Electromagnetic Forces on a Metallic Tokamak Vacuum Vessel Following a Disruptive Instability.

A Simplified Approach Assuming Quasistationary Vessel Surface Currents.

D. Eckhartt

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Abstract

During a "hard" disruptive instability of a Tokamak plasma the current-carrying plasma is lost within a very short time, typically a few milliseconds. If the plasma is contained in a metallic vacuum vessel, electric currents are set up in the vessel following the disappearance of the plasma current. These vessel currents together with the magnetic fields intersecting the vessel generate electromagnetic forces which appear as mechanical loads on the vessel.

In the following note it is assumed that the vacuum vessel is surrounded by an "outer equivalent" or "flux-conserving" shell having a characteristic time of magnetic field penetration which is long compared to the time of existence of the vessel currents. This property defines the distribution of vessel current densities (and hence the load distribution) without referring to the exact mechanism or time sequence of events by which the plasma current is lost.

Numerical examples of the electromagnetic force distribution from this model refer to parameters of the JET-device with the simplifying assumption of circular cross-sections for plasma current, vacuum vessel, and outer equivalent shell.

1. Introduction

When a current-carrying Tokamak plasma suffers from a hard disruptive instability the plasma current or a major part of it is commutated into the surrounding metallic vacuum vessel torus. If this vacuum vessel is composed of elements with different electric resistivity the vessel currents are split up into two parts: one part called "passing current" is flowing solely in longitudinal (or toroidal) direction, the other one is also flowing in direction of the minor circumference (poloidal direction) and is called "saddle current". These currents interact with the magnetic fields crossing the vacuum vessel, viz., the vertical maintaining field required for equilibrium of the unperturbed plasma and the main magnetic field in toroidal direction required for the stability of the plasma. Mechanical loads on the vacuum vessel arise from these interactions resulting in locally distributed compressive and shearing stresses. The summation of these loads leads to global effects such as

- a centripetal force which tends to crush the vacuum vessel in the same way as the outside atmospheric pressure, and
- a tilting moment which tends to rotate each element of constant electric resistivity out of the equatorial plane.

In order to assess the order of magnitude of these global effects let us consider for a moment the centripetal force resulting from the interaction between passing currents and the external vertical equilibrium field, $B_{\perp o}$ +). The centripetal force can be estimated from the Lorentz-force on a current-carrying plasma loop:

$$F_C \propto I_p \cdot 2\pi R_p \cdot B_{\perp o}$$

+) for a list of symbols used see Table I

With $B_{\perp o}$ /1/

$$B_{\perp o} = \frac{\mu_0 I_p}{4\pi R_p}\left(\ln\frac{8R_p}{a} + \beta_\vartheta + \frac{l_i - 3}{2}\right)$$

we obtain

$$F_c \propto \frac{I_p^2}{2}\left(\ln\frac{8R_p}{a} + \beta_\vartheta + \frac{l_i - 3}{2}\right)$$

This centripetal electromagnetic force can be compared with the force from the outside atmospheric pressure, p_{at}, acting in the same direction:

$$F_{at} = 2\pi^2 c^2 p_{at}$$

to arrive at a scaling

$$\frac{F_c}{F_{at}} \propto I_p \cdot \frac{I_p}{c^2}\left(\ln\frac{8R_p}{a} + \beta_\vartheta + \frac{l_i - 3}{2}\right)$$

When passing from present-day Tokamak experiments with tens of kiloamperes of plasma current to future experiments with plasma currents in the mega-ampere range, the plasma current density, $\frac{I_{pl}}{c^2}$, the inverse aspect ratio $\frac{R_p}{a}$, as well as the plasma parameters β_ϑ and l_i, are only slowly varying quantities. Hence the ratio $\frac{F_c}{F_{at}}$ roughly scales as the plasma current, I_{pl}, i.e., it increases by two orders of magnitude for the range of plasma currents considered above.

On the other hand, the quantity of material for the construction of the vacuum vessel and its supports has to be kept as small as possible for cost reasons. Hence the magnitude of the electromagnetic forces due to a disruptive instability becomes a decisive factor when designing the vacuum vessel for future large experiments.

When calculating the electromagnetic forces one has to remember that the series of events which take place during a disruptive instability and the accompanying loss of current-carrying plasma are not fully understood. Different models can be used, therefore, to describe these phenomena, and using a minimum number of disputable ad-hoc assumptions. In addition, several simplifications have to be made for an analytical treatment of the problem. On the other hand, in order to estimate the electromagnetic force distribution required for the stress analysis of the vacuum vessel, a first approach can be made by assessing the maximum values of these forces (their amplitudes) thus neglecting dynamic effects.

2. The Present Model

A simplified approach is used in the following by assuming that - as a result of the disrupted plasma current - a system of quasistationary currents flowing in the surrounding vacuum vessel is established. The interval of time during which these vessel currents are existing, is long compared to the duration of the disruptive event as well as to the time required for full penetration of magnetic fields through the metallic vacuum vessel. Thus no induction effects need to be considered and the vessel can be treated as "thin shell". The time of existence of the vessel currents is short, however, compared to the time of magnetic field penetration through an "outer equivalent shell" surrounding the vacuum vessel - this shell being formed by the system of poloidal field windings. Thus the outer equivalent shell is conserving the poloidal magnetic flux during the time of existence of the vessel currents. Hence all components of the magnetic field penetrating the shell before the disruption have to be retained during the time considered+). This magnetic field is composed of the maintaining field required for plasma equilibrium, plus the magnetic field generated by the plasma current before it disrupts. As the maintaining magnetic field is created by external sources not affected by the plasma disruption, it is the magnetic field of

+) Strictly speaking: it is only the magnetic field component normal to the surface of a super-conductor to which this consideration applies.

the disappearing plasma current which has to be "replaced" by the magnetic fields originating from the quasistationary vessel currents. This condition defines the distribution of vessel currents and hence, the electromagnetic forces on the vessel. In the present model the density of vessel currents is assumed to be composed of a homogeneous part plus a component varying periodically along the minor circumference (and producing a dipole-type magnetic field).

