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DIFFUSION IN TOKAMAKS WITH SHEAR

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Abstract

A $1/B^3$ scaling law is derived for anomalous diffusion produced by the dissipative trapped-ion instability in axisymmetric toroidal plasmas with shear. Our theory helps to explain why transport of this type has not been detected in the PLT heating experiment so far [9]. We prove, in addition, that the Kadomtsev-Pogutse diffusion formula [3] contradicts the trapped-fluid equations from which it was deduced [3], provided that the necessary boundary conditions are observed.

1. INTRODUCTION

Numerical results on anomalous diffusion caused by the dissipative trapped-ion instability have been published elsewhere [1], [2]. There the Kadomtsev-Pogutse trapped-fluid equations [3] were solved in a 2D slab model without taking shear effects into account. On the other hand, Gladd and Ross [4] have shown that, within the linearized theory, magnetic shear introduces new effects connected with strong Landau damping of the untrapped ions near mode-rational surfaces (where $K_{\parallel} = 0$). They find unstable trapped-ion modes which are either confined between two adjacent mode-rational surfaces ("localized modes") or may extend over many mode-rational surfaces, but have their radial nodes approximately there ("extended modes"). It seems reasonable to take this important shear effect into account by identifying the radial extent of the model slab with the distance Δa between properly chosen, adjacent mode-rational surfaces, rather than with the small plasma radius a . In this way the values of the electrostatic potential ϕ at mode-rational surfaces will approximately equal those at the radial boundaries of the slab, viz. $\phi = 0$.

In this paper we show that a similarity analysis of the trapped-fluid equations then leads to a new scaling law for the trapped-ion anomalous diffusion co-

efficient of a toroidal configuration with shear. This scaling law can be evaluated by feeding in the numerical results [2] obtained for the diffusion coefficient of a toroidal configuration without shear. The shear modifies the scaling of the diffusion coefficient to change from $1/B$ (Bohm-like) [2] to $1/B^3$. It reduces anomalous diffusion by a potentially large factor such that it falls below the Kadomtsev-Pogutse value [3].

While we take into account radial mode localization caused by strong Landau damping of untrapped ions near mode-rational surfaces, weak Landau damping of untrapped ions, far from mode-rational surfaces, and of trapped ions will be neglected. Here we call Landau damping strong or weak, respectively, for $v_{res} \sim v_{th}$ and $v_{res} \ll v_{th}$. The formulas for weak Landau damping [5] change their sign at $\eta_i \equiv d \ln T_i / d \ln N_p = 2/3$, so that Landau damping is negative for $\eta_i > 2/3$. On the other hand, Gladd and Ross [4] show that the total Landau damping (= sum of strong + weak L.D.) of untrapped ions may be positive up to $\eta_i \lesssim 2$. Considering these discrepancies, it seems best to discard any correction terms for weak Landau damping altogether, specifically in the case $\eta_i = \eta_e = 1$, which we shall treat exclusively. By dis-

carding weak Landau damping one gains because the problem depends on a smaller number of equilibrium parameters, while one apparently does not lose in accuracy as long as the determination of Landau damping effects is not in a better state than it is now.

It should be emphasized that the Kadomtsev-Pogutse fluid equations [3] and their consequences are only valid for a two-component plasma (one ion species and electrons), with an ion charge $Z_i = 1$. Occasionally, application of this theory to plasmas with impurities has been attempted by introducing an effective Z and modifying the collision frequencies accordingly, an approach which is open to serious doubts. Linear theories for multi-component plasmas also exist [6], [7], but calculations of anomalous transport were not made for these cases. Merely the ansatz $D \approx \gamma/K_{\perp}^2$ was employed [6], but this ansatz lacks sufficient theoretical foundation and, moreover, suffers from the uncertainty what value of K_{\perp} to choose.

