Lower-Hybrid Wave Absorption in the Presence of Simultaneous Density and Magnetic Field Gradients

Satish Puri

IPP 4/169 June 1978

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

8046 GARCHING BEI MÜNCHEN

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

Lower-Hybrid Wave Absorption in the Presence of Simultaneous Density and Magnetic Field Gradients

Satish Puri

IPP 4/169 June 1978

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

IPP 4/169

Lower-Hybrid Wave Absorption in the Presence of Simultaneous Density and Magnetic Field Gradients

Satish Puri

June 1978

Abstract

It is shown that for the lower-hybrid wave (the slower of the two cold-plasma modes propagating at an angle to the static magnetic field), the WKB requirements are satisfied in almost the entire region from the antenna upto and including the ion-cyclotron-harmonic resonance. Expressions for the electric field and the energy density in the WKB approximation are derived. Both the collisionless and collisional damping become significant near the resonance and the wave attenuation due to the two processes is estimated. From the differential euqation for the electric field valid near the resonance, it is found that the wave energy would be completely absorbed even in the absence of a damping mechanism. The effect of impurities, although negligible near the plasma edge, becomes important following wave conversions and a significant fraction of the wave energy may end up heating the impurity ions in the plasma interior. The implications of using the "local" dielectric tensor approximation in the above context are examined.

1. INTRODUCTION

Although impressive progress has been made in understanding the problems of accessibility[1-3], coupling[2-7] and conversion[8-13] of the lower-hybrid wave (the slower of the two cold-plasma modes propagating at an angle to the static magnetic field) to the "plasma" and the "electrostatic" waves respectively, the precise linear mechanism involved in its eventual absorption remains elusive. In this paper we examine the various absorption processes namely collisional, cyclotron-harmonic and "singular-turning-point" attenuation of the lower-hybrid wave.

Fig. 1 shows the computed dispersion characteristics of the lower-hybrid wave in the presence of simultaneous density and magnetic field gradients. After conversions to the "plasma" and "electrostatic" waves respectively, the wave encounters the ion-cyclotron-harmonic resonance. The fate of the wave as it approaches this resonance is the principal subject of study of this paper. It is assumed throughout that the plasma may be described by the "local" dielectric tensor. A posteriori justification for this assumption is presented.

An isotropic, Maxwellian plasma at temperature T, uniform in the y and z directions is taken to be immersed in a static magnetic field B_0 along the z-direction. The plasma density and B_0 increase in the x-direction; the effects due to the gradient drifts and the currents necessary to maintain the magnetic field gradients in a slab geometry are ignored. All field quantities are assumed to vary as expi $(k_z z-\omega t)$ with no variation in the y-direction.

1. LINEAR AMPLIFICATION

Assuming that both the density and magnetic field gradients are sufficiently weak to allow geometric optics description of the plasma, the Maxwell's equations may be written in the approximation form[14, 15]

$$\underline{\nabla} \times \underline{E} = - \mu_0 \frac{\partial \underline{H}}{\partial t} \qquad (1)$$

and
$$\nabla \times \underline{H} = \epsilon_0 \frac{\partial(\tilde{\underline{\underline{\epsilon}}} \cdot \underline{E})}{\partial t} \tag{2}$$

where $\tilde{\underline{\underline{\epsilon}}} = \underline{\underline{\epsilon}} + O(\delta)$, δ is a small parameter of the order of fractional change in $\underline{\underline{\epsilon}}$ over one wavelength, $\underline{\underline{\epsilon}}$ is the hot-plasma dielectric tensor for the "locally homogeneous" plasma and may be written as

$$\underline{\underline{\epsilon}} = \epsilon_0 \begin{pmatrix} \epsilon_x & \epsilon_y & 0 \\ -\epsilon_y & \epsilon_x & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix} \tag{3}$$

where

$$\epsilon_x = 1 + \sum_j \frac{\omega_p^2}{\omega\,\omega_c}\, \xi_0 \frac{e^{-\Lambda}}{\Lambda} \sum_{-\infty}^{\infty} p\, I_p(\Lambda)\, Z_p \tag{4}$$

$$\epsilon_y = i \sum_j s_j \frac{\omega_p^2}{\omega\,\omega_c}\, \xi_0\, e^{-\Lambda} \sum_{-\infty}^{\infty} I_p'(\Lambda)\, Z_p \tag{5}$$

$$\epsilon_z = 1 - \sum_j \frac{\omega_p^2}{\omega^2}\, \xi_0^2\, e^{-\Lambda} \sum_{-\infty}^{\infty} I_p(\Lambda)\, Z_p' \tag{6}$$

j represents the particle species, I_p is the modified Bessel function, Z_p is the plasma dispersion function defined by

$$Z_p = Z(\xi_p) = -i\pi^{1/2} e^{-\xi_p^2} + \pi^{-1/2} \int_{-\infty}^{\infty} \frac{e^{-t^2}\,dt}{t - \xi_p}, \quad \mathrm{Im}\ \xi_p < 0 \tag{7}$$

$$\xi_p = \frac{c}{n_z v_z}\left(1 - \frac{p\,\omega_{ci}}{\omega}\right) \tag{8}$$

$$\Lambda = (1/2)\, k_x^2 r_{ci}^2, \quad Z_p' = \partial Z_p/\partial \xi_p, \quad I_p' = \partial I_p/\partial \Lambda$$

and v_z is the ion thermal speed. In writing $\underline{\underline{\epsilon}}$ in the form (3) - (6), terms of the order of $(v_{th}/c)^2$, where v_{th} is the particle thermal speed have been neglected.[16] In confernity with the geometric optics assumption, all field quantities will be assumed to possess the WKB form

$$E_x(x) = (2\eta R)^{1/2} \varphi^{-1}(x) \exp i\int^x k_x(x)\, dx \qquad (9)$$

where $\eta = (\mu_0/\epsilon_0)^{1/2}$; R (= P+Q), $P = (1/2)(E_y H_z^* - E_z H_y)$ and $Q = -(\omega/2)\, \underline{E}^* \partial \underline{\underline{\epsilon}}/\partial k \cdot \underline{E}$ are the total, the electromagnetic and the kinematic energy fluxes respectively and