The above ordering of characteristic times appears to be justified in case of the JET-experiment /2/. In the following, all numerical evaluations refer to the parameters of the JET-machine, as listed in Table I, but simplified for circular cross-sections of plasma, vacuum vessel, and outer shell.

The above arguments as regards flux conservation are applied only to the fields generated by the passing currents. It has been shown /2/ that for the parameters of the JET-machine, the effects of passing currents and saddle currents can be decoupled in a first approximation. In other words, once the magnitudes of the passing currents have been established as described above, the saddle current density can be deduced afterwards. In particular, it is only the periodic component of the passing vessel currents which contributes to the saddle currents.

3. Current-Carrying Plasma Loop in Equilibrium

The poloidal and radial magnetic field components outside of a current carrying loop with circular cross-section are taken from Ref./3/ and refer to a (ϱ-ω) co-ordinate system centered on the (circular) axis of the loop (see Fig.1)

$$B_\omega = \frac{\mu_0 I}{2\pi\varrho} + \left[-\frac{\mu_0 I}{4\pi R}\ln\frac{8R}{\varrho} + \frac{1}{2\pi R}\left(C_2 - \frac{C_1}{\varrho^2}\right)\right]\cos\omega \qquad (1)$$

$$B_\varrho = \left[-\frac{\mu_0 I}{4\pi R}\left(\ln\frac{8R}{\varrho} - 1\right) + \frac{1}{2\pi R}\left(C_2 + \frac{C_1}{\varrho^2}\right)\right]\sin\omega \qquad (2)$$

We now apply these equations to the case of a current-carrying plasma loop in equilibrium with major radius $R = R_p$. Constants C_1 and C_2 are determined by the boundary conditions on the plasma surface ($\varrho = a$) /4/:

$$B_\varrho(a,\omega) = 0$$
$$B_\omega(a,\omega) = \frac{\mu_0 I_p}{2\pi a}\cdot\left(1 + \frac{a}{R_p}\Lambda\cdot\cos\omega\right)$$

where

$$\Lambda = \beta_\vartheta + \frac{l_i}{2} - 1 \tag{3}$$

leading to: $I = I_p$

$$C_1 = -\frac{\mu_0 I_p}{2}\, a^2\left(\Lambda + \frac{1}{2}\right) \tag{4}$$

$$C_2 = \frac{\mu_0 I_p}{2}\left(\ln\frac{8R_p}{a} + \Lambda - \frac{1}{2}\right) \tag{5}$$

Following Ref./5/ we now identify those terms in eqs. (1) and (2) which are determined by the constant C_2 (i.e., those which do not vanish for $\varrho\to\infty$) with the external maintaining field required for plasma equilibrium and directed perpendicular to the plane of the plasma loop: /1/

$$B_{\perp o} = \frac{\mu_0 I_p}{4\pi R_p}\left(\ln\frac{8R_p}{a} + \Lambda - \frac{1}{2}\right) \tag{6}$$

The remaining terms in eqs. (1) and (2) describe the magnetic field components generated by the plasma current itself

$$B_{p\omega} = \frac{\mu_0\cdot I_p}{2\pi\cdot\varrho} + \frac{\mu_0\cdot I_p}{4\pi R_p}\left[-\ln\frac{8R_p}{\varrho} + \frac{a^2}{\varrho^2}\left(\Lambda + \frac{1}{2}\right)\right]\cos\omega \tag{7}$$

$$B_{p\varrho} = \frac{\mu_0\cdot I_p}{4\pi R_p}\left[1 - \ln\frac{8R_p}{\varrho} - \frac{a^2}{\varrho^2}\left(\Lambda + \frac{1}{2}\right)\right]\sin\omega \tag{8}$$

In general, the circular axis of the toroidal plasma loop defined by $R = R_p$, does not coincide with the axis of the vacuum vessel, $R = R_o$ (being identical with the axis of the outer shell) but will be shifted by an amount Δ (see Fig.2).

$$R_p = R_o + \Delta = R_o\left(1 + \frac{\Delta}{R_o}\right), \quad \frac{\Delta}{R_o} < 1 \tag{9}$$

We therefore need to express the field components of eqs.(7) and (8) in terms of a (r,ϑ) co-ordinate system centered at the axis of the vacuum vessel at $R = R_o$. As seen from Fig.3, the following transformation formulae apply:

$$\begin{aligned} B_r &= B_\varrho - \frac{\Delta}{r} \cdot B_\omega \sin\vartheta \\ B_\vartheta &= B_\omega + \frac{\Delta}{r} \cdot B_\varrho \sin\vartheta \\ r \cdot \sin\vartheta &= \varrho \cdot \sin\omega \\ r \cdot \cos\vartheta &= \varrho \cos\omega + \Delta \end{aligned} \tag{10}$$

$$\frac{\varrho}{r} = 1 - \frac{\Delta}{r}\cos\vartheta \qquad \text{(neglecting } \left(\tfrac{\Delta}{r}\right)^2 \ll 1\text{)}$$

$$\sin\omega = \sin\vartheta + \frac{1}{2}\frac{\Delta}{r}\sin 2\vartheta$$

$$\cos\omega = \cos\vartheta + \frac{1}{2}\frac{\Delta}{r}\cos 2\vartheta - \frac{1}{2}\frac{\Delta}{r}$$

