Comparison of MHD pressure losses of liquid-lithium flows in coaxial and parallel ducts, passing through strong transverse magnetic fields

G. Trommer

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ABSTRACT

This report deals with theoretical calculations of MHD pressure losses of liquid-lithium flows in tubes of circular cross-section exposed to strong magnetic fields. Some simplifying assumptions were introduced, yielding an analytical solution which allows the pressure drop and losses in double tubes of coaxial geometry to be compared with those in normal flow pipes.

The investigations show that coaxial ducts require much more pumping power than normal ones under similar conditions. This great difference of the properties of the two duct types will decrease if the pipes are embedded in materials of good electrical conductivity. In this case the normal duct will afford a drastic increase in the pressure drop, while the coaxial one will be nearly unaffected. But even under these conditions the losses of the latter will dominate.

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1. INTRODUCTION

In a steady-state fusion reactor, as envisaged at present, a high-temperature reacting plasma will be confined by a magnetic field with toroidal geometry from which the plasma can only slowly leak by diffusion across field lines. One of the possible concepts for the nuclear blanket of a fusion reactor fuelled with a deuterium--tritium mixture would be the use of liquid lithium as both a tritium breeding and heat transfer medium.

The magnet coils used to generate the strong magnetic fields must be outside the heavy neutron flux regions. As a result, they have to be outside the lithium blanket. This, in turn, means that the Li flow pipes must pass inside the magnetic field region. In this way, there will be strong electric currents induced inside the Li flows which result in an enormous increase of the pressure losses. To avoid unrealistically high pumping power of the Li pumps, the losses have to be minimised by finding optimal pipe configurations inside the regions of strong magnetic field. These pressure losses are influenced to a great extent by the geometry and the material properties of the Li ducts, e.g. cross-section, thickness of duct wall, and electrical conductivity of the duct wall.

In order to find criteria for optimal Li ducts, two different concepts of conveying the Li to the blanket are compared in this paper:

(a) A single pipe of coaxial geometry is used to pass the Li from outside the magnetic coils to the blanket and back again (see Fig. 1).

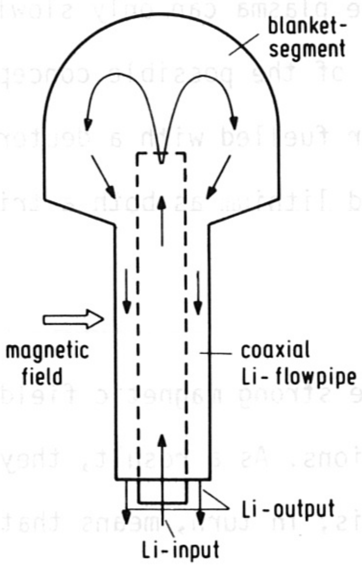


Fig. 1

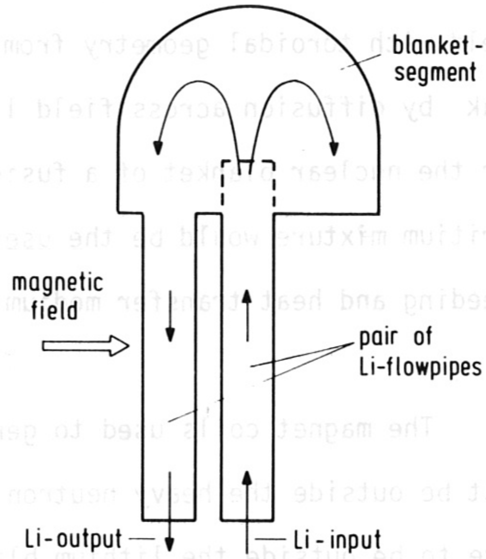


Fig. 2

(b) Two pipes of normal geometry are used for the Li circulation; they are placed close together, so that they are influenced by nearly identical magnetic fields (see Fig. 2).

In this investigation the MHD losses, which are created in the blanket segment itself, are neglected. The reason for the validity of this approach is the much greater flow velocity of the lithium inside the pipes compared with that inside the blanket segment with its wide cross-sections. As the extent of MHD losses is nearly proportional to the square of the flow velocity, all pressure drops created inside the Li ducts will largely dominate.

2. CALCULATION OF PRESSURE LOSSES IN FLOW PIPES WITH COAXIAL AND NORMAL GEOMETRY

2.1 Definitions and assumptions

With respect to the above-mentioned problems, the following question has to be answered:

We have a coaxial flow pipe of length L (see Fig. 3). In the inner part of the duct liquid-lithium flows with the mean velocity

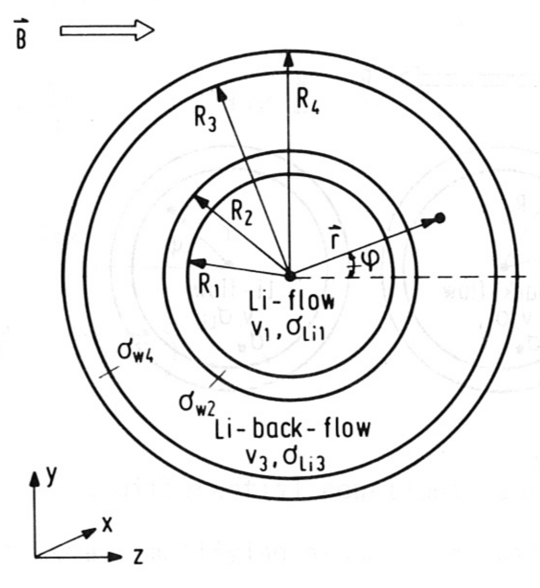


Fig. 3

v_1 in the x direction. The same amount of lithium streams back in the outer part of the duct with the mean velocity v_3 in the -x direction. The electric conductivity of the lithium is σ_{Li} , that of the duct walls σ_w. Along the whole length L the pipe is crossed by a strong, homogeneous magnetic field $\vec{B}$ pointing in the z direction: $\vec{B} = B\vec{e}_z$.

