Ohm's Law in Turbulent Plasmas
and Beta Limitations by Anomalous Diffusion.

Kurt Borrass

IPP 4/162 January 1978

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

8046 GARCHING BEI MÜNCHEN

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

Ohm's Law in Turbulent Plasmas and Beta Limitations by Anomalous Diffusion.

Kurt Borrass

IPP 4/162 January 1978

Paper presented at IAEA Workshop on Fusion Reactor Design, Madison, Wisconsin, 10 - 21 October 1977.

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

IPP 4/162 Kurt Borrass

Ohm's Law in Turbulent Plasmas and Beta Limitations by Anomalous Diffusion.

January 1978

Abstract

For axisymmetric diffusive equilibria a condition is derived by means of a generalized Ohm's law. It relates some effective outward particle flux to the toroidal current density. An approximate version of it requires that the corresponding effective diffusion velocity V_D^* must not exceed the poloidal magnetic diffusion velocity V_m. The simple version of Ohm's law as used in transport calculations only applies if $V_D^* \ll V_m$. A preliminary discussion is performed for the case of anomalous diffusion due to trapped particle instabilities.

OHM'S LAW IN TURBULENT PLASMAS AND BETA LIMITATIONS BY ANOMALOUS DIFFUSION

Kurt Borrass

Max-Planck-Institut für Plasmaphysik, 8046 Garching, F.R. of Germany

Abstract

For axisymmetric diffusive equilibria a condition is derived by means of a generalized Ohm's law. It relates some effective outward particle flux to the toroidal current density. An approximate version of it requires that the corresponding effective diffusion velocity V_D^* must not exceed the poloidal magnetic diffusion velocity V_m. The simple version of Ohm's law as used in transport calculations only applies if $V_D^* \ll V_m$. A preliminary discussion is performed for the case of anomalous diffusion due to trapped particle instabilities.

Introduction

The plasma physical data underlying current tokamak reactor studies are largely based on transport calculations. In this context the particle and heat balance equations are considered together with Maxwell's equations and Ohm's law, which is used in the form /1/, /2/

(1) $E_T = \eta \, j_T$, where j_T is the toroidal current density and E_T the toroidal electric field (In these preliminary considerations we confine ourselves to the usual cylinder approximation, $\beta \ll 1$ and assume $\eta_{\parallel} = \eta_{\perp} = \eta$).

With Maxwell's equations one obtains the equation for the poloidal field

(2) $$\frac{\partial B_P}{\partial t} = -\frac{c^2}{4\pi} \frac{\partial}{\partial r}\left(\eta \frac{1}{r} \frac{\partial}{\partial r}(r B_P)\right).$$

Equation (2) describes the skin penetration of B_p with the magnetic diffusion velocity $V_m \sim c^2 \eta / 4\pi L$. (Here L is some characteristic length of the order of a).

In other applications Ohm's law is assumed to be of the form

(3) $\vec{E}_T = \eta \vec{j}_T - \frac{1}{c} \vec{V} \times \vec{B}$. Its toroidal component

(4) $E_T = \eta j_T - \frac{1}{c} V_r \cdot B_p$ results in the equation

(5) $\frac{\partial B_p}{\partial t} = - \frac{c^2}{4\pi} \frac{\partial}{\partial r} \left(\eta \frac{1}{r} \frac{\partial}{\partial r} (r B_p) \right) + \frac{\partial}{\partial r} (V_D B_p)$ for B_p.

In eq. (5) we have used the fact that V_r, the velocity normal to the magnetic surface, is the diffusion velocity V_D. The second term on the right side of eq. (5) describes the freezing of the poloidal field.

A rough order of magnitude estimate of eq. (5) shows that stationarity requires that

(6) $V_D \lesssim V_m$.

Condition (6), which is a consequence of the $\vec{V} \times \vec{B}$ term in Ohm's law, means that B_p must penetrate faster than it is carried out by the diffusing plasma to achieve a stationary state.