In carrying through this transformation we neglect terms of the order $\left(\frac{\Delta}{r}\right)^2$ as well as higher harmonics ($\cos 2\vartheta$, $\sin 2\vartheta$,...) to arrive at

$$\begin{aligned} B_{p\vartheta} = {} & \frac{\mu_0 I_p}{2\pi r}\left(1 - \frac{\Delta}{2R_p}\left\{\ln\frac{8R_p}{r} - 1\right\}\right) + \\ & + \frac{\mu_0 I_p}{4\pi R_p} \cdot \left[-\ln\frac{8R_p}{r} + \frac{a^2\left(\Lambda + \frac{1}{2}\right) + 2\Delta \cdot R_p}{r^2}\right]\cos\vartheta \end{aligned} \tag{11}$$

$$B_{pr} = \frac{\mu_0 I_p}{4\pi R_p}\left[1 - \ln\frac{8R_p}{r} - \frac{a^2\left(\Lambda + \frac{1}{2}\right) + 2\Delta \cdot R_p}{r^2}\right]\sin\vartheta \tag{12}$$

The first term in eq.(11) can be simplified by developing $\ln \frac{r}{8R_p}$ and neglecting $(\frac{r}{8R_p})^2$ and higher orders:

$$1 - \frac{\Delta}{2R_p}\left\{\ln\frac{8R_p}{r} - 1\right\} \cong 1 + \frac{\Delta \cdot r}{16 R_p^2} \tag{13}$$

The second term in eq.(13) turns out to be small compared to unity if Δ is sufficiently small+). In this particular case eqs.(11) and (12) become

$$B_{p\vartheta} = \frac{\mu_0 I_p}{2\pi r} + \frac{\mu_0 I_p}{4\pi R_p}\left[-\ln\frac{8R_p}{r} + \frac{a^2(\Lambda+\frac{1}{2}) + 2\Delta \cdot R_p}{r^2}\right]\cos\vartheta \tag{14}$$

$$B_{pr} = \frac{\mu_0 I_p}{4\pi R_p}\left[1 - \ln\frac{8R_p}{r} - \frac{a^2(\Lambda+\frac{1}{2}) + 2\Delta \cdot R_p}{r^2}\right]\sin\vartheta \tag{15}$$

It should be noted that, in eqs.(14) and (15), the functional dependencies on r and ϑ are the same as on ϱ and ω, resp., in eqs.(7) and (8).

4. Magnet Fields of a Toroidal Current-Carrying Thin Shell

We now derive the magnetic field components produced by currents flowing in toroidal direction on the surface of a toroidal thin shell with major radius R_o and minor radius c with no external current sources. The surface current density is assumed to be composed of a homogeneous part, j_o, and a first harmonic with amplitude j_1

$$j_V = j_o + j_1 \cos\vartheta \tag{16}$$

+) with the numbers of Table I for the case of JET this term is less than 10^{-3}

such that the total net current becomes

$$I_V = \int_0^{2\pi} j_V \cdot c \cdot d\vartheta = 2\pi c \cdot j_0 \tag{17}$$

For the magnetic fields outside the shell eqs.(1) and (2) apply with C_2=0 (no external current sources)

$$B_{a\vartheta} = \frac{\mu_0 I_V}{2\pi r} + \left[-\frac{C_1}{2\pi R_0}\frac{1}{r^2} - \frac{\mu_0 I_V}{4\pi R_0} \ln\frac{8R_0}{r} \right] \cos\vartheta \tag{18}$$

$$B_{ar} = \left[\frac{C_1}{2\pi R_0}\frac{1}{r^2} - \frac{\mu_0 I_V}{4\pi R_0} \left(\ln\frac{8R_0}{r} - 1 \right) \right] \sin\vartheta \tag{19}$$

It can be seen that the dipole-type contribution (i.e. influence of j_1) is included in the constant C_1 /6/.

The field inside the current carrying shell is homogeneous and directed perpendicular to the equatorial plane of the toroidal shell /7/. This can be derived from eqs.(1) and (2) by considering that the inside field components stay finite for $r \rightarrow 0$ only if all terms except the one with C_2 are dropped. We can therefore write

$$\left.\begin{aligned} B_{i\vartheta} &= B_i \cdot \cos\vartheta & (20) \\ \\ B_{ir} &= B_i \cdot \sin\vartheta & (21) \end{aligned}\right\} \quad B_i = \text{const.}$$

By applying the boundary conditions at the surface of the shell (r = c) we can express B_i and j_1 by means of the constant C_1 in eqs.(18) and (19). These boundary conditions are

- continuity of the normal field component $B_{ar}(c) = B_{ir}$
- the jump of the tangential components is equal to the surface current density $B_{a\vartheta}(c) - B_{i\vartheta} = \mu_0 (j_0 + j_1 \cos\vartheta)$