What is calculated is the pressure drop between the ends of the pipe, which is the sum of the MHD losses caused by $\vec{j} \times \vec{B}$ forces in the inner part of the duct and those of the outer region.

This pressure drop, that must be forced by the lithium pumps, is compared with the pressure losses of normal flow pipes which, of course, are of double length to allow for the fact that the lithium must be forced into and back from the blanket segment (see Fig. 4).

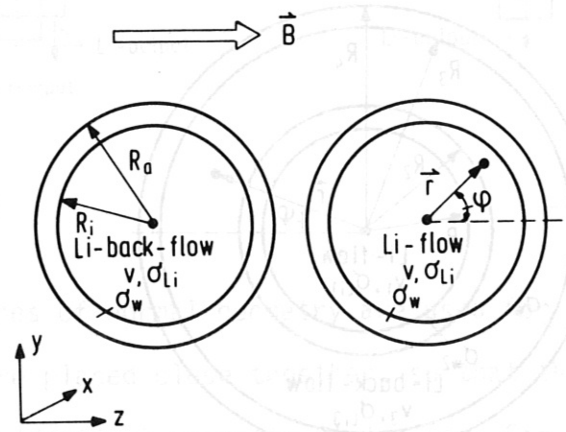


Fig. 4

The equations necessary for solving the problem stated are as follows:

Continuity equation

$$\frac{\partial \varrho}{\partial t} + \operatorname{div}(\varrho \vec{v}) = 0 \quad , \tag{1}$$

momentum balance

$$\varrho \frac{\partial \vec{v}}{\partial t} + \varrho(\vec{v} \cdot \operatorname{grad}) \vec{v} = - \operatorname{grad} p - \operatorname{div} \underline{\underline{\tau}} + \vec{j} \times \vec{B} \quad , \tag{2}$$

Maxwell's equations

$$\operatorname{rot} \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \quad , \quad \operatorname{div} \vec{D} = \varrho_{el.} \quad , \tag{3}$$

$$\operatorname{rot} \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad , \quad \operatorname{div} \vec{B} = 0 \quad , \tag{4}$$

and finally Ohm's law

$$\vec{j} = \sigma (\vec{E} + \vec{v} \times \vec{B}) \quad . \tag{5}$$

This system of coupled differential equations cannot be solved analytically, but some simplifying assumptions will lead to some differential equations with easy analytical solutions.

It can be shown that the velocity profile over the cross--section of the lithium flow will approach a rectangular profile when subjected to a strong external magnetic field. With magnetic fields of several tesla, as in our case and as expected inside the blanket region of a fusion reactor, we can assume the velocity profile of the lithium to be ideally rectangular. The error of the calculated pressure losses that would arise from this assumption together with the necessary neglect of the friction term div $\underline{\underline{\tau}}$ will be less than 1 % under the above-mentioned conditions if the magnetic field is homogenous along the duct. In the case of strong gradients of the magnetic field ($\frac{1}{B}\frac{dB}{dx} \approx 1$), the additional error caused by taking the mean value of the magnetic field along the duct length would be less than 30 %, as is shown in /1/.

As these errors will tend in the same direction in both lithium pipe configurations (coaxial and normal tube), they will not be so serious for the comparison of the losses of the two types of ducts. For the purposes of this paper these uncertainties will be tolerable. The errors given above are valid if we exclude the cases of isolating duct walls and Hartmann numbers below 1000.

For the following analysis we make the following assumptions:

a) stationary conditions

$\partial / \partial t = 0,$

b) isothermal conditions which lead to

$\sigma_W = \text{const},$

$\sigma_{Li} = \text{const},$

$\varrho_{Li} = \text{const},$

c) electrical charge neutrality

$\varrho_{el.} = 0$

d) Magnetic field homogeneous and directed perpendicular to the lithium flow direction

e) induced inner magnetic field is negligible relative to the external one, the Hartmann number R_m thus being much less than unity

f) lithium flow velocity having only one component in the direction of the tube axis and being constant across the length and cross-section (rectangular velocity profile which is valid for strong magnetic fields leading to H > 1000),

g) neglect of the friction term div$\underline{\underline{\tau}}$.

With these assumptions eqs. (1-5) reduce to the following:

$$\text{grad } p = \vec{j} \times \vec{B} \quad , \tag{6}$$

$$\vec{j} = \sigma (\vec{E} + \vec{v} \times \vec{B}) \quad , \tag{7}$$

$$\text{div} \vec{j} = 0 \quad , \tag{8}$$

$$\vec{E} = - \text{grad} \phi \quad . \tag{9}$$

2.2 Solution for the coaxial flow pipe

2.2.1 Derivation of the electric potential Ø

To find a solution for the potential Ø we use eqs. (7-9), from which we get

$$\mathrm{div}(-\mathrm{grad}\,\phi + \vec{v}\times\vec{B}) = 0 \;, \qquad (\mathrm{grad}\,\sigma = 0) \;. \tag{10}$$

Because of the spatial constance of $\vec{v}$ and $\vec{B}$ we have inside each of the four zones of the duct

$$\mathrm{div}(\vec{v}\times\vec{B}) = 0 \;. \tag{11}$$

We thus find Laplace's equation in cylindrical coordinates:

$$\Delta\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2} = 0 \;. \tag{12}$$

The influence of the $(\vec{v} \times \vec{B})$ term will enter via the boundary conditions.

