

Energy balances of fusion power plants  
as the basis of systems studies

J. Raeder

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**MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK**

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Abstract

The different types of fusion power plants are described by an energy flow oriented system of components which is not unduly complicated. The individual plant components are characterized by efficiencies and energy ratios which are called "component energy parameters". The "plant energy parameters": critical energy amplification  $Q_c$ , net plant efficiency  $\eta$ , and circulating energy ratio  $C$  are expressed by the component energy parameters. In a parametric study the latter are subjected to systematic variations with respect to reference conditions. These describe a high- $Q$  and a low- $Q$  system, which may be exemplified by a tokamak-like and a mirror-like device. The parametric variations show a great impact of nearly all component energy parameters on the plant energy parameters. This general sensitivity leads to the conclusion that the parametric treatment is not sufficient but is the framework for systems studies which have to be based on physical modelling of the plant components and the energy flows between them.

In two appendices simple physical models are used to establish relations between plant energy parameters and physical parameters such as the energy confinement time  $\tau_E$ , the plasma burn time  $\tau_b$ , and  $\tau_b/\tau_E$  as functions of the net plant efficiency  $\eta$  and plasma temperature  $T$  as well as diagrams showing the influences of the plasma  $Q$  and required pulsed magnetic energy on  $\eta$  and  $C$ .

## CONTENTS

Notations relating to the plant energy balance	1
1. Introduction	3
2. Structure of fusion power plants	4
3. Quantitative treatment by energy balances	6
3.1 Characterization of the plant components by "component energy parameters"	6
3.2 Energy balance of complete plants leading to "plant energy parameters"	10
3.3 Parametric sensitivity study	13
3.3.1 Reference conditions	14
3.3.2 Results of parametrically varying the component energy parameters $\eta_h$ , $\eta_a$ , $\eta_m$ , $\eta_d$ , $\epsilon_{mp}$ , $\epsilon_m$ , $\epsilon_{dt}$ , $\epsilon_a$ , and $Q$	16
4. Conclusions drawn from the parametric study	22
Appendix A	24
Appendix B	31
References	36



### Notations relating to the plant energy balance

Because the number of notations introduced in Section 3 to describe the plant energy flow diagram is very large these notations are collected here for easy reference. Most of them are also shown in Fig. 1.

$E_c$	energy circulated back from the plant exit to its entrance
$E_h$	energy input to the heating device
$E_{hf}$	energy delivered by the heating device to the fusion device
$E_{pa}$	heating energy absorbed by the plasma
$E_p$	energy input to the pulsed magnetic field device
$E_{pf}$	energy delivered by the pulsed magnetic field device to the fusion device
$E_{fp}$	energy recovered by the pulsed magnetic field device from the fusion device
$E_a$	energy consumption of the auxiliary systems (except heating and pulsed magnetic field devices)
$E_f$	energy delivered by plasma fusion reactions and blanket nuclear reactions
$E_{ft}$	thermal energy delivered by the fusion device
$E_{fd}$	energy delivered by the fusion device which can be converted directly
$E_{et}$	electric energy delivered by the thermal converter
$E_{ed}$	electric energy delivered by the direct converter
$E_g$	gross output energy of the plant
$E_n$	net output energy of the plant

$$\eta_h = E_{hf}/E_h$$

$$\eta_m = E_{pf}/(E_p + E_{fp})$$

$$\epsilon_m = E_{fp}/E_{pf}$$

$$\epsilon_a = E_a/(E_{ft} + E_{fd})$$

$$\eta_a = E_{pa}/E_{hf}$$

$$\epsilon_{mp} = E_{pf}/E_{pa}$$

$$Q = E_f/E_{pa}$$

$$\epsilon_{dt} = E_{fd}/(E_{fd} + E_{ft})$$

$$\eta_{th} = E_{et}/E_{ft}$$

$$\eta_d = E_{ed}/E_{fd}$$

$$\eta = E_n/E_f$$

$$C = E_c/E_n$$

$$Q_c = (E_f/E_{pa})_{\eta=0}$$



## 1. Introduction

The term "systems studies" though at present very frequently used is not well defined. Our understanding of it in the context of fusion power will therefore be described in the following.

Because no hardware realization of a fusion power plant exists as yet one is dealing with a hypothetical system. To learn something about such a system, one has to resort to models, which means that one has to study how the model behaves when subjected to variations of its parameters. By necessity such models have to be mathematical ones. Depending on the degree of sophistication and disaggregation, it may sometimes be possible to set up and study models analytically. In general, however, it will be necessary to make use of numerical work and computers.

A system usually consists of a number of subsystems coupled to each other. If a subsystem cannot be subdivided further or if this is not useful for the case considered, it is called a component. Systems studies thus include the modelling of components and their couplings as the first step and the investigation of the models' behaviour as the second step. This second step will obviously be done by applying variations of model parameters and checking the response.

The preceding description of systems studies disqualifies the term for referring to design work, which means engineering based on a set of virtually fixed parameters (so called "point designs"). In general, some kind of systems studies will be needed to produce just this parameter set.

Obviously, the discrimination between systems studies and design work should not be overstressed, because, for example, some minimum amount of design has to be done before a component can be modelled. Systems studies and design work will therefore mostly be used iteratively. This has also been the case in the field of fusion. The early (1954) stellarator fusion reactor study by Spitzer et al /1/, which was impressively complete if one takes into account its early date, mainly included the mathematical description of roughly designed components because it was the starting point in this field and therefore could

not enlist previous work. After a long phase concerned mainly with mathematical modelling of components and treating isolated problems /2 - 7/ the progress in confinement physics stimulated a great deal of design work during the first half of the seventies /8 - 23/. It now seems necessary to proceed to the modelling of complete fusion power plants.

The purpose of this report is to describe the basis of the IPP fusion systems studies. The scope will be restricted to plants delivering electric energy. Other options such as the production of synthetic fuels or the generation of heat will not be included. These would introduce further dimensions of speculation and ambiguity.

## 2. Structure of fusion power plants

For the systematic investigations to be performed the power plant will be described as a system of interlinked components which convert, transfer or consume different types of energy. A rather unique feature of the fusion power plant is the necessity of investing a non-negligible amount of heating energy into the plasma to ignite once for one burn cycle the reactions which deliver the fusion energy ("ignited devices") or to keep these reactions going during one burn cycle ("driven devices"). A fusion power plant will therefore work as an energy amplifier with finite amplification, its reservoir being the energy stored in the fusion fuels. In addition to the heating energy for the plasma, energy also has to be invested to power the auxiliary installations, but this is common to all types of power plants. At least for the purpose of modelling we may assume that the energies for heating and for auxiliary devices are recycled from the output side to the input side of the plant, thus forming a feedback system. The margin between the plant delivering net energy and consuming energy is marked by an energy amplification of unity along the closed feedback loop (output-input-output).

For a quantitative treatment we use the plant energy flow diagram shown in Fig. 1, which only comprises the most essential components and connections. At least part of the energies flowing between the plant components will vary with time. We shall therefore often talk



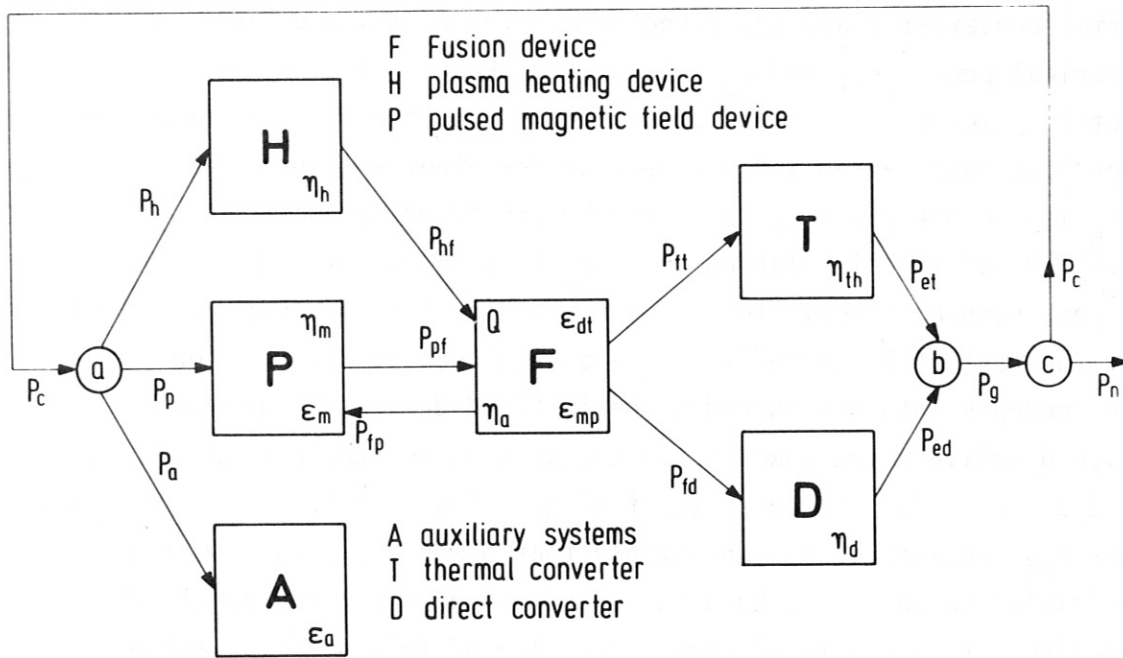


Fig. 1

about powers. If in connection with pulsed operation of the fusion device F the use of energies is appropriate, we get them by integrating the respective powers over the pulse length. We have always to keep in mind, however, that a power plant with a pulsed fusion reactor in its core has to deliver a smooth energy output to the grid. This means that at least the components which convert fusion energy (heat, charged particle energy) to electricity have to include storage devices. The blanket of a fusion device will have some smoothing effect because of its heat capacity. In the following, we shall use the term "energy" as well as the term "power", depending on the case considered.