We obtain

$$\frac{C_1}{2\pi R_0}\frac{1}{c^2} - \frac{\mu_0 I_V}{4\pi R_0}\left(\ln\frac{8R_0}{c} - 1\right) = B_i$$

$$\frac{I_V}{2\pi c} = j_0$$

$$-\frac{C_1}{2\pi R_0}\frac{1}{c^2} - \frac{\mu_0 I_V}{4\pi R_0}\ln\frac{8R_0}{c} - B_i = \mu_0 j_1$$

Hence

$$B_i = \frac{C_1}{2\pi R_0}\frac{1}{c^2} - \frac{\mu_0 I_V}{4\pi R_0}\left(\ln\frac{8R_0}{c} - 1\right) \tag{22}$$

$$-\mu_0 j_1 = \frac{2C_1}{2\pi R_0}\frac{1}{c^2} + \frac{\mu_0 I_V}{4\pi R_0} \tag{23}$$

As seen from eqs.(18)-(23), the magnetic field outside and inside the shell is fully determined if the number values of I_V plus the constant C_1 are given. This can be done, for example, by prescribing the field components at a particular location (e.g. the surface of the outer equivalent shell).

5. Quasistationary Currents on the Vacuum Vessel Surface after a Disruptive Instability

Prior to a disruptive instability the plasma current loop is centered at a major radius $R = R_p$ which is shifted by a small length Δ relative to the common axis of vacuum vessel and outer equivalent shell located at $R = R_0$ (Fig.4). The plasma current loop is kept in equilibrium by a homogeneous magnetic field $B_{\perp 0}$ (eq.(6)) directed normal to the equatorial plane of the plasma loop in such a way that it weakens the field of the plasma current at the inner side (smaller major radius) and strengthens it at the outside. The plasma current magnetic field components intersecting the outer equivalent shell are found from eqs. (14) and (15) by inserting r = b.

After the plasma current has disappeared (the exact mechanism not being referred to) a system of longitudinal currents is set up in the vacuum vessel thin shell. It is distributed in such a way that the magnetic field given by eqs. (18) and (19), in particular the normal component, generated by the vessel currents at the outer shell (r = b) is equal to the magnetic field of the plasma current prior to its disappearance. In comparing the two sets of equations for the magnetic fields of plasma current and vessel currents, resp., we are led to the identifications:

$$\frac{I_V}{R_o} = \frac{I_P}{R_P}$$

$$\frac{C_1}{2\pi R_o} = -\frac{\mu_0 I_P}{4\pi R_P}\left(a^2\left(\Lambda+\frac{1}{2}\right)+2\Delta\cdot R_P\right)$$

Neglecting the relative displacement $\frac{\Delta}{R_o}$ as small compared to unity+) these two relations become

$$I_V = I_P \tag{24}$$

$$\frac{C_1}{2\pi R_o} = -\frac{\mu_0 I_P}{4\pi R_o}\left(a^2\left(\Lambda+\frac{1}{2}\right)+2\Delta\cdot R_o\right) \tag{25}$$

Upon accepting this approximation not only do the two normal components at the outer shell become equal but throughout the whole interspace between vacuum vessel and outer equivalent shell the field components and hence the magnetic surfaces remain unchanged. Moreover, no "image currents" appear at the inner surface of the outer shell (at r = b) since the tangential field components remain unchanged.

+) for the numbers of the JET-device in Table I, $\frac{\Delta}{R_o}$ becomes 1.8×10^{-2}

Upon inserting relations (24) and (25) into eqs. (17) and (23) we obtain for the first harmonics of the vessel current surface density (with $\mathcal{E}_c = \frac{c}{R_o}$)

$$j_0 = \frac{I_P}{2\pi c}, \quad j_1 = j_0 \cdot \mathcal{E}_c \left(\frac{a^2(\Lambda + \frac{1}{2}) + 2\Delta \cdot R_o}{c^2} - \frac{1}{2} \right) \tag{26}$$

and for the magnetic field generated by the surface currents inside the vacuum vessel (see eq.(22)) /8/

$$B_i = -\frac{\mu_0 I_P}{4\pi R_o} \left(\ln \frac{8R_o}{c} - 1 + \frac{a^2(\Lambda + \frac{1}{2}) + 2\Delta \cdot R_o}{c^2} \right) \tag{27}$$

Hence the total field inside the vacuum vessel after the disruption becomes

$$B_i + B_{\perp o} = \frac{\mu_0 I_P}{4\pi R_o} \left((\Lambda + \tfrac{1}{2})\left(1 - \frac{a^2}{c^2}\right) + \ln \frac{c}{a} - \frac{2\Delta \cdot R_o}{c^2} \right) \tag{28}$$

Upon multiplying this expression with $\cos\vartheta$ and $\sin\vartheta$, we arrive at the ϑ- and r-components, resp., of the total inside field. Similarily the total magnetic field outside the vessel is obtained by inserting relations (24) and (25) into eqs.(18) and (19) and superimposing $B_{\perp o}$, as given by eq.(6):

$$\left(B_a + B_{\perp o}\right)_\vartheta = \tag{29}$$

$$\frac{\mu_0 I_P}{2\pi r} + \frac{\mu_0 I_P}{4\pi R_o} \left[\Lambda - \frac{1}{2} + \ln \frac{r}{a} + \frac{a^2(\Lambda + \frac{1}{2}) + 2\Delta \cdot R_o}{r^2} \right] \cos\vartheta$$

$$\left(B_a + B_{\perp o}\right)_r = \frac{\mu_0 I_P}{4\pi R_o} \left[\Lambda + \frac{1}{2} + \ln \frac{r}{a} - \frac{a^2(\Lambda + \frac{1}{2}) + 2\Delta \cdot R_o}{r^2} \right] \sin\vartheta \tag{30}$$

6. Electromagnetic Pressure Components Due to Vessel Currents

The electromagnetic pressure force $\vec{p}$ loading the vacuum vessel as a result of the interaction between surface currents $\vec{j}_v$ and magnetic fields $\vec{B}$ intersecting the vessel, can be computed from the vector product

$$\vec{p} = 1{,}02 \left[\vec{j}_v \times \vec{B}\right] \tag{31}$$

Here, p is in kp/cm^2 if j_v and B are inserted in kA/cm and Tesla, resp., ($1T = 10^4$ Gauss).