Under the condition of constant $\vec{B}$ field and the further assumption that no external electric field ($E_x = 0$) exists, the potential Ø will be independent of the variable x because of symmetry. Equation (12) thus reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi(r,\varphi)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi(r,\varphi)}{\partial\varphi^2} = 0 \;. \tag{13}$$

If we set

$$\phi(r,\varphi) = R_s(r)\sin\varphi + R_c(r)\cos\varphi \tag{14}$$

we get two identical differential equations for $R_s(r)$ and $R_c(r)$:

$$\frac{d^2R_c(r)}{dr^2} + \frac{1}{r}\frac{dR_c(r)}{dr} - \frac{1}{r^2}R_c(r) = 0 \; ,$$

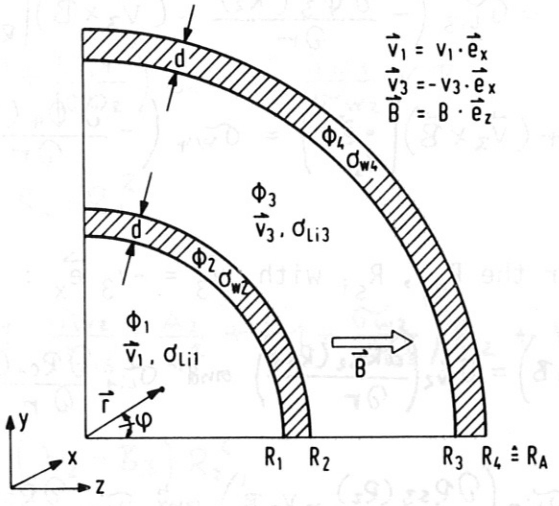
$$\frac{d^2R_s(r)}{dr^2} + \frac{1}{r}\frac{dR_s(r)}{dr} - \frac{1}{r^2}R_s(r) = 0 \; . \tag{15}$$

The general solution of this Euler-type differential equation is

$$R(r) = \frac{A}{r} + B\cdot r \; . \tag{16}$$

The eight coefficients, A_{s1}, A_{s2}, A_{s3}, A_{s4}, B_{s1}, B_{s2}, B_{s3}, B_{s4}, of the $R_s(r)$ and the eight A_{c1}, A_{c2}, A_{c3}, A_{c4}, B_{c1}, B_{c2}, B_{c3}, B_{c4} of the $R_c(r)$ are determined by the following boundary conditions (see Fig. 5):

Fig. 5



a) finite value of $\emptyset$ in the tube centre

$r = 0 : \quad |\emptyset\,(r = 0)| < \infty$

b) continuity of $\emptyset$ at the points:

$r = R_1,\ r = R_2,\ r = R_3$

for all φ and x :

$\emptyset_1\,(R_1) = \emptyset_2\,(R_1)$,

$\emptyset_2\,(R_2) = \emptyset_3\,(R_2)$,

$\emptyset_3\,(R_3) = \emptyset_4\,(R_3)$;

which leads to:

$R_{s1}\,(R_1) = R_{s2}\,(R_1)$ and $R_{c1}\,(R_1) = R_{c2}\,(R_1)$,

$R_{s2}\,(R_2) = R_{s3}\,(R_2)$ and $R_{c2}\,(R_2) = R_{c3}\,(R_2)$,

$R_{s3}\,(R_3) = R_{s4}\,(R_3)$ and $R_{c3}\,(R_3) = R_{c4}\,(R_3)$;

c) continuity of radial current density at the points $r = R_1$, $r = R_2$, $r = R_3$ for all φ and x:

$$\sigma_{Li1}\left(-\frac{\partial \phi_1(R_1)}{\partial r} + (\vec{V}_1 \times \vec{B})\Big|_{R_1} \circ \vec{e}_r\right) = \sigma_{W2}\left(-\frac{\partial \phi_2(R_1)}{\partial r}\right),$$

$$\sigma_{W2}\left(-\frac{\partial \phi_2(R_2)}{\partial r}\right) = \sigma_{Li3}\left(-\frac{\partial \phi_3(R_2)}{\partial r} + (\vec{V}_3 \times \vec{B})\Big|_{R_2} \circ \vec{e}_r\right),$$

$$\sigma_{Li3}\left(-\frac{\partial \phi_3(R_3)}{\partial r} + (\vec{V}_3 \times \vec{B})\Big|_{R_3} \circ \vec{e}_r\right) = \sigma_{W4}\left(-\frac{\partial \phi_4(R_3)}{\partial r}\right);$$

from this we get for the R_{ci}, R_{si} with $\vec{V}_3 = -V_3\,\vec{e}_x$:

$$\sigma_{Li1}\left(\frac{\partial R_{s1}(R_1)}{\partial r} + V_1 B\right) = \sigma_{W2}\left(\frac{\partial R_{s2}(R_1)}{\partial r}\right) \text{ and } \sigma_{Li1}\frac{\partial R_{c1}(R_1)}{\partial r} = \sigma_{W2}\frac{\partial R_{c2}(R_1)}{\partial r},$$

$$\sigma_{W2}\left(\frac{\partial R_{s2}(R_2)}{\partial r}\right) = \sigma_{Li3}\left(\frac{\partial R_{s3}(R_2)}{\partial r} - V_3 B\right) \text{ and } \sigma_{W2}\frac{\partial R_{c2}(R_2)}{\partial r} = \sigma_{Li3}\frac{\partial R_{c3}(R_2)}{\partial r},$$

$$\sigma_{Li3}\left(\frac{\partial R_{s3}(R_3)}{\partial r} - V_3 B\right) = \sigma_{W4}\left(\frac{\partial R_{s4}(R_3)}{\partial r}\right) \text{ and } \sigma_{Li3}\frac{\partial R_{c3}(R_3)}{\partial r} = \sigma_{W4}\frac{\partial R_{c4}(R_3)}{\partial r};$$

d) the radial current density must vanish at the tube edge $r = R_4$ for all φ and x, which leads to

$$\frac{\partial \Phi_4(R_4)}{\partial r} = 0 \quad ,$$

and so one gets for the R_{c4}, R_{s4}

$$\frac{\partial R_{s4}(R_4)}{\partial r} = 0 \quad , \quad \frac{\partial R_{c4}(R_4)}{\partial r} = 0 \quad .$$

These sixteen equations resulting from the boundary conditions a) - d) completely determine the sixteen coefficients A_{si}, B_{si}, A_{ci}, B_{ci}. It turns out that all A_{ci}, B_{ci} must vanish to satisfy the boundary conditions so that only the R_{si} terms(representing the sinφ-part) will remain in the potential function Ø.