With the formal relation $V_m \beta \sim 2 V_c$, where $V_c \sim c^2 \eta p/B^2 L$ is the classical diffusion velocity, eq. (6) can be transformed into a beta limiting condition

(7) $\beta \lesssim 2 \frac{V_c}{V_D}$.

The importance of eq. (6) and (7) is that, in accordance with current diffusion formulae, one has $V_m \sim V_D$ for small-size tokamaks, but $V_m \ll V_D$ for reactor-size tokamaks, indicating that eqs. (6) and (7) would impose drastic restrictions on large tokamaks with anomalous diffusion if Ohm's law were of the form of eq. (4) /3/.

Generally Ohm's law is essentially the force balance equation for the electrons. It contains, besides the $\vec{V} \times \vec{B}$ term, others such as pressure gradient and pressure anisotropy terms and it is not at all clear when, if at all, the simple version given by eq. (1) is a reasonable approximation.

This problem is studied here for arbitrary axisymmetric hydrogen-like plasmas by considering the equations of motion for electrons and ions together with Maxwell's equations. We derive a general form of Ohm's law which generalizes eq. (4) as well as a condition for stationary diffusive equilibria which generalizes condition (6). This discussion is largely based on ref. /3/.

Basic equations

The equations of motion for the two-particle species of a hydrogen-like plasma read /4/

$$(8)\quad \frac{\partial}{\partial t}(m^e n \vec{V}^e) + \nabla \overleftrightarrow{S}^e + en\vec{E} + \frac{e}{c} n \vec{V}^e \times \vec{B} = \vec{R}$$

$$(9)\quad \frac{\partial}{\partial t}(m^i n \vec{V}^i) + \nabla \overleftrightarrow{S}^i - en\vec{E} - \frac{e}{c} n \vec{V}^i \times \vec{B} = -\vec{R}$$

where $\overleftrightarrow{S}^{e/i}$ is the stress tensor and $\vec{R}$ is the electron ion momentum exchange due to collisions. Equations (8) and (9) comprise the turbulent case. Denoting the usual ensemble average by $\sim$ and decomposing each quantity into $f=\tilde{f} + \delta f$, where δf is the fluctuating part, the averages of eqs. (8) and (9) read

$$(10)\quad \frac{\partial}{\partial t}(m^e \widetilde{n\vec{V}^e}) + \nabla(m^e \tilde{n} \tilde{\vec{V}}^e \tilde{\vec{V}}^e) + \nabla \widetilde{\overleftrightarrow{\Pi}}^e + \nabla \tilde{p}^e + e\tilde{n}\tilde{\vec{E}} + \frac{e}{c}\widetilde{n\vec{V}^e \times \vec{B}} + e\,\widetilde{\delta n\,\delta \vec{E}} + \frac{e}{c}\,\widetilde{\delta(n\vec{V}^e)\times\delta\vec{B}} = \tilde{\vec{R}}$$

$$(11)\quad \frac{\partial}{\partial t}(m^i \widetilde{n\vec{V}^i}) + \nabla(m^i \tilde{n} \tilde{\vec{V}}^i \tilde{\vec{V}}^i) + \nabla \widetilde{\overleftrightarrow{\Pi}}^i + \nabla \tilde{p}^i - e\tilde{n}\tilde{\vec{E}} - \frac{e}{c}\widetilde{n\vec{V}^i \times \vec{B}} - e\,\widetilde{\delta n\,\delta \vec{E}} - \frac{e}{c}\,\widetilde{\delta(n\vec{V}^i)\times\delta\vec{B}} = -\tilde{\vec{R}}$$

Here we have introduced the generalized pressure tensors $\widetilde{\overleftrightarrow{\Pi}}^{e/i} + \mathbb{1}\tilde{p}^{e/i} = \widetilde{\overleftrightarrow{S}}^{e/i} - m^{e/i}\tilde{n}\tilde{\vec{V}}^{e/i}\tilde{\vec{V}}^{e/i}$, where $\widetilde{\Pi}^{e/i}$ is the anisotropic part and $\tilde{p}^{e/i}$ the isotropic part. The second terms in eqs. (10) and (11) are small and will be neglected. As to Maxwell's equations, only their averaged form is required:

(12) $\nabla \times \widetilde{\vec{E}} = -\frac{1}{c}\frac{\partial}{\partial t}\widetilde{\vec{B}}$, (13) $\nabla \times \widetilde{\vec{B}} = \frac{4\pi}{c}\widetilde{\vec{j}}$,

(14) $\nabla \cdot \widetilde{\vec{B}} = 0$.