$$\varphi^2(x) = -\Big[\Big\{ \frac{n_x \epsilon_y^2}{(n^2 - \epsilon_x)^2} + \frac{n_x n_z^2 \epsilon_z}{(n_x^2 - \epsilon_z)^2} \Big\}$$

$$+ k_0 \Big\{ \frac{\partial \epsilon_x}{\partial k_x} - \frac{\epsilon_y}{n^2 - \epsilon_x} \frac{\partial \epsilon_y}{\partial k_x} + \frac{n_x n_z}{n_x^2 - \epsilon_z} \frac{\partial \epsilon_z}{\partial k_z} \Big\}\Big] \qquad (10)$$

is chosen so as to conserve the total energy flux and is equivalent to the statement of energy conservation[15]. In Ref. 15 it is shown that the electric field amplitude obtained in the above manner is accurate to the lowest order expansion in δ. The refractive indices $n_x = k_x/k_0$ and $n_z = k_z/k_0$ where $k_0 = \omega/c$ are the values for the "locally homogeneous" plasmas. In the slab-plasma model, nz, the refractive index in the direction of the static magnetic field is chosen so as to be compatible with the requirements of accessibility and coupling[1-7] while n_x is given by

$$p_0(\underline{\underline{\epsilon}})\, n_x^4 + r_0(\underline{\underline{\epsilon}})\, n_x^2 + t_0(\underline{\underline{\epsilon}}) = 0 \qquad (11)$$

where

$$p_0(\underline{\underline{\epsilon}}) = \epsilon_x \qquad (12)$$

$$r_0(\underline{\underline{\epsilon}}) = \epsilon_x^2 + \epsilon_y^2 + \epsilon_x\epsilon_z - \epsilon_x n_z^2 - \epsilon_z n_z^2 \qquad (13)$$

and $$t_o(\underline{\underline{\epsilon}}) = \epsilon_z(\epsilon_x^2 + \epsilon_y^2 - 2\epsilon_x n_z^2 + n_z^4) . \tag{14}$$

For the validity of the geometric optics description one requires that

$$\delta_1 = |(\partial\epsilon_i/\partial x)/k_x\epsilon_i| << 1 \tag{15}$$

for each element i of the dielectric tensor $\underline{\underline{\epsilon}}$, plus the WKB requirements

$$\delta_2 = |(\partial\varphi/\partial x)/k_x\varphi| << 1, \tag{16}$$

and $$\delta_3 = |(\partial k_x/\partial x)/k_x^2| \ll 1. \tag{17}$$

Near the plasma edge, $R \simeq P$ and it can be readily shown from (15) - (17) that for the lower-hybrid wave, the WKB condition requires that

$$\omega_{pe}^2(x)/\omega^2 >> 1,$$

a condition fulfilled easily close to the edge of the plasma; and perhaps at the antenna surface itself in case a tenuous plasma extends upto the antenna.

Near the cyclotron-harmonic resonance, $n_x >> 1$, $R \simeq Q$ so that

$$\varphi^2(x) \simeq -k_o\,\partial\epsilon_x/\partial k_x \tag{18}$$

and $$\delta_2 = (\partial^2\epsilon_x/\partial k_x\partial x)/(2k_x\,\partial\epsilon_x/\partial k_x) . \tag{19}$$

From (11), one obtains the approximate dispersion relation

$$\epsilon_x \simeq 0 \text{ as } n_x >> 1. \tag{20}$$

This is the familiar dispersion relation for the electrostatic Gross[17]-Bernstein[18] waves with finite value of k_z. The successive conversions of the lower-hybrid wave to the "plasma" and "electrostatic" waves near $\Lambda \sim 1$ has been treated in detail by several authors[8-13]. In a narrow range defined by

$$0 \lesssim |\xi_p| \lesssim \xi_{max} = \varepsilon (c/n_z v_z) \tag{21}$$

in the vincinty of the pth cyclotron harmonic, (20) simplifies to

$$\epsilon_x \simeq \alpha + \frac{p \beta \gamma Z_p}{k_x^3 \, r_{ci}^3} = 0 \tag{22}$$

where

$$\alpha = 1 + (\omega_{pe}/\omega_{ce})^2 \tag{23}$$

$$\beta = 2 \pi^{-1/2} (\omega_{pi}^2 / \omega \omega_{ci}) \tag{24}$$

and $$\gamma = c/n_z v_z \, . \tag{25}$$

In (21) ε should be sufficiently small so that the pth term in the summation in (4) dominates. For the case of a single ion-species plasma, this requirement would be satisfied for $\varepsilon \lesssim 0.1$. Therefore we take

$$\xi_{max} = 0.1 \, (c/n_z v_z) \, . \tag{26}$$

In the case of a multispecies plasma or for an impurity harmonic the condition for the dominance of the pth term in (4) becomes

$$\left| \frac{\omega_{pi}^{*2}}{\omega_{ci}^*} \frac{e^{-\Lambda^*}}{\Lambda^*} \frac{p^*}{v_{zi}^*} I_{p^*}(\Lambda^*) Z_{p^*} \right| \gg \left| \frac{\omega_{pj}^2}{\omega_{cj}} \frac{e^{-\Lambda_j}}{\Lambda_j} \frac{p_j}{v_{zj}} I_{p_j}(\Lambda_j) Z_{p_j} \right| \tag{27}$$

where the asterik denotes impurity ion contribution and the right-hand side represents the maximum of contributions from among the neighbouring cyclotron harmonics. From (22)

$$k_x = -r_{ci}^{-1} (\alpha^{-1} \beta \gamma p Z_p)^{1/3} \tag{28}$$

$$\frac{\partial k_x}{\partial x} = \frac{2}{3} \frac{k_x}{R} \gamma \frac{p\omega_{ci}}{\omega} \frac{1 + q_p Z_p}{Z_p} \tag{29}$$

$$\frac{\partial \epsilon_x}{\partial k_x} \simeq \frac{3\alpha}{k_x} \tag{30}$$

and $$\frac{\partial^2 \epsilon_x}{\partial k_x \, \partial x} \simeq - \frac{2\alpha\gamma}{k_x R} \frac{p\omega_{ci}}{\omega} \frac{1 + q_p Z_p}{Z_p} \tag{31}$$

