Current Driven Drift Instability
in the W VII-A Stellarator

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Abstract

The instability region and growth rates of current driven drift
modes in the W VII-A stellarator are calculated.

Several theoretical results (ref. 4-7) are evaluated for specific
temperature and density profiles. It is found that in the outer
region of the plasma column (r > 6 cm) collisional drift waves
with wavelengths $k_\perp^{-1}$ = 0.13 - 0.3 cm exist. In this region also
the electron thermal conductivity determined experimentally
appears to be large.

Introduction

Recent experimental data concerning anomalous transport
phenomena in Tokamaks and Stellarators showed [1-4] that
the anomalous heat transport could be related to current
driven drift waves. The measurements in W VII-A concerning
the confinement time indicate also a dependence on the
drift parameter $\xi \sim I/n T_e^{1/2}$. This dependence can be related
to the increase of the thermal conductivity with the current
intensity, which could be explained with the existence of
the current driven drift waves. In this paper, we used the
most important results of the existing theory [4-7] to
estimate the possibilities of appearance of current driven
drift waves in the plasma with a density and temperature
profile which is typical for the W VII-A stellarator. The
temperature and density profiles used for numerical calcu-
lations are represented in Fig. 1. Although the majority of
the theoretical results consider the slab model of an inhomo-
geneous plasma, they give information on the probability
of the appearance of the instabilities.

1) Influence of the Ion Viscosity

The calculations made by Ellis and Motley [4] for the case
that only the density gradient and the driving current are
considered yield a first estimate of the unstable region for
a collisional plasma. It was shown that a critical current
exists which can be expressed for the parameter $\xi = -\dfrac{j}{nev_a} = \dfrac{u}{v_e}$
in the following form:

$$\xi_{cr} \approx \xi_1 - \xi_2 \tag{1}$$

where

$$\xi_1 = A_1 \frac{k_\perp^4 k_\parallel}{k_y} \quad , \quad \xi_2 = A_2 \frac{k_\perp^2 k_y}{k_\parallel}$$

$$A_1 = \frac{3}{10} \left(\frac{m_e}{2 m_i} \right)^{1/2} \frac{\tau_n v_e \rho_i^2}{\Omega} \quad \text{and} \quad A_2 = \frac{3 \Omega \rho_i^4}{\tau_n v_e}$$

Here we used the following notations: j is the longitudinal
current density in the torus, n = electron (or ion) density,
e = electric charge of the proton, u = longitudinal mean
velocity of the electrons, m_e = mass of the electrons,
m_i = mass of the ions, Ω = ion cyclotron frequency,
ϱ_i = ion Larmor radius, $r_n^{-1} = \frac{1}{n}\frac{dn}{dx}$ and the components of
the wave vector are k_x, k_y and $k_{\parallel}$. We used also $k_{\perp} = \sqrt{k_x^2 + k_y^2}$.
The x-axis is oriented in the direction of -grad n.

The instability appears if

$$\xi > \xi_{cr} \tag{2}$$

A critical current exists if

$$\frac{\xi_1}{\xi_2} = C\,\frac{k_{\perp}^2\,k_{\parallel}^2}{k_y^2} > 1$$

with

$$C = \frac{A_1}{A_2} = \frac{1}{10}\left(\frac{m_e}{2\,m_i}\right)^{1/2}\frac{r_n^2\,v_e^2}{\varrho_i^2\,\Omega^2}$$

The numerical values of C for the temperature and density
profiles represented in Fig. 1 are given in Table 1.

Table 1

r cm	O	1,2	2,4	3,6	4,8	6	7,2	8,4	9,6	10,8	12
C cm^2	-	$9\cdot10^6$	4900	800	230	80	35	19	13	10	14

For perturbations with

$$\frac{k_y^2}{k_{\perp}^2\,k_{\parallel}^2} > C \tag{3}$$

the critical current is zero. Every current of an arbitrary in-
tensity leads to the appearance of instabilities.