The plasma under consideration is in equilibrium. Hence the force balance equation must be satisfied. It is the sum of eqs. (10) and (11), the time derivatives being neglected. We confine ourselves to cases where the terms $\frac{1}{c}\widetilde{\vec{j}} \times \widetilde{\vec{B}}$ and $\nabla \widetilde{p}\,(p = p^e + p^i)$ are the dominant ones so that the force balance equation has its familar form

(15) $c\,\nabla \widetilde{p} = \widetilde{\vec{j}} \times \widetilde{\vec{B}}$ and the equilibrium configuration is determined as usual. Since we consider axisymmetric configurations, the average magnetic field can then be expressed by

(16) $\widetilde{\vec{B}}_T = f(\psi)\nabla\varphi = \widetilde{B}_\varphi \vec{e}_\varphi$, (17) $\widetilde{\vec{B}}_p = \nabla\psi \times \nabla\varphi = \widetilde{B}_p \cdot \vec{e}_p$,

defining the rectangular coordinates ψ, ℓ, φ, where $2\pi\psi$ is the average poloidal magnetic flux, ℓ the distance in the poloidal direction, and φ the large azimuth. $\vec{e}_\varphi$ and $\vec{e}_p$ are unit vectors in the respective directions. Since $\vec{e}_\varphi = R\,\nabla\varphi$, where R is the distance from the axis of symmetry, we have

(18) $R \cdot \widetilde{B}_\varphi = f(\psi)$.

Equilibrium conditions and Ohm's law for the toroidal direction

In what follows we shall chiefly consider eqs. (10) and (15). For the light electrons the inertial term in eq. (10) is small compared with the momentum exchange term and can be neglected.

Multiplying the φ component of eq. (10) by R, we now obtain the rate equation for the angular momentum density in the direction of the axis of symmetry:

(19) $$0 = \widetilde{B}_p \frac{\partial}{\partial\psi}(\widetilde{\Pi}^e_{\varphi\psi} R^2) + \widetilde{B}_p \frac{\partial}{\partial\ell}(\widetilde{\Pi}^e_{\varphi p} R/\widetilde{B}_p) + \frac{\partial}{\partial\varphi}\widetilde{\Pi}^e_{\varphi\varphi} + \frac{\partial}{\partial\varphi}\widetilde{p}^e$$
$$+ e R \widetilde{n} \widetilde{E}_\varphi + \frac{e}{c} R \widetilde{n V^e_\psi} \widetilde{B}_p + e R \widetilde{\delta n \delta E_\varphi} + \frac{e}{c} R \widetilde{\delta(n V^e_\psi) \cdot \delta B_p} - R \widetilde{R}_\varphi$$

Applying the operator $\langle\cdots\rangle = \int_0^{2\pi} d\varphi \oint dl\, \widetilde{B}_P^{-1} \cdots$ to eq.(19), we get

(20) $$0 = \langle \widetilde{B}_P \frac{\partial}{\partial\psi}(\widetilde{\Pi}^e_{\varphi\psi} R^2)\rangle + e\langle R\widetilde{n}\widetilde{E}_\varphi\rangle + \frac{e}{c}\widetilde{\Gamma}_\psi + e\langle R\,\widetilde{\delta n \delta E_\varphi}\rangle + \frac{e}{c}\langle R\,\widetilde{\delta(nV^e_\psi)\cdot\delta B_P}\rangle - \langle R\widetilde{R}_\varphi\rangle ,$$

where $\widetilde{\Gamma}_\psi = \int_0^{2\pi} d\varphi \oint dl\, R\, \widetilde{nV^e_\psi}$ is the average electron flux through a magnetic surface. The first term in eq.(20) is a force due to toroidal momentum transfer perpendicular to the magnetic surface. It is small compared with the momentum exchange term if the momentum transfer in the ψ direction is essentially determined by the diffusion velocity and will be neglected.

