Midplane Flux Distributions in Highly Elongated Axisymmetric MHD Equilibria

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Abstract

The minimum half-axis ratio $(\frac{b}{a})_{min}$ for all flux surfaces in a strongly elongated plasma is derived for which the Grad-Shafranov equilibrium equation reduces to the 1D-cylindrical slab equation in the midplane (z = o). $(\frac{b}{a})_{min}$ is found to be about $\sqrt{5A}$ where A is the aspect ratio. For $\frac{b}{a} \gtrsim (\frac{b}{a})_{min}$ poloidal flux distributions in the midplane $\psi(r,o)$ are determined by the toroidal current density $j_\varphi(r,\psi)$ and boundary conditions $\psi(r_{in},0) = \psi(r_{ou},0) = \psi_c$ alone, whereas they do not depend on the detailed shape of the plasma surface. These results are confirmed by numerical 2D-equilibrium calculations with "flat" $(j_\varphi = c_{op} r + c_{oF} \frac{1}{r})$ and "parabolic" $(j_\varphi = (c_{1p} r + c_{1F} \frac{1}{r})(\psi - \psi_c))$ current distributions. A determination of $j_\varphi(r,\psi)$ from experimental ψ (r,o) -profiles is proposed.

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Conclusive information on cross-sectional shape and current density in highly elongated axisymmetric plasmas can be gained by fitting magnetic probe measurements and total and poloidal beta values with results of free-boundary MHD equilibrium codes /1/. The free-boundary contour $\psi(r,z)=\psi_c$ and the toroidal current density $\dot{j}_\varphi(r,\psi)$, which define the equilibrium configuration, are generally not known and have to be determined by a trial-and-error procedure. One approach to find the solutions more systematically was to approximate the equilibria by a cylindrical slab model /2/. In the present paper we propose to determine $\dot{j}_\varphi(r,\psi)$ and β_p from midplane flux distributions $\psi(r,z=0)$ which do not depend on the free-boundary contour if all flux surfaces have half-axis ratios above $(\frac{b}{a})_{min}$. The value of $(\frac{b}{a})_{min}$ will be derived. One significant advantage of this procedure is that the detailed plasma shape has not to be known. The experimental $\psi(r,o)$ can, for instance, be determined from magnetic probe measurements of $B_p = -\frac{1}{r}\frac{\partial\psi}{\partial r}$ in the midplane like those usually carried out in belt-pinches. Once $\dot{j}_\varphi(r,\psi)$ has been found it can be used as an input function for 2D computations with trial surface contours, which are chosen to be consistent with measured ψ or B_p-profiles outside the vacuum vessel. For that a free-boundary equlibrium code is needed which works with rather general input current densities.

For the midplane and symmetry plane (z = o) the following one-dimensional ordinary differential equation in cylindrical coordinates r, φ and z

$$\frac{d^2\psi}{dr^2} - \frac{1}{r}\frac{d\psi}{dr} \simeq -\frac{4\pi}{c}\, r\, \dot{j}_\varphi(r,\psi) \qquad (1)$$

with boundary conditions $\psi(r_{in},0) = \psi(r_{ou},0) = \psi_c$ is obtained, if the term $\frac{\partial^2\psi}{\partial z^2}$ in the Grad-Shafranov equilibrium equation becomes negligibly small. The magnitude of this term can be calculated by approximating a given flux contour with half-axes a and b in the z = o vertex points by an ellipse with half-axes a and and b. The projection of $\vec{\nabla}\psi$ in a point r,z on the z-direction reads

$$\frac{\partial\psi}{\partial z} \approx \frac{\partial\psi}{\partial\rho}\,\frac{z}{\rho_c}$$

where $\rho_c = b^2/a$ is the radius of curvature of the ellipse and ρ is a radial coordinate with the origin at the centre of the circle of curvature. After differentiating with respect to z one obtains for the midplane

$$\left|\frac{\partial^2\psi}{\partial z^2}\right| \approx \left|\frac{\partial\psi}{\partial\rho}\right| \frac{a}{b^2} \approx r\frac{a}{b^2}\,\frac{1}{r}\left|\frac{\partial\psi}{\partial r}\right|$$