where R is the scale length of variation of B_0. From (1) (17) and (28) - (31) one finally obtains

$$\delta_1 \simeq \left| \frac{\alpha - 1}{n_z^2 - 1} \frac{m_e}{m_i} \frac{k_x}{k_o^2 R} \right| \tag{32}$$

$$\delta_2 \simeq \left| \frac{1}{3} \left(\frac{\pi}{4}\right)^{1/6} \alpha^{1/3} \gamma^{2/3} p^{-2/3} Z_p^{-1/3} \frac{r_{ci}}{R} \left(\frac{\omega_{ci}^2}{\omega \omega_{pi}}\right)^{2/3} \frac{1 + q_p Z_p}{Z_p} \right. \tag{33}$$

and $$\delta_3 = \left| \frac{2}{3} \frac{\gamma}{R k_x} \frac{p\omega_{ci}}{\omega} \frac{1 + q_p Z_p}{Z_p} \right| . \tag{34}$$

In deriving (28) - (31) it was assumed that the quantities $\lambda, \alpha, \beta, \gamma$ and r_{ci} possess derivatives small compared to that of Z_p which is given by

$$\frac{\partial Z_p}{\partial x} \simeq \frac{2}{R} \gamma \frac{p\omega_{ci}}{\omega} (1 + q_p Z_p) .$$

For the assumed reactor parameters of Table I, $\delta_1 \lesssim 10^{-2}$, $\delta_2 \lesssim 10^{-4}$ and $\delta_3 \lesssim 10^{-3}$, i.e. the WKB approximation is valid from close to the plasma edge upto and including the resonant layer. A similar test in the wave-conversion region where $\Lambda \sim 1$ would show that δ << 1 and the WKB assumption is valid. This implies that the lower-hybrid wave propagates without reflection and the entire wave energy coupled into the plasma by the launching antenna is somehow assimilated into the plasma. The particular manner of energy assimilation in the plasma will be examined in the next three sections.

From (9), (28) and (30) the magnitude of the electric field in the plasma is given by

$$E_x = \left(\frac{\lambda_0 \eta R}{3\pi r_{ci}} \right)^{1/2} \alpha^{-2/3} \left(\beta \gamma p Z_p \right)^{1/6} . \tag{34}$$

For an energy flux of 1 Watt/cm^2, E_x attains the value of approximately 450 volt/cm near the cyclotron-harmonic resonance for the parameters of Table I. The corresponding energy density in the wave

$$W_0 = \frac{1}{4} \underline{E}^* \cdot \frac{\partial}{\partial \omega} (\omega \underline{\underline{\epsilon}}) \cdot \underline{E} \simeq \frac{\epsilon_0 E^2}{2} \alpha \gamma \frac{p \omega_{ci}}{\omega} \frac{1 + \varrho_p Z_p}{Z_p} \tag{35}$$

is about 10^{-4} J/cm^3. Not until the energy flux is increased to about 1 kW/cm^2 with a corresponding vacuum field of over 1 kV/cm near the launching antenna, does the energy density at the resonance approach the value of the thermal energy density of 0.1 J/cm^3.

3. WAVE ABSORPTION

Form (28) using (7) one obtains the imaginary part of k_x as

$$k_{xi} = r_{ci}^{-1} \left(\alpha^{-1} \beta \gamma p \right)^{1/3} \left[\pi^{-1/6} e^{-(1/3)\varrho_p^2} - \frac{2^{1/3}}{3} (1 - \alpha^{-1}) \frac{\nu_{ei}}{\omega} Z_p^{1/3} \right] \tag{36}$$

where the two terms in the parenthesis arise from the cyclotron-harmonic and collisional dissipation respectively. The electron-ion momentum transfer collisions have been simulated by replacing the electron mass m_e by m_e $(1+ i\nu_{ei}/\omega)$ in (28). The total wave attenuation in the WKB approximation is given by $e^{-\chi}$, where

$$\chi = \int^{x} k_{xi}\, dx \simeq \frac{2R}{r} \frac{\omega}{p\omega_{ci}} \int_{0}^{\xi_{max}} k_{xi}\, d\xi_p \; . \tag{37}$$

Treating the two absorption terms in (36) separately one obtains

$$\chi_{CH} \simeq \frac{3^{1/2}\, \pi^{1/3}}{2} \frac{R}{r_{ci}} (\alpha^{-1} \beta p)^{1/3} \frac{\omega}{p\omega_{ci}} \left(\frac{n_z v_z}{c}\right)^{2/3} \tag{38}$$

and $$\chi_{coll} \simeq (20)^{-2/3} \frac{R}{r_{ci}} \left(\frac{\beta \omega}{\omega_{ci}}\right)^{1/3} (1-\alpha^{-1}) \frac{\nu_{ei}}{\omega} \; . \tag{39}$$

In obtaining (38) from (37), the integration limit ξ_{max} was replaced by ∞ in view of the rapidly decaying exponent $\exp(-\xi_p^2)$ in the integrand. Also since most of the contribution in (39) comes from values of $\xi_p > 1$, Z_p in (37) was approximated by $-\xi_p^{-1}$ while the relatively minor contribution from the lower limit in the integral was ignored.

Note that $\chi_{coll} \to 0$ as $\nu_{ei} \to 0$ but is independent of n_z. χ_{CH}, on the other hand, vanishes as $n_z^{2/3}$ with $n_z \to 0$. For the parameters of Table I, $\chi_{CH} \simeq 380$, $\chi_{coll} \simeq 4.6 \times 10^5$ (ν_{ei}/w), and together they would ensure complete absorption of the rf energy in the vicinity of the cyclotron-harmonic resonance in a fusion type machine.

For the smaller machines currently in operation, however, $\chi \lesssim 1$ and the wave energy is only partially absorbed. Yet δ in (32) - (34) is still small enough to allow WKB propagation without reflection. What happens to the wave energy in such a case warrants a closer look at the wave propagation in the vicinity of the resonance. We proceed to resolve this problem in the next section.