To get from eq. (20) an equation of the type of an Ohm's law, $\vec{R}$ must be specified. In this paper we confine ourselves to cases where at least approximately

(21) $$\vec{R} = \vec{R}_\perp + e\widetilde{n}\,\widetilde{\eta}_\parallel\,\widetilde{\vec{j}}_\parallel .$$

Equation (21) states that the average parallel electron ion momentum exchange due to collisions is essentially of a frictional nature. (Non-frictional contributions will be omitted here for convenience).A previous paper treats the assumption that the total electron ion momentum exchange including that due to turbulent fields should be of the frictional type /3/. Eliminating $\widetilde{R}_\varphi$ and $\widetilde{j}_p$ between eqs. (15), (20) and (21), we get

(22) $$\langle R\widetilde{n}\widetilde{E}_\varphi\rangle = \langle \widetilde{n}\,\widetilde{\eta}_\parallel R\widetilde{j}_\varphi\rangle - \frac{1}{c}\widetilde{\Gamma}_\psi - \langle R\,\widetilde{\delta n \delta E_\varphi}\rangle - \frac{1}{c}\langle R\,\widetilde{\delta(nV^e_\psi)\delta B_P}\rangle + \langle R\widetilde{n}\,\widetilde{\eta}_\parallel \frac{\widetilde{B}_P^2}{\widetilde{B}^2}\frac{\partial\widetilde{p}}{\partial\psi}\rangle - \frac{1}{e}\langle R\widetilde{R}_{\perp\varphi}\rangle ,$$

which can be considered as one form of Ohm's law in the toroidal direction. Further information can be obtained by considering one more component of eq.(10). By multiplying eq.(10) by $\widetilde{\vec{B}}$ and applying oparation $\langle\cdots\rangle$ we get, together with eq.(21),

(23) $$\langle \widetilde{\vec{B}}\nabla\widetilde{\Pi}^e\rangle + e\langle\widetilde{n}\,\widetilde{\vec{B}}\widetilde{\vec{E}}\rangle + e\langle\widetilde{\vec{B}\,\delta n\,\delta\vec{E}}\rangle = e\langle\widetilde{n}\,\widetilde{\eta}_\parallel\widetilde{j}_P\frac{\widetilde{B}^2}{\widetilde{B}_P}\rangle - ec\langle\widetilde{n}\,\widetilde{\eta}_\parallel\frac{\partial\widetilde{p}}{\partial\psi}R\widetilde{B}_\varphi\rangle .$$

Elimination of $\tilde{j}_\varphi$ between eqs.(22) and eq.(15) gives

$$(24)\ \langle R\tilde{n}\tilde{E}_\varphi\rangle = \langle \tilde{n}\tilde{\eta}_\parallel R\frac{\tilde{B}_\varphi}{\tilde{B}_p}\tilde{j}_P\rangle - \frac{1}{c}\tilde{\Gamma}_\psi - \langle R\,\widetilde{\delta n\,\delta E_\varphi}\rangle$$
$$-\frac{1}{c}\langle R\,\widetilde{\delta(nV_\varphi^c)\,\delta B_p}\rangle - c\langle R^2\tilde{n}\tilde{\eta}_\parallel\frac{\tilde{B}_\varphi^2}{\tilde{B}^2}\frac{\partial\tilde{p}}{\partial\psi}\rangle + \frac{1}{e}\langle R\tilde{R}_{\perp\varphi}\rangle.$$