2. SIMILARITY ANALYSIS

In [2] a similarity analysis ("dimensional analysis") was applied to the trapped-fluid equations in a slab geometry. As a result, the anomalous diffusion coefficient (defined as the ratio of the radial particle flux divided by the gradient of the equilibrium density N_p) was shown to obey

$$D = \delta_o a v_o g(c_1, c_2). \quad (1)$$

Here a is the minor plasma radius and the radial extent of the slab, $v_o = (\delta_o c T) / [2 e B r_n (1-\delta_o)]$, N_p = plasma density, n_o = trapped particle density, $\delta_o = n_o/N_p$, $r_n = n_o/n'_o$ (prime = radial derivative), B = magnetic field, $T = 2 T_i T_e / (T_i + T_e)$ = effective temperature [erg], g is an unspecified function of two dimensionless variables, viz. $c_1 = v_i/v_e$, $c_2 = v_o/(v_e a)$, v_j = effective collision frequencies of trapped particles. The density length scale r_n and the angular extent b of the slab had both been set equal to a . For $b \neq a$, eq. (1) must be replaced by

$$D = \delta_o \frac{a^2 v_o}{b} g(c_1, c_2') \quad (2)$$

with $c_2' = v_0 / (v_e b)$. For $b > a$ this yields a smaller diffusion than according to eq. (1), if g is a non-increasing function of c_2' (see below).

We shall now modify the above formulas in order to include the shear effect mentioned above. To this end the radial extent of the slab, now called Δa , is identified with the distance between two properly chosen mode-rational surfaces (see below); i.e. we have $\Delta a \leq a$, with $a =$ plasma radius. Two radial length scales, viz., Δa and r_n , the scale of the radial density variation, must be distinguished. This does not hamper the similarity analysis because the density $n_0(x)$ itself does not enter the equations for the perturbations $\tilde{n}_j = n_j - n_0$, but only its gradient $n_0'(x)$. The similarity analysis is again straight-forward (see [2]) so that it suffices to list the units employed, viz. v_e^{-1} for t , Δa for x (radial coordinate), b for y (the angular coordinate), and $\langle n_0 \rangle \cdot \frac{\Delta a}{r_n}$ for the density perturbations \tilde{n}_j . The result is

$$D_s = \delta_0 (\Delta a)^2 \frac{v_0}{b} g(c_1, c_2') \quad (3)$$

again with $c_1 = v_i / v_e$; $c_2' = v_0 / (v_e b)$; $v_0 = (\delta_0 c T) / [2 e B r_n (1 - \delta_0)]$, but now $\Delta a \neq r_n$,

with r_n of order a . It follows from the dimensional analyses that the g -functions in eqs. (1), (2), (3) are identical. The result is remarkably simple in that D_s is obtained from D of eq. (2) by multiplying by $(\Delta a/a)^2$. That is, shear is shown to reduce anomalous diffusion whenever $\Delta a < a$.

Comparison of eq. (2) or (3) with Kadomtsev and Pogutse's [3] formula for trapped-ion anomalous diffusion, viz. $D_{KP} = \delta_o v_o^2 / (2 v_e)$, shows that the Kadomtsev-Pogutse formula is generally in conflict with these two equations and, hence, also with the underlying trapped-fluid equations from which the Kadomtsev-Pogutse diffusion formula was originally deduced [3]. This is so because the necessary boundary conditions are taken into account in our study, whereas Kadomtsev and Pogutse have failed to observe these boundary conditions. Consequently, the Kadomtsev-Pogutse diffusion formula [3] appears to lack theoretical foundation. Further use of this formula in numerical transport codes is deemed unjustified.

The validity of the above results rests upon several assumptions [2]. These are:

- (1) D and D_s are assumed to be independent of a representative class of initial conditions of the plasma.
- (2) The linearized version of the quasineutrality condition is used.
- (3) The approximations $\nabla \delta_o = 0$, $\nabla n_o = \text{const}$, $N_p/T = \text{const}$, i.e. $\eta = 1$, with $\eta = d \ln T / d \ln N_p$, and $\nabla v_j = 0$ are used (the latter being somewhat in conflict with $N_p/T = \text{const}$ for a real plasma).
- (4) Weak Landau damping, far from the mode-rational surfaces, is neglected. This would be exact for $\eta = 2/3$ rather than $\eta = 1$ if the pertinent formulas used [5] could be considered to be correct.
- (5) In accordance with numerical results [2] anomalous diffusion is assumed to depend mainly on the long-wavelength part of the spectrum and only negligibly on the short-wavelength cutoffs at $K_x R_{Bi} \gtrsim \pi$ and $\omega_K \gtrsim \omega_{bi}$, with R_{Bi} = ion banana width, ω_{bi} = ion bounce frequency. This can only be valid if the distance Δa between the adjacent mode-rational surfaces contains several ion banana widths.