Thus for $r\frac{a}{b^2} \lesssim 0.2$ and z = o the term $\frac{\partial^2\psi}{\partial z^2}$ is negligibly small so that Eq.(1) may be used. For a given flux surface this result corresponds to $(\frac{b}{a})_{min} \approx \sqrt{5A}$ where A is the aspect ratio. If all flux surfaces have half-axis ratios above $(\frac{b}{a})_{min}$ one can conclude that practically identical midplane distributions of ψ and B_p result, if the same toroidal current density $j_\varphi(r,\psi)$ and boundary conditions $\psi(r_{in},0)=\psi(r_{ou},0)=\psi_c$ are prescribed and that the surface contour of the plasma does not enter.

In the asymptotic case $\frac{b}{a}\rightarrow\infty$ (cylindrical slab) Equation (1) becomes exact $(\frac{\partial^2\psi}{\partial z^2}=0)$, so that its solutions may be regarded as cylindrical slab solutions for arbitrary current densities. For special $j_\varphi(r,\psi)$ analytic solutions are known, e.g. for "flat" current profiles of type $c_{op}r+c_{oF}\frac{1}{r}$ /2/ or for "parabolic" distributions of the form $(c_{1p}r+c_{1F}\frac{1}{r})(\psi-\psi_c)$, where c_{op}, c_{oF}, c_{1p} and c_{1F} are constants.

In order to check the above conclusions two-dimensional numerical equilibria are compared. Solutions for $j_\varphi = c_{op}r + c_{oF}\frac{1}{r}$ with fixed values A = 4.4 and β_p = 2.5 but different surface contours are presented in Fig. 1. Since the flux contours are symmetric with respect to z = 0, the midplane distributions can be easily compared by combining one upper (reference case) and one lower half of surface plots. From Fig. 1a it is obvious that the flux contours coincide in the midplane if all flux surfaces have half-axis ratios larger than $(\frac{b}{a})_{min} \simeq 4.7$. Note that an identical ψ(r,o) is obtained analytically by inserting the parameters of this case in the cylindrical slab solution given in Ref. /2/. In contrast to that a different ψ(r,o)-profile results for a half-axis ratio small compared with $(\frac{b}{a})_{min}$ (see Fig. 1b).

Corresponding results are expected for arbitrary current densities $j_\varphi(r,\psi)$. This is confirmed for the special case of a parabolic distribution with A = 4.4 and β_p = 2.5 as demonstrated in Fig. 2.

We conclude that midplane flux distributions are determined by the toroidal current density $j_\varphi(r,\psi)$ and boundary conditions $\psi(r_{in},0)=\psi(r_{out},0)=\psi_c$ alone if all flux contours have half-axis ratios above $(\frac{b}{a})_{min} \approx \sqrt{5A}$. The detailed shape of the plasma surface does not enter. Consequently, without knowing or making assumptions on the free-boundary contour $j_\varphi(r,\psi)$ can be determined by matching the experimental $\psi(r,o)$ with analytical or numerical solutions of a one-dimensional ordinary differential equation (Eq. (1)). On the other hand for $\frac{b}{a} > (\frac{b}{a})_{min}$ it is not possible to draw conclusions on the plasma shape from measured $\psi(r,o)$ or B_p (r,o)-profiles even if $j_\varphi(r,\psi)$ is known.

Midplane flux distributions can also be applied to find the flux surface structure of special equilibria without carrying out 2D equilibrium calculations. Two examples are elliptical surfaces with flat and racetrack-shaped surfaces with parabolic current profiles which both exhibit an approximately fixed half-axis ratio for all flux contours.

References

/1/ Becker, G., Lackner, K., Nucl. Fusion 17 (1977) 903.

/2/ Becker, G., Nucl. Fusion 18 (1978) 9.