From $\tilde{\vec{B}}_T = f(\psi)\nabla\varphi$ and the poloidal component of eq.(13) it follows that

$$(25)\ \tilde{\vec{j}}_P = \frac{4\pi}{c}f'\nabla\psi\times\nabla\varphi = \frac{4\pi}{c}f'\tilde{\vec{B}}_P$$

Hence one gets

$$(26)\ \langle\tilde{\eta}_\parallel\tilde{n}\frac{\tilde{B}^2}{\tilde{B}_P}\tilde{j}_P\rangle = \langle\tilde{\eta}_\parallel\tilde{n}\tilde{B}^2\rangle\langle\tilde{\eta}_\parallel\tilde{n}R\tilde{B}_\varphi\rangle^{-1}\langle\tilde{\eta}_\parallel\tilde{n}\tilde{j}_P\frac{R\tilde{B}_\varphi}{\tilde{B}_P}\rangle.$$

We now confine ourselves to stationary situations so that

$$(27)\ \nabla\times\tilde{\vec{E}} = -\frac{1}{c}\frac{\partial\tilde{\vec{B}}}{\partial t} = 0$$

. From eq.(27) it follows that

$$(28)\ \tilde{\vec{E}} = \frac{V_a}{2\pi R}\vec{e}_\varphi - \nabla\tilde{\Phi}$$

, where V_a is the external voltage and $\tilde{\Phi}$ the self-consistent average potential. Eliminating $\tilde{j}_p$ by means eq.(26) between eqs.(23) and (24), we get with eq.(28) an expression for $\tilde{\Gamma}_\psi$

$$(29)\ \tilde{\Gamma}_\psi = \frac{cV_a}{2\pi}\left\{\langle\tilde{n}\frac{\tilde{B}_\varphi}{R}\rangle\frac{\langle\tilde{\eta}_\parallel\tilde{n}R\tilde{B}_\varphi\rangle}{\langle\tilde{\eta}_\parallel\tilde{n}\tilde{B}^2\rangle} - \langle\tilde{n}\rangle\right\}$$
$$+\frac{c}{e}\langle R\tilde{R}_{\perp\varphi}\rangle$$
$$-c^2\frac{\partial\tilde{p}}{\partial\psi}\left\{\langle\tilde{\eta}_\parallel\tilde{n}R^2\frac{\tilde{B}_\varphi^2}{\tilde{B}^2}\rangle - \langle\tilde{\eta}_\parallel\tilde{n}R\tilde{B}_\varphi\rangle\frac{\langle\tilde{\eta}_\parallel\tilde{n}R\tilde{B}_\varphi\rangle}{\langle\tilde{\eta}_\parallel\tilde{n}\tilde{B}^2\rangle}\right\}$$
$$+c\frac{\langle\tilde{\eta}_\parallel\tilde{n}R\tilde{B}_\varphi\rangle}{\langle\tilde{\eta}_\parallel\tilde{n}\tilde{B}^2\rangle}\left\{\frac{1}{e}\langle\tilde{\vec{B}}\nabla\tilde{\overleftrightarrow{\Pi}}^c\rangle - \langle\tilde{n}\tilde{\vec{B}}\nabla\tilde{\Phi}\rangle\right\}$$
$$-c\langle R\,\widetilde{\delta n\,\delta E_\varphi}\rangle - \langle R\,\widetilde{\delta(nV_\varphi^c)\,\delta B_p}\rangle + c\frac{\langle\tilde{\eta}_\parallel\tilde{n}R\tilde{B}_\varphi\rangle}{\langle\tilde{\eta}_\parallel\tilde{n}\tilde{B}^2\rangle}\langle\widetilde{\vec{B}\,\delta n\,\delta\vec{E}}\rangle.$$

The relation between $\tilde{\Gamma}_\psi$ as given by eq.(29) and the usual expressions for diffusion is as follows: In the stable case the terms in the last row vanish. In addition it holds that $\vec{R}_\perp \approx en\,\eta_\perp\,\vec{j}_\perp$, and hence it follows that $\frac{c}{e}\langle RR_{\perp\varphi}\rangle = -c^2\frac{\partial p}{\partial\psi}\langle\eta_\perp n\,R^2\frac{B_P^2}{B^2}\rangle$, which is the classical contribution in this geometry. The terms in the third and fourth rows describe neoclassical effects including pressure anisotropy and self-consistent E-field contributions. In the case of a turbulent plasma, anomalous contributions may result from the terms in the last row and from anomalous modifications of, for instance, $\overleftrightarrow{\Pi}^e$ or $\overrightarrow{\tilde{R}}$.