4. ENERGY ACCUMULATION AT THE SINGULAR TURNING POINT

Near the resonance since $n_x >> 1$, one obtains from (1) and (2) after neglecting terms of the order n_x^{-1} compared to unity

$$\frac{\partial^2 E_x}{\partial x^2} + 2\,\frac{\tilde{\epsilon}_x'}{\tilde{\epsilon}_x}\,\frac{\partial E_x}{\partial x} + \frac{k_o^2}{\tilde{\epsilon}_x}\left(\tilde{\epsilon}_y^2 - n_z^2\,\tilde{\epsilon}_z\right) E_x = 0 \tag{40}$$

Introducing

$$\mathcal{E}_x(x) = \tilde{\epsilon}_x\, E_x(x) \tag{41}$$

in (40) and neglecting terms containing $\tilde{\epsilon}_x''$, one obtains

$$\frac{\partial^2 \mathcal{E}_x}{\partial x^2} + \frac{k_o^2\left(\tilde{\epsilon}_y^2 - n_z^2\,\tilde{\epsilon}_z\right)}{\tilde{\epsilon}_x}\,\mathcal{E}_x = 0 \tag{42}$$

Near the resonance writing $\tilde{\epsilon}_x(x) = \tilde{\epsilon}_x(o) + x\tilde{\epsilon}_x'(o)$, and $\xi = [\tilde{\epsilon}_x(o)/\tilde{\epsilon}_x'(o)] + x$, (42) gives

$$\frac{\partial^2 \mathcal{E}_x}{\partial \xi^2} + \xi^{\mu}\,\psi^2(x)\,\mathcal{E}_x = 0 \tag{43}$$

where $\mu = -1$ and,

$$\psi^2(x) = \frac{k_o^2}{\tilde{\epsilon}_x'(o)}\left(\tilde{\epsilon}_y^2 - n_z^2\,\tilde{\epsilon}_z\right). \tag{44}$$

We assume that $\psi^2(x)$ is almost constant in a narrow neighbourhood of the resonance; the justification for this assumption occurs later in the section. For $\psi^2(x)$ constant (43) has the solution

$$\mathcal{E}_x(x) = \xi^{1/2} H_1^{(1,2)} (2\psi\xi^{1/2}) \tag{45}$$

which together with (41) gives

$$E_x(x) = \tilde{\epsilon}_x^{-1} \xi^{1/2} H_1^{(1,2)} (2\tilde{k}_x \xi) \tag{46}$$

where $H_1^{(1,2)}$ are the Hankel functions of kind 1 and 2 and of order unity and

$$\tilde{k}_x^2 = k_o^2 \frac{\tilde{\epsilon}_y^2 - n_z^2 \tilde{\epsilon}_z}{\tilde{\epsilon}_x(x)} . \tag{47}$$

For $2\tilde{k}_x \xi >> 1$ i.e. for

$$\left|\frac{x}{\lambda_o}\right| >> \left|\frac{x_o}{\lambda_o}\right| = \frac{1}{8\pi^2(n_z^2-1)} \frac{m_e}{m_i} \frac{\lambda_o}{R} \sim 10^{-6} \tag{48}$$

we may use the asymptotic form of (46) and obtain,

$$E_x(x) = \epsilon_x^{-1} \xi^{1/4} (2k_x)^{-1/2} \left\{ A e^{i2k_x\xi} + B e^{-i2k_x\xi} \right\}, \tag{49}$$

where the tildas have been dropped from $\underline{\underline{\epsilon}}$ and k_x because away from the resonance $\tilde{\underline{\underline{\epsilon}}} \to \underline{\underline{\epsilon}}$ and $\tilde{k}_x \to k_x$. For $x \to 0$, k_x has the form (see Appendix A)

$$k_x \sim C + iD , \qquad D > C > 0 . \tag{50}$$

In order that E_x remains finite for $x >> x_0$, $B \equiv 0$ in (49). This implies the absence of a reflected wave[19, 20]. In Ref. 19 it is explained that in such a case the energy transported by the incoming wave would simply accumulate in the region of low group velocity near the resonance in the absence of dissipative processes. In obtaining the above result, it is not required to assume that $|k_x| \to \infty$ at the resonance; it is only necessary that (i) for large x, the waves possess propagating and evanescent character for $x < o$ and $x > o$ respectively and (ii) the asymptotic expansion (49) be valid in the region of interest. This latter condition is satisfied for $\psi^2(x)$ = constant for $-2 < \mu < 0$. One may readily verify that in the narrow range $|x| \lesssim |x_0|$, the condition $\psi^2(x) \approx$ constant is indeed well satisfied. The results of this section are a generalization of the "singular turning point" analysis of Budden[19] to the case where there is no actual resonance of the type $|k_x| \to \infty$ at some physical location in the plasma. Since an indefinite energy accumulation in a finite region of space is contrary to the spirit of the linearized theory, the problem has to be reformulated allowing for the existence of non-linear processes.

5. IMPURITY ABSORPTION

During its passage from the edge towards the plasma interior, the lower-hybrid wave is intercepted by a number of ion-cylcotron harmonics (Fig. 1) including those of the impurity ions. Prior to wave conversions, $\Lambda \simeq 1$, and the primarily electromagnetic wave is correctly described only by the full dispersion relation (11). As the wave crosses a cyclotron harmonic, part of the wave energy is removed by the particular ion species. This appears as a perturbation δn_x (or δk_x) in n_x and one obtains from (11)

$$\delta k_x = f\,\delta\epsilon_x + g\,\delta\epsilon_y + h\,\delta\epsilon_z \qquad (51)$$

where

$$f = -k_0 \left[n_x^4 + n_x^2 (2\epsilon_x + \epsilon_z - n_z^2) + 2\epsilon_z (\epsilon_x - n_z^2) \right] / \Delta \tag{52}$$

$$g = -k_0 \, 2\epsilon_y (n_x^2 + \epsilon_z) / \Delta \tag{53}$$

$$h = -k_0 \left[n_x^2 (\epsilon_x - n_z^2) + \epsilon_x^2 + \epsilon_y^2 - 2\epsilon_x n_z^2 + n_z^4 \right] / \Delta \tag{54}$$

and $\Delta = 4 p\, n_x^3 + 2 r\, n_x \, .$ (55)