The fusion device F delivers the thermal power  $P_{ft}$ , which originates from moderated neutrons and from absorbed particle and radiation energies (these particles and the radiation may partly originate from heating energy not absorbed by the plasma but by walls, divertor plates etc.).  $P_{ft}$  also includes the energy delivered by exothermic blanket reactions. For some concepts it is conceivable that power is also delivered as kinetic energy of charged particles, such as fuel particles, reaction product particles and particles from beam heating not absorbed by the plasma. Furthermore, work of the plasma against the magnetic field may be converted directly by induction to electric energy. For the total power that can be converted directly we use the

term  $P_{fd}$ . The powers  $P_{ft}$  and  $P_{fd}$  are converted to electricity by the thermal converter T and the direct converter D, which deliver the electrical powers  $P_{et}$  and  $P_{ed}$ , these together forming the gross electric power  $P_g$ . From this gross output a portion  $P_c$  ("circulating power") is subtracted and fed back to the plant entrance. The rest of  $P_g$  is the net power  $P_n$  delivered by the plant to the grid. A portion  $P_a$  of the circulating power  $P_c$  is used to power the auxiliary devices, such as pumps, steady state magnetic fields, control systems etc. The rest of  $P_c$  is fed into the plasma heating device H and in some concepts into the pulsed magnetic field device P. The heating device H delivers the power  $P_{hf}$  to the plasma, and the pulsed magnetic field device P is coupled to the fusion device F by the pulsed magnetic power  $P_{pf}$ . Because it is conceivable that power flows to F during the initial phase of the burn pulse and may be recovered partly by induction during the final phase, the flow of this power is assumed to be bidirectional ( $P_{pf}$  and  $P_{fp}$ ).

In some cases the boundaries between different components are not clear a priori and therefore have to be fixed by definition in our present global treatment. A typical example would be a tokamak reactor where the pulsed magnetic field device induces the plasma ring current, which primarily establishes the plasma equilibrium, but which also heats the plasma by the ohmic losses. In spite of this side effect we do not include P in H and describe by H only the devices necessary to heat the plasma after the initial ohmic phase to burn conditions by the main heating methods (e.g. neutral beam injection, RF heating, propagating thermonuclear burn). In more detailed representations of fusion power plants most of the problems associated with defining boundaries disappear owing to the higher degree of disaggregation.

### 3. Quantitative treatment by energy balances

#### 3.1 Characterization of the plant components by "component energy parameters"

For quantitative treatment such as was first performed systematically by Nozawa and Steiner /25/ we have to characterize the energy handling properties of the components. This is done in terms of energy ratios, which relate energies entering, leaving or flowing inside a component.



Very often these ratios are efficiencies as commonly used in connection with power plants, but not always. In the following we shall list these "component energy parameters".

The energies  $E$  always result from the corresponding powers  $P$  by integration over the puls length.

The fusion device  $F$  is characterized by:

$$Q = E_f/E_{pa} \text{ ("energy multiplication factor")} \quad (1)$$

$$\eta_a = E_{pa}/E_{hf} \text{ ("absorption efficiency")}, \quad (2)$$

$$\epsilon_{mp} = E_{pf}/E_{pa}, \quad (3)$$

$$\epsilon_{dt} = E_{fd}/(E_{ft} + E_{fd}) \quad (4)$$

$E_f$  = nuclear energy delivered by plasma fusion and blanket reactions (in general  $E_f \neq E_{ft} + E_{fd}$  !),

$E_{pa}$  = heating energy absorbed by the plasma,

$E_{hf}$  = heating energy delivered to the plasma,

$E_{pf}$  = pulsed magnetic energy delivered by  $P$  to  $F$ ,

$E_{fp}$  = pulsed magnetic energy recovered by  $P$  from  $F$ ,

$E_{fd}$  = energy delivered by  $F$ , that is concerted directly to electricity,

$E_{ft}$  = thermal energy delivered by  $F$ .

In connection with the energy multiplication factor  $Q$  care is necessary because of the different definitions used in the literature. We adopt the definition [eq. (1)] given in /25/ but want to mention other definitions as well. An energy amplification factor  $R$  was introduced by Lawson /26/. This factor  $R$  is identical with  $Q$  used here, only if the energy containment is infinitely good as was assumed in /26/. For the purpose of compiling and comparing the results of various fusion reactor studies /27/ the authors used a  $Q$ -definition /28/ which reads in our notation

$$Q_{PLH} = (E_{ft} + E_{fd})/E_{hf}.$$

For ideal conditions ( $\eta_a = 1$ , total recovery of the pulsed magnetic energy) this leads to

$$(Q_{PLH}) = 1 + Q_{ideal},$$

which reflects that the definition of  $Q_{PLH}$  includes the heating energy, whereas  $Q$  only accounts for the energy released by nuclear reactions. Very often - mainly in connection with mirror reactors -  $Q$  is defined as the nuclear energy divided by the heating energy input to the plasma, which in our notation leads to  $E_f/E_{hf}$  as definition of  $Q$ . The absence of nuclear reactions in the ideal case therefore leads to  $(Q_{PLH}) = 1$ , whereas we get the more suggestive value  $Q_{ideal} = 0$ .

It is important to keep in mind that  $E_{pa}$  is the total energy absorbed by the plasma during the heating phase and not the plasma energy content at a certain instant in time (for example at the end of heating). This instantaneous plasma energy  $E_p$  is smaller than  $E_{pa}$  if the heating time  $\tau_h$  is comparable with or larger than the energy confinement time  $\tau_{Eh}$  which is characteristic of the heating phase. The ratio  $\epsilon_{mp}$  for a tokamak reactor, for example, is therefore not only proportional to  $\beta_{pol} \cdot V_m/V_p$  ( $V_m$  = pulsed magnetic field volume,  $V_p$  = plasma volume), but also depends on  $\tau_h$  and  $\tau_{Eh}$ .

The ratios  $\beta$  and  $\beta_{pol}$  are commonly defined as the ratios of the space averaged plasma pressure to the magnetic pressure averaged over the plasma surface:

$$\beta \approx \frac{\langle 2nkT \rangle}{\langle B^2/2\mu_0 \rangle}, \quad (5)$$

$$\beta_{pol} \approx \frac{\langle 2nkT \rangle}{\langle B_{pol}^2/2\mu_0 \rangle} \quad (6)$$

( $B$  = main field induction,  $B_{pol}$  = poloidal field induction).

These relations are usually only approximately valid because they imply  $T_e = T_i$ ,  $p = nkT$ , and  $n_e = n_i$ , which is not valid for every case.



To fit the  $\beta$ 's into our scheme of energy ratios, we can use the fact that  $nkT$  is the thermal energy density in the two degrees of freedom normal to the magnetic induction  $\vec{B}$ , and that  $B^2/2\mu_0$  is the energy density of this field.

The thermal energy converter  $T$  and the direct energy converter  $D$  are characterized by the efficiencies with which the input energies are converted to electrical energy:

$$\eta_{th} = E_{et}/E_{ft}, \quad (7)$$

$$\eta_d = E_{ed}/E_{fd}, \quad (8)$$

$E_{ft}$  = thermal energy delivered by  $F$ ,

$E_{fd}$  = energy delivered by  $F$  which can be converted directly

$E_{et}$  = electric energy delivered by  $T$ ,

$E_{ed}$  = electric energy delivered by  $D$ .

The heating device  $H$  converts its input energy  $E_h$  with efficiency  $\eta_h$  to the heating energy  $E_{hf}$  delivered to the plasma:

$$\eta_h = E_{hf}/E_h. \quad (9)$$

The total efficiency of the heating process is given by  $\eta_h \eta_a = E_{pa}/E_h$ .

The pulsed magnetic field device  $P$  converts its input energy ( $E_p + E_{fp}$ ; see Fig. 1) to the magnetic energy  $E_{pf}$  transferred to the fusion device  $F$ . The overall efficiency of conversion and transfer to the plasma is  $\eta_m$ :

$$\eta_m = E_{pf}/(E_p + E_{fp}). \quad (10)$$

The fraction of  $E_{pf}$  that can be transferred back to  $P$  after the burn pulse is denoted by  $\epsilon_m$ :

$$\epsilon_m = E_{fp}/E_{pf}. \quad (11)$$

The energy consumption  $E_a$  of the auxiliary systems A is characterized by the fraction

$$\epsilon_a = E_a / (E_{ft} + E_{fd}). \quad (12)$$

This definition is used because one can expect the auxiliary energy required to be roughly proportional to the energy delivered by the fusion device F, because this is a measure of the plant size. This definition of  $\epsilon_a$  is fundamentally different from those used in /25/ for the corresponding parameter a, which gives the energy  $E_a$  in units of  $(P_h + P_p)$ . Whereas  $\epsilon_a$  has a limited range of variations, the parameter a strongly depends on Q. If for given gross output, for example, Q tends to large values, a has the same tendency. Therefore, a cannot be treated as virtually constant but has to be adjusted to Q.

### 3.2 Energy balance of complete plants leading to "plant energy parameters"

The following fourteen energies are involved in the energy balance of the plant according to Fig. 1:

$$E_c, E_h, E_p, E_a, E_{hf}, E_{pa}, E_{pf}, E_{fp}, E_{ft}, E_{fd}, E_{et}, E_{ed}, E_g, E_n.$$

These energies were already introduced and defined in Section 3.1, with the exception of  $E_g$ ,  $E_n$ , and  $E_c$ , which are not characteristic of a single component but of the plant as a whole.  $E_g$  is the gross energy delivered by the two energy conversion systems T and D,  $E_n$  is the net energy delivered to the grid after the energy  $E_c$  has been subtracted to be circulated back to the plant entrance.