We are dealing here only with the effects of the passing currents j_v which, in our notation, are counted positive if they flow along the z-direction of a (r, ϑ, z) system of co-ordinates (Fig.4). They give rise to pressure components (p_n, p_z) in the directions normal and tangential to the vacuum vessel surface. p_n will be counted positive if it acts as a compressive force (i.e. pointing in the negative r-direction).

Following Ref./9/ we write the poloidal field components on the vessel surface (r=c) in the general form

$$B_n = B_{n1} \sin\vartheta = B_r(c) \tag{32a}$$

$$B_t = B_{t0} + B_{t1} \cdot \cos\vartheta = B_\vartheta(c) \tag{32b}$$

similar to the expression for the vessel current j_v (eq.(16)). The normal field component remaining continuous at r=c, the presence of the vessel currents, eq.(16), leads to a discontinuity in the tangential field component. In order to calculate the normal component of the electromagnetic pressure we have to use the mean value, $\overline{B_t}$, of the ϑ-components inside and outside the vessel shell, r=c, (eqs.(28) and (29), resp.). Thus eq.(32b) has to be replaced by

$$\overline{B_t} = \overline{B_{t0}} + \overline{B_{t1}} \cdot \cos\vartheta \tag{32c}$$

Having expressed the currents and fields involved as harmonic series the pressure components can be written as follows /9/

$$p_n \propto \dot{j}_v \cdot \overline{B_t} = \sum_{i=0}^{2} p_{ni} \cdot \cos(i\vartheta)$$
$$p_t \propto \dot{j}_v \cdot B_n = \sum_{t=0}^{2} p_{ti} \cdot \sin(i\vartheta) \qquad (33)$$

Using the expressions for the magnetic field components (eqs.(32)), and for the vessel currents (eq.(16)) one gets for the pressure amplitudes in (33) (see Ref./9/)

$$p_{n0} = 1{,}02\left(\dot{j}_0 \cdot \overline{B_{t0}} + \dot{j}_1 \cdot \overline{B_{t1/2}}\right)$$
$$p_{n1} = 1{,}02\left(\dot{j}_0 \cdot \overline{B_{t1}} + \dot{j}_1 \cdot \overline{B_{t0}}\right)$$
$$p_{n2} = 1{,}02\, \dot{j}_1 \cdot \overline{B_{t1/2}} \qquad (34)$$
$$p_{t1} = 1{,}02\, \dot{j}_0 \cdot B_{n1}$$
$$p_{t2} = 1{,}02\, \dot{j}_1 \cdot \frac{B_{n1}}{2}$$

With this formulation of harmonic components of the electromagnetic pressure forces in normal and tangential direction, p_n and p_t, resp., the global effects such as the centripetal force can be calculated. This centripetal force, like the outside atmospheric pressure p_{at}, tends to compress the vacuum vessel in its major radius R towards the axis of symmetry. The pressure components are combined into a centering pressure, p_R, directed parallel to the midplane of the torus /10/

$$p_R = \cos\vartheta \sum_{i=0} p_{ni} \cos(i\vartheta) + \sin\vartheta \sum_{i=0} p_{ti} \sin(i\vartheta)$$

The centripetal force per angular unit in toroidal direction is then found by integration over the vessel surface

$$F_\varphi = \int_0^{2\pi} R \cdot p_R \cdot c \cdot d\vartheta$$

$$F_\varphi = R_o \cdot c \int_0^{2\pi} p_R (1 + \varepsilon \cos\vartheta)\, d\vartheta$$

$$= \pi \cdot R_o \cdot c \left[p_{n1} + p_{t1} + \varepsilon_c \left(p_{no} + p_{at} + \frac{p_{n2} + p_{t2}}{2} \right) \right] \qquad (35)$$

F_c in kp if R_o and c are given in cms.

Inserting the numbers for the JET device (Table I), together with the assumptions about the plasma state prior to a hard disruption as used in Ref./11/ into the equations for the vessel currents (eq.(26)) and the magnetic field components (eqs.(28)-(30) for r=c), one gets the following results

$$\dot{j}_o = 4{,}68 \ \frac{kA}{cm} \qquad p_{no} = 1{,}430 \ \frac{kp}{cm^2}$$

$$j_1 = 0{,}37 \ \frac{kA}{cm} \qquad p_{n1} = 0{,}770 \ \frac{kp}{cm^2}$$

$$B_{n1} = 0{,}115 \ T \qquad p_{n2} = 0{,}026 \ \frac{kp}{cm^2}$$

$$\overline{B}_{to} = 0{,}294 \ T \qquad p_{t1} = 0{,}549 \ \frac{kp}{cm^2}$$

$$\overline{B}_{t1} = 0{,}138 \ T \qquad p_{t2} = 0{,}022 \ \frac{kp}{cm^2}$$

$$B_{\perp o} = 0{,}499 \ T \qquad F_\varphi = 431 \ \text{tons}$$

References

/1/ L.A.Artsimovich:"Tokamak Devices", Nuclear Fusion 12 (1972),p.218, eq.3.6.