The wave attentuation due to the pth impurity harmonic is given by $\exp(-\chi_{imp})$, where

$$\chi_{imp} = \int^{x} \delta k_{xi} \, dx$$

$$= \frac{R}{\gamma^*} \int^{\xi_p} \frac{\omega}{p^* \omega_{ci}^*} \delta k_{xi} \, d\xi_{p^*} \tag{56}$$

the asterik denotes the impurity contribution and the integration is carried across the harmonic over the region where δk_{xi} becomes significant. From (7), one observes that $\mathrm{Im}\,\delta\epsilon_z \sim -2\pi^{-1/2} \xi_p e^{-\xi_p^2}$ is an odd function of ξ_p; (54) – (56) show that its net contribution to χ_{imp} vanishes. One can also show that the attenuation due to the term containing $\delta\epsilon_y$ in (51) is small compared to the contribution from the term with $\delta\epsilon_x$ so that (56) becomes

$$\chi_{imp} \simeq \frac{R}{\gamma^*} \int \frac{\omega}{p^* \omega_{ci}^*} f \, \delta\epsilon_{xi}^* \, d\xi_{p^*}$$

which from (4) and (7) gives

$$\chi_{imp} \simeq \pi^{-1/2} f R \left(\frac{\omega_{pi}^*}{\omega_{ci}^*} \right)^2 \frac{e^{-\Lambda^*} I_p(\Lambda^*)}{\Lambda^*} \int_{-\infty}^{\infty} e^{-\xi_{p^*}^2} d\xi_p$$

$$= \frac{fR}{2}\left(\frac{\omega_{pi}^{*}}{\omega_{ci}^{*}}\right)^{2} \frac{e^{-\Lambda^{*}} I_p(\Lambda^{*})}{\Lambda^{*}} . \qquad (57)$$

In obtaining (57) from (56) all quantities varying slowly with x were taken out of the integrand while the limits of integration were extended to $\pm\infty$ in view of the fast decaying function exp $(-\xi_p^{*2})$.

For $\Lambda^* \lesssim 1$ corresponding to the region before wave conversions, $I_p^*(\Lambda^*) \lesssim 10^{-20}$, $f \lesssim 10^5$, $(\omega_{pi}^*/\omega_{ci}^*)^2 \lesssim 10^5$ while $R \sim 1$ so that there is negligible wave attenuation due to the impurity ions or for that matter by the ions of the majority species itself.

Following wave conversions, however, Λ^* increases rapidly and the wave is subject to significant damping at the impurity cyclotron harmonics. In the event when ω_{pi}^{*2} is large enough to satisfy (27), the impurity absorption is no longer a perturbation and the rf wave will be fully absorbed by the impurity ions. This could happen for relatively small values of ω_{pi}^{*2} . Thus even minor impurity concentrations in the plasma interior could rob the wave of a considerable fraction of its energy.

This picture changes somewhat if one were to attempt plasma heating at a low cyclotron harmonic e.g. at $\omega = \omega_{ci}$ using the lower-hybrid wave (Fig. 2). The wave conversions, in this case, occur close to the plasma edge (or alternatively if the antenna protrudes past the wave conversion region, the electrostatic ion-cyclotron wave would be directly launched by the antenna) and for small values of p^*, χ_{imp} becomes significant at the plasma edge itself.

5. THE LOCAL APPROXIMATION

An important assumption in the foregoing analysis involved treating the plasma as "locally" homogeneous. Near the plasma edge where the local wavelength $\lambda(x) \gg r_{ci}(x)$ corresponding to $\Lambda(x) \ll 1$, the plasma acts like a cold medium in which the "local" approximation is rigorously valid even when the relations (15) - (17) are not satisfied. In the interior of a thermonuclear plasma, the gradient length $R \gg r_{ci}$ and once again little error is incurred in describing the plasma by the local dielectric tensor (3).

In the sensitive region surrounding the resonance we have already seen that the geometric optics conditions (15) - (17) are easily satisfied. However, since $\underline{\underline{\epsilon}}^A$ becomes comparable to $\underline{\underline{\epsilon}}^H$ (where A and H denote the antihermetian and hermetian components respectively) near the resonance one may sense an apparent conflict with the usage of the ray theory. This problem, however, is only superficial. One may derive the results pertinent to the usage of "local" approximation by retaining $\underline{\underline{\epsilon}}^A$ along with $\underline{\underline{\epsilon}}^H$ in the total $\underline{\underline{\epsilon}}$ as a zeroeth order quantity in a treatment paralleling that of Ref. 15; so that only the derivatives of $\underline{\underline{\epsilon}}$ and the electric field multiplied with the zeroeth order quantities appear as higher order terms. Note further that, in any case, the important result in Sec. 4 that the wave is totally absorbed at the resonance is insensitive to the precise detail at the resonance; only requiring the propagation constant to have propagating and evanescent character respectively away from the resonance plus the requisite conditions for the validity of the asymptotic expansion (49). The quantitative results (38) and (39) should, in any case, be viewed as estimates because of the approximations used in evaluating the integral in (37).

Another approximation in using the "local" $\underline{\underline{\epsilon}}$ near the resonance arises from the rapid variation of the form $\exp(-\xi_p^2)$ of plasma dispersion function $Z(\xi_p)$ and hence of k_x. Physically, the imaginary part of Z_p arises from an energy transfer from the wave to the group of particles resonating at their doppler shifted frequency. In a uniform static magnetic field, the particle velocity in the linearized theory in an electric field $E = E_o \exp i(k_x x + k_z z - \omega t)$ is given by

$$v(x) \sim \sum_{-\infty}^{\infty} J_p(\Lambda) \frac{e^{i(k_z v_z + p\omega_{gi} - \omega)t}}{k_z v_z + p\omega_{gi} - \omega} \tag{58}$$

where ω_{gi}, the particle gyrofrequency, is identical to ω_{ci}, the "local" cyclotron frequency, in a homogeneous plasma. For a given k_z near the pth cyclotron harmonic, the group of particles with the parallel velocity

$$v_z = \frac{\omega - p\omega_{gi}}{k_z} \tag{59}$$

resonates with the wave. If ω_{ci} varies with x, ω_{gi} departs slightly from ω_{ci}. Since v_z varies rather rapidly involving a different group of particles from within the velocity distribution function in order to keep in step with the resonance, this slight shift in ω_{ci} causes a large change in the energy transfer to the particles due to the exponentially varying energy distribution function of the Maxwellian as exhibited by the exponent $\exp(-\xi_p^2)$ in the imaginary part of $Z(\xi_p)$. Due to the systematic shift of ω_{gi} from ω_{ci}, this subtle error excapes detection by the tests (15) - (17).