The fourteen energies are linked by the following thirteen equations:

$$E_c = E_h + E_p + E_a, \quad (13)$$

$$E_{hf} = \eta_h E_h, \quad (14)$$

$$E_{pa} = \eta_a E_{hf}, \quad (15)$$

$$E_{pf} = \eta_m (E_p + E_{fp}), \quad (16)$$

$$E_{fp} = \epsilon_m E_{pf}, \quad (17)$$

$$E_{pf} = \epsilon_{mp} E_{pa}, \quad (18)$$

$$E_a = \epsilon_a (E_{ft} + E_{fd}), \quad (19)$$

$$E_{ft} + E_{fd} = Q E_{pa} + (1 - \eta_a) E_{hf} + (1 - \epsilon_m) E_{pf} + \eta_a E_{hf}, \quad (20)$$

$$E_{fd} = \epsilon_{dt} (E_{ft} + E_{fd}), \quad (21)$$

$$E_{et} = \eta_{th} E_{ft}, \quad (22)$$

$$E_{ed} = \eta_d E_{fd}, \quad (23)$$

$$E_g = E_{et} + E_{ed}, \quad (24)$$

$$E_c = E_g - E_n. \quad (25)$$

Equations (14), (15), (16), (17), (18), (19), (21), (22), and (23) emerge from the definitions of  $\eta_h$ ,  $\eta_a$ ,  $\eta_m$ ,  $\epsilon_m$ ,  $\epsilon_{mp}$ ,  $\epsilon_a$ ,  $\epsilon_{dt}$ ,  $\eta_{th}$ , and  $\eta_d$  in Section 3.1. Equations (13), (24), and (25) obviously follow from balancing the energy flows at the nodes a, b, and c in Fig. 1. Equation (20) reflects the fact that the energy delivered by the fusion device is the sum of the reaction energy delivered by the plasma and blanket, the heating energy not absorbed by the plasma, the pulsed magnetic energy not transferred back to P, and the heating energy originally absorbed by the plasma but lost during or after the burn to the reactor structure. By using eqs. (13) to (25), we can calculate the "plant energy parameters"  $Q_c$ ,  $\eta$ , and  $C$  which are characteristic of the plant as a whole:  $Q_c$  is the critical value of  $Q$  necessary for the plant to sustain itself energetically,  $\eta$  is the net plant efficiency, and  $C$  is the ratio of circulated energy  $E_c$  to the net energy  $E_n$ . They are functions of the component energy parameters introduced in Section 3.1

$$Q_c = \left( \frac{E_f}{E_{pa_c}} \right) = \frac{1}{\epsilon \eta_a \eta_{eff}} - [1 + \eta_a \epsilon_{mp} (1 - \epsilon_m)], \quad (26)$$

$$\eta = \frac{E_n}{E_f} = (1 - Q_c/Q) \eta_{eff}, \quad (27)$$

$$C = \frac{E_c}{E_n} = \frac{1 + \epsilon \epsilon_a [Q \eta_a + 1 + \eta_a \epsilon_{mp} (1 - \epsilon_m)]}{\epsilon \eta_a \eta_{eff} (Q - Q_c)} \quad (28)$$

with the following abbreviations

$$\epsilon = \frac{E_{hf}}{E_h + E_p} = \frac{\eta_h \eta_m}{\eta_m + \epsilon_{mp} \eta_a \eta_h (1 - \epsilon_m \eta_m)}, \quad (29)$$

$$\eta_{eff} = (\eta_{th} - \epsilon_a) + \epsilon_{dt} (\eta_d - \eta_{th}). \quad (30)$$

The parameter  $\epsilon$  is the ratio of the heating energy delivered to the plasma to the sum of the energies fed into the heating device H and the pulsed magnetic field device P;  $\eta_{eff}$  is the effective energy conversion efficiency of the plant. It depends on the types of energy converters used via  $\eta_{th}$  and  $\eta_d$  but also on the type of fusion device via  $\epsilon_{dt}$  and finally on the energy consumption of the auxiliary devices via  $\epsilon_a$ .

The critical amplification  $Q_c$  quantifies the imperfection with respect to energy handling of the components surrounding the fusion device F. If there were no energy losses or demands made by these components ( $\eta_h = 1$ ,  $\eta_m = 1$ ,  $\eta_a = 1$ ,  $\eta_{th} = 1$ ,  $\eta_d = 1$ ,  $\epsilon_a = 0$ ,  $\epsilon_m = 1$ ), the critical amplification  $Q_c$  would be zero. This means that in the ideal case no additional energy such as fusion energy is necessary to keep the plant itself running.  $Q_c$  provides a yardstick for assessing plasma quality. This is best done in terms of the ratio  $Q_c/Q$ . Only if  $Q_c/Q$  is sufficiently small, say less than 0.1, can the net efficiency  $\eta$  come close to its upper limit  $\eta_{eff}$  and C close to its lower limit  $\epsilon_a/\eta_{eff}$  as shown by eqs. (27) and (28) respectively.

The net efficiency  $\eta$  is the most commonly used figure of merit for all types of power plants. The obvious implications concerning resources, plant economy and environmental aspects need not be repeated here.

The circulating energy ratio C measures the energy consumed by the plant in units of the net energy delivered to the consumer. Values of C up to several per cent are usual for fossil and fission power plants. A unique feature of fusion power plants lies in the danger of needing C values closer to or larger than unity than values close to those of present-day power plants. The reasons for this are the need for complicated systems with presumably moderate efficiencies to produce



and maintain fusion plasmas. An additional reason are the large energies, such as pulsed magnetic field energy (described by  $\epsilon_{mp}$ ), which possibly have to be transferred internally. If such transfers could be performed without losses (e.g.  $\epsilon_m = 1$ ), their influence on  $Q_c$  and  $C$  would disappear as shown by eqs. (26) and (28). Large values of  $C$  mean large installations with the corresponding negative impact on installation and maintenance costs. Even for ideal internal energy transfers (e.g.  $\epsilon_m = 1$ ) high values of these energies (e.g.  $\epsilon_{mp} \gg 1$ ) lead to high costs because of the size of the installations needed. This effect is not displayed by  $C$  but by parameters such as  $\epsilon_{mp}$ .

For the hypothetical case  $Q \rightarrow \infty$  we read from eqs. (27) and (28)

$$\eta \rightarrow \eta_{eff}, \quad (31)$$

$$C \rightarrow \epsilon_a / \eta_{eff}. \quad (32)$$

This limiting case confirms the importance of the definition of  $\epsilon_a$  used in this report: only for  $\epsilon_a \rightarrow 0$  would one approach the ideal value  $C = 0$ , but  $Q \rightarrow \infty$  is not sufficient to reach this goal.

To show the connection between fusion physics and the formal energy balances treated in this section, we present in Appendix A a calculation of  $Q$  based on an extremely simplified plasma model similar to that used by Lawson /26/.

### 3.3 Parametric sensitivity study

To draw essential conclusions from a parametric study of component energy parameters and plant energy parameters, we have to assume that the latter represent real figures of merit which quantify important physical, technological, and economic aspects. Furthermore, we have to assume that also in the future economic and ecological criteria will decide on the introduction of new energy systems and not only the mere need of energy. This does not mean, however, that a decision between future systems can be made on the basis of present cost estimates because the extrapolation in time is hardly possible and because costs are only one ingredient of energy

prices, which are also influenced by economic and political boundary conditions. On the other hand, it is absolutely necessary to optimize each energy system separately with respect to costs and environmental standards because otherwise resources or money would be wasted.

### 3.3.1 Reference conditions

In the preceding section the plant energy parameters  $Q_c$ ,  $\eta$ , and  $C$  are given in terms of the component energy parameters which describe the individual plant components. We shall use eqs. (26), (27), and (28) to assess the sensitivity of the plant energy parameters with respect to changes of the component energy parameters. This is necessary because only some of these parameters are known, whereas the rest are speculative, particularly for the unconventional components such as the fusion device  $F$ , the heating device  $H$ , the pulsed field device  $P$ , and the direct converter  $D$ . We thus intend to separate parameters with strong impact on the plant energy parameters from those of only moderate influence.

To have a starting point, we choose two sets of reference values, one describing a high  $Q$  system which may be realized by a system with good particle confinement, (e.g. a "closed system" such as a tokamak-like device) and one for a system with a low  $Q$ , such as a device with poor particle confinement (e.g. an "open system" such as a mirror-like device). To be more precise, we should compare the two systems on the basis of their values for  $Q/Q_c$  corresponding to a given  $\eta$  instead of their  $Q$  values. Therefore, our terms "high  $Q$ " and "low  $Q$ " have to be interpreted as "high  $Q/Q_c$ " and "low  $Q/Q_c$ ". The two sets of energy balance parameters chosen are shown in Table 1.

The high- $Q$  system is supposed to operate in a pulsed mode. The values chosen for  $\eta_h$ ,  $\eta_m$ ,  $\eta_a$ ,  $\epsilon_m$ , and  $\epsilon_a$  are intended to be realistic or even somewhat pessimistic.  $\epsilon_{dt} = 0$  describes the fact that the whole output of the fusion device is in the form of heat. The value of  $\eta_d$  therefore has no influence (we have arbitrarily chosen  $\eta_d = 0$ ). The value of  $\epsilon_{mp}$  is rather arbitrary. We have chosen  $\epsilon_{mp} = 5$ , which is close to the value for the UWMAK II design /8/ (poloidal field energy/absorbed energy = 5.2), which uses poloidal field windings inside the toroidal field coils. For the poloidal field coils outside the toroidal coils used in the UWMAK I design the poloidal field energy is larger by a factor of five. For other concepts

High-Q system (e.g. tokamak system)	Low-Q system (e.g. mirror system)
$\eta_h = 0.5$	$\eta_h = 0.5$
$\eta_m = 0.8$	$\eta_m = 1$
$\eta_a = 0.5$	$\eta_a = 0.5$
$\eta_{th} = 0.36$	$\eta_{th} = 0.36$
$\eta_d = 0$	$\eta_d = 0.7$
$\epsilon_{mp} = 5$	$\epsilon_{mp} = 0$
$\epsilon_m = 0.5$	$\epsilon_m = 1$
$\epsilon_{dt} = 0$	$\epsilon_{dt} = 0.5$
$\epsilon_a = 0.03$	$\epsilon_a = 0.03$
$Q = 233.58$	$Q = 17.5$

Table 1

such as the reversed field pinch where all magnetic fields are pulsed and produced by normal-conducting coils  $\epsilon_{mp}$  assumes very high values, which indeed may not be tolerable.

