Estimates on the Acceleration of Pellets by Gasdynamic and Electrostatic Means.

L.L. Lengyel
W. Riedmüller

IPP 4/171 Juli 1978

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Abstract

Three acceleration methods are considered: acceleration by gas-dynamic drag, pneumatic acceleration, and electrostatic acceleration. The maximum pellet velocities attainable are estimated by means of simple mathematical models, and the technical feasibility of the acceleration methods is discussed briefly. The results of the estimates show that pellet acceleration by gasdynamic drag is limited to velocities of a few 100 m/s. Pneumatic acceleration of single pellets may prove possible up to velocities of a few 1000 m/s. If the pellets could be charged electrostatically up to the field emission limit (or mechanical strength limit), electrostatic acceleration up to velocities of several 1000 m/s would become possible and technically feasible.

Introduction

Pellet injection, proposed originally by Spitzer [1], seems to be one of the prospective methods of refueling large-scale magnetic confinement configurations. Replanishing particle losses and increasing the plasma density by means of gas puffing, as is done with considerable success in present tokamak experiments, may prove to be impractical with larger plasma dimensions, particularly in the case of divertor--type tokamaks.

The necessary injection velocities predicted by the available ablation models (see, for example, [2] and [3]) are between a few 100 m/s and 10^4 m/s, depending upon the ablation model, the required penetration depth, and the plasma temperatures (and densities) assumed. The first ablation experiments [4, 5, 6] carried out at relatively low plasma temperatures are in agreement with the calculated values. More elaborate time-dependent calculations based on the coupling of ablation models with transport codes [7, 8] yield required injection velocity values which lie in the range given above and depend upon the assumed pellet size, plasma parameters, etc.

The acceleration of pellets to high velocities within reasonable acceleration lengths is limited by the relatively low mechanical strength of hydrogen isotope pellets. Assuming a pellet cross-section a, a pellet height h, a pellet density ρ, and a tensile strength σ, the maximum acceleration g_{max} the pellet can withstand is given by

$$g_{max} = \sigma/h\rho \quad (m_{pellet} \cdot g_{max} = a\sigma).$$

According to Bolshutkin et al. [9, 10], $\sigma = 5.2 \times 10^5$ N/m^2. The minimum acceleration time $t = v_p/g_{max}$ and the minimum acceleration length $L = v_p^2/2\, g_{max}$ corresponding to a given pellet velocity v_p can thus be calculated by assuming a constant (maximum) acceleration g_{max} value. Some representative L and t values computed for a pellet size of $h = 10^{-3}$ m are given in Table 1 and Fig. 1.

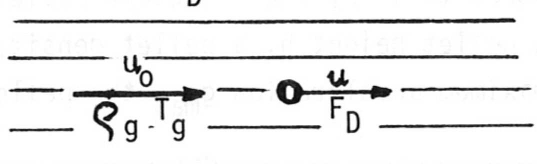
TABLE 1

Material	ρ (kgm/m^3)	g_{max}[m/sec^2]	v [m/s] for L=1 m	L [m] for v=1000 $\frac{m}{s}$	t [s] for v= 1000 $\frac{m}{s}$
H_2	88,3	5.9×10^6	3440	8.5×10^{-2}	1.7×10^{-4}
D_2	200	2.6×10^6	2280	2×10^{-1}	3.9×10^{-4}
DT	258	2.0×10^6	2000	2.5×10^{-1}	5×10^{-4}

Acceleration by Gasdynamic Friction

This acceleration method can be described as follows: The pellets are introduced into a quasi-stationary (pulsed) gas stream and, owing to gasdynamic drag, accelerated to a velocity less than or equal to the velocity of the gas stream. Before the pellet enters the plasma chamber, the carrier gas must be pumped off. There are two kinds of forces acting on the pellet during its residence in the gas stream: surface tension (pressure and shear) and wake resistance. Both of these effects are accounted for by means of the drag coefficient C_D:

$$F_D = C_D \frac{1}{2} \rho_g (\Delta u)^2 A_p = M_p \frac{du}{dt},$$



$$\Delta u = u_o - u,$$

where F_D is the drag force acting on the pellet, M_p is the pellet mass, u and u_o are the pellet velocity and the free stream velocity, respectively, ρ_g, Tg are the gas density and gas temperature, and A_p is the projection of the pellet surface on a plane normal to the flow. The drag coefficient C_D is, in general, a function of the Reynold's number (see, for example, Baily [11], J. Fluid Mech. 65 (1974), 401-410): its value ranges from 0.1 to 0.5 for a Reynolds number variation from $R_e \approx 10^3$ to $\approx 10^6$. Assuming a certain value for C_D and constant pellet mass, the acceleration range corresponding to a given (final) pellet velocity $u_p \leq u_o$ can be obtained by integrating the above expression ($A_p = \pi r_p^2$, $M_p = 4\pi r_p^3 \varrho_p/3$, $\varrho_p \approx 200\ \mathrm{kgm/m^3}$, $C_D \approx$ const, and r_p denotes the pellet radius):

$$\ell = C \cdot f(u_p/u_o), \text{ where } C = \frac{8}{3}\,\frac{r_p}{C_D}\,\frac{\varrho_p}{\varrho_g}, \text{ and}$$

$$f(u_p/u_o) = \frac{u_p/u_o}{1-u_p/u_o} + \ln(1-u_p/u_o).$$

The acceleration time and the amount of carrier gas streaming through a control-section of radius R during the acceleration time (this number may be useful for estimating the required pumping capacities; R is the radius of the accelerator channel) are found to be

$$\Delta t = \frac{C}{u_o}\,\frac{u_p/u_o}{1-u_p/u_o}, \text{ and}$$

$$M_g = \pi R^2\, u_o \varrho_g \Delta t = \frac{8}{3}\pi R^2\, \frac{r_p \varrho_p}{C_D}\,\frac{u_p/u_o}{1-u_p/u_o}.$$

As can be seen, in this approximation the characteristic carrier gas mass M_g is not a function of gas state parameters ϱ_g, T_g; it is determined solely by the pellet characteristics and the given u_p/u_o ratio. Some characteristic acceleration range and acceleration time values computed for a pellet diameter of 1 mm, a pellet velocity of 1000 m/s, and $C_D = 0.3$ are given in Tables 2 to 2b. H_2 has been assumed as carrier gas and R = 2.5 mm. The temperature $T(u_o = c_o)$ appearing in the tables is the gas temperature at which u_o is equal to the respective sonic velocity c_o. Since, owing to the known supersonic flow phenomena (shock wave formation, wave interference, pellet wave interaction), pellet acceleration by gasdynamic drag in supersonic flows may be difficult to realize, the carrier gas velocities appearing in Table 1a are limited to sonic velocity.

