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The possibility of heating the plasma of the W VII A experiment by coupling to the plasma a fast Alfred wave at $\omega = 2\Omega_{oi}$ is investigated. The power adsorbed by the plasma is evaluated, taking into account the inhomogeneity of the static magnetic field and the effect of the rotational transform. The R.F. power absorbed by the plasma is then compared with the power lost by ohmic heating of the wall and of the coupling structure. It is found that a large amount of R.F. power can be coupled to the plasma (up to 90 %), if the plasma parameters are such that toroidal eigenmodes set up, in spite of the fact that the coupling structure in the W VII A experiment must be very close to the conducting wall.

Recent experiments on Ion Cyclotron Heating (ICH) in toroidal devices have shown that coupling efficiencies up to 90 % and heating efficiencies of 20 % to 40 % can be achieved by this method[1,2,3,4]. The heating efficiency is relatively low as compared to the coupling efficiency (it is nevertheless one of the highest obtained by RF heating methods) only due to the particular conditions in which the ICH experiments were performed rather than to be peculiar of any such experiment. In fact, in all these experiments (with exception of the TFR experiment where, on the other hand, very low power levels were used, so that the heating efficiency could not be measured), the RF frequency was relatively low (21 - 25 MHz), so that to meet the condition $\omega = 2\Omega_i$ the magnetic field had to be reduced to less than 16 kΓ and a deuterium plasma had to be used.

Under these conditions the banana orbits for energetic ions were quite large and eventually could become comparable with the plasma radius. The poor heating efficiency and the observed influx of impurities at high power levels could then be explained by the bad confinement of the energetic ions. Moreover, a few percent of resonant hydrogen ions ($\omega = 2\Omega_D = \Omega_H$), which are always present in a deuterium plasma, could absorb a large amount of RF power and be accelerated to very high energies. This fact has been clearly shown in the Kurchatov experiment where, by means of a mass-energy analyzer, it was found that the tail of the distribution function after ICH was occupied exclusively by hydrogen ions.

A further complication arising from the use of a deuterium plasma is that, owing to the inhomogeneity of the static magnetic field and to the presence of a minority hydrogen ions, there is inside the plasma a surface where the conditions for the two-ions hybrid resonance are met. At this surface mode conversion can occur and the behaviour of the wave will strongly depend on the concentration of the minority hydrogen ions.

All these difficulties could be avoided in the WVIIa Stellarator if a pure hydrogen plasma is to be heated (a small quantity of residual deuterium plasma will not be relevant, because the frequency $\omega = 2\Omega_H$ is everywhere in the plasma different from the resonance frequency of deuterium, ω_D) and if the frequency of the RF power is chosen appropriately (f $\approx$ 100 MHz) in order to meet the condition $\omega = 2\Omega_H$ at high field strength (B $\approx$ 30 KΓ). An advantage of WVIIa with respect to other devices is the large aspect ratio R/r $\gtrsim$ 15, since the RF power absorbed by the plasma is, as we shall see, roughly proportional to R/r. Moreover, as the toroidal eigenmodes are very close to each other in WVIIa, overlapping of modes can occur and the coupling efficiency becomes less sensitive to changes in the plasma density. In the following we evaluate first the RF power absorbed by the plasma taking into account both the inhomogeneity of the state magnetic field and the effect of the rational transform, and then the resistive loading of a coupling structure.

Basic equations

We approximate in the following the toroidal plasma by a straight cylinder confined by a magnetic field of the form

$$(1) \qquad \begin{cases} B_z = B_0 \left(1 - (r \cos\vartheta)/R\right) \\ B_\vartheta = B_0\, r/qR \end{cases} \qquad |B_\vartheta| \ll B_0$$

and impose that all quantities are periodic in z with period $2\pi R$.

To evaluate the characteristics of the Vlasov equation we derive first the motion of a particle in the magnetic field given by eq. (1). After introducing the variables

$$(2) \qquad \begin{cases} v_\xi = \dfrac{B_z}{|B|}\, v_\vartheta - \dfrac{B_\vartheta}{|B|}\, v_z \\[2ex] u = \dfrac{B_\vartheta}{|B|}\, v_\vartheta + \dfrac{B_z}{|B|}\, v_z \end{cases}$$

we obtain, neglecting terms of the order $\rho \dfrac{d}{dr} \left(B_\vartheta / B_z\right)$:

$$(3) \qquad \begin{cases} \dot{v}_r = - v_\xi\, \Omega \\[1ex] \dot{v}_\xi = v_r\, \Omega \\[1ex] u = \text{const.} \end{cases}$$

with $\Omega = \dfrac{e}{mc}\, |B|$.

Eq.(3) is of course incorrect for particles which are trapped or quasi trapped; however, owing to the large aspect ratio of W7, their number is quite small, so that we can assume that their contribution to the plasma currents can be neglected.

Now we approximate the value of the magnetic field encountered by a particle at a point (r, ϑ, φ) by the value at the gyrocenter $(\bar{r}, \bar{\vartheta}, \bar{\varphi})$, whose equations of motion are assumed to be

$$(4) \qquad \begin{cases} \bar{r} = r_0 \\ \bar{\vartheta} = \vartheta_0 + ut/qR_0 \end{cases}$$

After introducing rotating coordinates $v^{\pm} = v_r \pm i v_\xi$ we obtain for the characteristics the expressions

$$(5) \qquad \begin{cases} v^{\pm\prime} = v^{\pm} e^{\mp i \psi} \\ u' = u \\ r' = r - \left[v^{+}(e^{-i\psi} - 1) - v^{-}(e^{i\psi} - 1)\right]/2\Omega_0 \\ \vartheta' = \vartheta + \frac{u}{qR}(t'-t) + \left[v^{+}(e^{-i\psi} - 1) + v^{-}(e^{i\psi} - 1)\right]/2r\Omega_0 \\ z' = z + u\,(t'-t) \end{cases}$$

where $\psi = (t'-t)\Omega_0 - \frac{qr}{u}\Omega_0\left[\sin\left(\vartheta + \frac{u}{qR}(t'-t)\right) - \sin\vartheta\right]$

In deriving eqs.(5) we have neglected the radial shifts due to toroidal drifts although they are of the same order of magnitude, i.e. ρ_i/r, of the terms retained. We shall however show in the appendix that this is allowed, when evaluating the power absorbed by the plasma.

Since $(r - v_\xi/\Omega_o)$ is a constant of the motion, as eq.(5) shows, we can choose $f_{oj} = \frac{n}{\pi^{3/2} v_j^3}\, g(r - v_\xi/\Omega_{oj})\, e^{-(|\underline{v}|/v_j)^2}$ as the zero order distribution function; n is the plasma density (we assume $n_i = n_e$) and v_j is the thermal velocity of the j-th species.