The value  $\eta_{th} = 0.36$  is a reasonably realistic mean value for a thermal conversion system using steam turbines at moderate live steam conditions.

For the low-Q system we assume continuous operation and have chosen  $\eta_m = 1$ ,  $\epsilon_{mp} = 0$ ,  $\epsilon_m = 1$  to describe formally the fact that no pulsed magnetic fields are present. The losses due to stationary fields are included in  $\epsilon_a$ . The value of  $\eta_d = 0.7$  is rather speculative, but certainly not over pessimistic. With  $\epsilon_{dt} = 0.5$  we have adopted a value which is characteristic of the classical mirror machine (see Appendix B). In fact,  $\epsilon_{dt}$  decreases with increasing Q. In our parametric study we do not want to account for this coupling. Therefore  $\epsilon_{dt} = 0.5$  represents an academic reference value. New concepts such as the tandem mirror /29/ would have values of  $\epsilon_{dt}$  appreciably lower than 0.5.

The  $Q$  values for both systems are not realistic, pessimistic or optimistic but required to get a net efficiency  $\eta = 0.3$ . This value seems to be a target figure which one should not fall short of. The high value  $Q = 233.58$  is only conceivable if ignition (see Appendix A) can be achieved. The value  $Q = 17.5$  for the low- $Q$  system cannot be reached in a classical mirror machine which are characterized by  $Q$  near unity. Whether new concepts such as the tandem mirror can lead to  $Q = 17.5$ , which we have characterized as "low", is still open.

The values of the plant energy parameters  $Q_c$ ,  $\eta$ , and  $C$  and the value of  $\eta_{eff}$  corresponding to the reference values of the energy balance parameters given in Table 1 are collected in Table 2.

High-Q system	Low-Q system
$Q_c = 21.235$	$Q_c = 7.00$
$\eta = 0.30$	$\eta = 0.30$
$C = 0.2125$	$C = 0.8733$
$\eta_{eff} = 0.33$	$\eta_{eff} = 0.50$

Table 2

Table 2 shows the characteristic difference between a high- $Q$  system (only heat delivered) and a low- $Q$  system (heat and directly convertible energy delivered): the low- $Q$  system can reach the margin of net energy output with much less amplification  $Q_c$  than the closed system. The  $Q_c$  values

differ by a factor of 3 in the reference case. The much less stringent requirements concerning plasma performance in the low- $Q$  case have to be paid for by a large circulating power ratio  $C$ , which in the reference case amounts to 4.1 times the high- $Q$  system value. This high amount of energy circulating inside the plant is also reflected by the large difference between the effective efficiency  $\eta_{eff} = 0.50$  and the net efficiency  $\eta = 0.30$ . The large value of  $C$  and the need for using two different energy conversion systems presumably increases the installation costs.

### 3.3.2 Results of parametrically varying the component energy parameters $\eta_h, \eta_a, \eta_m, \eta_d, \epsilon_{mp}, \epsilon_m, \epsilon_{dt}, \epsilon_a$ and $Q$

In this section we shall describe the effect of varying the component energy parameters (see Table 1) on the plant energy parameters  $Q_c$ ,  $\eta$ , and  $C$ . The plant energy parameters are given in units of their reference



values according to Table 2. The only energy balance parameter not varied is the thermal efficiency  $\eta_{th}$ , which we assume to be reasonably well known. According to the characteristics of the two systems, some of their parameters will not be varied. The parameters to be varied are shown in Table 3.

high-Q system	$\eta_h$	$\eta_a$	$Q$	$\epsilon_a$	$\eta_m$	$\epsilon_{mp}$	$\epsilon_m$	not varied
low-Q system	$\eta_h$	$\eta_a$	$Q$	$\epsilon_a$	not varied			$\eta_d$ $\epsilon_{dt}$

Table 3

The variations of  $Q_c$ ,  $\eta$ , and  $C$  for the two systems are shown in Figs. 2, 4, 6, and 3, 5, 7 respectively. Each curve is marked by the component energy parameter which varies along the curve, the variation with respect to its reference value being given by the abscissa values. In some cases the extent of the variations is academic. Examples are  $\eta_d < \eta_{th} = 0.36$ , which is not of practical interest, or  $\epsilon_m = 1$ , which is not possible physically because of coil losses and plasma resistivity.

For systematically assessing the sensitivity of the plant energy parameters with respect to variations of the component energy parameters we use the following procedure, which admittedly is somewhat ambiguous: We vary the component energy parameters in the range of 1/2 to 2 times their reference values and classify the resulting variations of the plant energy parameters according to the sector scheme shown in Fig. 8. The classification ranges from the obvious "no influence" over "weak", "moderate", "strong" to "very strong". The classification "very strong" means that the resulting relative variation of a plant energy parameter is stronger than the original relative component energy parameter change, so that the line between "strong" and "very strong" marks the boundary between the amplifying and damping action of the system. The results of the classification are shown in matrix form in Fig. 9.

It is obvious that generally the low-Q system is much more sensitive than the high-Q system. The reason for this behaviour lies in the fact that the net output energy  $E_n$ , the gross energy  $E_g$  and the circulated energy  $E_c$  are of the same order. Difference machines of this kind are always very sensitive to parameter variations. A well-known example is the gas turbine, the output of which is the difference between the gross output energy and compression work, both being of the