The eight A_{si}, B_{si} must be found by solving the remaining eight coupled linear equations of a) - d). The following relations (from now on the A_{si}, B_{si} are replaced by the shorter notation A_i, B_i):

$$A_1 = 0, \tag{16}$$

$$B_2 = \frac{1}{2}\left(1 + \frac{\sigma_{Li1}}{\sigma_{W2}}\right) B_1 + \frac{1}{2}\frac{\sigma_{Li1}}{\sigma_{W2}} V_1 B \quad , \tag{17}$$

$$A_2 = (B_1 - B_2) R_1^2, \tag{18}$$

$$B_3 = \frac{1}{2}\left[\left(1 - \frac{\sigma_{W2}}{\sigma_{Li3}}\right)\frac{A_2}{R_2^2} + \left(1 + \frac{\sigma_{W2}}{\sigma_{Li3}}\right) B_2 + V_3 B\right], \tag{19}$$

$$A_3 = A_2 + (B_2 - B_3) R_2^2, \tag{20}$$

$$B_4 = \frac{1}{2}\left[\left(1-\frac{\sigma_{Li3}}{\sigma_{W4}}\right)\frac{A_3}{R_3^2} + \left(1+\frac{\sigma_{Li3}}{\sigma_{W4}}\right)B_3 - \frac{\sigma_{Li3}}{\sigma_{W4}}V_3 B\right], \tag{21}$$

$$A_4 = A_3 + \left(B_3 - B_4\right)R_3^2, \tag{22}$$

and

$$A_4 = B_4\, R_4^2. \tag{23}$$

All these coefficients can be calculated successively if B_1 is known. It can be expressed by

$$B_1 = \frac{B_{z1}}{B_{n1}} \tag{24}$$

with

$$\begin{aligned} B_{z1} = {} & \frac{1}{2}\frac{\sigma_{Li1}}{\sigma_{W2}}V_1 B R_2^2 + \frac{\frac{1}{2}\left(R_3^2+R_4^2\right)\frac{\sigma_{Li3}}{\sigma_{W4}}V_3 B}{1-\frac{1}{2}\frac{R_3^2+R_4^2}{R_3^2}\left(1-\frac{\sigma_{Li3}}{\sigma_{W4}}\right)} - \\ & -\frac{1}{2}\left[\left(1+\frac{\sigma_{W2}}{\sigma_{Li3}}\right)\left(\frac{1}{2}\frac{\sigma_{Li1}}{\sigma_{W2}}V_1 B\right)+V_3 B\right]\cdot\left[R_2^2 + \frac{\frac{1}{2}\left(R_3^2+R_4^2\right)\left(1+\frac{\sigma_{Li3}}{\sigma_{W4}}\right)-R_3^2}{1-\frac{1}{2}\frac{R_3^2+R_4^2}{R_3^2}\left(1-\frac{\sigma_{Li3}}{\sigma_{W4}}\right)}\right] + \\ & +\left(\frac{1}{2}\frac{\sigma_{Li1}}{\sigma_{W2}}V_1 B R_1^2\right)\cdot\left[\frac{1}{2R_2^2}\left(1-\frac{\sigma_{W2}}{\sigma_{Li3}}\right)\cdot\left(R_2^2 + \frac{\frac{1}{2}\left(R_3^2+R_4^2\right)\left(1+\frac{\sigma_{Li3}}{\sigma_{W4}}\right)-R_3^2}{1-\frac{1}{2}\frac{R_3^2+R_4^2}{R_3^2}\left(1-\frac{\sigma_{Li3}}{\sigma_{W4}}\right)}\right)-1\right] \end{aligned} \tag{25}$$

and

$$\begin{aligned} B_{n1} = {} & \left[1-\frac{1}{2}\left(1+\frac{\sigma_{Li1}}{\sigma_{W2}}\right)\right]R_1^2\cdot\left[\frac{1}{2R_2^2}\left(1-\frac{\sigma_{W2}}{\sigma_{Li3}}\right)\left(R_2^2 + \frac{\frac{1}{2}\left(R_3^2+R_4^2\right)\left(1+\frac{\sigma_{Li3}}{\sigma_{W4}}\right)-R_3^2}{1-\frac{1}{2}\frac{R_3^2+R_4^2}{R_3^2}\left(1-\frac{\sigma_{Li3}}{\sigma_{W4}}\right)}\right)-1\right] + \\ & +\left[\frac{1}{2}\left(1+\frac{\sigma_{W2}}{\sigma_{Li3}}\right)\frac{1}{2}\left(1+\frac{\sigma_{Li1}}{\sigma_{W2}}\right)\right]\cdot\left[R_2^2 + \frac{\frac{1}{2}\left(R_3^2+R_4^2\right)\left(1+\frac{\sigma_{Li3}}{\sigma_{W4}}\right)-R_3^2}{1-\frac{1}{2}\frac{R_3^2+R_4^2}{R_3^2}\left(1-\frac{\sigma_{Li3}}{\sigma_{W4}}\right)}\right] - \\ & -\left[\frac{1}{2}\left(1+\frac{\sigma_{Li1}}{\sigma_{W2}}\right)R_2^2\right]. \end{aligned} \tag{26}$$

The $R_{si}(r)$ are thus determined for the four zones i = 1 to 4:

$$R_{s1}(r) = B_1 r \quad , \tag{27}$$

$$R_{s2}(r) = B_2 r + A_2/r \ , \tag{28}$$

$$R_{s3}(r) = B_3 r + A_3/r \ , \tag{29}$$

$$R_{s4}(r) = B_4 r + A_4/r \ . \tag{30}$$

2.2.2 Pressure losses and optimal ratio of duct radii

The power $\mathcal{P}_{coax}$ lost inside the coaxial tube because of the $\vec{j} \times \vec{B}$ forces of the induced electric currents, which must be compensated by the lithium pumps, can be expressed by /1/

$$\mathcal{P}_{coax} = + \sigma_{Li1} \pi L \int_0^{R_1} \left(\frac{\partial R_{s1}(r)}{\partial r} + \frac{R_{s1}(r)}{r} + 2 v_1 B \right) v_1 B \, r dr + \\ + \sigma_{Li3} \pi L \int_{R_2}^{R_3} \left(\frac{\partial R_{s3}(r)}{\partial r} + \frac{R_{s3}(r)}{r} - 2 v_3 B \right) \left(- v_3 B \right) r dr \ . \tag{31}$$