In the present context, fortunately, we are not so concerned with k_{xi} but only with its integral $\int k_{xi}\,dx$ which is insensitive to the precise variations of ξ_p with x because ultimately all the particles get their chance to partake of the wave energy. Therefore, these modifications of the "local" dielectric tensor may though lead to incorrect values of k_{xi} in an inhomogeneous plasma, the integrated attenuation χ found in Sec.(2-4) is still valid.

We thus conclude that the "local" dielectric tensor approximation though crucial, is not critical in the present context.

7. DISCUSSION AND CONCLUSIONS

We saw in the last section that the slab inhomogeneity merely caused the local gyrofrequency ω_{gi} to depart slightly from the local cyclotron-frequency ω_{ci} causing a minor shift in the absorption profile without affecting the total wave attenuation. In reality, however, the toroidal drifts cause the particles guiding centers to move through a varying magnetic field so that ω_{gi} is a function of time as well. Naturally the particles' phase relationship with the wave changes constantly both by the finite gyroradius as well as its guiding center drift. The latter effect may be simulated by introducing an element of stochasticity in the energy absorption process represented by an effective collision frequency $\nu_s = v_{thi}/\pi QRS$ there Q, R and S are the safety factor, plasma radius and the torus aspect ratio respectively. In practice, ν_s is low enough to leave the absorption results of Sec. 2 - 4 substantially unaffected. Another effect of the particle's drift would be to distribute the wave energy over a larger number of particles and to cause a spatial spread in the absorption profile. Both these effects, even when small,will tend to limit the non-linear buildup at the resonance.

In conclusion we note that within the context of the linearized theory, the lower-hybrid wave is fully absorbed at the ion cyclotron-harmonic resonance following wave conversion to the Gross-Bernstein electrostatic wave. The wave though immune from impurity resonances near the plasma edge may cause considerable heating of the impurity ions in the plasma interior.

ACKNOWLEDGEMENT

I wish to express my sincere appreciation to Prof. D. Pfirsch for his interest in this work and for several helpful discussions about the usage of the WKB approximation.

FIGURE CAPTIONS

Fig. 1 Lower-hybrid wave dispersion characteristics at fixed ω in the presence of simultaneous density and magnetic field gradients for $n_z = 1.5$, $T_e = T_i = 100$ eV, $B_o = 60$ kG at the lower-hybrid resonance layer and $\omega_{LH} = 3.5\omega_{ci}$. The density is assumed to vary linearly from $n_e = 0$ at $\omega/\omega_{ci} = 7$ to $n_e = n_{LH}$ at $\omega/\omega_{ci} = 3.5$. The solid curve is for ω/ω_{ci} versus $\log(k_{xr})$ and the dotted curve shows $\log(k_{xi})$ versus $\log(k_{xr})$. The $\log(k_{xi})$ scale is iterated for $4 \leq \log(k_{xi}) \leq 0$ between each pair of cyclotron harmonics.

Fig. 2 Same as Fig. 1 except that $\omega_{LH} = 1.5\omega_{ci}$.

TABLE CAPTIONS

Table I Assumed reactor parameters used for quantitative estimates in this paper.

REFERENCES

1 T.H. Stix, The Theory of Plasma Waves, McGraw Hill, N.Y. (1962).

2 V.E. Golant, Zh. Tekh. Fiz. 41, 2492 (1971); Sov. Phys. Tech. Phys. 16, 1980 (1972).

3 R. Parker, RLE-QPR No. 102, MIT, Cambridge, Mass. 97 (1971).

4 C.F.F. Karney, A. Bers and J.L. Kulp, APS Bulletin 18 (1973) Paper 2 B15.

5 P. Lallia, Symposium on Plasma Heating in Toroidal Devices, Varenna (1974), p. 120.

6 S. Puri, Second International Conference on Waves and Instabilities in Plasmas, Innsbruck (1975), p. P1.

7 M. Brambilla, Nucl. Fusion 16, 47 (1976).

8 T.H. Stix, Phys. Rev. Lett. 15, 878 (1965).

9 V.M. Glagolev, Plasma Phys. 14, 301 (1972).

10 S.S. Pesic, Plasma Phys. 15, 193 (1973).

11 M.D. Simonutti, Phys. Fluids 18, 1524 (1975).

12 K.C. Wong and T. Tang, Second International Conference on Waves and Instabilities in Plasmas, Innsbruck (1975), Paper H7.

13 M. Brambilla, Plasma Phys. 18, 669 (1976).

14 S. Weinberg, Phys. Rev. 126, 1899 (1962).

15 I.B. Bernstein, Phys. Fluids 18, 320 (1975).

16 S. Puri, F. Leuterer and M. Tutter, J. Plasma Phys. 14, 169 (1975).

17 E.P. Gross, Phys. Rev. 82, 232 (1951).

18 I.B. Bernstein, Phys. Rev. 109, 10 (1958).

19 K.G. Budden, Radio Waves in the Ionosphere, Cambridge University Press (1966).

20 S. Puri, Third Topical Conference on Radio-Frequency Plasma Heating, Pasadena (1978), p. E3-1. Despite some algebraic errors in Eqs. (5) - (7), the results in this paper are correct.