TABLE 2 Carrier gas characteristics required for reaching a pellet velocity of u_p = 1000 m/s

u_p/u_o	0.2	0.4	0.6	0.8
$f(u_p/u_o)$	0.027	0.156	0.584	2.39
u_o(m/s)	5000	2700	1700	1250
$T(u_o=c_o)$	3600	910	406	227
M_g(mgr)	4.28	11.6	25.8	68.3

TABLE 2a Acceleration by means of sonic flows ($T = T\ (u_o = c_o)$): required acceleration ranges and acceleration times.

(a) p_g = 45 torr				
$\Delta\ell$ (m)	59.1	87.	144.	330.
Δt (s)	0.1	0.14	0.22	0.44

(b) p_g = 152 torr				
$\Delta\ell$ (m)	17.5	25.6	42.8	97.9
Δt (s)	0.032	0.04	0.065	0.13

(c) p_g = 760 torr				
$\Delta\ell$ (m)	3.5	5.1	8.6	19.6
Δt (s)	0.007	0.008	0.013	0.026

TABLE 2b Acceleration by means of supersonic flows:
p_g = 304 Torr, T_g = 50 ^{o}K (C_o = 586 m/s, ρ_g = 0.194 kgm/m^3, C = 4.49)

u_p/u_o	0.2	0.4	0.6	0.8
$\Delta\ell$(m)	0.12	0.70	2.6	10.7
Δt(s)	2.2×10^{-4}	1.1×10^{-3}	4×10^{-3}	1.4×10^{-2}
M	8.52	4.6	2.9	2.1

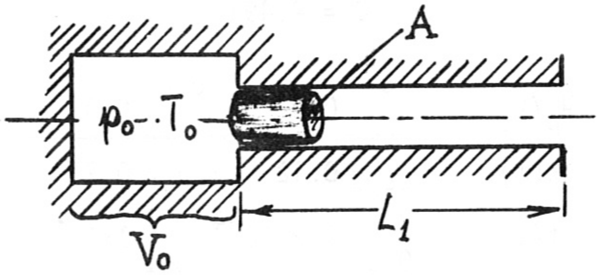
As can be seen from the values given in Table 2a, gasdynamic acceleration to pellet velocities of the order of 1000 m/s by means of subsonic flows is rather difficult,if not impossible. The required acceleration ranges are very long and the long contact with the hot carrier gas would vaporize the pellet long before it reached the plasma. The values displaced in Table 2b correspond to acceleration by means of cold supersonic stream (M denotes the Mach number). Although the computed $\Delta\ell$ and Δt values seem to be acceptable, the resulting Mach number values are high and may cause significant difficulties in the technical realization of this method. We may thus conclude that pellet acceleration by means of gasdynamic drag to velocities of the order of 1000 m/s and higher has little chance, if any, from the point of view of technical feasibility. However, the method is relatively simple and may readily be applied if only moderate pellet velocities (200 to 300 m/s) are desired.

Pneumatic Acceleration

The model used for estimating the output characteristics of gas-gun-type accelerators is as follows: the pellet to be accelerated (of cylindrical or spherical symmetry) is located in a cylindrical channel whose i.d. is equal to the pellet diameter. The pellet is accelerated by the pressure exerted by the driver gas on the surface of the pellet exposed to the gas. The pellet is accelerated into vacuum, wall frication and pellet vaporization effects shall be neglected in this first approximation. Since in this case the work done by the expanding gas is equal to the change of the kinetic energy of the pellet:

$$M_p \frac{d}{dt}\left(\frac{1}{2}u^2\right) = pdV, \text{ i.e.}$$

$$M_p \frac{du}{dt} = pA,$$



where M_p is the pellet mass, and Vo and L_1 denote the initial gas volume and the length of the accelerator barrel (A is the barrel and pellet cross-section). If one neglects the transient gradients and waves induced in the driver gas during the expansion process and replaces the actual expansion by a loss-free adiabatic process with $c_p/c_v = \gamma = \text{const}$ (ideal gas approximation), the following expression is obtained for the pellet velocity in terms of the initial parameters and the expansion ratio:

$$u = \left(2 e_o \frac{M_g}{M_p}\right)^{1/2} \left[1 - (V_o/V)^{\gamma-1}\right],$$

where $M_g = \rho_o V_o$ is the mass of the driver gas, and e_o is its initial specific internal energy. Since $V = V_o + A\int u dx$ and $u = dx/dt$, the above expression rewritten as

$$-\int_1^{c^*} c^{*-\frac{\gamma+1}{\gamma-1}} (1 - c^{*2})^{-1/2} dc^* = \int_o^{t^*} \left(\frac{\gamma-1}{2\gamma}\frac{Mg}{Mp}\right)^{1/2} dt^*$$

yields, for any given γ value, a unique relation between the dimensionless acoustic velocity $c^* \equiv c/c_o$ (or the dimensionless volume $V^* = c^{*-\frac{2}{\gamma-1}}$) and the reduced time t^* defined as $t^* \equiv c_o t\, A/V_o$:

for $\gamma = \frac{5}{3}$ $\quad (c^{*-2} - 1)(c^{*-2} + 2)^2 = \frac{9}{5}\frac{Mg}{Mp} t^{*2}$

$\gamma = \frac{7}{5}$ $\quad (c^{*-2} - 1)(13c^{*-2} + 2)^2 = \frac{225}{7}\frac{Mg}{Mp} t^{*2}$.