/2/ B.Streibl:"Eddy Currents and Magnetic Forces Originating in the JET Vaccum Vessel Due to Disruptive Instability", JET-Internal Report (1978).

/3/ V.S.Mukhovatov and VD.Shafranov:"Plasma Equilibrium in a Tokamak", Nuclear Fusion 11 (1971), p.608, eqs.(13) and (14).

/4/ ibid. p.609, eqs.(26) and (27)

/5/ ibid.,p.609, eq. (29)

/6/ ibid.,p.617, eqs.(92) and (93)

/7/ ibid.,p.621, eq.(152)

/8/ ibid.,p.621, eq.(152) noting that the variation in local resistance of the liner, characterized by λ , can be expressed by a variation in the local surface current density upon replacing λ by $\frac{-j_1}{j_o}$

/9/ B.Streibl:"Eddy Currents and Magnetic Forces Originating in the JET Vacuum Vessel Due to Disruptive Instability", JET-Internal Report (1978), paragraph 5.1.

/10/ ibid., paragraph 5.3

/11/ ibid., Table 7.1 "Status 1", $\Delta = 5{,}2$ cm, $\Lambda = \frac{1}{2}$, $I_p = 5$ MA

Table I:

List of Symbols used (if not explained in the text)

Symbol	Meaning	Numerical Values for the Circularised JET-Device/11/
R_o	major radius of vacuum vessel and equivalent shell	2 96 cm
a	minor radius of plasma loop	123,9 cm
b	minor radius of equivalent shell	270 cm
c	minor radius of vacuum vessel	170 cm
$\mathcal{E}_c$	aspect ratio of vacuum vessel	0,574
R_p	major radius of plasma loop	
$\Delta = R_p - R_o$	displacement of plasma loop	5,2 cm
I	total current of toroidal loop	
I_p	plasma current	5,0 MA
I_v	net vessel current	
j_v	vessel current density	
j_o	homogeneous part of vessel current density	
j_1	amplitude of first harmonic of vessel current density	
β_ϑ	ratio between average plasma pressure and energy density of the poloidal magnetic field at the plasma boundary	
ℓ_i	internal inductance of the plasma loop per unit length	
Λ	$\beta_\vartheta + \frac{\ell_i}{2} - 1$	$\frac{1}{2}$
$B_{\perp o}$	perpendicular magnetic field to keep the plasma loop in equilibrium	
F_c	total centering force	
μ_o		$4\pi \cdot 10^{-9}\ \frac{Vs}{Acm}$