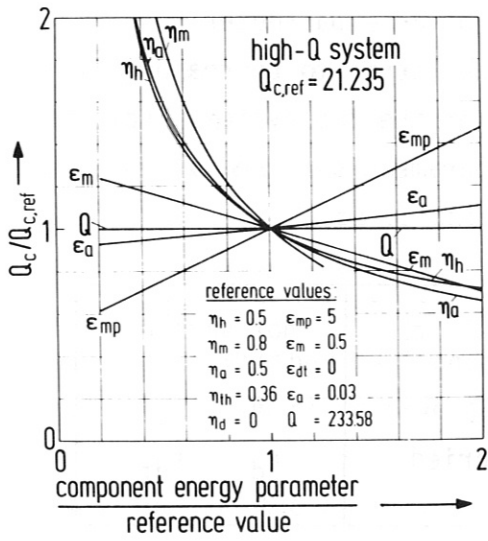


Fig. 2

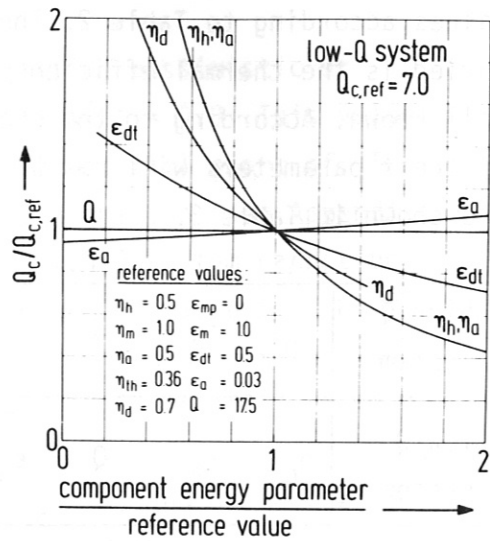


Fig. 3

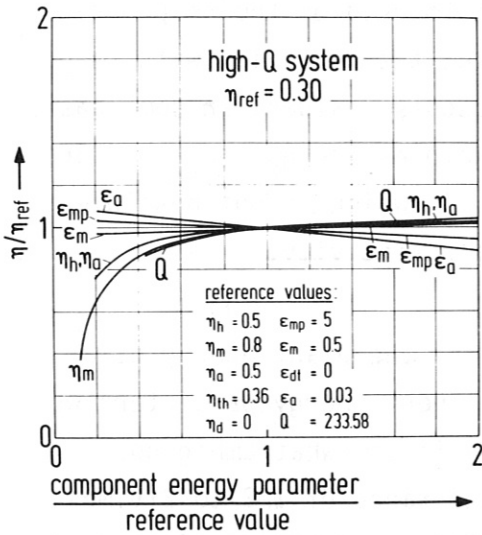


Fig. 4

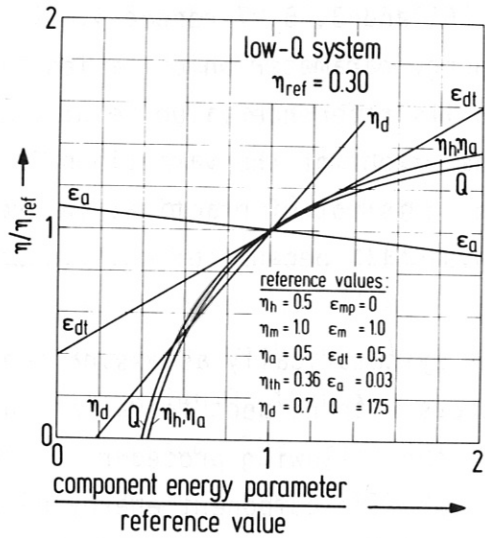


Fig. 5

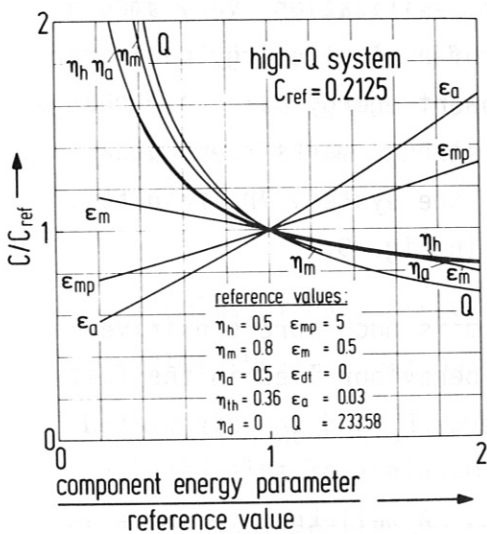


Fig. 6

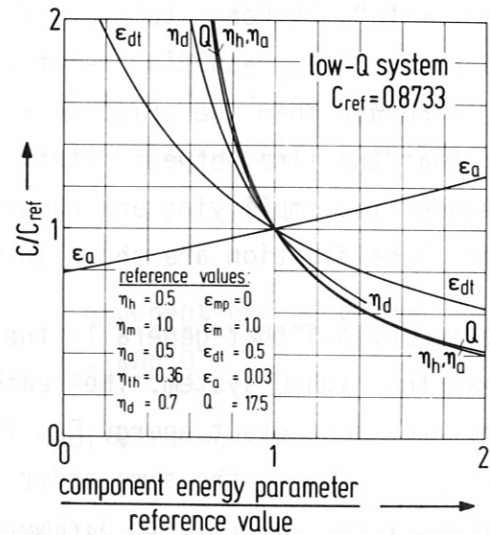


Fig. 7

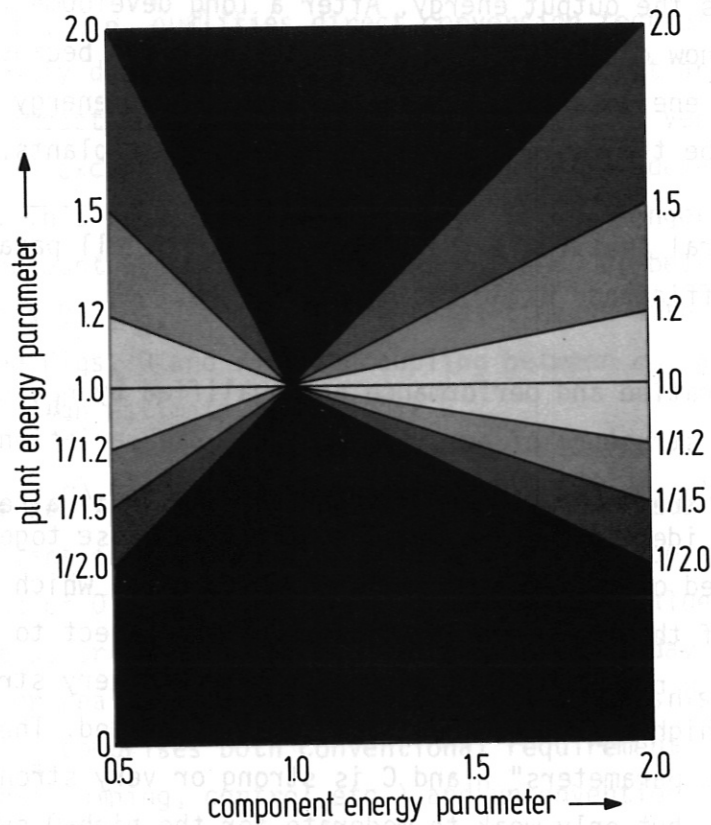


Fig. 8

		$\eta_h$	$\eta_a$	$Q$	$\epsilon_a$	$\eta_m$	$\epsilon_{mp}$	$\epsilon_m$	$\eta_d$	$\epsilon_{dt}$
high-Q System	$Q_c$									
	$\eta$									
	$C$									
low-Q System	$Q_c$									
	$\eta$									
	$C$									

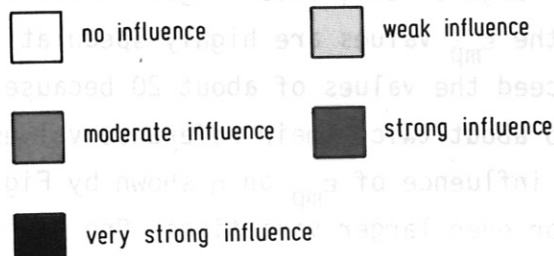


Fig. 9

same order as the output energy. After a long development the gas turbine can now cope with this disadvantage partly because the recirculated energy does not have to pass through energy conversion. This has to be the case, however, in fusion power plants.

Another general feature is the weak influence of all parameter changes on the net efficiency  $\eta$  for the high-Q system.

Plasma preparation and performance are qualified by  $\eta_h$ ,  $\eta_a$ , and  $Q$ . Owing to the structure of eqs. (26) to (29) the variations of  $Q_c/Q_{c,ref}$ ,  $\eta/\eta_{ref}$ , and  $C/C_{ref}$  with  $\eta_h/\eta_{h,ref}$  and  $\eta_a/\eta_{a,ref}$  are either analytically identical or at least numerically close together. We therefore need only discuss them in terms of  $\eta_h\eta_a$ , which is the efficiency of the whole heating process. With respect to  $Q_c$  the influence of  $\eta_h\eta_a$  ranges from strong (high-Q) to very strong (low-Q) which means high impact on the plasma quality needed. The impact on the "economic parameters"  $\eta$  and  $C$  is strong or very strong for the low-Q system, but only weak to moderate for the high-Q system. The influence of varying the plasma performance  $Q$  is strong to very strong (with the exception of  $\eta$  of the closed system).

The influence of pulsed magnetic field technology is described by  $\eta_m$ ,  $\epsilon_{mp}$ , and  $\epsilon_m$ . The influence of  $\eta_m$  on the critical amplification  $Q_c$  and on  $C$  is strong, whereas variations of  $\epsilon_m$  only cause weak to moderate changes. Within the range of variations shown in Figs. 2, 4, and 6 the influence of  $\epsilon_{mp}$  is weak to moderate. It may well be, however, that upper values for  $\epsilon_{mp}$  will not be restricted to twice the reference values ( $\epsilon_{mp} = 5$ ) but will amount to much larger values. This fact has already been mentioned at the end of Section 3.3.1 in connection with the reference value. Figures 2 and 6 show a linear increase of  $Q_c$  and  $C$  with  $\epsilon_{mp}$  of considerable slope. The weak to moderate influence of  $\epsilon_{mp}$  displayed by Fig. 9 can therefore change to a much more drastic or even intolerably large effect. This danger cannot be quantified at present because the  $\epsilon_{mp}$  values are highly speculative. At any rate  $\epsilon_{mp}$  should not exceed the values of about 20 because otherwise  $Q_c$  and  $C$  would amount to about twice their reference values, which are rather high anyhow. The influence of  $\epsilon_{mp}$  on  $\eta$  shown by Fig. 4 is weak and will remain so for even larger variations. One has to keep in mind, however, that the reference value  $\eta = 0.30$  is very low compared with those for present-day power plants.



The parameters  $\eta_d$  and  $\epsilon_{dt}$  are specific for low-Q systems especially for mirror devices;  $\eta_d$  qualifies direct conversion technology, whereas  $\epsilon_{dt}$  is strongly dependent on the confinement concept and its performance. The impact on the plant energy parameters is very strong or strong, with the exception of  $Q_c$  and  $C$ , which only moderately depend on  $\epsilon_{dt}$ . This latter fact would change to a stronger dependence if we took into account the coupling between  $\epsilon_{dt}$  and  $Q$ , because the slope of the functions  $Q_c(\epsilon_{dt})$  and  $C(\epsilon_{dt})$  becomes steeper with decreasing  $\epsilon_{dt}$  and  $Q$ . (see Figs. 3 and 7). The coupling between  $\epsilon_{dt}$  and  $Q$  is treated by a rough estimate in Appendix B.

Generally, one reads from Figs. 2 to 7 a weak to moderate influence of  $\epsilon_a$  on all plant energy parameters. This is mainly due to the low reference value of 0.03 adopted for  $\epsilon_a$ . Whether this value, which is about the same as or slightly higher than for present-day power plants, is optimistic or realistic is open to speculation. This is because of the fact that  $\epsilon_a$  comprises both conventional requirements (e.g. power for conventional pumping, control etc.) and unconventional ones (power for, for example, liquid metal and vacuum pumps, refrigeration of superconducting magnets etc.). Whether the unconventional contributions can amount to the same order as or even more than the conventional ones can only be decided on the basis of fully elaborated designs. Because these cannot exist at the present state of development a final assessment of the impact of  $\epsilon_a$  cannot be made. An upper value of  $\epsilon_a = 0.12$ , which is four times the reference value, would drastically increase  $C$ . This effect is most pronounced for the high-Q system, whose  $C$  value was a factor of four lower than for the low-Q system and therefore reacts more sensitively to additional power demands.  $Q_c$  would not react very sensitively for both systems. Formally the same is true of  $\eta$  but again one has to keep in mind in this context that  $\eta_{ref} = 0.3$  is rather low anyhow.

#### 4. Conclusions drawn from the parametric study

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The modelling of fusion power plants by systems of energy handling components with component energy parameters as input and with plant energy parameters as output provides very sensitive systems. This sensitivity affords the chance of obtaining important results already now where fusion systems studies are of necessity theoretical and cannot be supplemented by experiments in the near future.

The component energy parameters  $\eta_h$ ,  $\eta_a$ ,  $\eta_m$ ,  $Q$ , and  $\epsilon_{dt}$ , which describe the plasma type, preparation and performance are highly speculative at present or at least theoretical all have a strong or a very strong influence on the plant energy parameters. None of these component energy parameters can therefore be omitted even for very crude investigations of fusion power plants. The direct conversion efficiency  $\eta_d$  is very important for the prospects of mirror-type reactors if  $\epsilon_{dt}$  is large enough because it very strongly influences all plant energy parameters. Its impact on these parameters is stronger than the influence of  $\eta_m$ , which is specific to pulsed magnetic systems.