These approximations appear to be justified in that they yield an important reference case, with a particularly simple scaling law holding for D_S . Indeed, eqs. (1) to (3) are, in essence, one-parameter scaling laws because $c_1 = v_i/v_e$ is constant for a given ion species and a fixed value of T_i/T_e . Without the use of these approximations the resulting formula for D_S will involve several additional dimensionless parameters or profiles and, hence, be of restricted usefulness.

3. SHEAR MODEL AND ANOMALOUS DIFFUSION

In order to further evaluate D_s from eq. (3) the distance Δa between adjacent mode-rational surfaces must be known. It is [4]:

$$\Delta a \sim 1/(\ell q') \sim r_q/m \quad (4)$$

for two components (m, ℓ) and $(m+1, \ell)$ of an eigenmode, with m = poloidal wave number, ℓ = toroidal wave number, q = safety factor, $r_q = q/q'$ = radial length scale of the variation of q . Numerical results [2], [8] show that, in the majority of cases, at late times the m -spectrum is monochromatic in the sense that it virtually consists only of the components $m = 0$ and $m = m_{\text{final}}$, with $m_{\text{final}} \approx \max \{1, \alpha m_{\text{marg}}\}$. Here m_{marg} is the mode number margin of stability, viz.

$$m_{\text{marg}} \sim \sqrt{c_1/(2\pi c_2')} = b \sqrt{(v_e v_i)/(2\pi v_o)}, \quad (5)$$

and $\alpha \sim 4$ in the range $0.3 \lesssim m_{\text{marg}} \lesssim 2.0$. Remembering the condition $\Delta a \lesssim a$ eq. (4) is to be replaced by

$$\Delta a = \min \left\{ a, r_q, \frac{r_q}{\alpha m_{\text{marg}}} \right\} . \quad (6)$$

It is this expression for Δa which must be inserted into eq. (3) in order to obtain the anomalous diffusion coefficient in the presence of shear. The function g was determined numerically [2], viz. $g \approx 6 \times 10^{-2}$ in the range $0.3 \leq m_{\text{marg}} \leq 3.0$ for $c_1 = v_i/v_e = 1.65 \times 10^{-2}$. This value of v_i/v_e holds for a deuterium plasma with $T_i = T_e$ and $Z_i = 1$, if $v_i/v_e = \sqrt{(m_e/m_i)}$ is used. By optimizing several definitions used, the improved value $g \approx 5 \times 10^{-2}$ is obtained instead. We shall assume that this value is also an upper bound for $m_{\text{marg}} \geq 3.0$. If the minimum of the curly bracket of eq. (6) is given by \underline{a} or r_q a Bohm-like diffusion is obtained by inserting Δa and g into eq. (3). This agrees with our earlier numerical results [2]. A new and different diffusion formula is obtained, however, if the minimum mentioned is given by $r_q/(\alpha m_{\text{marg}})$. It is this case which shows the effects of shear most strongly and which is discussed exclusively in the following. Inserting Δa and g into eq. (3) then yields the anomalous diffusion in the presence of shear, viz.

$$D_s \approx 2.0 \delta_0 \frac{r_q^2}{\alpha^2 v_e v_i} \left(\frac{v_0}{b} \right)^3, \quad (7)$$

with $b = 2\pi r$, r = radial coordinate. Here the collision frequencies v_i and v_e are not independent of each other because $v_i/v_e = 1.65 \times 10^{-2}$ is implied.

It is interesting to compare the new formula with Kadomtsev-Pogutse diffusion [3], viz.

$$D_{KP} = \delta_o v_o^2 / (2v_e). \text{ This yields}$$

$$\frac{D_s}{D_{KP}} \approx \frac{0.50}{\alpha^2 m_{\text{marg}}}, \quad (8)$$

where the approximations $r_q = a$, $b = \pi a$ have been used. Equation (8) shows that D_s is usually much smaller than D_{KP} . Note that m_{marg} scales as

$$m_{\text{marg}} \propto \frac{B N_p (1-\delta_o) r_n r}{T^{5/2} \delta_o^3}. \quad (9)$$