If condition (3) is not satisfied, the maximal value of ξ_{cr}
will be ξ_1. This is always less than the experimentally measured
ξ values if

W VII A

Temperature
and
density profiles

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H₂

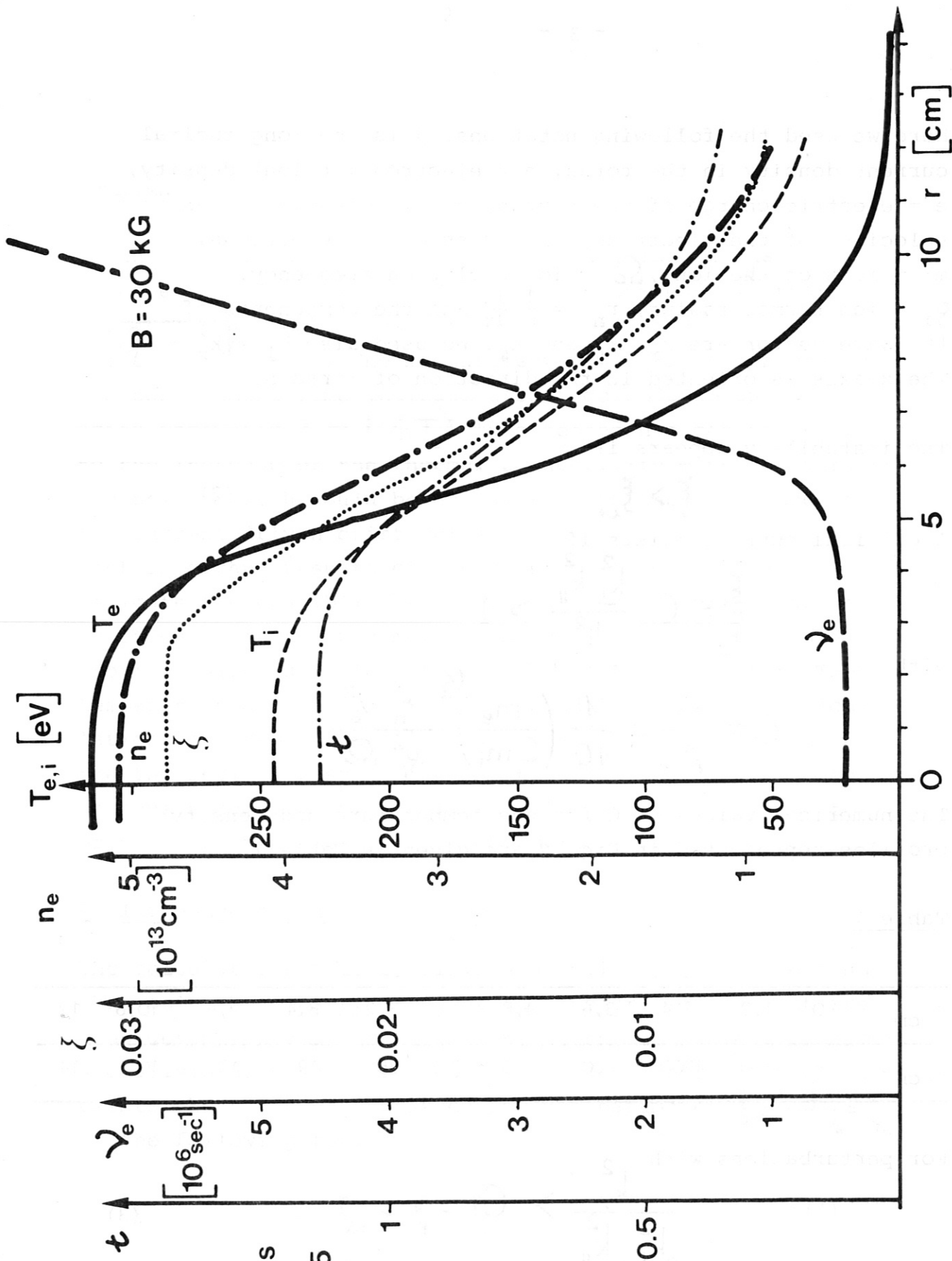$T_{e,i}$ [eV]

n_e [10^{13} cm^{-3}]

ξ

ν_e [10^6 sec^{-1}]

ι

B = 30 kG

Fig. 1

$$\frac{k_y}{k_\perp^4 \, k_\parallel} > \frac{3}{10} \left(\frac{m_e}{2 \, m_i}\right)^{1/2} \frac{\tau_n \, v_e \, \varrho_i^2}{\Omega \, \xi} \qquad (4)$$

It therefore results from the calculations of Ellis and Motley that the current driven drift instabilities cannot be stabilized by the ion viscosity in a collisional plasma if condition (3) or (4) is satisfied.