The component energy parameters  $\epsilon_{mp}$  and  $\epsilon_m$  describing the amount of pulsed magnetic energy needed and the efficiency of its recovery have not the strong influence sometimes intuitively suspected but act only weakly on the "economic parameters"  $\eta$  and  $C$  and moderately on the "physical parameter"  $Q_c$ . One has to keep in mind, however, that this situation changes if the actual value for  $\epsilon_{mp}$  becomes much larger than the reference value of 5, as was already discussed in Section 3.3.2.

An analogous uncertainty holds with respect to  $\epsilon_a$ . If  $\epsilon_a$  no longer remains small compared with unity, its influence may become rather large, as was already pointed out at the end of Section 3.3.2.

The potential influence of  $\epsilon_{mp}$  and  $\epsilon_a$  therefore lies in the fact that they do not have a theoretical upper bound ( $\epsilon_{mp}$ ) or that this bound is much larger than the reference value adopted on the basis of non-fusion power plants.

The above stated sensitivity of the system with respect to parameter changes comprises the risk of proving statements which would be correct only if the component energy parameters were independent of each other or not subject to limitations besides the trivial ones. In reality, this simple situation does not hold. To set up models of reasonable realism, one has to account for couplings (e.g.  $\epsilon_{dt}(Q)$ , see Appendix B) and limitations. In general, however, these act on the level of c o m p o n e n t p h y s i c s and not on the level of the highly aggregated component energy parameters (see, for example, Appendix B).

The necessity of accounting for mutual relations between the component energy parameters and the advantages of an energy flow diagram oriented system can be brought together by modelling the different components on a physical basis. This can be done by using the equations which describe the physics of the individual components to determine the component energy parameters. The couplings between the components have to be treated similarly to the components, which means that the abstract energy flows have to be replaced by flows of particles, radiation, heat and so on which physically carry the energy.

The representation of fusion power plants in this way should provide the models for the theoretical systems studies necessary in the course of fusion development. Because of the modular structure it is possible to adjust the models to different kinds of fusion reactors as well as to progress made in physics and technology of individual components.

## APPENDIX A:

### A condition for the reactor burn time based on a simple plasma model and the plant energy balance

To find a connection between the energy amplification factor  $Q$  and fundamental plasma parameters, we shall use the following strongly simplified model of a pulsed confinement system:

- a) DT plasma,
- b) temperature  $T$  ( $T_i = T_e = T$ ) and density  $n$  ( $n_i = n_e = n$ ) constant during plasma burn,
- c) after a burn time  $\tau_b$  the pulse terminates abruptly,
- d) the time  $\tau_h$  necessary to bring the plasma to ignition ( $\alpha$  self-heating) is small compared with  $\tau_b$ ,
- e) the losses during the heating time are described by the gross energy confinement time  $\tau_{Eh}$ ,
- f) the transport losses during the burn phase are described by the energy confinement time  $\tau_E$ ,
- g) the radiation losses during the burn phase are only due to electron bremsstrahlung.

The idealized function  $T(t)$  is shown in Fig. A1.

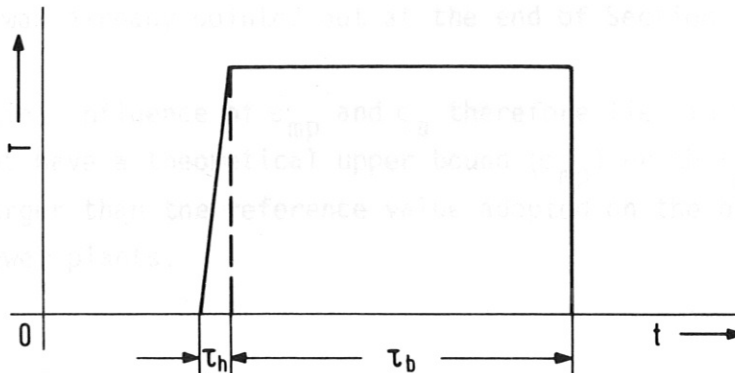


Fig. A1



In the following we shall use energies per unit volume ( $e$ ) instead of the energies ( $E$ ) used in Section B.

The energies delivered by fusion reactions in the form of neutron and  $\alpha$ -particle kinetic energies are given by

$$e_n = \frac{1}{4} n^2 \langle \sigma v \rangle E_n \tau_b \quad (A1)$$

$$e_\alpha = \frac{1}{4} n^2 \langle \sigma v \rangle E_\alpha \tau_b \quad (A2)$$

( $E_n = 14.06$  MeV,  $E_\alpha = 1/4 \cdot E_n = 3.52$  MeV).

To heat the plasma to the burn conditions, the energy  $e_{pa}$ , which is the sum of the following two energies, is necessary:

$$e_{th} = 3 nkT, \quad (A3)$$

$$e_{h,1} = (3 nkT/\tau_{Eh}) \cdot \tau_h, \quad (A4)$$

where  $e_{th}$  is the energy content of the plasma under burn conditions ( $T, n$ ), and  $e_{h,1}$  is the energy absorbed by the plasma but already lost again during the heating phase.

During the burn the energies  $e_{transp}$  and  $e_{rad}$  are lost by transport processes and radiation:

$$e_{transp} = (3 nkT/\tau_E) \tau_b, \quad (A5)$$

$$e_{rad} = g_b n^2 T^{1/2} \tau_b \quad (A6)$$

( $g_b$  = factor of proportionality).

The burn conditions ( $T, n, \tau_E$ ) are chosen such that the transport and radiation losses are just covered by the fusion energy carried by the  $\alpha$  particles:

$$e_{transp} + e_{rad} = e_\alpha. \quad (A7)$$

Together with eqs. (A2), (A5), and (A6) this leads to the so-called "ignition criterion"

$$n\tau_E = \frac{12 kT}{\langle \sigma v \rangle E_\alpha - 4g_b T^{1/2}}. \quad (A8)$$

The energy balance of the "ignited plasma" is visualized by Fig. A2.

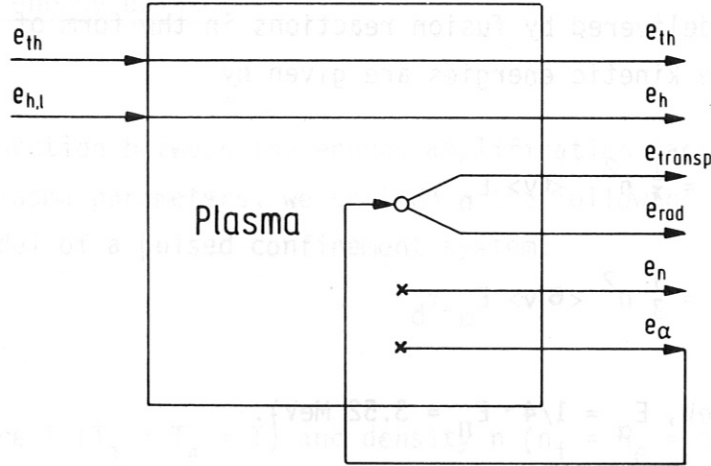


Fig. A2

By definition (1) the amplification factor  $Q$  is given as the energy delivered by nuclear reactions ( $e_f$ ) divided by the energy absorbed by the plasma ( $e_{pa}$ ). With

$$e_f = e_n + e_\alpha \quad (A9)$$

and

$$e_{pa} = e_{th} + e_{h,l} \quad (A10)$$

we get

$$Q = \frac{\frac{1}{4} n^2 \langle \sigma v \rangle (E_n + E_\alpha) \tau_b}{3 nkT (1 + \tau_h/\tau_{Eh})}. \quad (A11)$$

No amplification  $M$  by blanket reactions was included in (A11). From (A11) we obtain the condition

$$n\tau_b = \frac{12 kT (1 + \tau_h/\tau_{Eh})}{\langle \sigma v \rangle (E_n + E_\alpha)} Q, \quad (A12)$$

which always has to be supplemented during the burn by the "ignition criterion" (A8). By combining eqs. (A8) and (A12) we arrive at

$$\frac{\tau_b}{\tau_E} = Q (1 + \tau_h/\tau_{Eh}) \frac{\langle \sigma v \rangle E_\alpha - 4 g_b T^{1/2}}{\langle \sigma v \rangle (E_n + E_\alpha)}. \quad (A13)$$

Equation (A13) quantifies the ratio between the reactor burn time  $\tau_b$  and the transport energy confinement time necessary to achieve the energy amplification  $Q$  taking into account the burn temperature  $T$  and the losses during the heating phase.

By using eq. (27) we get from (A13) and (A12) respectively

$$\frac{\tau_b}{\tau_E} = \frac{Q_c}{1 - \eta/\eta_{eff}} (1 + \tau_h/\tau_{Eh}) \frac{\langle \sigma v \rangle E_\alpha - 4g_b T^{1/2}}{\langle \sigma v \rangle (E_n + E_\alpha)}, \quad (A14)$$

$$n\tau_b = \frac{Q_c}{1 - \eta/\eta_{eff}} (1 + \tau_h/\tau_{Eh}) \frac{12 kT}{\langle \sigma v \rangle (E_n + E_\alpha)}. \quad (A15)$$

It is reasonable to evaluate eqs. (A14) and (A15) for the assumption  $\tau_h \sim \tau_{Eh}$ , which means that the heating power has to be  $\sim 3 nkT/\tau_{Eh}$ . Figures A3 and A4 show  $\tau_b/\tau_E$  and  $n\tau_b$  vs.  $T$  for the marginal case  $\tau_h/\tau_{Eh} = 1$ , with  $\eta/\eta_{eff}$  as parameter. For  $Q_c$  we have chosen 21.235,