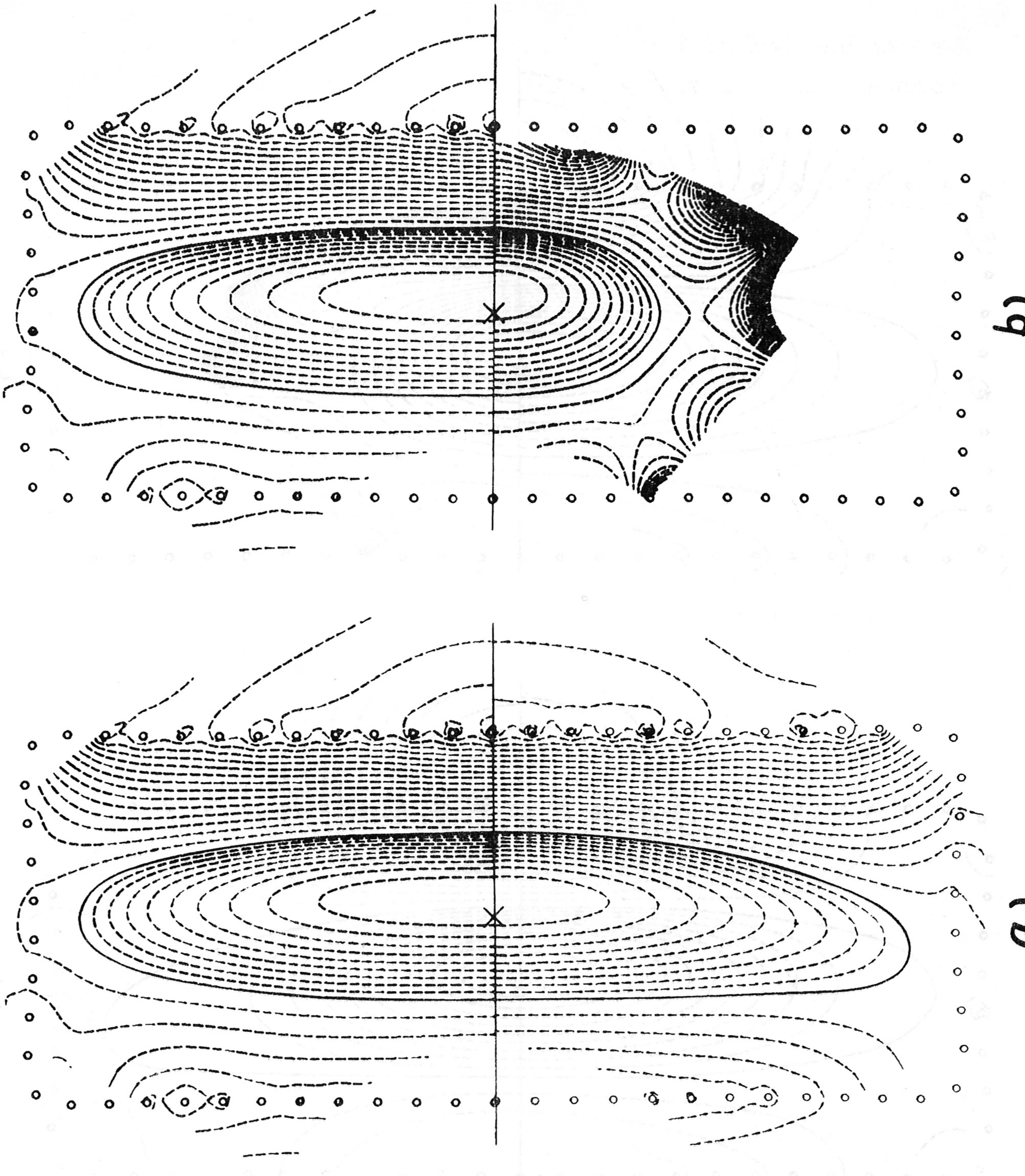

Fig. 1 Equilibrium solutions with $j_\varphi = c_{0p} r + c_{0F} \frac{1}{r}$,
A = 4.4, β_p = 2.5 and $(\frac{b}{a})_{min} \simeq 4.7$ for
various surface contours (reference case in
upper half). $\frac{b}{a}$ of all flux surfaces
a) above $(\frac{b}{a})_{min}$, b) below $(\frac{b}{a})_{min}$.