An estimate of the scaling, in terms of experimental parameters, of the anomalous diffusion in the presence of shear can be obtained by replacing the \approx sign in eq. (7) by \sim . Then

$$D_s \propto \frac{T^6 \delta_o^8 r_q^2}{N_p^2 B^3 (1-\delta_o)^3 r_n^3 r^3}. \quad (10)$$

For small β -values, the approximation $\delta_o \sim \sqrt{(r/R)}$ yields the simplified scaling formula

$$D_s \propto \frac{T^6 r_q^2 r}{N_p^2 B^3 (1-\delta_o)^3 r_n^3 R^4}. \quad (11)$$

Of course, eqs. (10) and (11) hold only in the regime where $\min \left\{ a, r_q, r_q / (\alpha m_{\text{marg}}) \right\} = r_q / (\alpha m_{\text{marg}})$.

4. DISCUSSION OF THE RESULTS

It is important to discuss several points related to evaluating the anomalous diffusion coefficients for specific plasma configurations and to questions of validity. Equation (11) demonstrates the strong temperature dependence of D_s of eq. (7). Consequently, if T is not known accurately, a large uncertainty obtains for D_s . On the other hand, if plasma heating is considered, the T^6 -dependence of D_s provides that a rather accurate equilibrium temperature can be obtained from an energy balance consideration. Consider also the dependence of D_s on the radial coordinate. If the density and temperature profiles are approximately triangular-shaped, a case found for instance in the PLT heating experiment [9] (see below), then the radial dependence is comparatively weak, viz.

$$D_s \propto \frac{(1-\rho)\rho}{\left[1 - \sqrt{(\rho/A)}\right]^3}, \quad (12)$$

with the definition $\rho = r/a$, $A =$ aspect ratio, and the approximation $r_q = \text{const.}$ In other cases, however, the radial dependence may be strong. Moreover, on varying the radius, D_s may change its scaling according to eq. (6), and the validity conditions to be listed below may add further variations with r . Be-

cause of this potential sensitivity of D_s it will usually be advisable to evaluate D_s by introducing it in a self-consistent transport code that allows a unified treatment of the various regimes. A related point concerns the explicit way in which eqs. (3) and (6) can be evaluated. A simple, straight-forward way is to evaluate "locally", i.e. for a definite set of radii, $\{ r_K \}$. In this case, however, the intervals $\Delta a(r_K)$ centered around the various radii r_K will overlap each other, which ought not to happen in a physically adequate model. Hence one will prefer a "global" evaluation, where one may start out by, for instance, determining the maximum of $\Delta a(r)$ and then continue by repeatedly joining appropriate intervals $\Delta a(r_v)$ left and right till one reaches the boundaries. As a last point, eqs. (3), (6), (7) hold, of course, only if a set of existence conditions for the dissipative trapped-ion instability are all satisfied. They are: $2 \pi v_j < \omega_{bj}$, $\omega < \omega_{bj}$, $K_y R_i < \pi$ and $K_x R_{Bi} < \pi$, i.e. $R_{Bi} < \Delta a$ [4], with ω_{bj} = bounce frequencies of the trapped ions and electrons, R_i = gyro-radius of ions, R_{Bi} = banana width of trapped ions. If any of these conditions is violated, then the approximation $D_s = 0$ should be used instead of eqs. (3), (6), (7). In addition, Kadomtsev and Pogutse [3] indicate that a transition to the collision-

less trapped-ion instability should occur whenever $v_e \lesssim k_y v_0$ or, equivalently, $m \lesssim m_{\text{marg}} (v_e/v_i)^{1/2}$. For this case Kadomtsev and Pogutse suggest a Bohm-like diffusion [3].