The solution of the linearized Vlasov equation can then be written

$$f_{1j} = \frac{z_j e_j}{\pi^{3/2} m_j v_j^{5/2}}\, e^{-(|\underline{v}|/v_j)^2} \Big[g(r + v_\vartheta/\Omega_{oj}) \int_0^t e^{\nu(t'-t)}$$

$$(6) \quad (v^{+\prime} E_+^{-\prime} + v^{-\prime} E_+^{+\prime} + 2E_u^{\prime} u^{\prime})\, dt' - \frac{v_j^2}{\Omega_{oj}} \frac{\partial g}{\partial r} \int_0^t E_\xi^{\prime}\, dt' \Big] +$$

$$+ \left[f_{1j} \right]_{t=0} e^{-\nu t} ,$$

where f_{1j}, e_j, m_j are the first order distribution function, the charge and the mass of the j-th species; $v^{\pm}$, $E^{\pm}$ are the rotating components of $\underline{v}_\perp, \underline{E}_\perp$; u, E_u are the components of $\underline{v}$ and $\underline{E}$ parallel to $\underline{B}$; ν is the collision frequency and the prime indicates that a quantity is taken along the characteristics.

As the magnetic field does not depend on z, we can write the electric field in the form

$$\underline{E} = \mathcal{E}(r, \vartheta)\, e^{i(\frac{N}{R_o} z - \omega t)} \qquad (N = 0, \pm 1, \ldots)$$

After a time larger than $1/\nu$ we can neglect the term

$$\left[f_{1j}\right]_{t=0} e^{-\nu t} \qquad \text{in eq. (6),}$$

so that the plasma currents are

$$j_\alpha = -\sum_j \frac{\omega_{pj}^2}{4\pi^{5/2}\, v_j^{\,5}}\; e^{i\left(\frac{N}{R_0} z - \omega t\right)} \left\{ \int d\underline{v}\; v_\alpha\; e^{-(|\underline{v}|/v_j)^2} \right.$$

$$(7) \quad \int_0^t \left[\left(v^{-\prime} \mathcal{E}_+^{+\prime} + v^{+\prime} \mathcal{E}_+^{-\prime} + 2\mathcal{E}_u^{\prime} u^{\prime} \right) g + \frac{1}{2i} \left(\mathcal{E}^{+\prime} - \mathcal{E}^{-\prime} \right) \frac{\partial g}{\partial v_\xi} \right]$$

$$\left. e^{i\left(\frac{N}{R_0} u - \omega - i\nu\right)(t' - t)} \quad dt' \right\}$$

Since the integrands are, for $\Omega_{oj} \gg \nu$ and $\Omega_{oj} \gg v_j / qr$, rapidly oscillating functions of t', the largest contribution to the integral in eq.( ) will come from the neighbourhoods of the time t' where the phase is stationary, i.e. where

$$(8) \quad \omega \pm \Omega_{oj} \left[1 - \frac{r}{R_0} \cos\left(\vartheta + \frac{u v_j}{q R_0} t' \right) \right] = 0 ,$$

or from the neighbourhoods of the integration limit t'=t. When $\omega = 2\Omega_{oi}$ eq.(8) is never fulfilled; we can then expand the exponents of the integrand around t'=t and obtain an expression for the plasma currents to lowest order in ρ_i/r_i :

$$j^{\pm} = \frac{i}{4\pi} e^{i\left(\frac{N}{R_0} - \omega t\right)} g \left[\frac{\omega_{pi}^2}{\omega \mp \Omega_{oi}\left(1 - \frac{r}{R_0}\cos\vartheta\right)} \mp \frac{\omega_{pe}^2}{\Omega_e\left(1 - \frac{r}{R_0}\cos\vartheta\right)} \right] \mathcal{E}^{\pm}$$

$$\text{(9)} \quad \approx \pm \frac{i}{4\pi} \frac{\omega_{pi}^2 \, \omega}{\Omega_{oi}\left(1 - \frac{r}{R_0}\cos\vartheta\right)} g e^{i\left(\frac{N}{R_0} z - \omega t\right)} \frac{\mathcal{E}^{\pm}}{\omega \mp \Omega_{oi}\left(1 - \frac{r}{R_0}\cos\vartheta\right)}$$

$$j_u = \frac{i}{4\pi} \frac{\omega_{pe}^2}{\omega} e^{i\left(\frac{N}{R_0} z - \omega t\right)} g \, \mathcal{E}_u .$$

Power absorption

The RF power absorbed by the plasma is given by

$$\text{(10)} \quad P = \frac{1}{2} \operatorname{Re} \left\{ \underline{j} \cdot \underline{E}^* \right\} = \frac{1}{4} \operatorname{Re} \left\{ j^+ \mathcal{E}^{+*} + j^- \mathcal{E}^{-*} + 2 j_u \mathcal{E}_u^* \right\} .$$

If we neglect the RF power absorbed via collisions or electron Landau damping it is evident from eq.(9) that the lowest order in ρ_i/r the absorbed power is zero. Only to a higher order in ρ_i/r the plasma current has a component in phase with the electric field and contributes to the power absorption.

In order to evaluate the higher order plasma current $\underline{j}_1$, we assume that $\underline{\mathcal{E}}(r,\vartheta)$ and $g(r+v_\perp/\Omega_{oi})$ do not change appreciably over a distance comparable with an ion Larmor radius, so that we can write

$$\mathcal{E}(r_i', \vartheta_i') \simeq \underline{\mathcal{E}} + \Delta r \frac{\partial}{\partial r} \underline{\mathcal{E}} + \Delta\vartheta \frac{\partial}{\partial \vartheta} \underline{\mathcal{E}} + \frac{(\Delta r)^2}{2} \frac{\partial^2}{\partial r^2} \underline{\mathcal{E}} +$$

$$+ \frac{(\Delta\vartheta)^2}{2} \frac{\partial^2}{\partial \vartheta^2} \underline{\mathcal{E}} + \Delta r \, \Delta\vartheta \frac{\partial^2}{\partial r \partial \vartheta} \underline{\mathcal{E}}$$

(11)

$$g(r + v_\xi/\Omega_{oi}) \approx g(r) + \frac{v_\xi}{\Omega_{oi}} \frac{\partial}{\partial r} g(r),$$

where

$$\Delta r = \frac{i}{2\Omega_{oi}} \left[v_i^+ (e^{-i\psi} - 1) - v_i^- (e^{i\psi} - 1) \right]$$

(12)

$$\Delta\vartheta = \frac{i}{2\Omega_{oi} r} \left[v_i^+ (e^{-i\psi} - 1) + v_i^- (e^{i\psi} - 1) \right]$$

and $\underline{\mathcal{E}} = \underline{\mathcal{E}}(r, \vartheta + u\tau/qR_o)$.