APPENDIX A

From (4), the approximate dispersion relationn $\epsilon_x = 0$ near the pth cyclotron harmonic may be expressed as

$$\frac{\Lambda e^{\Lambda}}{I_p(\Lambda)} \simeq -p\gamma \frac{\omega_{pi}^2}{\omega\,\omega_{ci}} Z_p \ . \tag{A1}$$

Above the harmonic $\varphi_p > 0$ so that Re $Z_p < 0$ and from (A1) for $\varphi_p \gg 1$

$$\frac{\Lambda e^{\Lambda}}{I_p(\Lambda)} = |\Theta_+| + i\,\Phi_+ \ , \ |\Theta_+| \gg |\Phi_+| \tag{A2}$$

Since $|\Lambda| \gg 1$ near the harmonic

$$I_p(\Lambda) \sim \frac{e^{\Lambda}}{(2\pi\Lambda)^{1/2}} \ , \ |\arg \Lambda| < \frac{\pi}{2} \tag{A3}$$

which gives from (A2)

$$(2\pi)^{1/2}\,\Lambda^{3/2} = |\Theta_+| + i\,\Phi_+ \tag{A4}$$

or

$$\Lambda \sim |\Theta_+| + i\,\phi_+ \ , \ |\Theta_+| \gg |\phi_+| \tag{A5}$$

which is consistent with the region of validity of the asymptotic expansion (A3).

Below the harmonic $\varphi_p < 0$, Re $Z_p > 0$ and (A1) gives for $|\varphi_p| \gg 1$

$$\frac{\Lambda e^{\Lambda}}{I_p(\Lambda)} \simeq -|\Theta_-| + i\,\Phi_- \ , \ |\Theta_-| \gg |\Phi_-| \tag{A6}$$

In this case it is not possible to find an asymptotic solution for (A6) consistent with the region of validity in the manner of (A5). Writing

$$I_p(\Lambda) = I_p(-\Lambda e^{i\pi}) = e^{i\pi p} I_p(-\Lambda) \tag{A7}$$

gives from (A6),

$$\frac{\bar{\Lambda} e^{-\bar{\Lambda}}}{e^{i\pi p} I_p(\bar{\Lambda})} = |\Theta_-| - i\,\Phi_- \tag{A8}$$

where $\bar{\Lambda} = -\Lambda$. Using the asymptotic expansion

$$I_p(\bar{\Lambda}) \sim \frac{e^{\bar{\Lambda}}}{(2\pi\bar{\Lambda})^{1/2}} \quad , |\arg \bar{\Lambda}| < \frac{\pi}{2} \tag{A9}$$

one gets

$$(2\pi)^{1/2}\, \bar{\Lambda}^{3/2}\, e^{-(2\bar{\Lambda} + i\pi p)} = |\Theta_-| - i\,\Phi_- \tag{A10}$$

which allows a solution of the form

$$\Lambda = -|\Theta_-| + i\,\phi_- \tag{A11}$$

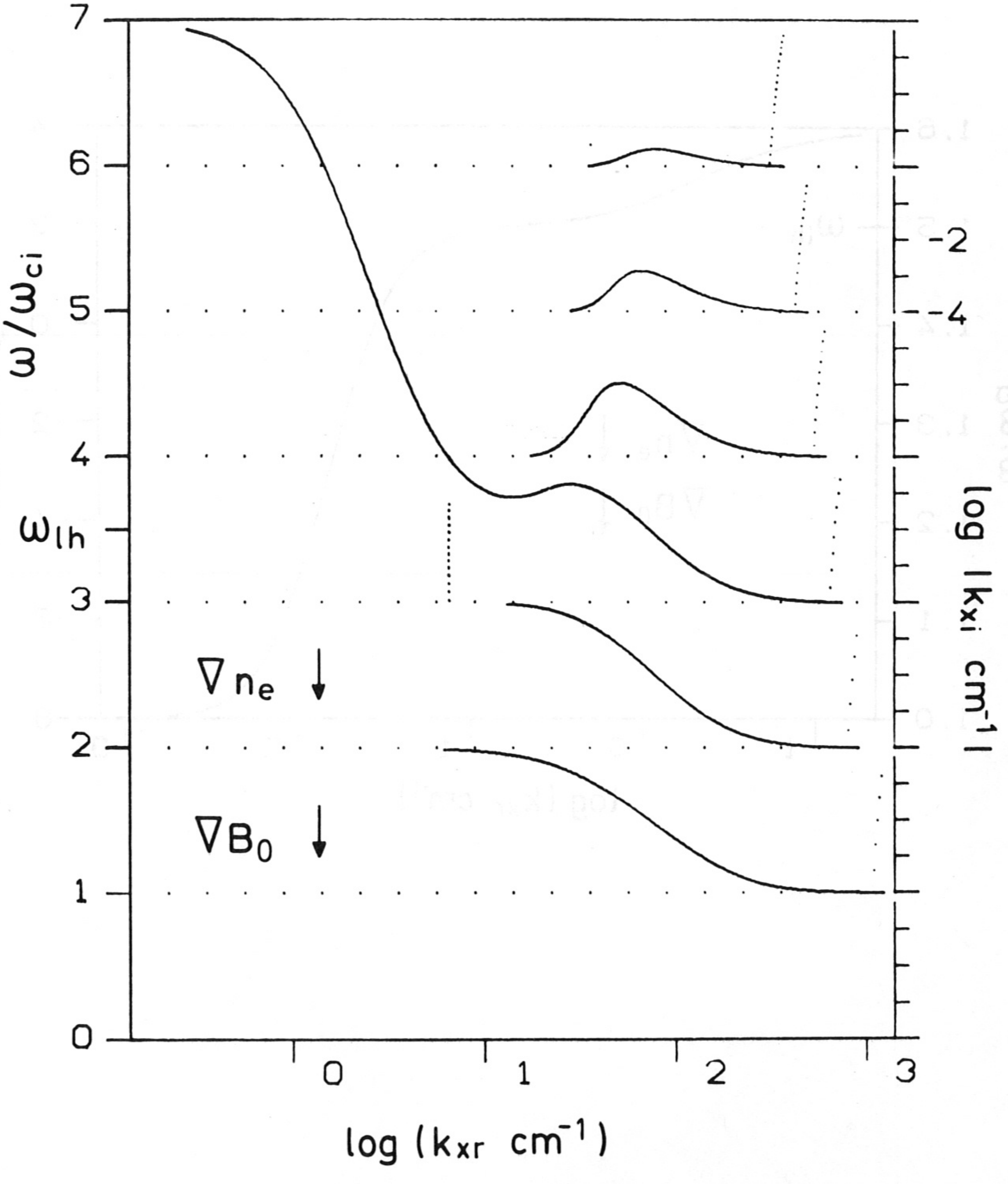
consistent with the requirement that $|\arg(-\Lambda)| < \pi/2$. Finally from (A11) we observe that below the harmonics the wave has the evanescent character of (50).