We wish also to comment on the validity of the mode-rational surface, shear model of Gladd and Ross [4], in the form in which we have used it here. First, one must be sure that for a given poloidal mode number $m = m_{\text{final}}$ the q -profile $q(r)$ allows one to find a set of appropriate toroidal mode numbers ℓ such that the consequent intervals Δa between mode-rational surfaces (m, ℓ) and $(m \pm 1, \ell)$ cover the whole radial extent of the plasma, where ℓ is any member of the set. It is easily shown that the q -interval $1 \leq q \leq q_{\text{max}}$ with

$$q_{\text{max}} = (m+1) / \left\{ \frac{m-1}{2} \right\}_+ \quad (13)$$

can in fact be covered by overlapping intervals Δa in this way. Here the bracket symbol $\{ \}_+$ designates the smallest integer \geq the argument. Secondly, we have to ask about the role of mode components (m, ℓ) which do not have any mode-rational surfaces in the volume. Such modes have not been investigated by Gladd and Ross [4]. It is easily seen, however, that usually, for $|q_{\text{max}} - q_{\text{min}}| > 1$ and $m \gg 1$, such modes

lead to $|m - \ell q| \gg 1$ in the bulk of the volume, i.e. they are not flute-like. They can therefore be expected to have smaller growth rates than the flute-like ones [10], [11], a fact that may justify omission of these modes in the theory of anomalous trapped-ion diffusion.

5. COMPARISON WITH THE LITERATURE

The method of introducing an effective slab volume limited by adjacent mode-rational surfaces in order to take into account strong Landau damping near mode-rational surfaces as an important shear effect has been used before in 2D, nonlinear, trapped-ion transport calculations. Sugihara and Ogasawara [12] have simplified the Kadomtsev-Pogutse trapped-fluid equations by keeping only the most secular nonlinear terms and introducing a turbulent collision frequency. They do not give a scaling law for the anomalous diffusion, but compute D and D_s for only a single set of plasma parameters. Sugihara and Ogasawara claim agreement of their results with the results of [2]. Numerical comparison shows, however, that the diffusion coefficient without shear of [12] is a factor of ~ 70 larger than the diffusion coefficient that is deduced from [2] with the $b > a$ correction taken into account. It is surmised that the rough approximation of the nonlinear terms in [12] is mainly responsible for this large discrepancy. On the other hand, if shear is taken into account, the diffusion coefficient of [12] is only a factor of ~ 7 larger than our value deduced from eq. (7). We do not know why these two factors differ so much from each other. It may well

be that the k_y -spectrum of [12] differs from ours, and in this way different effective values of Δa turn up in the two theories.

On the other hand, Cohen and Tang [5] have recently published a 2D trapped-fluid theory that also contains several additional microscopic effects. Their calculations of saturated wave spectra, however, use restricted spectra and interactions only. For an assumed three-wave interaction their result is inconclusive because the equilibrium spectrum turns out to be unstable. No scaling law is given in this case. For an assumed four-wave, one-mode self-interaction, the result is applicable only for very small saturation amplitudes close to marginal stability. Here a scaling law is given, but that result cannot be compared with ours because taking the limit of vanishing weak Landau damping (letting $\eta_i \rightarrow 2/3$) in their formula [5] would yield an infinitely large diffusion coefficient.

6. PRELIMINARY APPLICATION

Our new diffusion formulas are to be inserted in a numerical transport code [13]. In the mean-time it is useful, as an illustration, to consider the magnitude of the anomalous diffusion coefficient and of the resulting anomalous energy transport coefficient in a simple example. We choose the PLT heating experiment [9] because some publicity was given the fact that trapped-ion anomalous energy flow could not be detected there. At the highest temperatures the PLT heating experiment shows $Z_{\text{eff}} > 1$ [9], a fact which by itself may suffice to stabilize the dissipative trapped-ion modes under these conditions. Rather than address ourselves to this question, which cannot be decided anyhow merely on the evidence of Z_{eff} , we shall demonstrate that the PLT heating experiment would show an undetectable anomalous trapped-ion energy flow (i.e., below the neoclassical value of heat transport) even if one had $Z_{\text{eff}} = 1$ in this experiment. As mentioned above, it is only for the case of $Z_i = Z_{\text{eff}} = 1$ that a nonlinear theory of the dissipative trapped-ion instability and the consequent transport has been developed up to now.