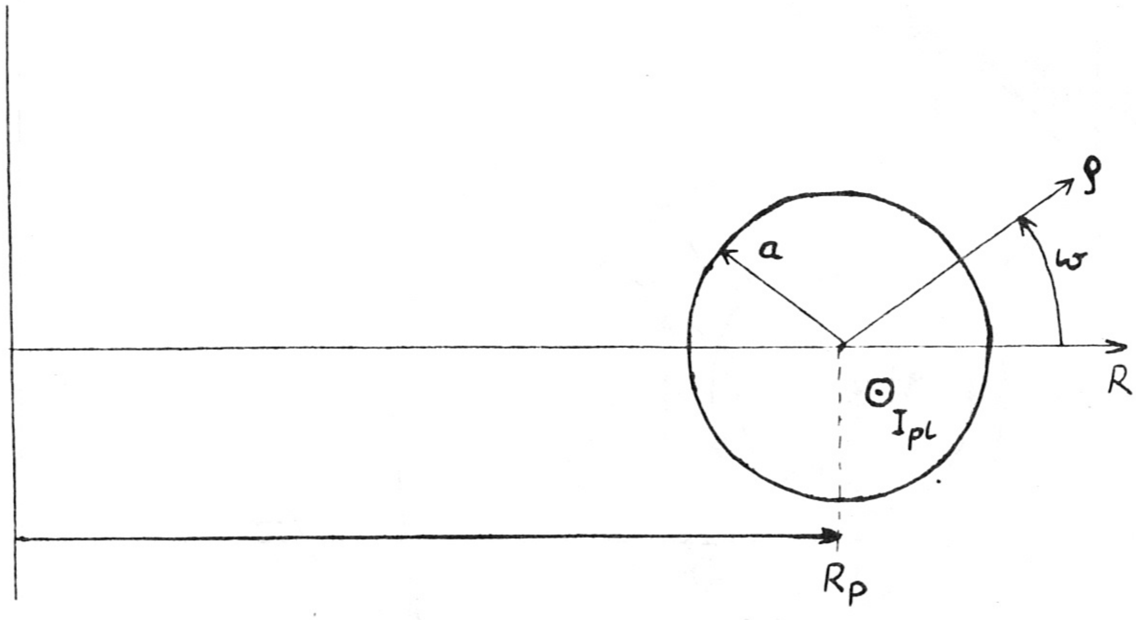

a
ρ
ω
R
I_{pl}
R_p

FIG. 1

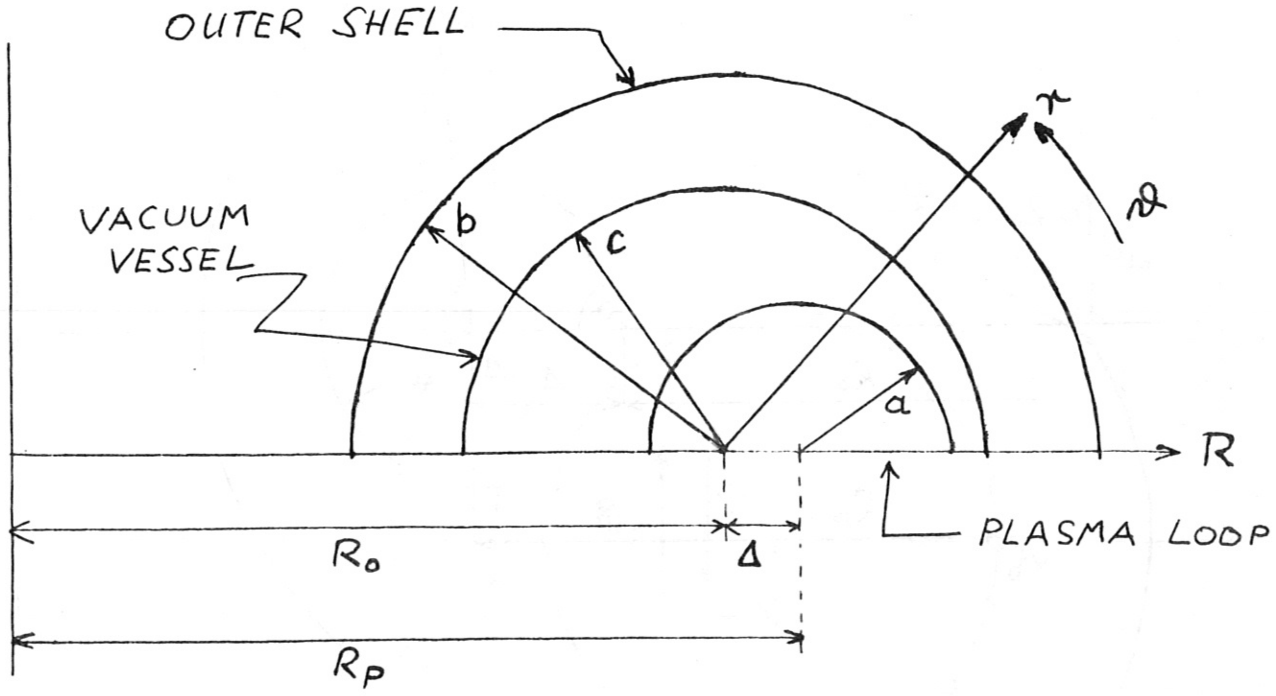
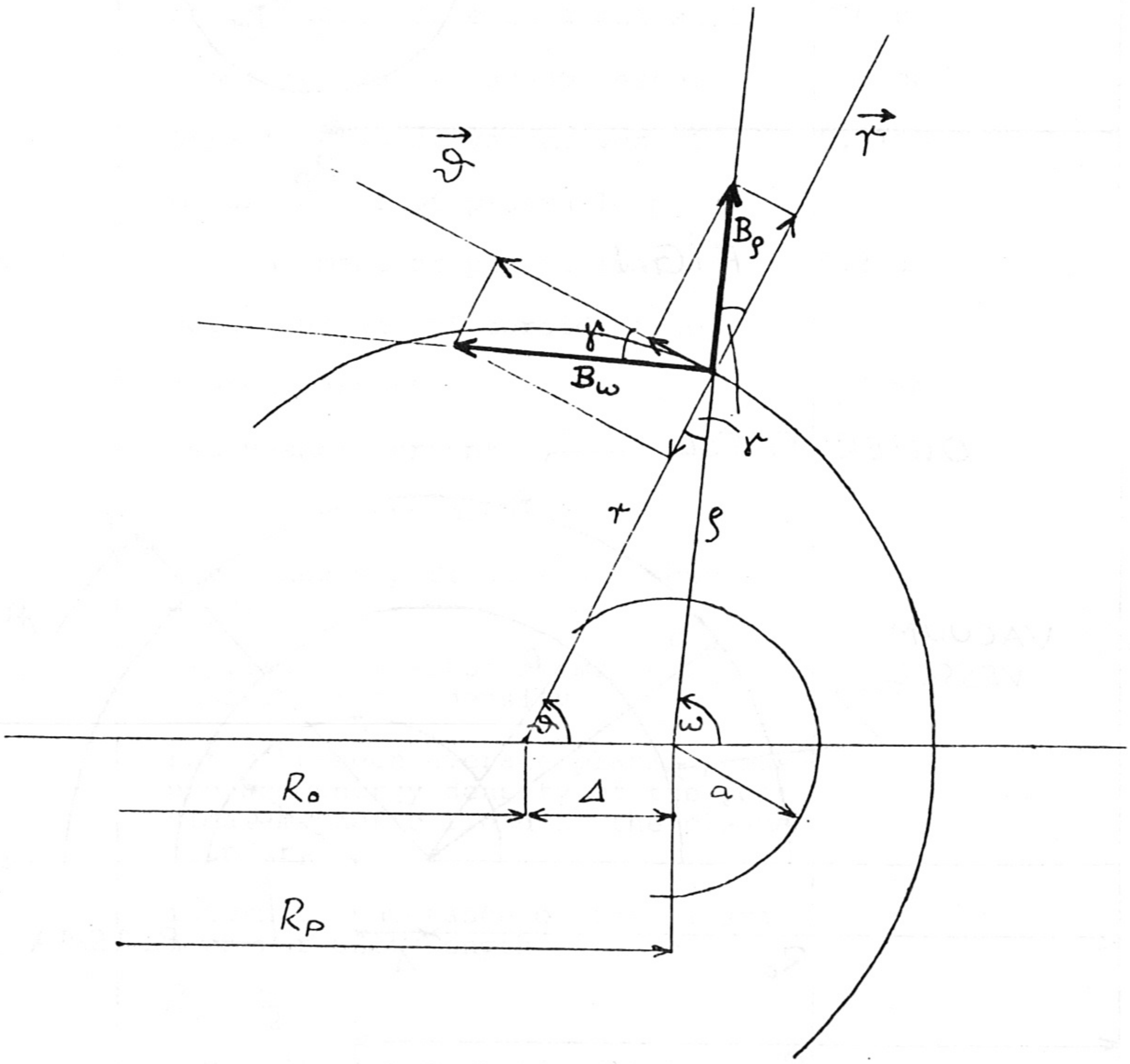