That (50) indeed represents evanescence rather than absorption is clear from the fact that it is substantially the same as would be the case if $k_z = 0$ in which case the perpendicularly propagating Gross-Bernstein waves are strictly lossless. For finite k_z the evanescent waves would possess a small lossy component.

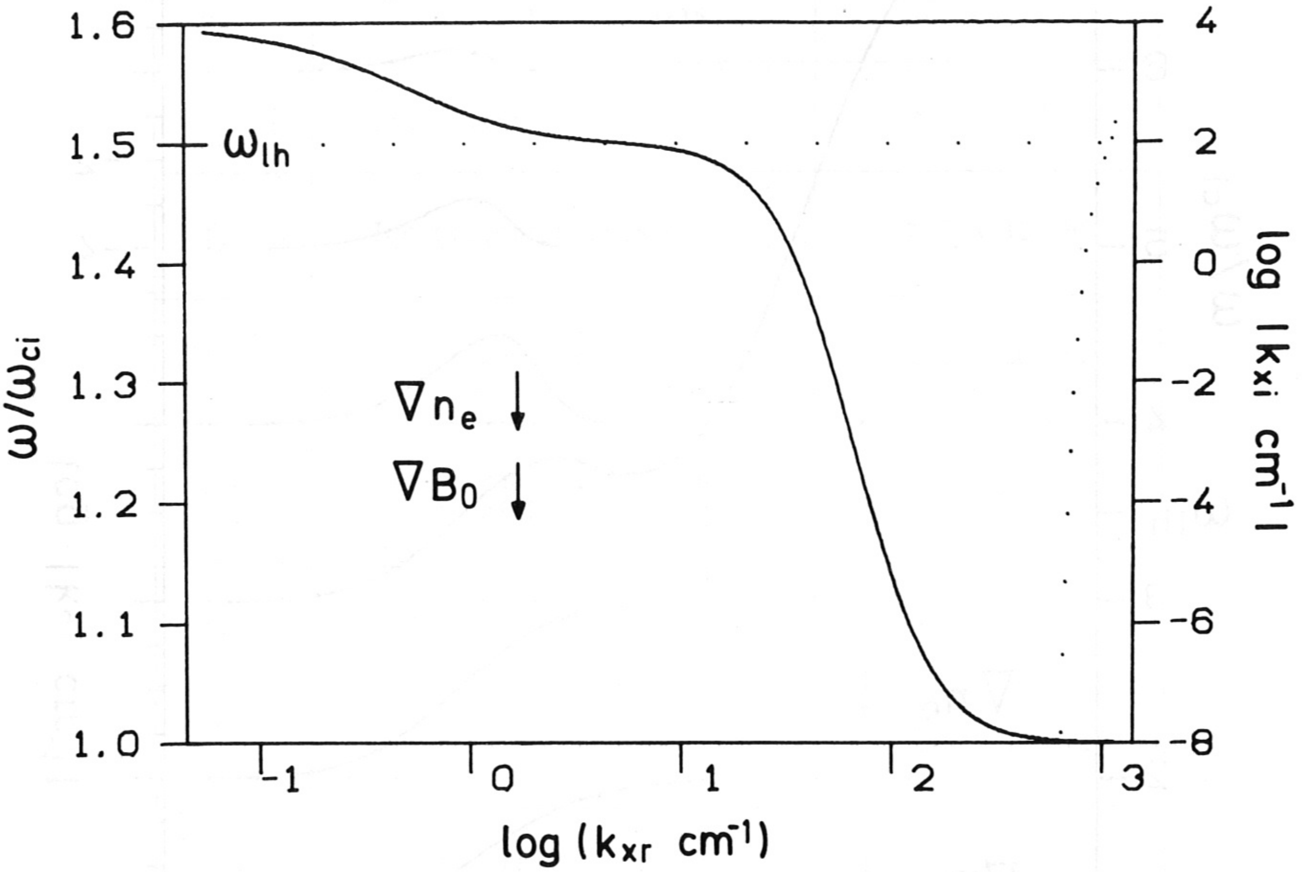
In Ref. 6 it is further pointed out that even for the case of strictly perpendicular propagation, the dielectric tensor component ϵ_x is complex in the region of complex waves so that $\underline{\underline{\epsilon}}$ is non-hermetian i.e. the non-hermiticity of $\underline{\underline{\epsilon}}$ does not necessarily imply a lossy medium in a hot plasma.

TABLE I

n_e	Electron density	10^{15} cm^{-3}
B_o	Magnetic field on axis	100 kG
f_{pi}	Ion plasma frequency	4.7×10^9 Hz
f_{ci}	Ion cyclotron frequency	7.7×10^7 Hz
f	Wave frequency	3.3×10^9 Hz
R	Plasma radius, gradient length of B_o	200 cm
λ_o	Free space wavelength	9 cm
r_{ci}	Ion cyclotron radius	6.4×10^{-2} cm
$\alpha = n_z$	$1 + (f_{pe}/f_{ce})^2$	2
β	$\frac{2}{\sqrt{\pi}} \quad \frac{\omega_{pi}^2}{\alpha\omega_{ci}}$	97
γ	$c/n_z v_z$	480
T	Plasma temperature	10^7 ^oK
v_z	Parallel ion thermal speed	3.2×10^7 cm/s
þ	f/f_{ci}	43



7
6
5
4
ω_{lh}
3
2
1
0
ω/ω_{ci}
∇n_e
∇B_0
0
1
2
3
log (k_{xr} cm^{-1})
-2
-4
log |k_{xi} cm^{-1}|


1.6
1.5
1.4
1.3
1.2
1.1
1.0
ω/ω_{ci}
ω_{lh}
∇n_e
∇B_0
4
2
0
-2
-4
-6
-8
$\log |k_{xi}\ cm^{-1}|$
-1
0
1
2
3
$\log (k_{xr}\ cm^{-1})$