2) Influence of Toroidal Geometry

An estimate of the influence of the <u>toroidality</u> of the plasma on <u>current driven drift waves</u> in the presence of <u>density gradients</u> in a <u>collisional plasma</u> was made by Horton and Varma [4]. If $T_i/T_e > \frac{1}{2}$, a small stabilization is possible. The corresponding shift of the critical ξ value due to the toroidality is

$$\Delta \xi_{cr, tot.} \approx \frac{\omega_n^* \, \tau_n^2}{t \, \tau R \, v_e \, k_y} \left(1 + \frac{T_e}{T_i}\right) \left[3 \frac{T_i}{T_e} + 1 + \frac{\tau}{\tau_n}\left(\frac{T_i}{T_e} - \frac{1}{2}\right)\right] \qquad (5)$$

Here we introduced the characteristic parameters of a toroidal plasma. $t = \frac{R \, B_\vartheta}{\tau \, B_\phi}$ with B_ϑ and B_ϕ as toroidal components of the magnetic field. $\omega_n^* = -\frac{c \, k_y \, T}{e \, B \, n} \frac{dn}{dx}$ is the drift frequency.

Comparing the calculated $\Delta \xi_{cr, tot}$ with the measured ξ values for the W VII-A stellarators, the values given in Table 2 are found.

Table 2

r_{cm}	0	1,2	2,4	3,6	4,8	6	7,2	8,4	9,6	10,8	12
$\Delta\xi_{cr,tot}/\xi$	-	27	0,01	0.003	0.002	0,003	0.004	0.007	0.01	0.02	0.03

As it can be seen, the shift of ξ due to the toroidality is negligible in our case.

3) Influence of Simultaneous Temperature and Density Gradients

In the case of an inhomogeneous collisional plasma with density and temperature gradient the growth rate of current driven drift waves depends on the parameter

$$\eta = \frac{\frac{\partial}{\partial r} \ln T}{\frac{\partial}{\partial r} \ln n} = \frac{r_n}{r_T}$$

Horton and Varma showed that if temperature and density gradients are oriented in the same direction, a stabilizing effect appears. The current driven drift instability can develop only if

$$\xi > \xi_{cr} = \frac{3}{2} \frac{\eta_e v_d}{v_e} \frac{k_y}{k_{\parallel}} \qquad (v_d = \text{drift velocity of electrons})$$

Therefore condition

$$\frac{k_y}{k_{\parallel}} < \frac{2 \xi v_e}{3 \eta v_d} \qquad (6)$$

limits the possible dimensions of the unstable perturbations.

4. Influence of Shear

In the limit of the slab model shear has a stabilizing effect. Changing the direction of the magnetic field, the $k_{\perp}$ and $k_{\parallel}$ components are changed keeping k = const. This can lead to a transition of the perturbations in the stable region. For the largest scale perturbations admitted by the finite dimensions of the experimental system an estimate of shear stabilization was made by Mikhailovskii [5]. The following condition results (vol. 2, p. 166):

$$\theta \gtrsim \frac{\left(\xi v_e v_e^2 k_{\perp} \right)^{1/3}}{\Omega} \qquad (7)$$

(Here we used the following notations: $\theta = \frac{r\, r_n}{R} \left| \frac{d\ell}{dr} \right|$ is the shear and v_e the electron-ion collision frequency). This condition limites the maximal value of $k_{\perp}^{-1}$, which can lead to the appearance of the current convective instability. This value is represented in Fig. 2 by curve a). In the same figure, we

have also represented the minimal values of $k_\perp^{-1}$ which result
from eqs. (3), (4), and (6). These values correspond to curve b).
As an illustration, we represented also the Larmor radius of
the ions as a function of r (curve c)). We can see that curve b)
lies in the region $k_\perp \varrho_i < 1$ of applicability of the fluid
theory. Curves a) and b) circumscribe the region in the $(r, \frac{1}{k_\perp})$
plane in which the perturbations lead to instability. In this
figure, the stability region is also indicated. In the same
figure we have plotted also curve d) for which the growth rate
was calculated analytically by Kadomtsev and Pogutse [6]
(v. 5, p. 295). In this case, the stability region is given by

$$x_\perp \approx \left[\sqrt{\frac{2 m_e T_i}{m_i T_e}} \; \frac{\xi \, \tau_n}{\varrho_i \, \tilde{\theta}} \right]^{1/2} \frac{\tau_n \nu_e}{\nu_e \, \tilde{\theta}} \quad \text{with} \quad \tilde{\theta} = \frac{2 \pi \tau \theta}{\tau_n} \tag{8}$$