When we insert expressions (11) in the integrand of eq.(10), the terms proportional to $e^{i(\omega - 2\Omega_{oi})\tau}$ oscillate much slowlier than the other terms in the integrand when $\omega = 2\Omega_{oi}$, and therefore give the largest contribution to the integral (this will be clear in the following). Retaining therefore only terms proportional to $e^{2i\psi}$ we obtain

$$j_1^{\pm} \approx \frac{\omega_{pi}^2}{4\pi^{5/2} v_i^5 \Omega_{oi}} e^{i\left(\frac{N}{R_o}\xi - \omega t\right)} \int d\underline{v} \; v_i^{\pm} e^{-(|\underline{v}|/v_i)^2}$$

(13)

$$g(r + v_\xi/\Omega_{oi}) \int_{-t}^{0} e^{-i\omega\tau} e^{\nu\tau} e^{2i\psi} e^{i\frac{N}{R_o}u\tau} v^{-2} \left[-i \frac{\partial}{\partial r} \mathcal{E}_-^+ \right.$$

$$\left. - \frac{1}{r}\frac{\partial}{\partial \vartheta} \mathcal{E}^+ + \frac{i v^-}{\Omega_{oi}} \frac{\partial^2}{\partial r \partial \vartheta} \mathcal{E}^+ - \frac{v_-^+ v^-}{2\Omega_{oi}} \frac{\partial^2}{\partial r^2} \mathcal{E}^+ - \frac{1}{r^2} \frac{v_+^+ v^-}{2\Omega_{oi}} \frac{\partial^2}{\partial \vartheta^2} \mathcal{E}^+ \right] d\tau$$

Taking into account that, after integration over $\underline{v}$, for symmetry reasons only the terms proportional to $(v^+v^-)^2$ are not zero and that $v_\xi = -\frac{v^+-v^-}{2}$, we get

$$j_1^- \approx 0$$

$$(14) \quad j_1^+ \approx -\frac{\omega_{pi}^2}{8\pi^{3/2}} \frac{\rho_i^2}{v_i} e^{i\left(\frac{N}{R_0}z-\omega t\right)} \int_{-\infty}^{+\infty} e^{-u^2/v_i^2} du \int_{-t}^{0} e^{2i\psi} \cdot$$

$$e^{i\left(\frac{N}{R_0}u-\omega-i\nu\right)\tau}\left[\frac{\partial}{\partial r}\left(g\frac{\partial \mathcal{E}^+}{\partial r}\right) - i\frac{\partial g}{\partial r}\frac{\partial}{\partial \vartheta}\mathcal{E}^- + \frac{g}{r^2}\frac{\partial^2}{\partial \vartheta^2}\mathcal{E}^-\right] d\tau .$$

The power absorbed by the plasma per unit volume will then be given by

$$P = \frac{1}{4}\,\mathcal{R}e\left\{j_1^+\,\mathcal{E}^{+*}\right\} =$$

$$(15) \quad = \frac{\omega_{pi}^2}{32\pi^{3/2}} \frac{\rho_i^2}{v_i}\,\mathcal{R}e\left\{\int_{-\infty}^{+\infty} e^{-u^2/v_i^2} du \int_{0}^{-t} e^{2i\psi} e^{i\left(\frac{N}{R_0}u-\omega-i\nu\right)\tau}\right.$$

$$\left.\mathcal{E}^+\left[\frac{\partial}{\partial r}\left(g\frac{\partial}{\partial r}\mathcal{E}^+\right) - \frac{i}{r}\frac{\partial g}{\partial r}\frac{\partial}{\partial \vartheta}\mathcal{E}^- + \frac{g}{r^2}\frac{\partial^2}{\partial \vartheta^2}\mathcal{E}^-\right] d\tau\right. .$$

Now we have to evaluate the self-consistent electric field $E^{\pm} = E_r \pm i\,E_\theta$; note that in order to evaluate the power absorbed by the plasma it is sufficient to insert in the Maxwell equations the zero order plasma currents, given by eq.(9), since the first order correction of the plasma currents, $\underline{j}_1$, will only slightly affect the form of the electric field and will give no appreciable contribution to the power absorbed by the plasma, as given by eq.(15).

When terms of the order r/R are neglected, the poloidal modes are decoupled, so that for the poloidal mode m=0, the only one we shall consider here, we have

$$\bar{k}^2 \mathscr{E}_r + i\bar{k}\frac{\partial}{\partial r}\mathscr{E}_z = \frac{4\pi i\omega}{c^2} j_r$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\mathscr{E}_\vartheta)\right) - \bar{k}^2\mathscr{E}_\vartheta = -\frac{4\pi i\omega}{c^2} j_\vartheta$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\mathscr{E}_z\right) - i\frac{\bar{k}}{r}\frac{\partial}{\partial r}(r\mathscr{E}_r) = -\frac{4\pi i\omega}{c^2} j_u$$

where, from eq. (9):

$$j_r = i\frac{c^2}{4\pi\omega}\sigma\left(\mathscr{E}_r + i\frac{\omega}{\Omega_{oi}}\mathscr{E}_\vartheta - i\frac{\omega}{\Omega_{oi}}\frac{r}{qR_o}\mathscr{E}_u\right)$$

$$j_\vartheta = i\frac{c^2}{4\pi\omega}\left[\sigma\left(\mathscr{E}_\vartheta - i\frac{\omega}{\Omega_{oi}}\mathscr{E}_r\right) + \frac{r}{qR_o}\frac{\omega_{pe}^2}{c^2}\mathscr{E}_u\right]$$

(16) $$j_z = i\frac{c^2}{4\pi\omega}\left[\sigma\frac{r}{qR_o}\left(\mathscr{E}_\vartheta - i\frac{\omega}{\Omega_{oi}}\mathscr{E}_r\right) + \frac{\omega_{pe}^2}{c^2}\mathscr{E}_u\right]$$

with : $$\bar{k}^2 = \frac{N^2}{R_o^2} - \frac{\omega^2}{c^2}$$

$$\sigma = \frac{\omega_{pi}^2}{c^2}\frac{\omega}{\Omega_{oi}}\frac{\omega^2}{\omega^2 - \Omega_{oi}^2}$$

$$\mathscr{E}_u \approx \mathscr{E}_z + \frac{r}{qR_o}\mathscr{E}_\vartheta .$$

As $\omega_{pe}^2/c^2 \gg \sigma$, $|\mathscr{E}_u| \ll (|\mathscr{E}_r|, |\mathscr{E}_\vartheta|)$, so that neglecting again terms of higher order in r/R we obtain:

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\mathcal{E}_\vartheta)\right)+\left(k_\perp^2+\frac{\omega}{\Omega_{oi}}\frac{Nr}{q^2R_o^2}\frac{\partial}{\partial r}\left(\frac{\sigma q}{k_\parallel^2+\sigma}\right)\right)\mathcal{E}_\vartheta=0$$

(17) $$\mathcal{E}_r=-\frac{i}{k_\parallel^2+\sigma}\frac{\omega}{\Omega_{oi}}\sigma\,\mathcal{E}_\vartheta$$

where

(18) $$k_\perp^2=\frac{\omega^2}{\Omega_{oi}^2}\frac{\sigma^2}{k_\parallel^2+\sigma}-(k_\parallel^2+\sigma).$$

The last term in eq.(17) is, for the parameters of WVIIa very small and can be neglected. This term however is proportional to the azimuthal number N and will change the form of the electric field according to the sign of N. This effect is responsible for the splitting of the toroidal modes observed in some experiments.