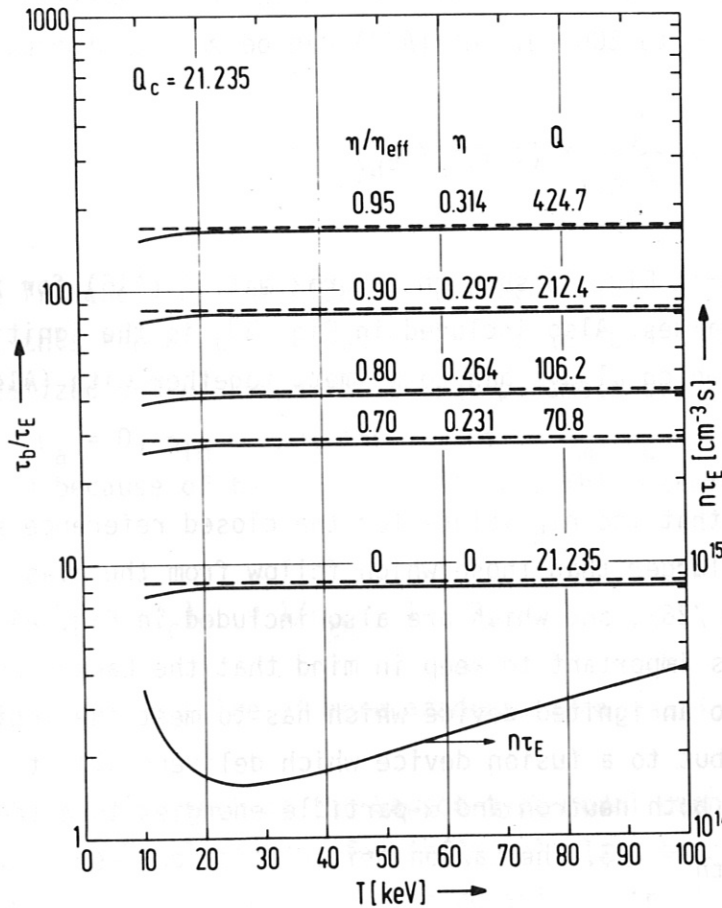


Fig. A3

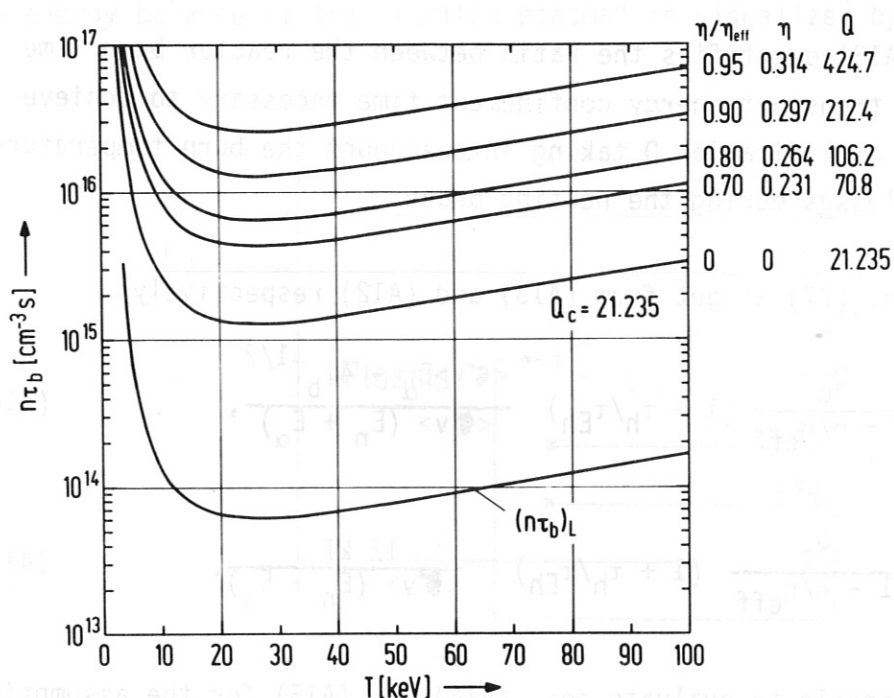


Fig. A4

which is the reference value for the closed system described in Section 3.3. Because  $4f_b T^{1/2} \ll \langle \sigma v \rangle E_\alpha$  at least for temperatures in the range of 10 to 100 keV eq. (A14) can be well approximated by

$$\frac{\tau_b}{\tau_E} \approx \frac{1}{5} \cdot \frac{Q_c}{1 - \eta/\eta_{eff}} \cdot (1 + \tau_h/\tau_{Eh}). \quad (A16)$$

The dashed lines in Fig. A3 show the approximation (A16) for the above used parameter values. Also included in Fig. A3 is the ignition criterion (A8), which always has to be met, together with (A14) and (A15).

Figure A4 shows that the  $n\tau_b$  values for the closed reference system have to be much larger than those which follow from the classical Lawson criterion [26], and which are also included in Fig. A4 for comparison. It is important to keep in mind that the Lawson criterion does not refer to an ignited device which has to meet the ignition criterion (A8), but to a fusion device which delivers all its output energy including both neutron and  $\alpha$ -particle energies to a thermal converter with  $\eta_{th} = 1/3$ . The Lawson criterion describes the margin

between net output and net consumption of a very simple plant model: the whole output of the energy converter is fed back to the plasma to maintain the plasma temperature against the losses due to electron bremsstrahlung (transport losses and losses during plasma heating and energy necessary for refuelling are not accounted for). This simple scheme is visualized by Fig. A5.

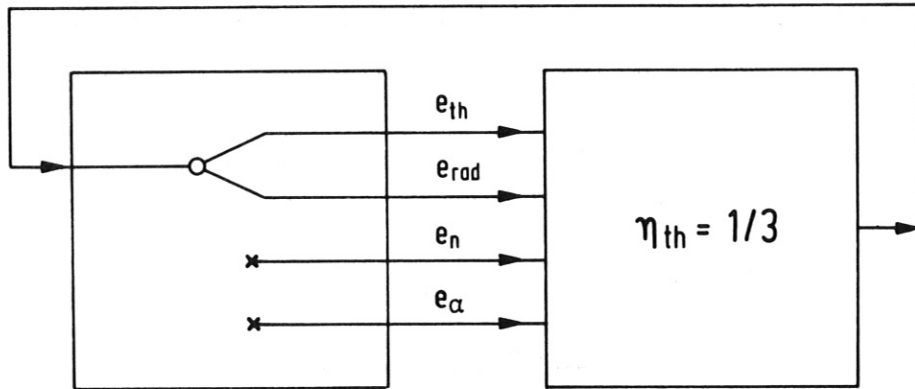


Fig. A5

The product  $(n\tau)_L$  for this case is given by

$$(n\tau_b)_L = \frac{12 \text{ kT}}{\langle \sigma v \rangle (E_n + E_\alpha) \eta_{th} / (1 - \eta_{th}) - 4g_b T^{1/2}} \quad (\text{A17})$$

Within the framework of the plant scheme according to Fig. 1 used throughout this report the Lawson case is described by the following highly idealized set of component energy parameters:  $\eta_h = \eta_a = 1$ ,  $\epsilon_{mp} = \epsilon_{dt} = \epsilon_a = 0$ ,  $\eta_{th} = 1/3$ ; the values of  $\eta_m$ ,  $\eta_d$ , and  $\epsilon_m$  need not be specified because of the simplifications. This parameter set leads to

$$Q_c = (Q_c)_L = 1/\eta_{th} - 1 = 2, \quad (\text{A18})$$

which is obviously a value of more academic than practical interest.

A not ignited device such as the Lawson-type model cannot lead to infinitely large  $Q$  values even for infinitely long burn times  $\tau_b$ . The  $Q$  values attainable depend on the details of the losses as well



as on the energy requirements of plasma heating and refuelling. For a burn temperature of 25 keV a rough upper limit is given by the ratio of fusion energy (17.6 MeV) to the thermal energy of the two ions and the two electrons involved ( $E_{th} = 4 \times 25 = 100$  keV). This simple estimate leads to  $Q_{max} \approx 175$ .

## APPENDIX B:

### A simple estimate for the relation between $\epsilon_{dt}$ and Q for mirror-like devices

In an open-ended device the amplification factor Q can be enhanced by decreasing the loss of particles which carry energy out of the ends of the device. This is inevitably accompanied by a decrease of  $\epsilon_{dt}$  because less energy is delivered to the direct converters.

The ratio  $\epsilon_{dt}$  was defined by eq. (4) as

$$\epsilon_{dt} = \frac{E_{fd}}{E_{fd} + E_{ft}} \quad (B1)$$

We consider a device (e.g. the central part of a tandem mirror) whose ion and electron temperatures and the density are kept constant. Radiation losses will be neglected.

The power density lost with particles is given by

$$p_p = \frac{n_i}{\tau_p} (\overline{E}_{i,1} + \overline{E}_{e,1}) \quad (B2)$$

( $n_i \approx n_e$ ,  $n_i$  = ion density,  $n_e$  = electron density,  $\overline{E}_{i,1}$  = average energy of the lost ions,  $\overline{E}_{e,1}$  = average energy of the lost electrons,  $\tau_p$  = particle confinement time). The power loss due to heat conduction is expressed by using the ion and electron energy confinement times  $\tau_{Ei}$  and  $\tau_{Ee}$ :

$$p_c = \overline{E}_i / \tau_{Ei} + \overline{E}_e / \tau_{Ee} \quad (B3)$$

( $\overline{E}_i$  and  $\overline{E}_e$  are the average ion and electron energies of the plasma). The fusion power delivered by neutrons will be moderated and appears as heat:

$$p_n = \frac{1}{4} n_i^2 \langle \sigma v \rangle E_n \quad (B4)$$

( $E_n = 14.06$  MeV). The fusion power delivered by  $\alpha$  particles leaves the system via particle losses and heat conduction and is therefore included in  $p_p$  and  $p_c$ .