Integration leads to

$$\mathcal{P}_{coax} = \pi L \left[\sigma_{Li1} (B_1 + v_1 B) v_1 B R_1^2 + \sigma_{Li3} (-B_3 + v_3 B) v_3 B (R_3^2 - R_2^2) \right] \ . \tag{32}$$

The pressure difference ΔP_{coax} which must be established by the lithium pumps to force a given mass input per time $\mathring{m}$ of lithium through the pipe is given by /1/

$$\Delta P_{coax} = \frac{\rho_{Li}}{\dot{m}} P_{coax} \quad , \tag{33}$$

where ρ_{Li} is the mass density of lithium. The fact that $\dot{m}$ in eq. (33) appears in the denominator does not mean a decrease of ΔP_{coax} with increasing $\dot{m}$ because in eq. (32) $\dot{m}$ is a quadratic factor hidden in that equation so that ΔP_{coax} is, at least, about linearly proportional to $\dot{m}$.

We are now able to find the optimal values of R_1 to R_4 which give minimal pressure losses for a given outer radius $R_A = R_4$, wall thickness d and lithium mass flow rate $\dot{m}$.

To do this, a parameter α, which characterizes the relative positions of all radii (see Fig. 5) is defined:

$$R_1 = \alpha R_A \quad , \tag{34}$$

$$R_2 = R_1 + d \quad , \tag{35}$$

$$R_3 = R_A - d \quad , \tag{36}$$

$$R_4 = R_A \quad . \tag{37}$$

The equation of continuity (1) gives for our case of the same lithium mass flow in the inner $0 < r < R_1$ and outer sections $R_2 < r < R_3$ if ρ_{Li} = const

$$v_1 R_1^2 = v_3 (R_3^2 - R_2^2) \quad . \tag{38}$$

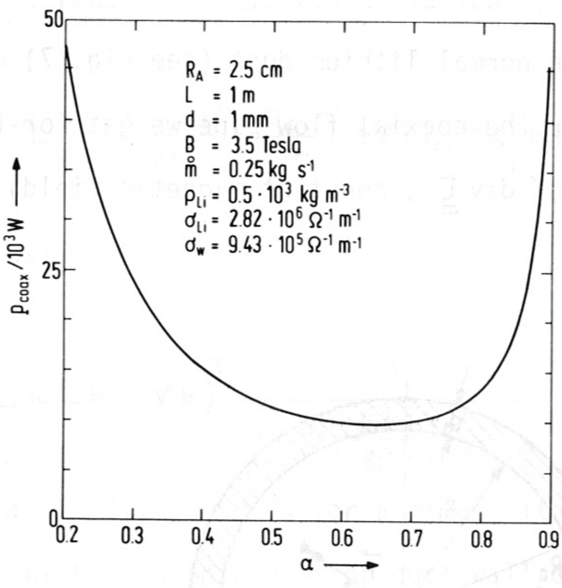
This yields the two velocities v_1 and v_3 as functions of α, R_A, d and $\mathring{m}$:

$$v_1 = \frac{\mathring{m}}{\rho_{Li}\,\pi} \cdot \frac{1}{\alpha^2 R_A^2} \quad , \tag{39}$$

$$v_3 = \frac{\mathring{m}}{\rho_{Li}\,\pi} \cdot \frac{1}{R_A^2(1-\alpha^2) - 2R_A d(1+\alpha)} \quad . \tag{40}$$

The dependence of the power P_{coax} on α required to maintain stationary lithium flow for given lithium mass input per time is shown in Fig. 6.

Fig. 6



To determine the minimum of the losses, the parameter α_{min} has to be adjusted according to

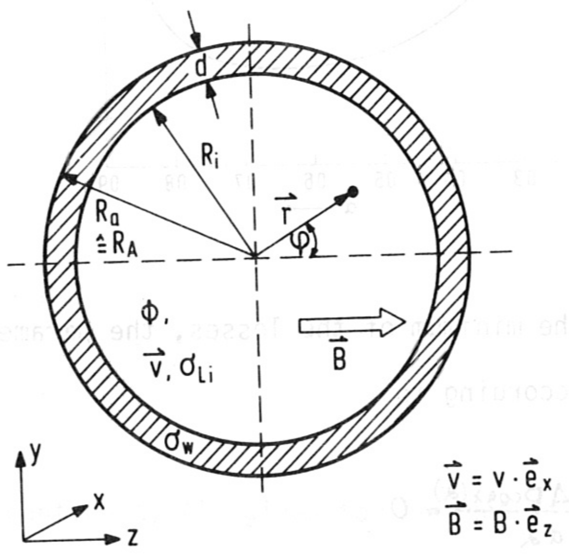
$$\frac{d P_{coax}(\alpha)}{d\alpha} = \frac{d\,\Delta p_{coax}(\alpha)}{d\alpha} = 0 \quad . \tag{41}$$

In the following investigations the optimal value α_{min} is calculated numerically. It turns out that the optimum is reached when the magnitudes of the two velocities are about equal. This holds if the thickness of the wall is small compared with the cross-sectional dimensions of the duct and if the ratio of the electrical conductivity of the duct wall and lithium is about unity.

2.3 Solution for normal flow pipe

In the case of a normal lithium duct (see Fig. 7) under the same conditions as for the coaxial flow pipe we get for the electric potential $\emptyset$ (neglect of div $\underline{\underline{\tau}}$, constant magnetic field) using the results of /1/.