The expression relating the pellet velocity u and the instantaneous expansion ratio V_o/V indicates that there exists a minimum initial gas temperature for any given expansion ratio $1 > V_o/V_1 \gtrsim 0$ (V_1 denotes the gas volume at the end of the acceleration process):

$$T_{o\,min} \equiv \frac{1}{2}\frac{u^2}{c_v}\frac{M_p}{M_g} \leq \frac{1}{1}\frac{u^2}{c_v}\frac{M_p}{M_g}\ [1 - (V_o/V_1)^{\gamma-1}]^{-2}$$

which is required for reaching the specified pellet velocity. Hence if the initial gas temperature T_o is given ($T_o \geq T_{o\,min}$), the following relations are valid:

$$V_1/V_o = (1 - T_{o\,min}/T_o)^{\frac{-1}{\gamma-1}},$$

$$T_1 = T_o - T_{o\,min},$$

where T_1 denotes the temperature of the driver gas at the end of the acceleration process. The length of the accelerator barrel is given as

$$L_1 = (V_1/V_o - 1)/(A/V_o).$$

Accelerator characteristics corresponding to given sets of pellet velocity u_p, driver gas to pellet mass ratio M_g/M_p, and initial gas temperature T_o are displayed in Table 3 (pellet data: r_p = 0.6 mm, $A = \pi r_p^2$, M_p = 0.177 mgr, $\rho_p \simeq$ 200 kgm/m^3, and c_v = 6.19 x 10^3 J/kgm ^{o}K).

TABLE 3 Pneumatic acceleration, accelerator characteristics.

u_p(m/s)	M_g/M_p	To(K)	$T_{o\,min}$(K)	T_1(K)	V_o(cm^3)	p_o(atm)	L_1(m)
10^3	1	131	81	50	1	0.88	2.86
-"-	-"-	161	-"-	80	1	1.09	1.64
-"-	-"-	-"-	-"-	-"-	0.5	2.18	0.82
10^3	4	80.2	20.2	60	1	2.15	0.48
-"-	-"-	80.2	-"-	60	2	1.08	0.96
-"-	-"-	160.2	-"-	140	2	2.13	0.39
5x10^3	4	1005	505	500	1	26.9	1.63
-"-	-"-	765	-"-	260	1	20.6	3.58
5x10^3	9	324.5	224.5	100	1	19.6	4.29
-"-	-"-	374.5	-"-	150	1	22.6	2.61

As can be seen from the tabulated data, the required accelerator lengths as well as the necessary initial gas parameter values p_0, T_0 are acceptable from the point of view of technical feasibility. According to these estimates, the acceleration of pellets by pneumatic means to velocities of the order of 1000 m/s requires a driver gas with $T_0 \simeq 100\ ^oK$, $p_0 \simeq 1$ atm. and an accelerator length of about 1 m. Of course, these values are only approximate; wall friction and pellet evaporation were not taken into account in the above estimates. In particular, extensive pellet evaporation and the associated gas leakage around the pellet may reduce the characteristics of this acceleration method to that of the gasdynamic drag method. However, the vaporization rate may be controlled to a certain extent by a suitable selection of the barrel temperature and the driver gas parameters. One may thus conclude that the pneumatic acceleration of pellets to velocities up to a few times 1000 m/s is relatively simple and technically feasible.

Electrostatic Acceleration of Pellets

Multi-stage linear electrostatic accelerators were developed to accelerate electrons and ions [12, 13]. Such accelerators were also built a few years ago for larger particles with radii of 0.1 - 1 μm with specific charges of 1 - 300 Cb/kg (micrometeorite simulation) [14, 15, 16]. Electrostatic acceleration of hydrogen pellets has meanwhile been proposed for cold refueling in fusion plasmas [17].

The attainable velocity v is given by

$$v = \left(2 \frac{Q}{M} U\right)^{1/2}, \qquad (1)$$

where U is the acceleration voltage and Q the charge, and the mass of the pellet M is given by

$$M = \frac{4}{3} r_p^3 \pi \varrho_p. \qquad (2)$$

The densities ρ_p for the appropriate materials are presented in the following table:

	Density ρ_p [kg/m^3]	Temp. [K]	Ref.
Hydrogen	88.3	4	[18]
Deuterium	204	4	[18]
Deuterium Tritium	258	19.4 (Trip.pt.)	[19]

It is not known at present what charge can be applied to a hydrogen pellet. A few limiting values can, however, be estimated.

1. Charging in the liquid phase (droplets)

The droplets are kept together by the surface tension σ, which is counteracted by the repulsive electric forces. According to Ref. [20, 21] the maximum electric charge at which destruction of the droplets occurs is given by

$$Q^1_{max} = 4\pi \sqrt{2\varepsilon_o \sigma \, r_p^3} \; .$$

It thus follows that the maximum specific charge is $\frac{Q^1_{max}}{M} = \frac{3(2\varepsilon_o\sigma)^{1/2}}{\rho_p \, r_p^{3/2}}$. This is presented in the following table for $r_p = 10^{-3}$ m and plotted in Fig. 1 as a function of the radius. The densities and surface tensions at the triple point are taken from Ref.[22].

	ρ [kg/m^3]	σ [N/m^2]	T [^{o}K]	$\frac{Q^1}{M}$ (r_p=10^{-3} m) [C/kg]
liquid H_2	77.3	3×10^{-3}	14	2.8×10^{-4}
liquid D_2	174	3.7×10^{-3}	18.7	1.4×10^{-4}
liquid DT	225	4.4×10^{-3}	19.7	1.2×10^{-4}

The velocities $v \approx 10^{-2} \sqrt{U}$ m/s thus attainable are extremely low, e.g. with 10^8 V just 200 m/s. With liquid nitrogen only 1/5 of the theoretical possible maximum specific charge was obtained experimentally [20].

2. Limitation due to field emission

At low temperatures the maximum electric field strength attainable at surfaces - before the onset of field emission - is directly proportional to the square of the binding energy of the charge carriers [23, 24]. The values for solid hydrogen are not known.

To allow an estimate to be made at all, we take as a basis the values measured for metals (e.g. tungsten). In the case of tungsten, the field emission for electrons sets in at a few times 10^8 V/m, and for ions at a few times 10^{10} V/m [23, 24]. The maximum charge as regards field emission is then given by

$$Q_{max}^2 = 4\pi\varepsilon_o r_p^2 \cdot E_{max} \text{ and} \qquad (4)$$

$$\frac{Q_{max}^2}{M} = 3\,\frac{\varepsilon_o}{\varrho}\,\frac{E_{max}}{r_p} .$$

The values for charging by electrons ($E_{max} = 10^8$ V/m) and ions ($E_{max} = 10^{10}$ V/m) are presented in the following table for $r_p = 10^{-3}$ m and plotted in Fig. 4 as a function of the radius:

	electron emission	Q/M [C/kg] for $r_p = 10^{-3}$ m	ion emission	Q/M [C/kg] for $r_p=10^{-3}$m
H_2	$3 \times 10^{-5}/r_p$	3×10^{-2}	$3.\times 10^{-3}/r_p$	3
D_2	$1.3 \times 10^{-5}/r_p$	1.3×10^{-2}	$1.3 \times 10^{-3}/r_p$	1.3
DT	$1 \times 10^{-5}/r_p$	1×10^{-2}	$1 \times 10^{-3}/r_p$	1