Eliminating $\tilde{\Gamma}_\psi$ between eqs.(29) and (22), again using eq.(28), now results in

$$(30)\quad \frac{V_a}{2\pi}\left\langle \tilde{n}\frac{\tilde{B}_\varphi}{R}\right\rangle \frac{\langle \tilde{\eta}_\parallel \tilde{n} R \tilde{B}_\varphi\rangle}{\langle \tilde{\eta}_\parallel \tilde{n} \tilde{B}^2\rangle} = \langle \tilde{n}\,\tilde{\eta}_\parallel R \tilde{j}_\varphi\rangle - \frac{1}{c}\tilde{\Gamma}^x_\psi$$

with the abbreviation

$$(31)\quad \tilde{\Gamma}^x_\psi = -c^2\frac{\partial\tilde{p}}{\partial\psi}\left\langle \tilde{\eta}_\parallel \tilde{n} R^2\frac{\tilde{B}_p^2}{\tilde{B}^2}\right\rangle$$
$$-c^2\frac{\partial\tilde{p}}{\partial\psi}\left\{\left\langle \tilde{\eta}_\parallel \tilde{n} R^2\frac{\tilde{B}_\varphi^2}{\tilde{B}^2}\right\rangle - \langle \tilde{\eta}_\parallel \tilde{n} R \tilde{B}_\varphi\rangle \frac{\langle \tilde{\eta}_\parallel \tilde{n} R \tilde{B}_\varphi\rangle}{\langle \tilde{\eta}_\parallel \tilde{n} \tilde{B}^2\rangle}\right\}$$
$$+c\,\frac{\langle \tilde{\eta}_\parallel \tilde{n} R \tilde{B}_\varphi\rangle}{\langle \tilde{\eta}_\parallel \tilde{n} \tilde{B}^2\rangle}\left\{\frac{1}{e}\langle \vec{\tilde{B}}\nabla\overleftrightarrow{\tilde{\Pi}}^e\rangle - \langle \tilde{n}\vec{\tilde{B}}\nabla\tilde{\phi}\rangle + \langle \vec{\tilde{B}}\,\widetilde{\delta n\,\delta \vec{E}}\rangle\right\}.$$

Equation (30) can be considered as one more form of the toroidal component of Ohm's law (but restricted to the stationary case). In the limit case discussed in the introduction eq.(30) obviously reduces to eq.(4) but with the effective diffusion velocity $\tilde{V}_D^x = \tilde{\Gamma}^x_\psi / \int_0^{2\pi} d\varphi \oint dl\, R\tilde{n}$ occurring instead of the real diffusion velocity and with $\tilde{\eta}_\parallel$ instead of η. Furthermore, it is easy to see from the left side of eq.(30) that

(32) $c\langle \tilde{\eta}_\parallel \tilde{n} R \tilde{j}_\varphi\rangle - \tilde{\Gamma}^x_\psi \geq 0$ must always be valid /3/, which is a necessary condition for a plasma under consideration to be in a stationary diffusive equilibrium. With the toroidal component of eq.(13) an order of magnitude estimate, if applied to eq.(32), gives

(33) $\tilde{V}_D^x \lesssim \tilde{V}_m'$, where $\tilde{V}_m' \sim \tilde{\eta}_\parallel c^2/4\pi L$ is the poloidal magnetic diffusion velocity. By analogy with the introduction we get from eq.(33) the equivalent condition

(34) $\beta \lesssim \dfrac{\tilde{V}_c'}{\tilde{V}_D^x}$, where $\tilde{V}_c' \sim 2c^2\tilde{p}\,\tilde{\eta}_\parallel/\tilde{B}^2\cdot L$

is the classical diffusion velocity, but computed with $2\tilde{\eta}_\parallel$ instead of $\tilde{\eta}_\perp$. Obviously condition (32) is the generalization of eqs.(6) and (7).