Note that expression (18) can become very large when $\bar{k}^2 + \sigma \approx 0$, i.e. when $(\omega^2/c^2) \approx \sigma + N^2/R_o^2$. This is possible only for small values of N and at very low plasma density ($n \lesssim 1.5\cdot 10^{11}$ cm^{-3} for WVIIa parameters). We do not consider here plasma regions of such low densities and assume that $\sigma \gtrsim \omega^2/c^2$ everywhere.

A solution of eq.(17), regular for $r \to 0$ can be obtained numerically, for a given density profile; if we call it $\bar{\mathcal{E}}_\vartheta$, the general form of the electric field in the plasma is $\mathcal{E}_\theta = A\bar{\mathcal{E}}_\vartheta$ where A is a constant determined by the boundary conditions (if the density were constant, it would be $\mathcal{E}_\theta = AJ_1(k_\perp r)$).

Resistive loading of an external coupling structure

We assume that at a radius $s > p$ (p = plasma radius) there is a sheet current distribution of the form

(19) $$J_{ext} = e^{-i\omega t}\, \delta(r-s) \sum_N J_N\, e^{i\frac{N}{R_0}z},$$

and at a radius $w > s$ a conducting wall. Then the electric field has the form

$$\mathcal{E}_\vartheta \equiv e^{-i\omega t} \sum_N e^{i\frac{N}{R_0}z}\, \mathcal{E}_\vartheta^N$$

with

(20)
$$\begin{array}{ll}
r \le p & \mathcal{E}_\vartheta^N \equiv A_N \bar{\mathcal{E}}_\vartheta^N \\
p \le r \le s & \mathcal{E}_\vartheta^N \equiv B_N I_1\left(\frac{N}{R_0}r\right) + C_N K_1\left(\frac{N}{R_0}r\right) \\
s \le r \le w & \mathcal{E}_\vartheta^N \equiv D_N I_1\left(\frac{N}{R_0}r\right) + F_N K_1\left(\frac{N}{R_0}r\right) \\
w \le r & \mathcal{E}_\vartheta^N \equiv G_N e^{-\sqrt{2i}\, r/\delta}
\end{array}$$

where $\delta = (c^2/2\pi\omega\sigma)^{1/2}$ is the skin depth of the conducting wall and we have assumed $w \gg \delta$. The boundary conditions for the electric field are

(21)
$$\begin{array}{ll}
\left[\mathcal{E}_\vartheta^N\right]_p = 0 & \left[\frac{\partial}{\partial r}\mathcal{E}_\vartheta^N\right]_p = 0 \\
\left[\mathcal{E}_\vartheta^N\right]_s = 0 & \left[\frac{\partial}{\partial r}\mathcal{E}_\vartheta^N\right]_s = -\frac{4\pi i\omega}{c^2} J_N \\
\left[\mathcal{E}_\vartheta^N\right]_w = 0 & \left[\frac{\partial}{\partial r}\mathcal{E}_\vartheta^N\right]_w = 0
\end{array}$$

where the brackets indicate the jumps at the boundary surfaces.

After straightforward calculations one obtains

(22) $$A_N = \frac{4\pi i \omega}{c^2} J_N \frac{s}{p} \frac{1}{\Delta_N} \left[K_1\left(\frac{N}{R_0}w\right) I_1\left(\frac{N}{R_0}s\right) - K_1\left(\frac{N}{R_0}s\right) I_1\left(\frac{N}{R_0}w\right) - \frac{i\delta}{2w} K_1\left(\frac{N}{R_0}s\right) / K_1\left(\frac{N}{R_0}w\right) \right]$$

$$B_N = - A_N \, p \, \alpha_N \qquad ; \qquad C_N = A_N \, p \, \beta_N$$

$$G_N = - \frac{2\pi \omega s}{c^2} J_N \frac{\delta e^{\sqrt{2i}\, w/\delta}}{\sqrt{w}} \frac{1}{\Delta} \left[\alpha_N I_1\left(\frac{N}{R_0}s\right) - \beta_N K_1\left(\frac{N}{R_0}s\right) \right]$$

where

$$\Delta_N = \alpha_N I_1\left(\frac{N}{R_0}w\right) - \beta_N K_1\left(\frac{N}{R_0}w\right) + i\delta\alpha_N / 2w \, K_1\left(\frac{N}{R_0}w\right)$$

$$\alpha_N = \left[\bar{\mathcal{E}}_\vartheta^N \frac{\partial}{\partial r} K_1\left(\frac{N}{R_0}r\right) - K_1\left(\frac{N}{R_0}r\right) \frac{\partial}{\partial r} \bar{\mathcal{E}}_\vartheta^N \right]_{r=p}$$

$$\beta_N = \left[\bar{\mathcal{E}}_\vartheta^N \frac{\partial}{\partial r} I_1\left(\frac{N}{R_0}r\right) - I_1\left(\frac{N}{R_0}r\right) \frac{\partial}{\partial r} \bar{\mathcal{E}}_\vartheta^N \right]_{r=p}$$

Whereas $\Delta_{RN} = \mathcal{R}e \left\{ \alpha_N I_1\left(\frac{N}{R_0}w\right) - \beta_N K_1\left(\frac{N}{R_0}w\right) \right\}$ can be obtained by solving eq. (17), the quantity $\Delta_{IN} = \mathcal{I}m \left\{ \alpha_N I_1\left(\frac{N}{R_0}w\right) - \beta_N K_1\left(\frac{N}{R_0}w\right) \right\}$ could be obtained by evaluating the imaginary part of the electric field, i.e. by taking into account the first order correction of the plasma currents, $\underline{j}_1$, which has been neglected in eq.(17). However, as $|\mathcal{I}m\, \bar{\mathcal{E}}_\vartheta^N| \ll |\mathcal{R}e\, \bar{\mathcal{E}}_\vartheta^N|$, in general it will be $|\Delta_{IN}| \ll |\Delta_{RN}|$ so that the contribution of Δ_{IN} to Δ_N becomes important only if, for particular values of the parameters, $|\Delta_{RN}|$ becomes small, i.e. if a toroidal eigenmode sets up. In this case Δ_{IN} determines the amplitude and the width of the eigenmode and can be evaluated without evaluating $\mathrm{Im}(\bar{E}_\vartheta^N)$ explicitely. In fact, Δ_{IN} can be obtained by equating the sum of the total power absorbed by the plasma plus the power absorbed by the wall, to the power lost by the external structure.