By using (B1) we get for  $\epsilon_{dt}$

$$\begin{aligned} \epsilon_{dt} &= \frac{p_p}{p_p + p_c + p_n} \\ &= \frac{n_i/\tau_p (\bar{E}_{i,l} + \bar{E}_{e,l})}{n_i/\tau_p (\bar{E}_{i,l} + \bar{E}_{e,l}) + \bar{E}_e/\tau_{Ee} + \bar{E}_i/\tau_{Ei} + \frac{1}{4} n_i^2 \langle \sigma v \rangle E_n}. \end{aligned} \quad (B5)$$

Depending on the confinement scheme the mean energy of the electrons lost is less than or approximately equal to the mean energy of the lost ions. The latter may be the case for good particle confinement. We thus have

$$\bar{E}_{e,l} \lesssim \bar{E}_{i,l} \quad (B6)$$

Furthermore, we assume that the mean energies of the plasma ions and the lost ions are approximately equal:

$$\bar{E}_i \approx \bar{E}_{i,l}. \quad (B7)$$

By using (B6) and (B7) together with the "fractional burn up"

$$f_b = \frac{1}{2} n_i \langle \sigma v \rangle \tau_p \quad (B8)$$

we get from (B.5)

$$\epsilon_{dt} = \frac{\bar{E}_i (1 + \bar{E}_{e,l}/\bar{E}_i)}{\bar{E}_i (1 + \bar{E}_{e,l}/\bar{E}_i) + \bar{E}_e (\tau_p/\tau_{Ee}) + \bar{E}_i (\tau_p/\tau_{Ei}) + \frac{1}{2} f_b E_n}. \quad (B9)$$

If we neglect radiation losses we may write for Q

$$Q = \frac{\frac{1}{4} n_i^2 \langle \sigma v \rangle (M E_n + E_\alpha)}{n_i \bar{E}_i/\tau_p \cdot (1 + \bar{E}_{e,l}/\bar{E}_i) + n_i \bar{E}_i/\tau_{Ei} + n_i \bar{E}_e/\tau_{Ee}} \quad (B10)$$

( $E_\alpha = 3.56$  MeV,  $M$  = energy multiplication by blanket reactions). The full inclusion of  $E_\alpha$  in the numerator of (B10) is not completely correct because most of the  $\alpha$ -heating is used to cover the power losses which are described by the denominator of (B10).

By using  $E_n = 4 E_\alpha$  and (B.8) we get from (B10)

$$Q = \frac{4M+1}{8} \frac{f_b E_n}{\bar{E}_i (1 + \bar{E}_{e,1}/\bar{E}_i + \tau_p/\tau_{Ei}) + \bar{E}_e (\tau_p/\tau_{Ee})} \quad (B11)$$

From (B9) and (B11) we finally get

$$\epsilon_{dt} = \frac{1 + \bar{E}_{e,1}/\bar{E}_i}{1 + \bar{E}_{e,1}/\bar{E}_i + \tau_p/\tau_{Ei} + (\bar{E}_e/\bar{E}_i) (\tau_p/\tau_{Ee})} \cdot \frac{4M+1}{4Q + 4M + 1} \quad (B12)$$

Exact determination of the first factor of (B12) is only possible within the framework of a detailed plasma model. In the case of very poor plasma containment and  $\bar{E}_e$  not much greater or less than  $\bar{E}_i$  the factor is about unity. For good particle confinement the case of particle loss by diffusion instead of pitch angle scattering is approached. In this limiting case (corresponding more or less to a toroidal trap) we may assume  $1/\tau_p \approx 1/\tau_{Ei} + 1/\tau_{Ee}$ , which is valid if energy and particle losses are caused by the same collision mechanism. Together with, for example,  $\bar{E}_{e,1} \ll \bar{E}_{i,1}$  and  $\bar{E}_i \approx \bar{E}_e$  this leads to the factor 1/2 in (B12). We can therefore expect that

$$\epsilon_{dt} \approx (1/2 + 1) \cdot \frac{4M + 1}{4Q + 4M + 1} \quad (B13)$$

is a reasonable estimate for the influence of  $Q$  on  $\epsilon_{dt}$  and hence on  $\eta_{eff}$  down to  $Q \approx 1$ , as is characteristic of the classical mirror machine. Our definition of  $Q$  according to (1) includes the blanket reactions. The value of  $Q$  therefore increases with  $M$ . For very good particle confinement  $Q$  is proportional to  $M$ . Thus, in general,  $\epsilon_{dt}$  according to (B13) depends only weakly on  $M$ .

To get an impression of the effect on the plant energy parameters produced by the relation between  $\epsilon_{dt}$  and  $Q$ , we shall calculate  $\eta$  and  $C$  as functions of  $Q$  and  $\epsilon_{mp}$ , the rest of the component energy parameters being fixed to the values:

$\eta_h$	=	0.75
$\eta_m$	=	0.80
$\eta_a$	=	0.75
$\eta_{th}$	=	0.36
$\eta_d$	=	0.70
$\epsilon_m$	=	0.50
$\epsilon_a$	=	0.03

Table B1

For  $\epsilon_{dt}(Q)$  we use

$$\epsilon_{dt} = 0.5 \frac{1}{1 + 2/3 \cdot Q}, \quad (B14)$$

which corresponds to  $M = 1.25$ .

Figure B1 gives an - at least rough - impression how  $\eta$  and  $C$  vary if we go from a low- $Q$  system to a high- $Q$  system and take into account the fact that the possibility of direct conversion decreases smoothly with increasing  $Q$ .

The values shown in Table B1 as a whole represent an optimistic set of parameters. The values of  $\eta$  and  $C$  shown in Figs. B1a and B1b therefore also represent quite an optimistic view of the power plant as a whole.

For  $\epsilon_{mp} = 0$ , which corresponds to mirror-like or stellarator-like devices, one has to reach at least  $Q \approx 40$  to get  $\eta \approx 0.3$ . Such values of  $Q$  are hardly conceivable for mirror devices, on the one hand, and lead to such small values of  $\epsilon_{dt}$ , on the other hand, that direct energy conversion becomes useless. High values of  $\epsilon_{mp}$  (e.g.  $\epsilon_{mp} = 5.2$  for the UWMAK II design) lead to very high  $Q$  values necessary to reach  $\eta = 0.3$ .  $Q$  has to be 145 for  $\epsilon = 5$  and  $\eta = 0.3$ . Most probably these values call for an ignited device with a long burn time.



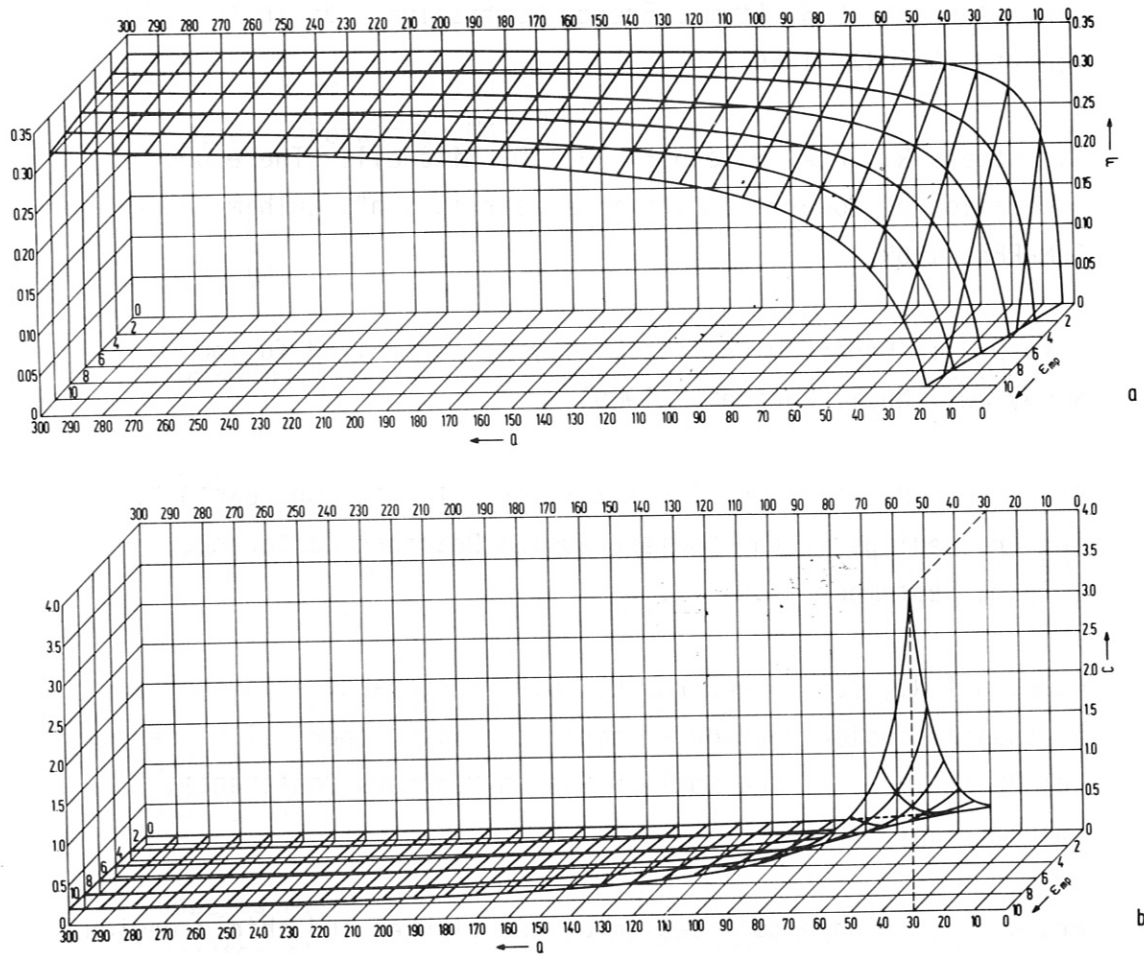


Fig. B1

The drastic effect of small  $Q$  values on the circulating energy ratio is demonstrated by Fig. B1b.

In passing we note that  $Q_c$  can be read from Fig. B1a. It is represented by the line of intersection between the surface  $\eta(Q, \epsilon_{mp})$  and the  $Q-\epsilon_{mp}$  plane.

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