We now return to the questions that were raised in the introduction. First we note that the simple form (1) of Ohm's law only applies when the system is far from the limit set by condition (32), (33) or (34). These are related to the usual bootstrap condition. Indeed, in the stable case, when all terms which contain fluctuating quantities disappear, one concludes from eqs. (29) and (31) and the above remarks about $\frac{c}{e}\langle R\,\tilde{R}_{\perp\varphi}\rangle$ in the stable case that $\tilde{V}_D \approx \tilde{V}_D^{*}$ is valid. Thus, confining attention to, for instance, the banana regime, where $\tilde{V}_D \sim \tilde{V}_c' q^2 A^{3/2}$, it follows from eq.(34) that

(35) $\beta \lesssim 1/q^2 A^{3/2}$ or $\beta_P \lesssim A^{1/2}$,

which is the well-known bootstrap condition within a numerical factor of the order of unity.

We are chiefly interested in the question whether any anomalous enhancement of $\tilde{\Gamma}_\psi$ enhances $\tilde{\Gamma}_\psi^{*}$ too, thus leading to sharper restrictions than in the stable case owing to relation (32). The following general statements can be made:

Since $\vec{R}_\perp$ is no longer contained in $\tilde{\Gamma}_\psi^{*}$, any anomaly of $\vec{R}_\perp$, though it may enhance $\tilde{\Gamma}_\psi$, does not affect $\tilde{\Gamma}_\psi^{*}$ and hence the relation (32). By analogy it is concluded that magnetic field fluctuations have no direct impact on $\tilde{\Gamma}_\psi^{*}$ and the relation (32).

To draw further conclusions, one must refer to the specific instability underlying the diffusion mechanism. We shall do this briefly for trapped particle instabilities, which are responsible for the predominant diffusion in usual low-beta tokamak reactors.

These instabilities can be considered as purely electrostatic so that $\delta\vec{B}=0$ and $\delta\vec{E} = -\nabla\delta\phi$, where $\delta\phi$ is the fluctuating part of the electric potential. For these modes it typically holds that

(36) $K_\parallel/K_\perp \ll 1$, where $K_\parallel$ and $K_\perp$ are the parallel and perpendicular wave vector components. It is rather obvious that the instabilities under

consideration should not significantly modify $\tilde{\overleftrightarrow{\Pi}}^e$ and $\tilde{\eta}_{\parallel}$. Hence, considering eq.(31), it follows that any anomalous enhancement of $\tilde{\Gamma}_\psi^x$ would result from the term containing density and electric field fluctuations. With the approximation of eq.(35) it holds that

$$c\langle R\,\widetilde{\delta n\,\delta E_\varphi}\rangle \approx -\int_0^{2\pi} d\varphi \oint dl\, R\,c\, \frac{\widetilde{\delta n\,(\delta\vec{E}\times\tilde{\vec{B}})_\psi}}{\tilde{B}^2} \tag{37}$$

which is the average electron flux due to the ExB drift caused by the field fluctuations. It is this contribution which is given by usual diffusion formulae. Owing to eq.(35) one has, furthermore,

$$\left|\frac{\langle\tilde{\eta}_{\parallel}\,\tilde{n}\,R\,\tilde{B}_\varphi\rangle}{\langle\tilde{\eta}_{\parallel}\,\tilde{n}\,\tilde{B}^2\rangle}\langle\tilde{\vec{B}}\,\widetilde{\delta n\,\delta\vec{E}}\rangle\Big/\langle R\,\widetilde{\delta n\,\delta E_\varphi}\rangle\right| \sim \frac{K_{\parallel}}{K_\varphi} \ll 1 .$$