From eq. (15) we have that the total power absorbed by the plasma is given by

$$P_{tot} = \int_0^{2\pi} d\vartheta \int_0^{2\pi} d\varphi \int_0^{\rho} r\, P\, dr$$

(23)

$$= \frac{\omega_p^2 R_0^2}{16\,\pi^{1/2}} \frac{\varrho_i^2}{v_i} \sum_N \int_0^{\rho} r\, \mathcal{E}_N^{+*} \frac{\partial}{\partial r}\left(g \frac{\partial}{\partial r} \mathcal{E}_N^{+}\right) H(r)\, dr$$

where

$$H(r) = \frac{1}{R_0} \mathcal{R}e \int_0^{2\pi} d\vartheta \int_{-\infty}^{+\infty} e^{-u^2/v_i^2} du \int_0^{-t} e^{2i\psi} e^{\nu\tau} e^{i\left(\frac{N}{R_0}u - \omega\right)\tau} d\tau .$$

The power lost on the wall is

$$P_w = \frac{1}{2}\int_0^{2\pi R_0} dz \int_0^{2\pi} d\vartheta \int_w^{\infty} \underline{E}^* \cdot \underline{\underline{\sigma}}\, \underline{E}\, r\, dr$$

(24) $$= \frac{2\pi^3 \omega s^2 R_0}{c^2} \frac{\delta}{w} \sum_N J_N^2 \frac{1}{|\Delta_N|^2} \left|\alpha_N I_1\left(\frac{N}{R_0}s\right) - \beta_N K_1\left(\frac{N}{R_0}s\right)\right|^2 .$$

We must now equate the sum of P_p and P_w to the power P_{ext} lost by the external structure:

$$P_{ext} = \frac{1}{2}\int_0^{2\pi R_0} dz \int_0^{2\pi} d\vartheta \int_0^{\infty} E_\vartheta^* J_{ext}\, r\, dr$$

(24) $$= \frac{8\pi^3 \omega s^2 R_0}{c^2} \sum_N J_N^2\, \mathrm{Im}\left\{\frac{1}{\Delta_N}\left[\alpha_N I_1\left(\frac{N}{R_0}s\right) - \beta_N K_1\left(\frac{N}{R_0}s\right)\right]\right.$$

$$\left.\left[K_1\left(\frac{N}{R_0}w\right) I_1\left(\frac{N}{R_0}s\right) - K_1\left(\frac{N}{R_0}s\right) I_1\left(\frac{N}{R_0}w\right) - i\delta K_1\left(\frac{N}{R_0}s\right)/2wK_1\left(\frac{N}{R_0}w\right)\right]\right\} .$$

As Δ_{IN} becomes important only when Δ_{RN} is of the same order of magnitude as Δ_{IN}, we can neglect the terms proportional to $\Delta_{RN}\, \mathrm{Im}\, \alpha_N$ or to $\Delta_{RN}\, \mathrm{Im}\, \beta_N$ in eq.(25). Furthermore, as the coefficient J_N can be varied arbitrarily without changing Δ_N, we can equate each term of the sum over N in eq.(23),(24) to the corresponding one of the sum in eq.(25). By noting finally that from eq.(17) for $\omega \approx 2\Omega_{oi}$ it follows

(26) $$\mathcal{E}^+ = -i\frac{\sigma - k_\parallel^2}{\sigma + k_\parallel^2}\mathcal{E}_\vartheta ,$$

we get

$$\Delta_{IN} = \frac{\omega_p^2}{4\pi^{3/2}} \frac{R_o \rho}{p^2 c^2} \frac{K_1(\frac{N}{R_o}w) I_1(\frac{N}{R_o}s) - K_1(\frac{N}{R_o}s) I_1(\frac{N}{R_o}w)}{\alpha_N I_1(\frac{N}{R_o}s) - \beta_N K_1(\frac{N}{R_o}s)} \cdot$$

$$(27) \qquad \int_0^p \frac{\sigma - k_{\shortparallel}^2}{\sigma + k_{\shortparallel}^2} \bar{\mathcal{E}}_\vartheta^N \frac{\partial}{\partial r}\left[g \frac{\partial}{\partial r}\left(\frac{\sigma - k_{\shortparallel}^2}{\sigma + k_{\shortparallel}^2} \bar{\mathcal{E}}_\vartheta^N \right)\right] H(r) r \, dr .$$

The function H(r) is evaluated in the appendix. When $\Delta\omega=0$, i.e. when the resonant plane coincide with the mid-plane of the plasma cylinder, one has

$$(28) \qquad H(r) \approx \frac{2\pi^2}{N} e^{-\frac{1}{2}(r/N\rho)^2} I_o\left(\tfrac{1}{2}\left(\tfrac{r}{N\rho}\right)^2\right).$$

We note here that if $\Delta\omega=0$ and if the power absorbed in the region $r \leq \rho N$ is neglected, we can write $H(r) \approx 2\pi^{3/2}\rho/r$ and eq.(23) for the power absorbed by the plasma reduces to

$$(29) \qquad P \approx \frac{\pi}{8} \frac{\omega_p^2 R_o^2 \rho^2}{\Omega_{oi}} \int_{r \gg N\rho}^{p} g \left| \frac{\partial \mathcal{E}^+}{\partial r} \right|^2 dr .$$

This expression is identical with the one given in ref.(6), only if there the total power absorbed by the plasma is considered.

We emphasize here that the power absorbed by the plasma per unit volume, $P = \frac{1}{2}$ ($\underline{j} \cdot \underline{E}$), can be negative in some interval of r (see eq.(23)); this behaviour however does not mean that the plasma is cooled in that region, as the heat transported by the plasma should also be included. Consequently, the expression $P = \frac{1}{2}$ ($\underline{j} \cdot \underline{E}$) should not be used to evaluate the local heating rate of the plasma; when integrated over r, however, it yields the correct value of the total power absorbed by the plasma.

For a more detailed discussion of this problem, see ref. (7), where the heating of a plasma by TTMP has been considered.

Conclusions

We have solved eq. (17) numerically for a density profile of the form

$$\begin{cases} n = n_o \ (1-r^2/p^2) + 1.15 \cdot 10^{11} \ cm^{-3} & r \leqslant p \\ n = 0 & r > p \end{cases}$$

We have assumed that the RF field is generated by a single turn coil and that the current in the coil has the form:

$$J_{ext} = J_o \ , \qquad |Z| \leqslant \Delta \qquad s-\delta \leq r \leq s+\delta$$

$$J_{ext} = 0 \qquad \text{otherwise,}$$

with $\Delta = 2.5$ cm; $\delta = C\ (2\pi\omega\sigma)^{-1/2}$ is the skin depth; the value $\sigma = 0.14 \times 10^7$ mhos/m corresponding to stainless-steel has been choosen both for the conducting wall and the RF coil.