Fig. 7



$$\phi = R_s(r)\sin\varphi \tag{42}$$

with

$$R_s = b_1 r \tag{43}$$

and

$$b_1 = vB\left(\frac{\frac{\sigma_w}{\sigma_{Li}}\left(1 - \frac{R_a^2}{R_i^2}\right)}{\frac{\sigma_w}{\sigma_{Li}}\left(1-\frac{R_a^2}{R_i^2}\right)-\left(1+\frac{R_a^2}{R_i^2}\right)} - 1\right) \tag{44}$$

The power P_{norm}, required to maintain stationary flow follows from

$$P_{norm} = \sigma_{Li}\pi L \int_0^{R_i}\left[\left(\frac{\partial R_s(r)}{\partial r} + \frac{R_s(r)}{r} + 2vB\right)vB\right] r\,dr \tag{45}$$

Integration gives

$$P_{norm} = \sigma_{Li}\pi L R_i^2 (vB)^2 \frac{1}{1-\frac{\sigma_{Li}}{\sigma_w}\frac{R_i^2+R_a^2}{R_i^2-R_a^2}} \tag{46}$$

The velocity of the lithium flow for a given lithium mass flow $\mathring{m}$, duct outer radius R_A and thickness of duct wall d is

$$v = \frac{\mathring{m}}{\rho_{Li}\pi}\frac{1}{(R_A-d)^2} \tag{47}$$

For a meaningful comparison of the MHD losses of this normal flow pipe with the coaxial one, the losses of the normal duct must be multiplied by a factor of 2 because the flow to the blanket segment and back again must be taken into account.

Finally, the pressure difference Δp_{norm} to be covered by the lithium pumps is

$$\Delta p_{norm} = \frac{\varrho_{Li}}{\dot{m}} P_{norm} = \frac{2}{\pi} \sigma_{Li} \frac{\dot{m}}{\varrho_{Li}} B^2 L \left(\frac{1}{(R_A - d)^2} \cdot \frac{1}{1 - \frac{\sigma_{Li}}{\sigma_W} \frac{(R_A-d)^2 + R_A^2}{(R_A-d)^2 - R_A^2}} \right) . \quad (48)$$

3. RESULTS

By using the eqs. (16) - (30) for the electric potential it is possible to calculate electric current flow and potential line patterns in the cross-section of a coaxial duct. Figure 8 shows the results under the following conditions:

$B = 3.5$ Tesla	$R_1 = 4.8$ cm	left: electric current
$V_1 = 7$ cm/s	$R_2 = 4.9$ cm	right: electric potential
$V_3 = -7$ cm/s	$R_3 = 6.9$ cm	
$\sigma_W = 9.43 \cdot 10^5 \Omega^{-1} m^{-1}$	$R_4 = 7.0$ cm	
$\sigma_{Li} = 2.82 \cdot 10^6 \Omega^{-1} m^{-1}$		

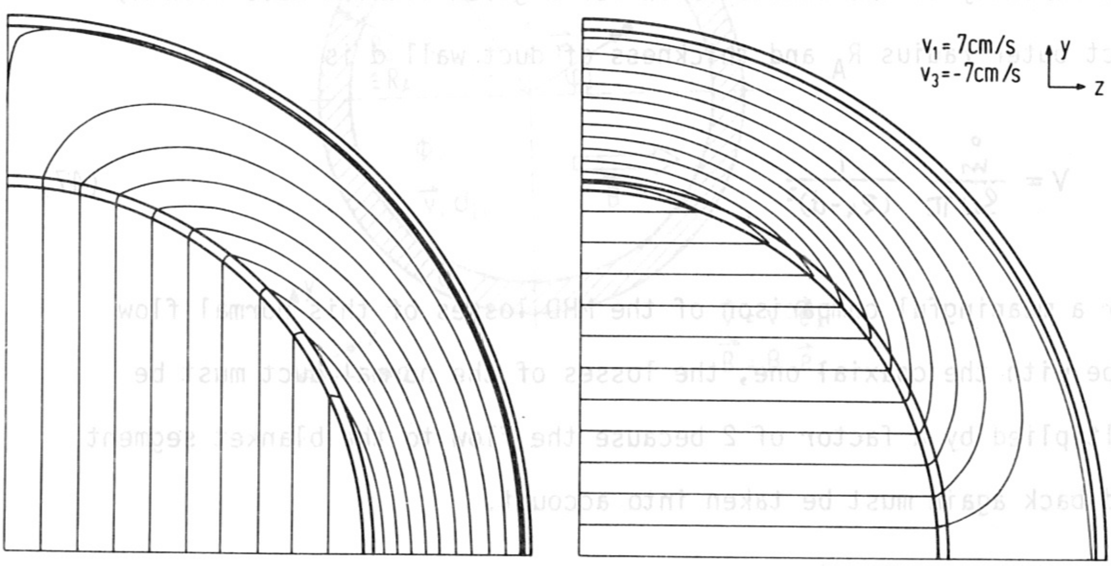


Fig. 8

The perpendicular distances of lines of constant electric potential are inversely proportional to the strength of the electric field, and the density of current lines reflect the local current density. Furthermore, Fig. 9 refers to the state of a duct identical to that above except for the fact that in this case there is a lithium flow only inside the inner part of the duct while in the outer part the lithium is at rest (v_1 = 7 cm/s, v_3 = 0 cm/s).