In conclusion, since the dominant term $c\langle R\,\widetilde{\delta n\,\delta E_\varphi}\rangle$ appears in eq.(29) but not in eq.(31), one has $\tilde{\Gamma}_\psi \gg \tilde{\Gamma}_\psi^x$, indicating that there will be no such dramatic consequence as in the case $\tilde{\Gamma}_\psi \sim \tilde{\Gamma}_\psi^x$ /3/. Unfortunately, the limit set by eq.(32) in the case of a stable plasma is rather low. Hence even a moderate enhancement of $\tilde{\Gamma}_\psi^x$ in the unstable case would be very unfavourable. This is not ruled out by the rough consideration above, and this point requires further discussion.

References

/1/ J.Kesner, R.W.Conn: Nuclear Fusion, Vol.16, Nr.3 (397)

/2/ P.H.Rutherford, D.F.Düchs: Princeton Plasma Physics Laboratory Report, MATT-1272

/3/ K.Borrass, D.Pfirsch: Max-Planck-Institut für Plasmaphysik Report, IPP 6/155

/4/ S.I.Braginskii: in Reviews of Plasma Physics, Vol. 1, Edit. M.A.Leontovich, New York 1965.

consideration should not significantly modify $\widetilde{\overleftrightarrow{\Pi}}^e$ and $\widetilde{\eta}_{\parallel}$. Hence, considering eq.(31), it follows that any anomalous enhancement of $\widetilde{\Gamma}_\psi^x$ would result from the term containing density and electric field fluctuations. With the approximation of eq.(35) it holds that

$$(37) \quad c\langle R\,\widetilde{\delta n\,\delta E_\varphi}\rangle \approx -\int_0^{2\pi} d\varphi \oint dl\, R c\, \frac{\widetilde{\delta n\,(\delta\vec{E}\times\widetilde{\vec{B}})_\psi}}{\widetilde{B}^2}$$

which is the average electron flux due to the ExB drift caused by the field fluctuations. It is this contribution which is given by usual diffusion formulae. Owing to eq.(35) one has, furthermore,

$$\left|\frac{\langle\widetilde{\eta}_{\parallel}\,\widetilde{n}\,R\widetilde{B}_\varphi\rangle}{\langle\widetilde{\eta}_{\parallel}\,\widetilde{n}\,\widetilde{B}^2\rangle}\langle\widetilde{\vec{B}}\,\widetilde{\delta n\,\delta\vec{E}}\rangle \Big/ \langle R\,\widetilde{\delta n\,\delta E_\varphi}\rangle\right| \sim \frac{K_{\parallel}}{K_\varphi} \ll 1 .$$

In conclusion, since the dominant term $c\langle R\,\widetilde{\delta n\,\delta E_\varphi}\rangle$ appears in eq.(29) but not in eq.(31), one has $\widetilde{\Gamma}_\psi \gg \widetilde{\Gamma}_\psi^x$, indicating that there will be no such dramatic consequence as in the case $\widetilde{\Gamma}_\psi \sim \widetilde{\Gamma}_\psi^x$ /3/. Unfortunately, the limit set by eq.(32) in the case of a stable plasma is rather low. Hence even a moderate enhancement of $\widetilde{\Gamma}_\psi^x$ in the unstable case would be very unfavourable. This is not ruled out by the rough consideration above, and this point requires further discussion.

References

/1/ J.Kesner, R.W.Conn: Nuclear Fusion, Vol.16, Nr.3 (397)

/2/ P.H.Rutherford, D.F.Düchs: Princeton Plasma Physics Laboratory Report, MATT-1272

/3/ K.Borrass, D.Pfirsch: Max-Planck-Institut für Plasmaphysik Report, IPP 6/155

/4/ S.I.Braginskii: in Reviews of Plasma Physics, Vol. 1, Edit. M.A.Leontovich, New York 1965.