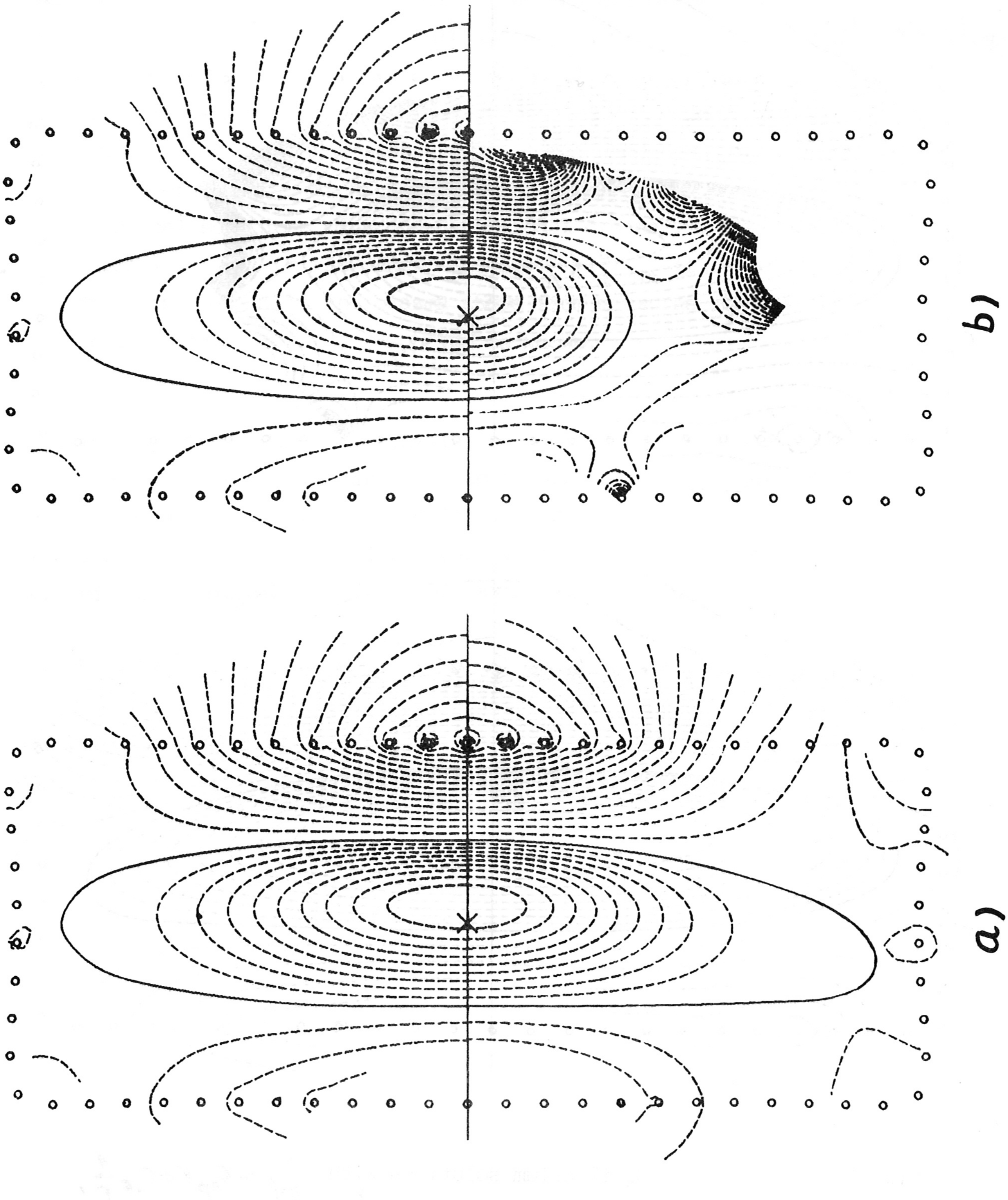

Fig. 2 as in Fig. 1, but with $j_\varphi = (C_{4p}\, r + C_{4F}\frac{1}{r})(\psi - \psi_c)$
and a) $\frac{b}{a}$ of the plasma surface above $(\frac{b}{a})_{min}$,
b) $\frac{b}{a}$ of all flux surfaces below $(\frac{b}{a})_{min}$.