3. Limitation due to mechanical strength

The mechanical stress imposed on a charged body is given in order of magnitude by

$$p = \frac{1}{2}\,\varepsilon_o\,E^2. \qquad (6)$$

The maximum tensile strength for solid deuterium, which is a very soft material, is given in Ref. [9]:

P $[10^5\ N/m^2]$	4.3	5.0	5.4	5.2	4.5	4.3	2.9	2.1
T $[^oK]$	1.4	3.0	4.2	5.4	8.0	11.6	15.6	16.4

At higher pressure the material is destroyed: for $T > 12$ K the material starts to creep, at $T < 12$ K it ruptures. The values for for hydrogen are of the same order of magnitude [10]..According to Souers [25] there is no reason why a newly frozen DT mixture should be much more resistent. After times of the order of minutes to hours he expects, however, the mechanical strength to be enhanced owing to changes in the crystal structure caused by radioactive decay of tritium.

It should be noted, however, that the above values of the mechanical strength refer to a very specific crystal structure (grain size 1-2 mm), and that the mechanical properties can vary as the crystal structure.

We take $p_{max} = 5 \times 10^5\ N/m^2$ and find the maximum field strength to be $E_{max} = \sqrt{2\ p_{max}/\varepsilon_o} = 3.36 \times 10^8$ V/m.

If it is not possible with electrons, this limit may possibly be achieved by charging with ions. In this case the maximum charge is

$$Q^3_{max} = 4\ \pi\ \varepsilon_o r_p^2\ E_{max} = 3,74 \cdot 10^{-2}\ r_p^2 \qquad (7)$$

and the maximum specific charge would be

$$\frac{Q^3_{max}}{M} = 3\ \frac{\varepsilon_o}{\rho}\ \frac{E_{max}}{r_p}. \qquad (8)$$

	Q/M [C/kg]	for $r_p = 10^{-3}$ m
H_2	$10^{-4}/rp$	10^{-3} C/kg
D_2	$4.4 \times 10^{-5}/rp$	$4.4 \cdot 10^{-2}$ C/kg
DT	$3.45 \times 10^{-5}/rp$	$3.45 \cdot 10^{-2}$ C/kg

These are also plotted in Fig. 2 as a function of r_p. The velocities $v = \sqrt{2U\ Q/M}$ thus attained are presented in the following table for various values of U and plotted in Fig. 3 as a function of the pellet radius:

Pellet radius		10^{-4}	3×10^{-4}	5×10^{-4}	10^{-3} [m]
$U = 10^5$ V	H_2	447	258	200	141
	D_2	297	171	133	94
	DT	263	152	117	83
$U = 1.5 \times 10^6$ V	H_2	1730	998	774	546
	D_2	1150	171	515	363
	DT	1020	152	453	321
$U = 10^8$ V	H_2	14 100	8150	6320	4460
	D_2	9 390	5400	4200	2900
	DT	8 310	4800	3700	2620

[m/sec]

In his proposal Mills [17] assumes for $r_p = 0.39$ mm a very high Q/M value of 1.73 C/kg, which he hopes to achieve by providing the pellet with a thin metal coating to ensure a uniform charge distribution and enhanced mechanical strength. This, of course, also yields higher final velocities with the same acceleration voltage. It is doubtful, however, whether metal-coated pellets are suitable for refueling purposes.

If one limits the chargeability by the mechanical strength, it is already possible with a single-stage accelerator (van de Graaff) of about 2 MV for hydrogen pellets with $r_p \sim 0.5$ mm to attain velocities of just under 1000 m/s. With a few times 10^8 V it might be possible to appraoch 10.000 m/s.

The influence of magnetic fields on the motion of pellets is very slight. The gyration radius r_G is

$$r_G = \frac{v}{\frac{Q}{M} \cdot B} \qquad (9)$$

which for hydrogen reduces to $r_G = \frac{v \cdot r_p}{10^{-4}\ B}$. With, for example, $B = 10$ T and $r_p = 10^{-3}$ one gets $r_G = 10^3$m.

The number of elementary charges on the pellet is n

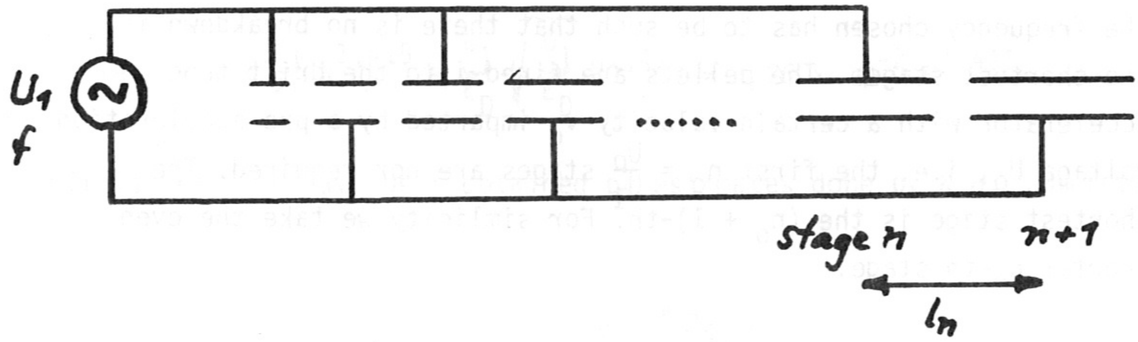
$$n = \frac{Q_{max}}{e} = \frac{3.7 \times 10^{-2}}{1.6 \times 10^{-19}}\ r_p^{\ 2} = 2.3 \times 10^{17}\ r_p^{\ 2}$$

Assuming a charging current i = 1 μA, the charging time is t = Q/i:

pellet radius r_p	10^{-4}	3×10^{-4}	[m]
maximum charge Q_{max}	3.7×10^{-10}	3.3×10^{-9}	[C]
number of elem.charges n	2.3×10^9	2.1×10^{10}	-
charging time t with 1 μA	3.7×10^{-4}	3.3×10^{-3}	[s]
end potential = $E_{max} \cdot r_p$	33.6	101	[kV]

Presumably the charge carriers have to be put on the pellet by uniform bombardment of the whole surface since hydrogen is a very good insulator [25]. To prevent too much energy input to the pellet during bombardment, the charge carriers should impinge with only a low velocity during the charging process. Their initial velocity probably has to be matched to the potential, which rises with increasing charge.