To make things explicit, we list the physical parameters of the PLT heating experiment used by us. They are $r = a/2$, $B = 3.2 \times 10^4$ G, $a = 40$ cm, $R = 130$ cm, $N_p = 2.2 \times 10^{13}$ cm⁻³, $T_i = 2.75$ keV, $T_e = 1.25$ keV (all taken at $r = a/2$). In addition, we use the assumptions $r_q = a$, $r_n = r_T = a$, $q = 2$. The radial profiles of density and temperature are taken as linear functions of r in near agreement with experimental results [9]. It follows that $m_{\text{marg}} = 5.3$. We maximize the anomalous diffusion coefficient by choosing $\alpha = 1$, i.e. $m_{\text{final}} = m_{\text{marg}}$ rather than using the value $\alpha = 4$ introduced above. We checked numerically that in the present case, with $v_i/v_e = 5.0 \times 10^{-3}$, we still have $g \approx 5 \times 10^{-2}$. On using this value the anomalous trapped-ion energy transport coefficient becomes

$$\chi_i \approx D_S = 3.8 \times 10^2 \text{ cm}^2/\text{s} . \quad (14)$$

We compare this with the neoclassical value of the ion heat conductivity [14], viz.

$$\chi_i^{\text{NC}} = 0.4 \frac{v_{ic} R_i^2 q^2}{\delta_o^3} , \quad (15)$$

and obtain $\chi_i^{\text{NC}} = 370$ cm²/s, $\chi_i/\chi_i^{\text{NC}} \approx 1.0$. The Kadomtsev-Pogutse formula would have yielded instead $\chi_i^{\text{KP}} \approx D^{\text{KP}} = 2.2 \times 10^3$ cm²/s, $\chi_i^{\text{KP}}/\chi_i^{\text{NC}} = 6.1$. Note that

eq. (8) does not hold here for $\chi_i/\chi_i^{KP} \approx D_s/D_{KP} = 0.17$. The reason is that eq. (8) was derived for $v_i/v_e = 1.65 \times 10^{-2}$, while for the above PLT parameters one has instead $v_i/v_e = 5.0 \times 10^{-3}$. The validity conditions of Sec. 4 are satisfied for the above PLT values; hence the instabilities should be there for $Z_{eff} = 1$, but, as eq. (14) shows, with small amplitudes and small anomalous transport only. The calculated value of D_s [eq. (14)] is so small and the r -dependence of D_s is so comparatively weak [see eq. (12)] that evaluation of D_s for different values of r is not necessary in order to demonstrate our above assertion of undetectable anomalous energy transport from the dissipative trapped-ion instability in the PLT heating experiment [9].

7. SUMMARY

From a similarity analysis of the Kadomtsev-Pogutse trapped-fluid equations [3], by incorporating the mode-rational surface model of Gladd and Ross [4] and using earlier numerical results [2] we obtained the following results:

- (1) A rectangular slab, with $b > a$, yields a smaller diffusion than a quadratic one with $b = a$ (a fixed). This is an important point for numerical calculations [see eq. (2)].

- (2) Shear decreases the diffusion by the factor $(\Delta a/a)^2$ [see eq. (3)], where Δa is the distance of appropriate mode-rational surfaces. If $\min \{a, r_q, r_q/(\alpha m_{\text{marg}})\} = r_q/(\alpha m_{\text{marg}})$, then it follows for the anomalous diffusion coefficient with shear, D_s , that ordinarily $D_s \ll D^{\text{KP}}$ [see eq. (8)]. If $\min \{a, r_q, r_q/(\alpha m_{\text{marg}})\} = a$ or r_q , then Bohm-type diffusion prevails, in agreement with [2], where, however, corrections for $b > a$ must be introduced.

- (3) If shear is effective, i.e. $\min\{\dots\} = r_q (\alpha m_{\text{marg}})$, see above, then the diffusion coefficient obeys a $1/B^3$ scaling law [see eqs. (10), (11)]. Because the diffusion also scales as T^6 , the trapped-ion anomalous transport will make its appearance rather suddenly when T is raised. In the PLT heating experiment [9] the anomalous trapped-ion energy transport is still of the order of or smaller than the neoclassical ion heat conductivity.
- (4) The Kadomtsev-Pogutse diffusion formula [3] is shown to be in conflict with the underlying trapped-fluid equations if the necessary boundary conditions are observed. The use of that formula in numerical transport codes is therefore deemed inappropriate. It should be replaced by Bohm-type diffusion [2], corrected for $b > a$, or by the new formula [eq. (7)], depending on the value of $\min\{a, r_q, r_q/(\alpha m_{\text{marg}})\}$ [see eq. (6)].

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