For the WVIIa parameters

$$R_o = 200 \text{ cm} \ , \qquad p = 10 \text{ cm}$$
$$s = 14{,}5\text{cm} \ , \qquad w = 17{,}5 \text{ cm}$$
$$q = 2.5 \ , \qquad B = 30 \text{ kT}$$

we have then evaluated the RF power absorbed by the plasma and the ohmic losses in the wall and the RF coil.

Fig. 1 shows the equivalent series resistance for the sum of the power absorbed by the plasma and the ohmic losses in the wall and in the conducting coil, as a function of the plasma density, for two ions temperatures:

a) $T_i = 100$ eV

b) $T_i = 400$ eV.

Fig. 2 shows the fraction of the total RF power absorbed by the plasma (a), by ohmic losses in the coil (b), and in the wall (c) as a function of the plasma density. The ion temperature is $T_i = 100$ eV.

Fig. 3 shows the same quantities as fig. 2 for $T_i = 400$ eV.

Appendix 1

We shall evaluate the expression

$$\text{(1a)} \quad H(r) = \frac{1}{R_0} \, \mathrm{Re} \left\{ \int_0^{2\pi} d\vartheta \int_{-\infty}^{\infty} e^{-u^2/v_i^2} du \int_0^{-t} e^{2i\psi} e^{i\left(\frac{\Delta}{R_0} u - \omega\right)\tau} e^{\nu\tau} d\tau \right\}$$

in the limit of small ϱ/r.

We consider here the situation where the mean free path is much smaller than a connection length, i.e. $\nu \gg v_i / 2\pi q R_0$. Then $u\tau / q R_0 \ll 1$ and we can expand the expression

$$\psi = \Omega_{oi} \tau - \frac{qr}{u} \Omega_{oi} \left[\sin\left(\vartheta - \frac{u}{qr}\tau\right) - \sin\vartheta \right]$$

around $\tau = 0$; we obtain:

$$H \approx \frac{v_i}{R_0} \, \mathrm{Re} \int_0^{2\pi} d\vartheta \int_{-\infty}^{\infty} e^{-u^2} du \int_0^{\infty} e^{-\nu\tau} e^{i\tau\left(k_\parallel v_i u - \Delta\omega - \frac{2r}{R_0}\Omega_{oi}\cos\vartheta\right)}$$

(2a)

$$e^{i\frac{\tau^2}{2}\frac{v_i^2 r \sin\vartheta}{\varrho_i R_0^2 q} u} \, d\tau .$$

When $k_\parallel \ll \nu / v_i$ and $r < \left(\frac{2\pi q R_0 \nu}{v_i}\right)^2 \frac{\varrho_i}{q \sin\vartheta}$ the collisional damping dominates. In this case it is convenient to perform the integration over u first; we get

$$H \approx \sqrt{\pi} \frac{v_i}{R_0} \, \mathrm{Re} \int_0^{2\pi} d\vartheta \int_0^{\infty} e^{-\nu\tau} e^{-i\left(\Delta\omega + \frac{2r}{R_0}\Omega_{oi}\cos\vartheta\right)\tau} \cdot e^{-\frac{\tau^2}{4}\left(k_\parallel v_i + \tau \frac{v_i^2 r \sin\vartheta}{\varrho_i R_0^2 q}\right)^2} d\tau$$

$$\approx \sqrt{\pi} \frac{v_i}{R_0} \, \mathrm{Re} \int_0^{2\pi} d\vartheta \int_0^{\infty} e^{-\nu\tau} e^{-i\left(\Delta\omega + \frac{2r}{R_0}\Omega_{oi}\cos\vartheta\right)\tau} d\tau$$

$$= \sqrt{\pi} \frac{v_i}{R_0} \, \mathrm{Re} \left[\nu^2 - (\Delta\omega)^2 + \left(\frac{2r}{R_0}\Omega_{oi}\right)^2 + 2i\nu\Delta\omega \right]^{-1/2}$$

Note that for r $\approx \frac{1}{2} \Delta R = R \frac{\Delta\omega}{\Omega_{oi}}$ the function H has a maximum. When one of the two before mentioned conditions is not fulfilled (i.e. when either $r > \left(\frac{2\pi\nu q R_o}{v_i}\right)^2 \frac{\varrho_i}{\pi^2 q \sin\vartheta}$ or the parallel wavelength is shorter than the mean free path) the collisional damping can be neglected. In this case we write eq. (2a) in the form

$$H \approx \frac{v_i}{2R_o} \, \mathrm{Re} \int_0^{2\pi} d\vartheta \int_{-\infty}^{\infty} e^{-u^2} du \int_{-\infty}^{\infty} e^{i\tau\left(k_\parallel v_i u - \Delta\omega - \frac{2r}{R_o}\Omega_{oi}\cos\vartheta\right)} \, e^{i\tau^2 \frac{v_i^2 r u \sin\vartheta}{\varrho_i R_o^2 u}} d\tau.$$

After integration over τ we get

$$H \approx \frac{v_i}{2R_o} \, \mathrm{Re} \int_{-\infty}^{\infty} e^{-u^2} du \int_0^{2\pi} \frac{R_o}{v_i} \sqrt{\frac{i\pi q \varrho_i}{r u \sin\vartheta}} \, e^{-i\frac{q R_o^2 \varrho_i}{4 v_i^2 r u \sin\vartheta}\left(k_\parallel v_i u - \Delta\omega - \frac{2r}{R_o}\Omega_{oi}\cos\vartheta\right)^2} d\vartheta$$

For $\varrho/r \rightarrow 0$ the integrand is a rapidly oscillating function of ϑ , so that we can use again the method of the stationary phase for the integration over ϑ ; the points where the phase is stationary are given by

(3a) $$\cos\vartheta_o = \frac{R_o}{2r}\left(k_\parallel \varrho_i u - \Delta\omega/\Omega_{oi}\right).$$

For the ions with velocity such that $|k_\parallel \varrho_i u - \Delta\omega/\Omega_{oi}| > 2r/R_o$ i.e. for the ions which do not cross the resonant region, there are no points of stationary phase. For these ions the integral over ϑ is a factor $\sqrt{\varrho_i/r}$ smaller than the integral over ϑ for the ions which have points of stationary phase, i.e. which cross the resonant region. From the leading term of the integral over ϑ it follows

(4a) $$H \approx \pi \frac{\rho_i}{r} \int \frac{e^{-u^2}}{\sin \vartheta_0} \, du,$$

where the integration is over the values of u which satisfy eq.(3a). It is evident that eq.(4a) is the correct expression for H only if the bulk of the particles crosses the resonant region, i.e. if $r \gtrsim \Delta R$, otherwise the small contribution to the integral over ϑ coming from the integration limits $(0, 2\pi)$ should also be taken into account.