Estimate of the length of a drift tube accelerator



Notation:

v_o	initial velocity
$v1 = \sqrt{2 \frac{Q}{M} U_1}$	velocity after acceleration by u_1
v	final velocity
U_o	pre-acceleration voltage
U_1	voltage per stage
U	total acceleration voltage
f	frequency of voltage u_1
l_n	length of n-th stage
v_n	velocity after n-th stage
N	number of stages
L	total length
E_D	field strength up to which no el. breakdown occurs.

One has $l_n = \frac{v_n}{2f}$ and $v_n = \sqrt{2 \frac{Q}{M} U_1 \cdot n}$,
and so it follows that

$$l_n = \sqrt{2 \frac{Q}{M} U_1 n}/2f \qquad (10)$$

The total length L is thus $L = \frac{v_1}{2f} \sum_1^N \sqrt{n}$

$\sum_1^N \sqrt{n}$ can be replaced by $N^{1.415}$ in the range n = 25 ... 1000, the error remaining less than 10 %. The total number N is given by

$$N = \frac{U}{U_1} = \left(\frac{v}{v_1}\right)^2 \qquad (11)$$

and so it follows that

$$L = \frac{v_1}{2f} \left(\frac{v}{v_1}\right)^{2,83} . \qquad (12)$$

The frequency chosen has to be such that there is no breakdown in the shortest stages. The pellets are fired into the drift tube accelerator with a certain velocity v_o imparted by a pre-acceleration voltage U_o, i.e. the first $n_o = \frac{U_o}{U_1}$ stages are nor required. The shortest stage is the $(n_o + 1)$-th. For simlicity we take the even shorter n_o-th stage:

$$l(n_o) = \frac{v_1 \sqrt{n_o}}{2f} = \frac{v_o}{2f} .$$

Apart from correction factors allowing for the special geometry of drift tubes, this stage has to be so long that there is no electrical breakdown:

$$\frac{v_o}{2f} \geqq \frac{U_1}{E_D} .$$

For the frequency one obtains $f \leqq \frac{v_o}{2} \frac{E_D}{U_1}$, (13)

which is substituted in eq. (12) to yield

$$L \geqq \frac{U_1}{E_D} \frac{v_1}{v_o} \left(\frac{v}{v_1}\right)^{2,83} \quad (14)$$

or, with $v_1 = \sqrt{2 \frac{Q}{M} U_1}$, $v_o = \sqrt{2 \frac{Q}{M} U_o}$ and $\frac{Q}{M} = 3 \ \varepsilon_o \frac{E_{max}}{\varsigma \cdot r_p}$,

$$L = \frac{v^{2,83}}{U_1^{0,085} \ U_o^{0,5} \ E_D} \qquad \left(\frac{\varsigma \ r_{po}}{6 \varepsilon_o \ E_{max}}\right)^{1,415}$$

For the following example the situation is presented in the table below and the length L is plotted in Fig. 5 as a function of the pellet radius:

$$\left.\begin{array}{l} U_1 = 2 \times 10^5 \text{ V} \\ E_D = 5 \times 10^6 \text{ V/m} \end{array}\right\} \quad \frac{U_1}{E_D} = 0.04 \text{ m} \quad (= 1_{n_o})$$

pre acceleration voltage $U_o = 2$ MV $\curvearrowright$ $n_o = \frac{U_o}{U_1} = 10$

frequency $f = \sqrt{\frac{U_o}{U_1}} \cdot \frac{E_D}{U_1} \frac{1}{2} v_1 = 39.5 \ v_1$

$N = \left(\frac{v}{v_1}\right)^2$

length $L = \frac{v_1}{2f} N^{1,415} = \frac{U_1}{E_D} \sqrt{\frac{U_1}{E_D}} N^{1,415} = 1.26 \times 10^{-2} \ N^{1.415}$

Similar results have been obtained by estimates done by K.H. Schmitter.

	R	10^{-4}	3×10^{-4}	5×10^{-4}	10^{-3}	m
H_2	v_1	630	364	282	199	m/s
	v_o	1990	1150	890	630	m/s
	v	$10^3/10^4$	$10^3/10^4$	$10^3/10^4$	$10^3/10^4$	m/s
	N	[2.5]/250	[7.5]/755	12,6/1257	25/2525	—
	f	24.9	14.4	11.1	7.9	kHz
	L	[0.15]/31	[0.22]/149	0.45/306	1.20/821	m
D_2	v_1	419	241	188	132	m/s
	v_o	1325	760	595	420	m/s
	N	[5.7]/570	17/1722	28/2830	57/5739	—
	f	16.6	9.5	7.4	5.2	kHz
	L	[0.15]/100	0.7/478	1.40/965	3.84/2625	m

Conclusions

The electric chargeability of hydrogen is unknown. If hydrogen pellets can be charged to the limit of their mechanical strength, as given in a Soviet paper, it should be possible to accelerate the pellets to almost 1000 m/s with single-stage van de Graaff accelerators. Velocities of 10.000 m/s are basically attainable, but the acceleration paths required are rather long (of the order of a few 100 m). Since deuterium or DT have higher masses (but presumable the same mechanical strength and chargeability), multi-stage accelerators would already have to be used to attain 1000 m/s. The acceleration lengths required are a factor of about 3 as large.

