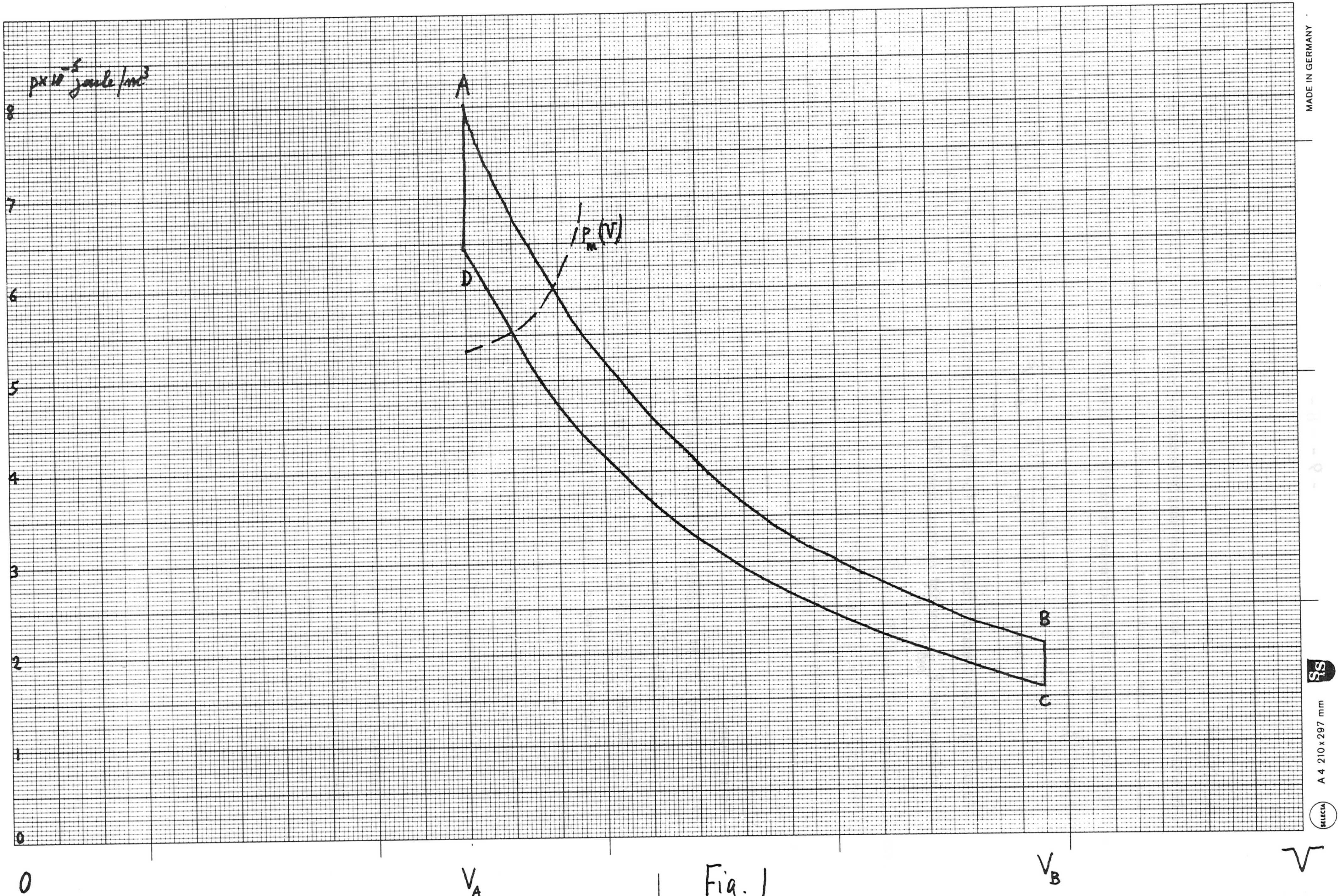

Fig. 1