Let us now use ϑ_0 as new integration variable, instead of u; then

$$H \approx \frac{\pi}{N} \int_0^{2\pi} e^{-(2r/N\rho_i)^2 (\cos\vartheta_0 - \Delta R/2r)^2} \, d\vartheta_0 .$$

In the interesting case Δ R=0 the integral can be evaluated exactly and one gets:

$$H \approx \frac{2\pi^2}{N} e^{-\frac{1}{2}(2r/N\rho_i)^2} I_0\left(\tfrac{1}{2}(2r/N\rho_i)^2\right).$$

When Δ R$\neq$0 the following approximations are valid:

a) $H \approx 2\pi^2/N$ for $r \ll N\rho_i$

b) $H \approx 2\pi^{3/2} \frac{\rho_i}{r} \left(1 - (\Delta R/2r)^2\right)^{-1/2}$, for $r \gg N\rho_i$.

In the neighbourhood of $r = \frac{\Delta R}{2}$ the last formula is not valid; for $r \approx \Delta R$ one gets directly

$$H \approx \frac{2\pi}{N} \int_0^\infty e^{-(2r/N\varrho_i)^2 \vartheta^4/4} \, d\vartheta$$

$$\approx 1.81\,\pi \left(\varrho_i / N r\right)^{1/2}$$

The results we have derived justify the assumption made earlier in deriving eq.(13), that the terms proportional to $e^{i(\omega - 2\Omega_{oi})\tau}$ are the most important; in fact, all other terms would have a much larger value of $\Delta\omega$, of the order Ω_{oi} or Ω_{oe}, so that eq.(3a) would be never fulfilled. The same is true for the terms which would come from the radial shifts due to toroidal drifts as they are small and oscillate too slowly as compared with the ion cyclotron frequency.

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Figure Captions

Fig.1 shows the equivalent series resistance for the sum of the power absorbed by the plasma and the ohmic losses in the wall and in the conducting coil, as a function of the plasma density, for two ions temperatures:

a) T_i = 100 eV
b) T_i = 400 eV.

Fig.2 shows the fraction of the total RF power absorbed by the plasma (a), by ohmic losses in the coil (b), and in the wall (c) as a function of the plasma density. The ion temperature is T_i = 100 eV.

Fig.3 shows the same quantities as Fig.2 for T_i = 400 eV.

Fig. 1

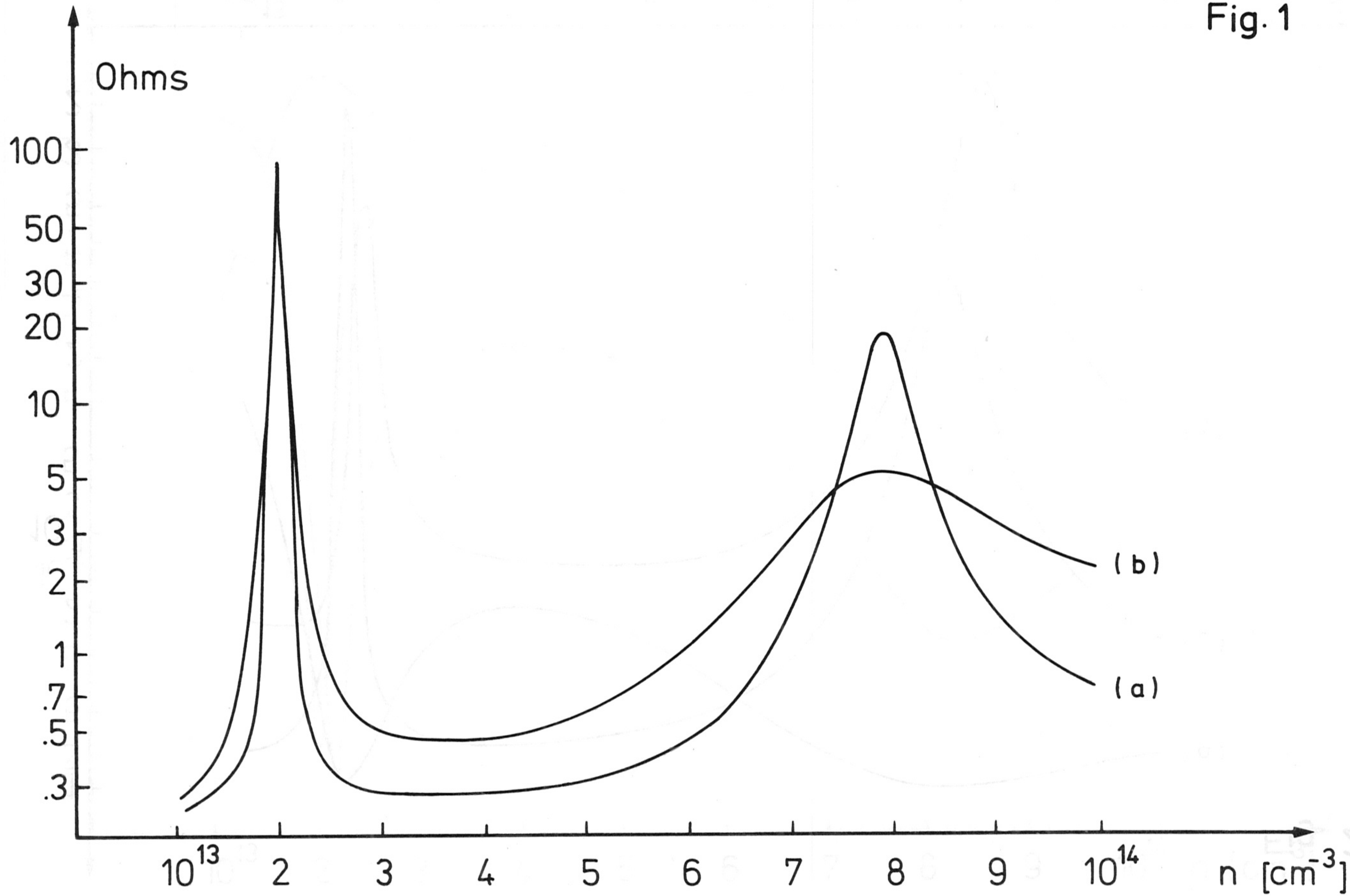
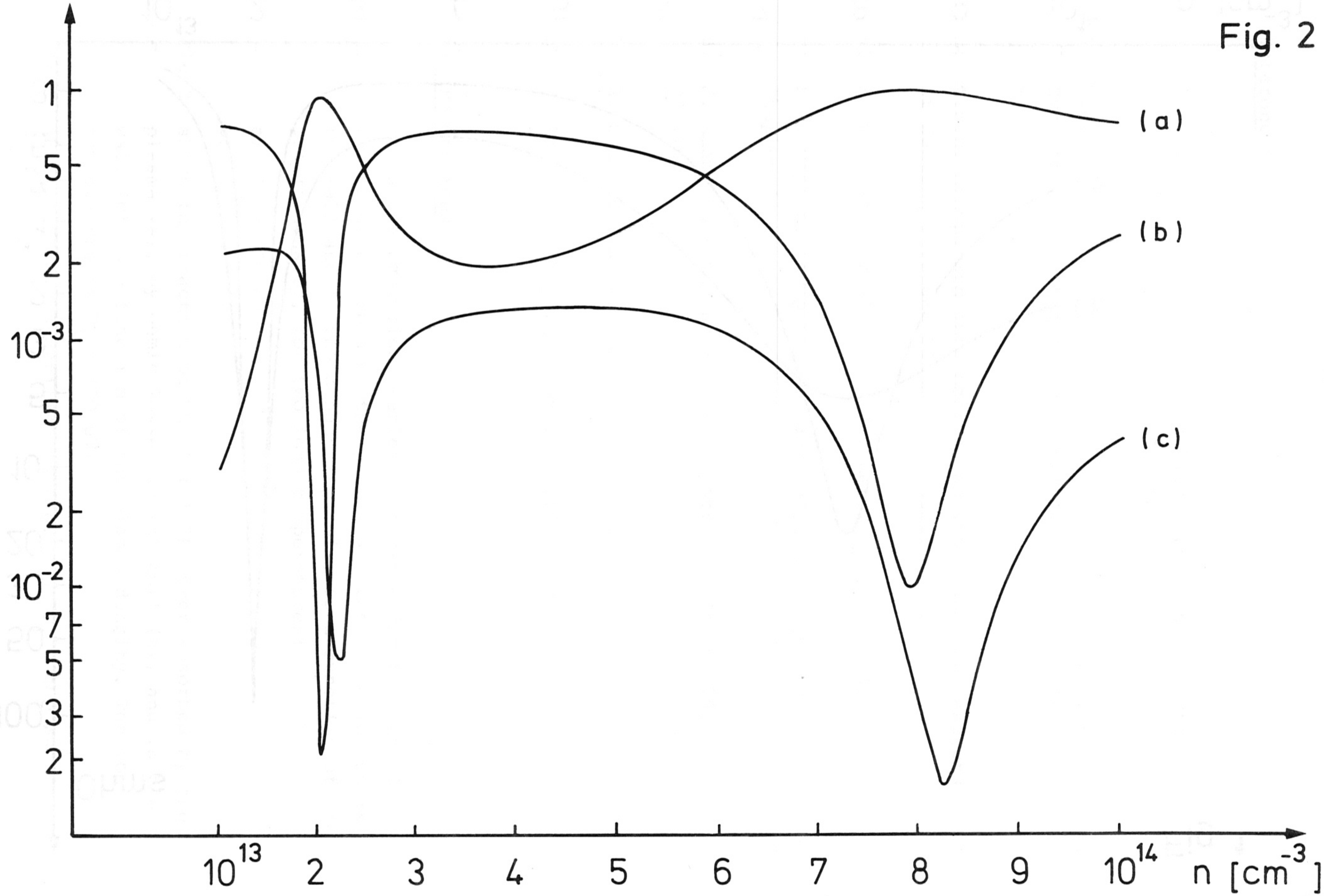