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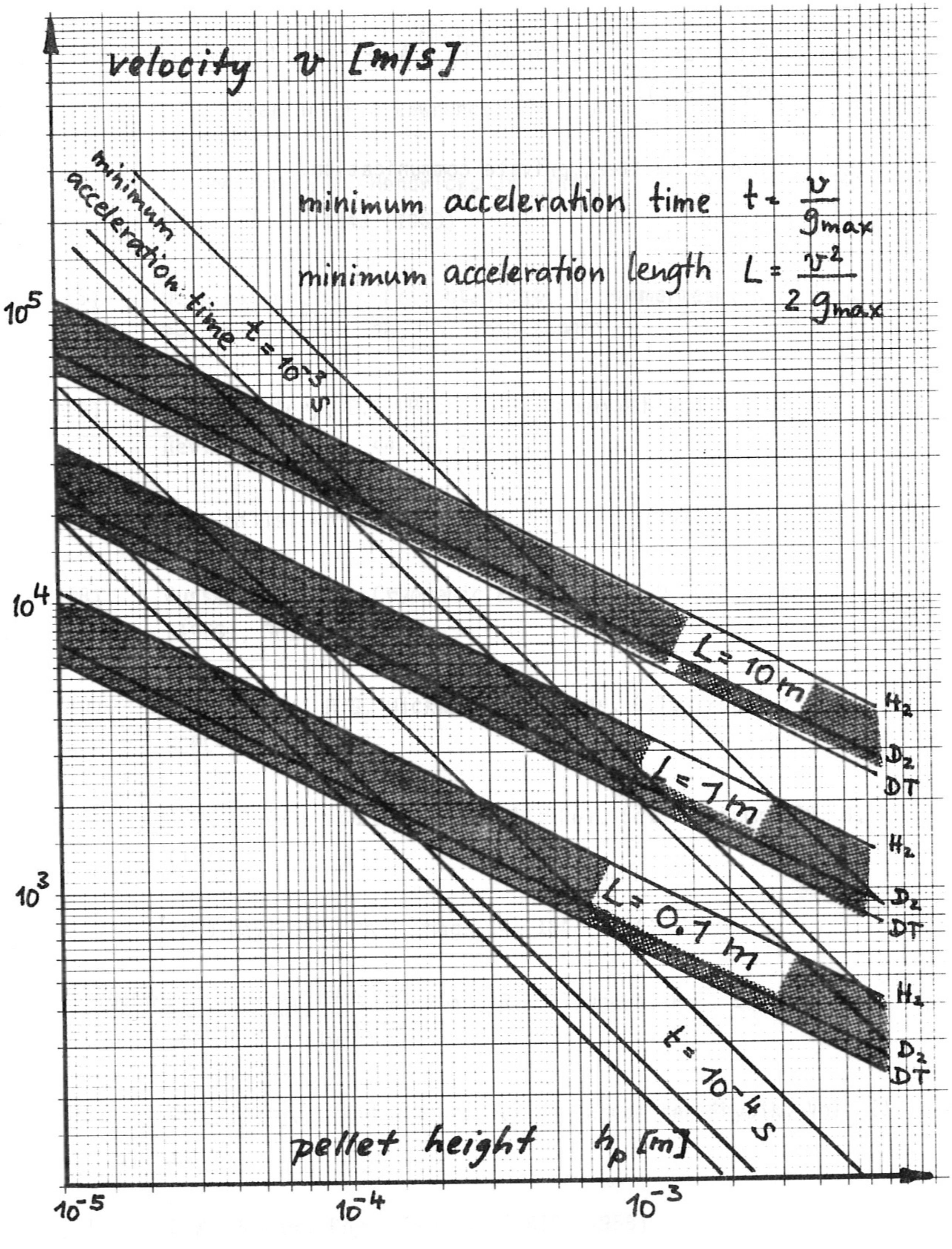

Fig. 1 Minimum acceleration time and minimum acceleration length as function of pellet height and velocity.

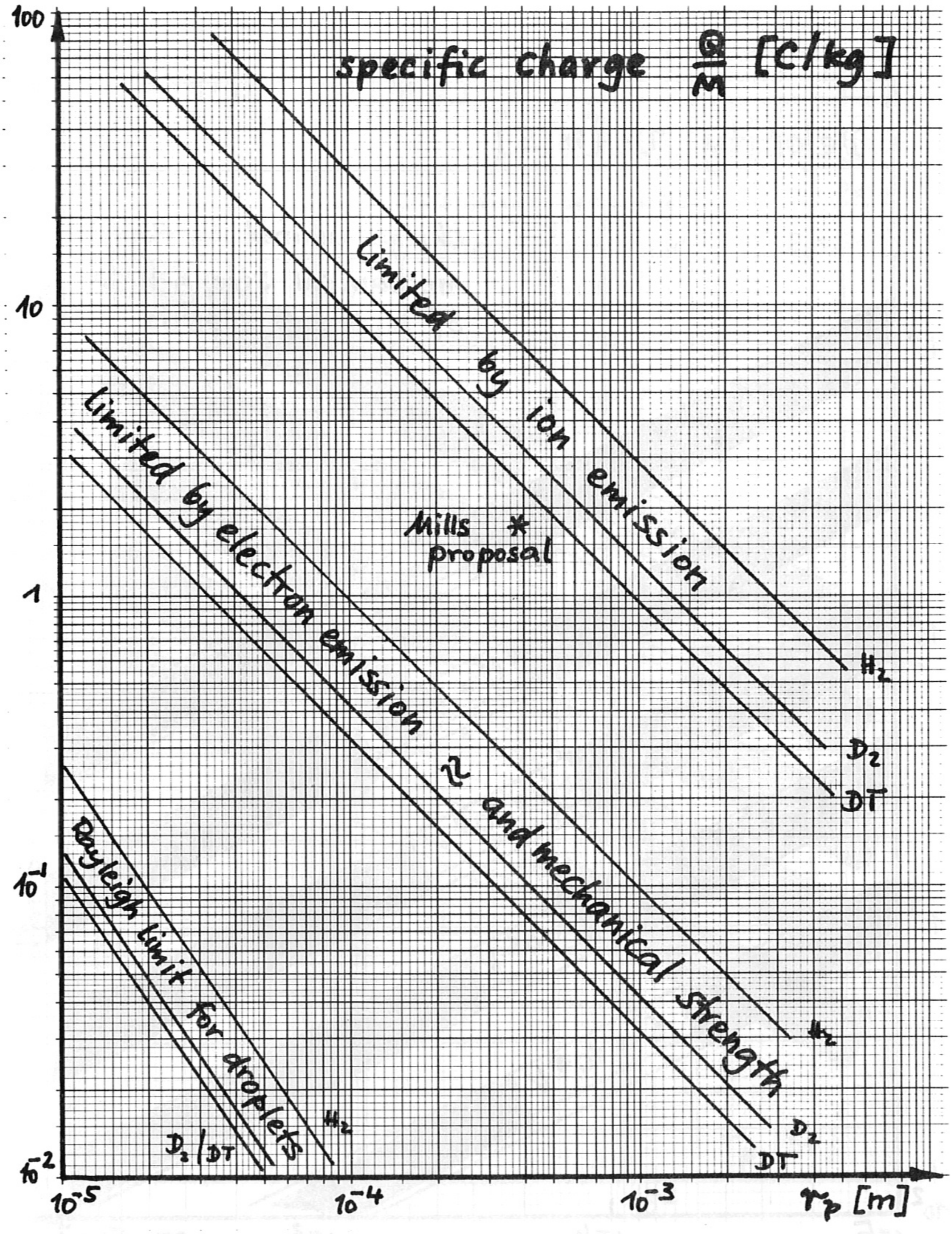

Fig. 2 Specific charge versus pellet radius for three different limits: ion emission, electron emission (approximatly equal to the mechanical strength limit) and Rayleigh limit for droplets.