OUTER SHELL
VACUUM
VESSEL
b
c
a
r
ϑ
R
PLASMA LOOP
R_o
Δ
R_p

FIG. 2



$$\omega = \vartheta + \gamma$$

$$\sin\gamma = \frac{\Delta}{r} \cdot \sin\vartheta$$

$$\cos\gamma \approx 1$$

FIG. 3

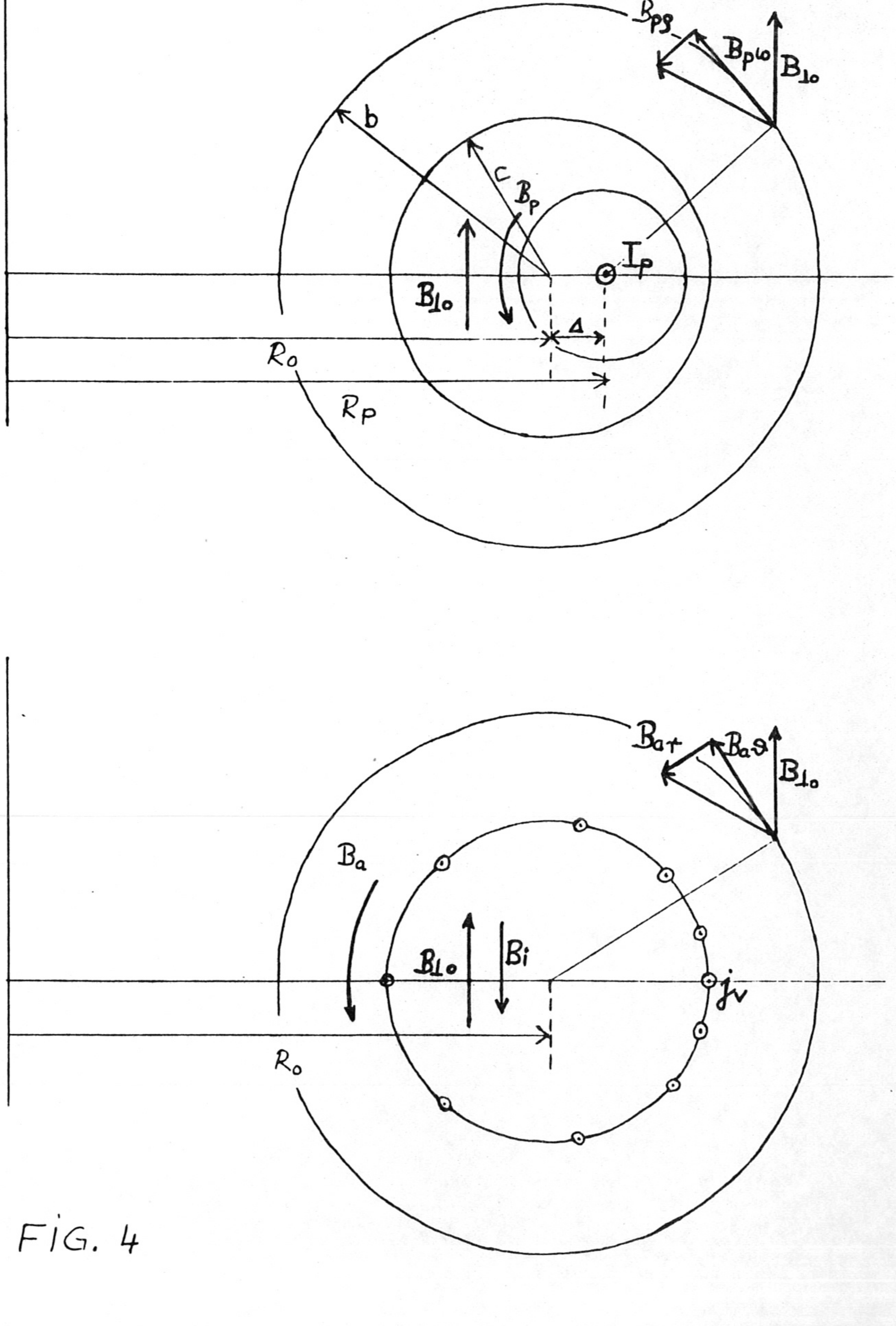


FIG. 4