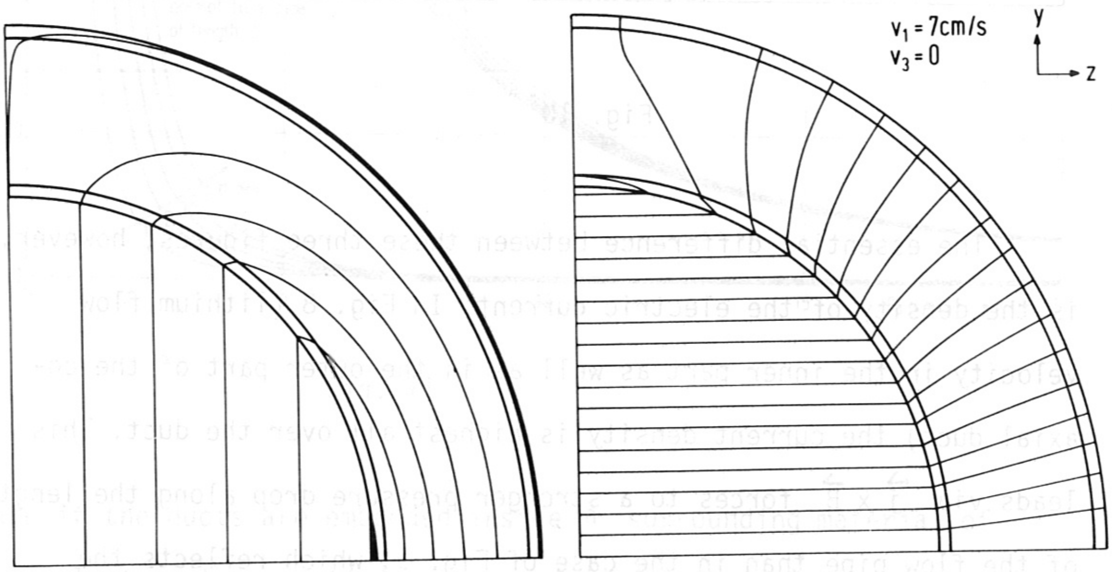


Fig. 9

By contrast, Fig. 10 shows the opposite case: now there is no flow inside the inner duct (v_1 = 0 cm/s), but in the outer part lithium flows with a velocity of v_3 = -7 cm/s.

In all these three cases the patterns of electric current are very similar. This shows that it is essentially the electric potential which will be influenced by the different $\vec{v} \times \vec{B}$ terms in such a way that the electric current may follow the paths of minimal net resistance.

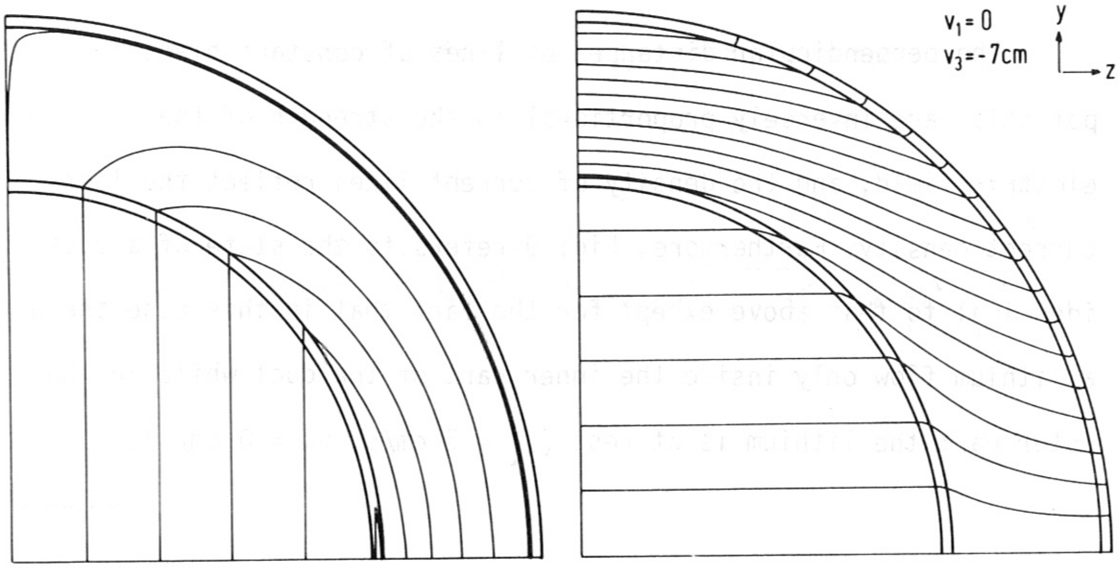


Fig. 10

The essential difference between these three figures, however, is the density of the electric current. In Fig. 8 (lithium flow velocity in the inner part as well as in the outer part of the coaxial duct) the current density is highest all over the duct. This leads via $\vec{j} \times \vec{B}$ forces to a stronger pressure drop along the length of the flow pipe than in the case of Fig. 9, which reflects the properties of a normal single duct with thick walls.

This behaviour is confirmed by Fig. 11, which shows the pressure losses of the two types of lithium flow pipes, depending on the outer radius and wall thickness of the duct for a given lithium mass flow $\mathring{m}$. The losses of the coaxial duct are those for the optimal ratio of the duct radii according to eq. (41).

It turns out that the pressure losses of coaxial flow pipes are much higher than those of normal single ducts. This obvious advantage of normal single flow pipes compared with coaxial ones will