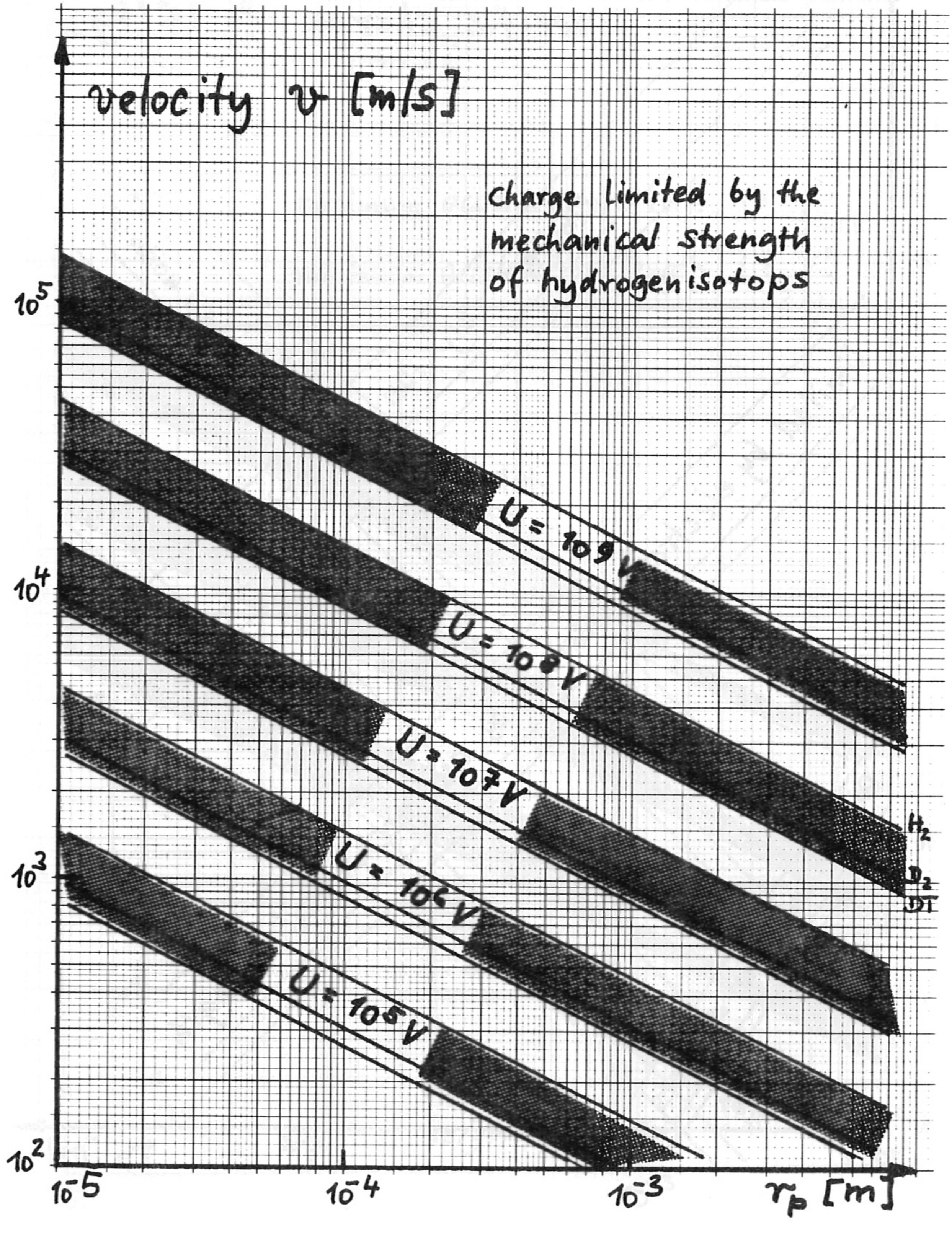

Fig. 3 Pellet velocity versus pellet radius for different acceleration voltages.