The growth rate is

$$\gamma \sim \left[\frac{\xi \tilde{\theta}}{\tau_n} \sqrt{\frac{m_i \varrho_i^2 T_e}{2 \, m_e T_i}} \right]^{1/2} \frac{\nu_i}{\tau_n} \tag{9}$$

and the real part of the frequency is

$$\omega \sim \left[\frac{\nu_e^2 \, \varrho_i^3 \, \tilde{\theta}^3}{\nu_e \, \xi \, \tau_n^5} \left(\frac{m_i}{m_e} \right)^{1/2} \left(\frac{T_e}{2 T_i} \right)^{3/2} \right]^{1/4} \frac{\nu_i}{\tau_n} \tag{10}$$

In Fig. 3 we can see that the appearance of collisional current
driven drift instability with $k_\perp^{-1} \gtrsim 0{,}4$ cm is to be expected
for $r > 8$ cm. This is exactly the region of the plasma column
in which the mean free path is less than the major radius of
the torus and a very high anomalous thermal conductivity appears.
The data shown in Fig. 2 are in agreement with the assumption
that the thermal conductivity and the anomalous diffusion are
produced by collisional current driven drift instabilities.

5) An Estimate od the Instability Region for Collisionless Current Driven Drift Waves

In the case of collisionless current driven drift waves a
limitation of k_y (the component of the wave vector in the
direction of the p-drift) appears. There are three factors
which limit the k_y^{-1} value:

Stability and instability regions in collisional regime

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STABILITY

INSTABILITY

a

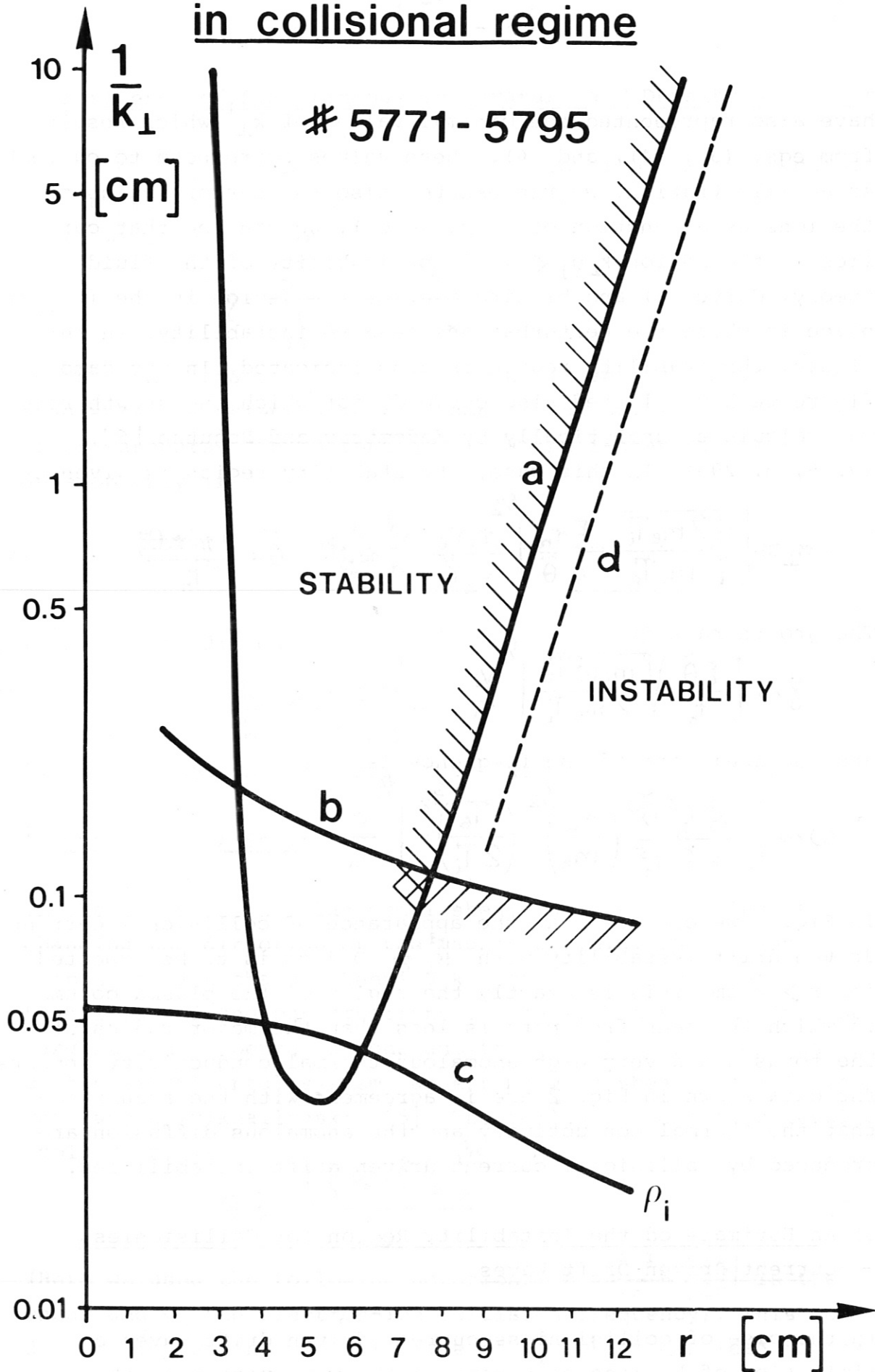d

b

c

ρ_i

$\frac{1}{k_\perp}$ [cm]

r [cm]

Fig. 2

a) The shear of the magnetic field lines. The current driven
   kinetic drift instabilities are stabilized by shear if
   ([5] vol. 2, p. 165)

$$\theta \gtrsim \left(\frac{m_e}{m_i}\right)^{1/2} \left(\xi \frac{v_e}{v_i}\right)^2 \frac{1}{\rho_i k_y} \tag{11}$$

b) The finite value of the major radius. Using the assumption
   that $k_{\parallel}^{-1}$ must be less than R, we get the condition
   ([5] vol. 2, p. 165)

$$k_y^{-1} < \frac{R \xi \rho_i v_e}{\tau_n v_i} \tag{12}$$

c) The existence of the ion Larmor radius as a limit for the
   applicability of the two fluid model.

Relations (11), (12), and the dependence of the Larmor radius
from r allows to determine the stability and the instability
regions in the $(r, \frac{1}{k_y})$ plane. They are represented in Fig. **3**.
Curve a) corresponds to the inequality (11), curve b) to (12),
and c) gives the values of the ion Larmor radius. These three
curves determine an instability region for the collisionfree
drift waves in the plasma column. This region lies near $r < 8$ cm.
It has practical importance only in the vicinity of $r \sim 6 \div 8$ cm.
For $r \sim 6 \div 8$ cm, we have a contact between the instability
regions of collisional and collisionless instabilities. So we
have an instability band around $k_{\perp}^{-1} \sim 0.13 - 0.3$ cm in both
collisional and collisionless regimes. Therefore, current
driven drift instability would be expected to appear in the
region $r > 6$ cm.

Conclusions

Our estimates show that the collisional current driven drift
waves are important in the region (r = 6 ÷ 8 cm) of high
thermal conductivity. It is possible to extend our cal-
culations to a variety of experimental situations.