Fig. 2



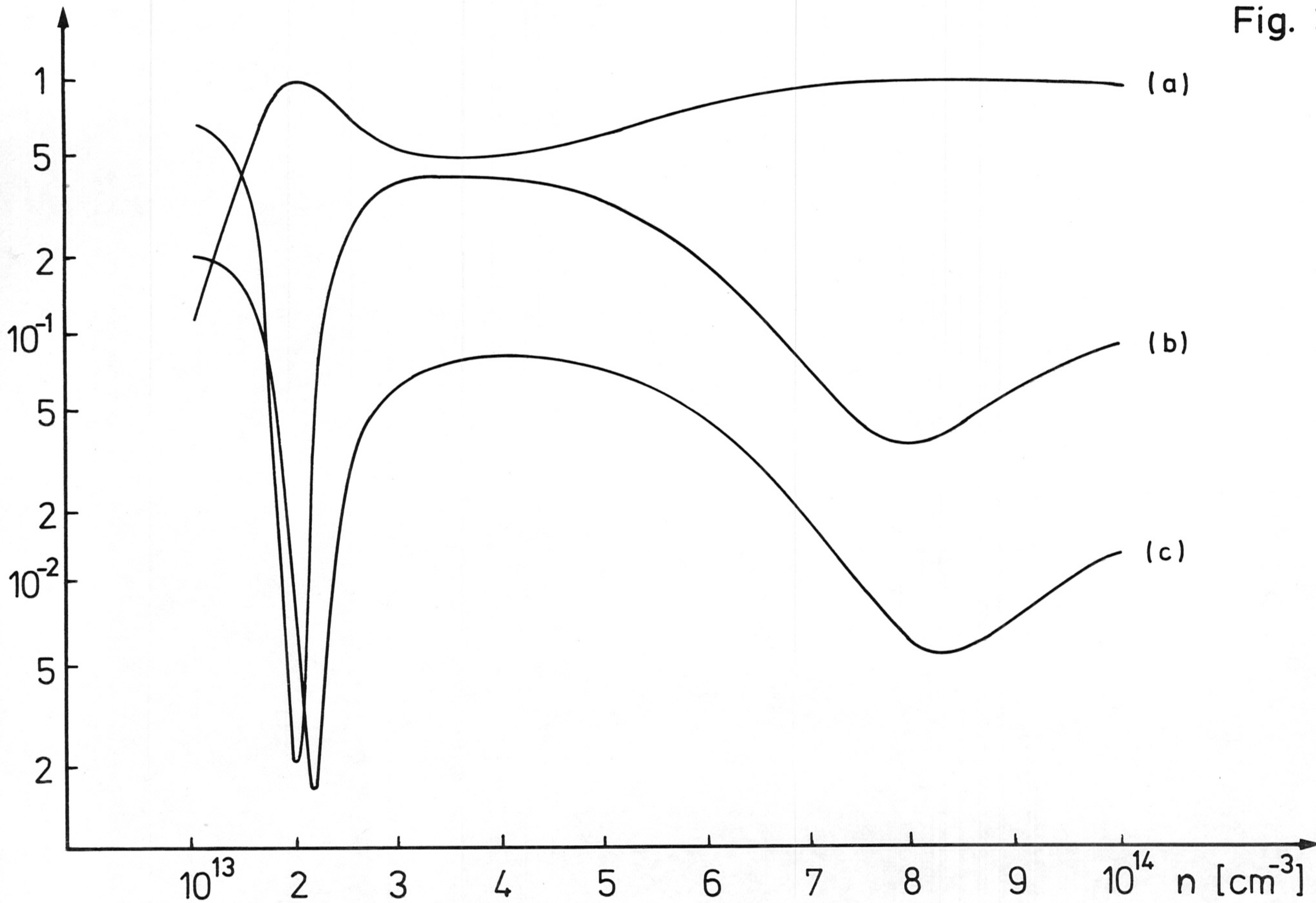


Fig. 3