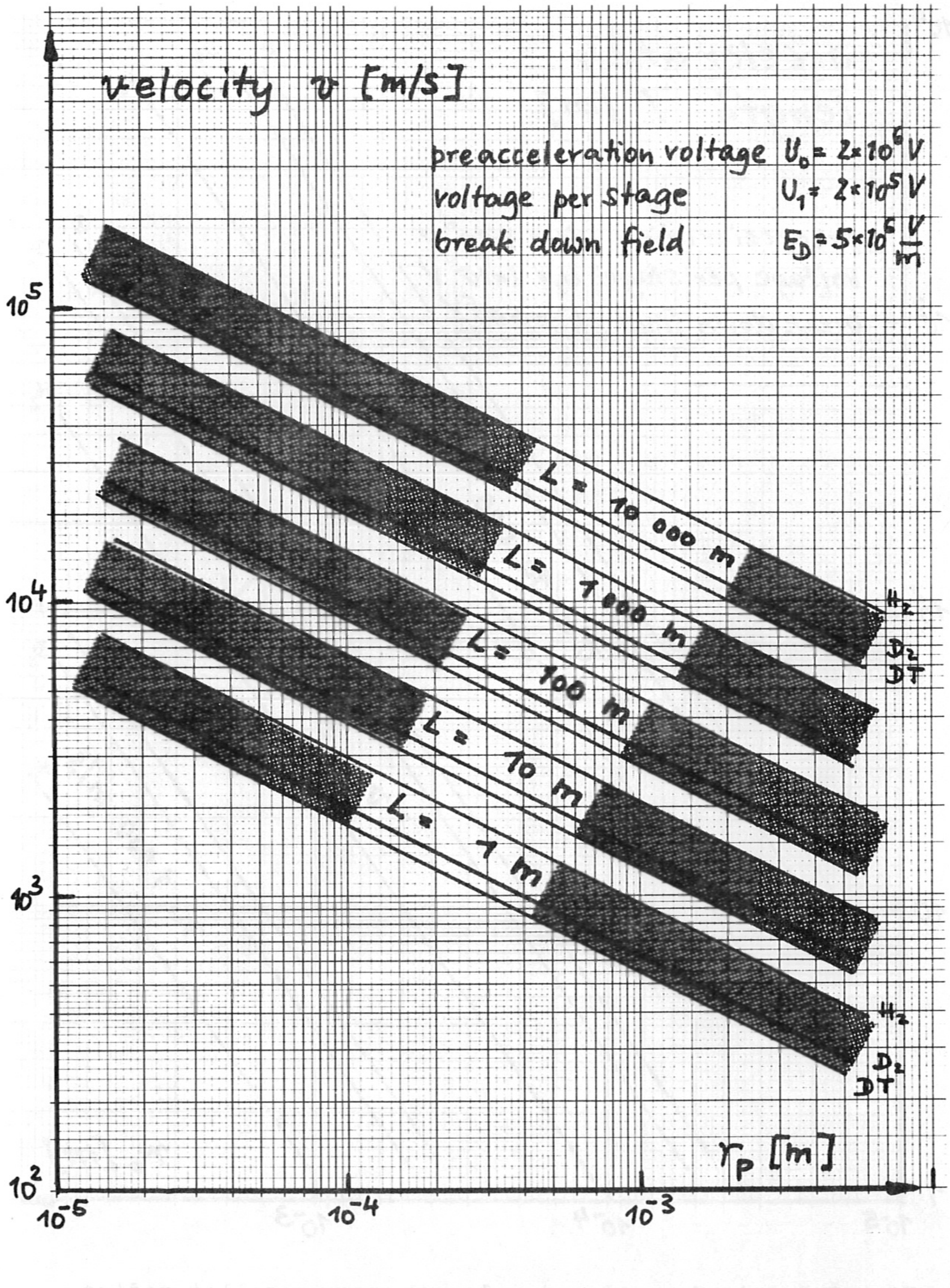

Fig 4 Pellet velocity versus pellet radius for different length of the drift tube accelerator. The voltage per stage is assumed to be 2×10^5 V.

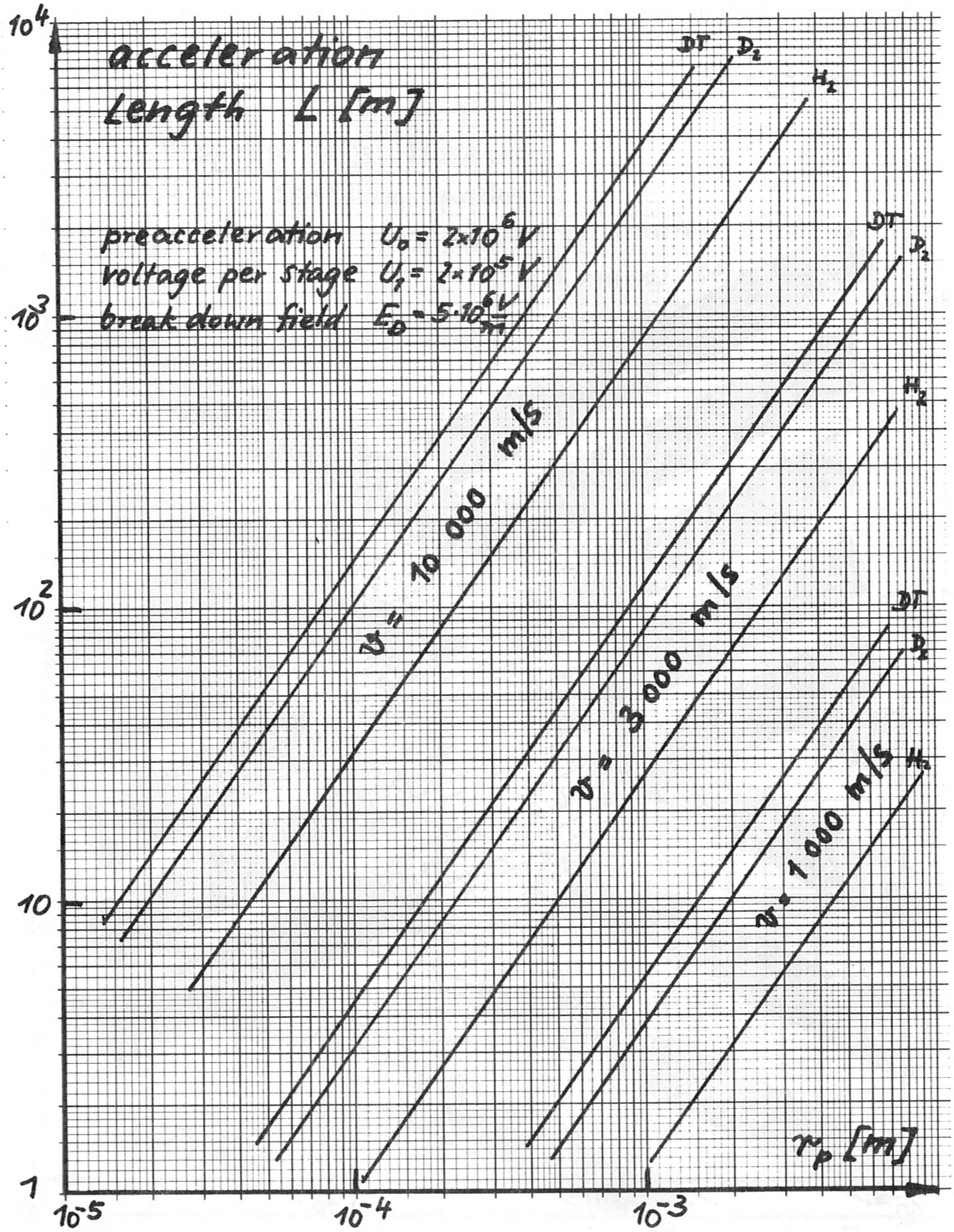

Fig. 5 Required accelerator length versus pellet radius to get 10 000, 3000 and 1000 m/s pellet velocity with an acceleration velocity of 2 x 10^5 V per stage.