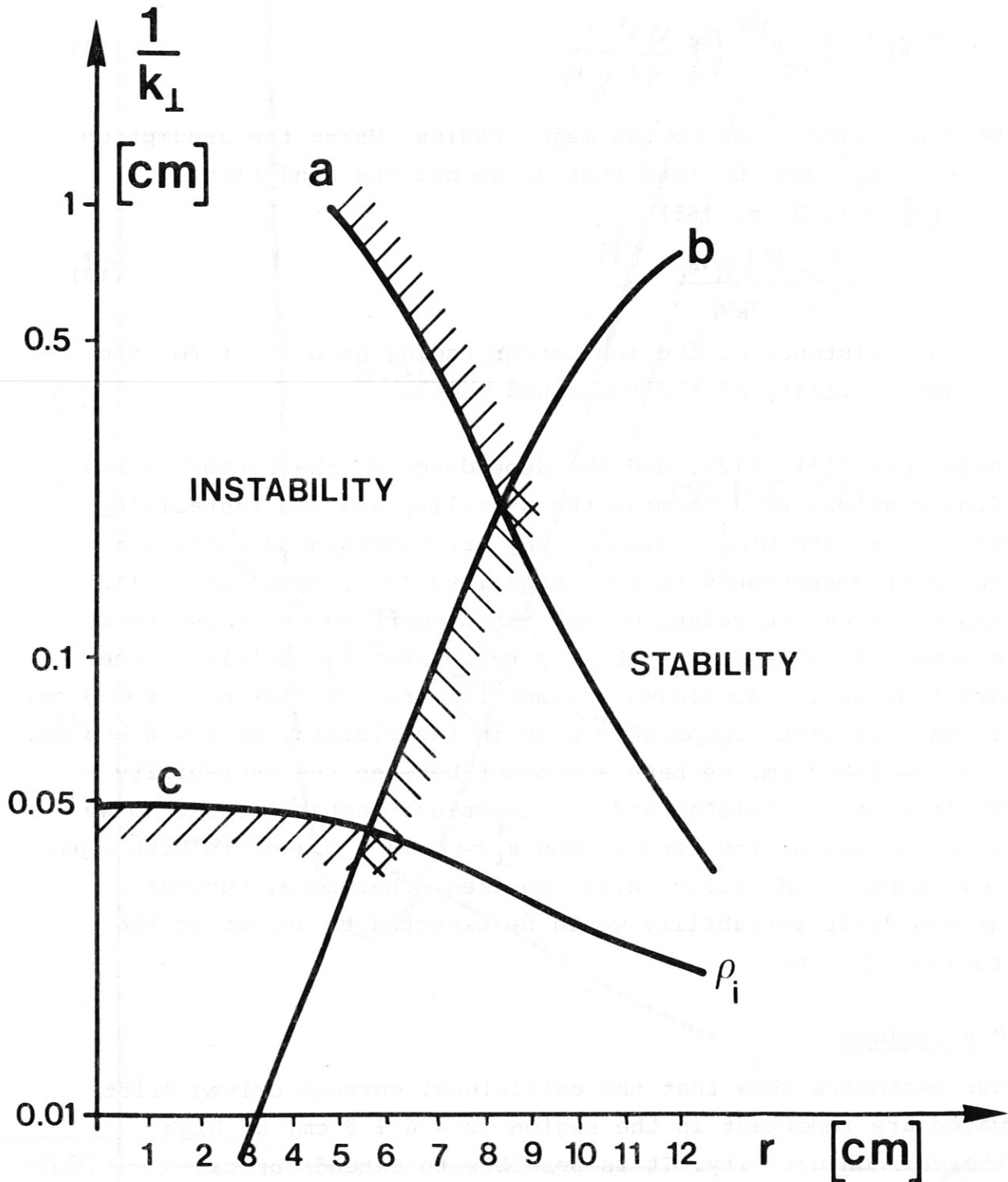# Stability and instability regions in collisionless regime

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Fig. 3

If we compare various situations with different discharges, different temperatures and densities, holding B, ξ, θ, and r_n constant, the following conclusions can be made:

For the collisional drift waves we get the proportionalities

$$\left(\frac{1}{k_\perp}\right)_{max} \sim Z\, n^2\, m_i^3\, T^{-5/2} \tag{13}$$

$$\left(\frac{1}{k_\perp}\right)_{min} \sim Z^{-3/4}\, m_i^{3/8}\, T^{1/2} \tag{14}$$

The relations show that

- if we increase the temperature, the instability region reduces,
- if we increase the density, the instability region increases,
- if we replace the hydrogen with helium, the instability region increases as an influence of the increase of the electric charge of the ions (Z).

If we make the same estimates for the collisionless drift waves, we see that the change of T and Z leads only to a shift of the instability region and only the influence of the ion mass leads to a reduction of the dimensions of the instability region.

The shear of the magnetic field has an essential stabilizing influence. A reduction of the dimensions of the instability region for collisional drift waves with the increase of the shear results from (7).For the dependence of the upper limit we obtain $(k_\perp^{-1})_{max} \sim \theta^{-3}$.

The change of the magnetic field intensity shifts the upper limit $(k_\perp^{-1})_{max}$ proportional to B^{-2} and the lower limit $(k_\perp^{-1})_{min}$ proportional to B^{-1}. Therefore, the increase of the magnetic field leads to a reduction of the dimensions of the instability region.

Here, a remark has to be made. Our estimates were made using results which neglect the variation of the plasma parameters along the magnetic field lines in the toroidal system. This effect could lead to an additional increase of the instability regions.

Acknowledgements

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