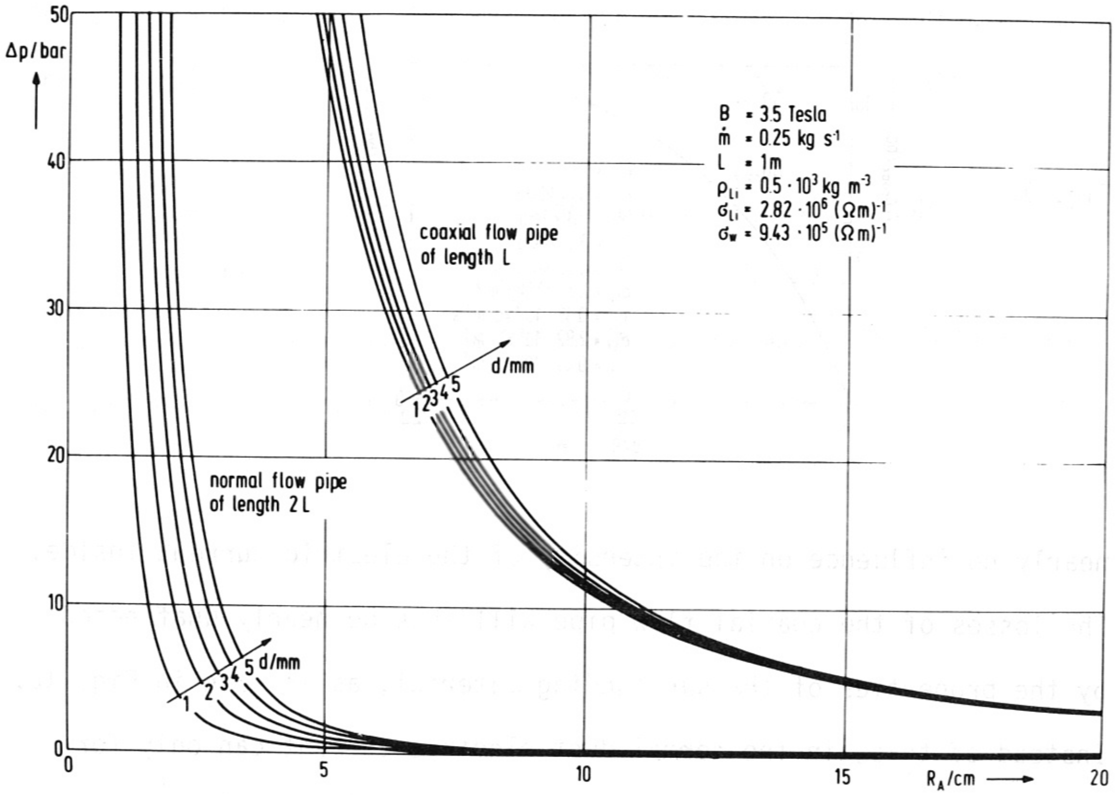


Fig. 11

fade if the ducts are embedded inside a surrounding material of good electrical conductivity.

It is obvious that each material of high electrical conductivity surrounding the lithium flow pipes is equivalent to an increase of the effective thickness of the outer duct wall. Figure 12 shows the influence of the ratio of the thickness d of the duct outer wall to the duct radius on MHD pressure losses for the two types of flow pipes. In the coaxial configuration nearly all electric currents will form closed loops in the inner region $0 < r < R_3$ (see Fig. 8) so that the thickness of the outer duct wall will have

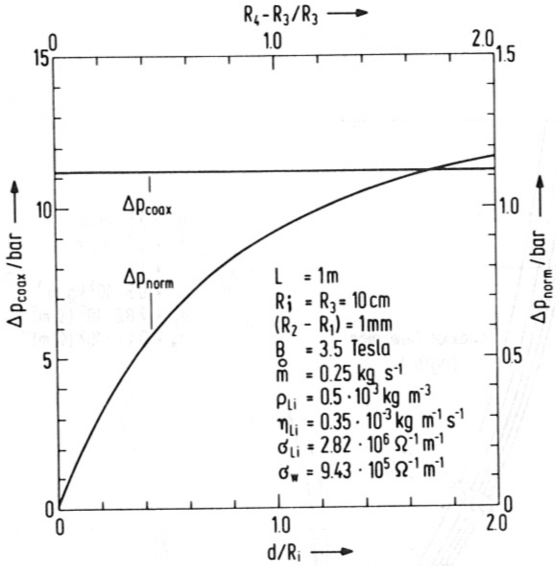


Fig. 12

nearly no influence on the intensity of the electric current inside. The losses of the coaxial flow pipe will thus be nearly unaffected by the properties of the surrounding material, as is seen in Fig. 12. Instead of this, in the normal duct electric current can only form closed loops by following the paths inside the duct walls (see Fig. 9). As a consequence pressure losses of normal flow pipes are strongly influenced by the geometry and electrical conductivity of surrounding materials.

4. CONCLUSIONS

It is thus concluded that the coaxial flow pipe will have much higher MHD pressure losses than the normal duct under similar conditions. The great difference in behaviour between the two duct types will get smaller if the ducts are embedded in surrounding leading structures of good electrical conductivity. In this case the normal flow pipe will suffer a drastic increase in the pressure drop, while the coaxial one will be nearly unaffected. But even under these conditions the losses of the latter one will dominate.

Consequently, the optimal configuration would be obtained by using thick flow pipes of normal geometry with nearly insulating walls that are as parallel as possible to the magnetic field lines and are electrically insulated against surrounding materials.

5. TABLE OF SYMBOLS AND DEFINITIONS

Vector quantities

$\vec{B}$ magnetic flux density
$\vec{H}$ magnetic field
$\vec{E}$ electric field
$\vec{j}$ current density
$\vec{v}$ fluid velocity

Scalar quantities

R_i, R_A, R_1, R_2	radii of circular duct
L	length of duct
d	thickness of duct wall
$\mathring{m}$	mass flow per time of liquid lithium
ϱ_{Li}	mass density of lithium
η_{Li}	viscosity of lithium
σ_{Li}	electrical conductivity of lithium
σ_w	electrical conductivity of duct wall
μ_{Li}	magnetic permeability of lithium
$\emptyset$	electric potential

Non-dimensional quantities

H Hartmann number
(electromagnetic stress/viscous stress)$^{1/2}$ $= R_i\, B \sqrt{\frac{\sigma_{Li}}{\eta_{Li}}}$

R_m Magnetic Reynolds Number
(induced magnetic field/imposed magnetic field) $= R_i\, v\, \sigma_{Li}\, \mu_{Li}$

C Conductance ratio
(conductance of duct wall/conductance of fluid) $= \frac{\sigma_w\, d}{\sigma_{Li}\, R_i}$

MKSA units are used throughout the paper.

6. REFERENCES

/1/ G. Trommer: "Theoretische Untersuchungen zu den MHD-Verlusten in einem Lithium-gekühlten Fusionsreaktormantel"
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