Numerical Calculations of the
Ion Motion in the Early Phase
of Accelerated Electron Rings

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Abstract

The numerical calculations of the ion motion in the initial
phase of accelerated electron rings describe (at least) quali-
tatively the experimental results very well. The ion energy
is found to be gained mainly while the ions are still captured
by the electron ring. The mean ion energy, the acceleration,
the transverse ion "temperature", and the ion loss are calcu-
lated for different initial ion loading fractions and differ-
ent slopes of the radial magnetic field component, that pro-
vides for the axial acceleration.

I. Introduction

The efficiency of the collective ion accelerator with relativistic electron rings (ERA) is limited by the electron ring quality. The acceleration limit is given by the holding power E_H of the ring [1,2], which - due to the polarization of the ring during its acceleration - is only a fraction of the maximum electric field strength of the electron ring before its acceleration. If the ring acceleration exceeds this limit, the ions fall out of the electron potential well and will not further be accelerated appreciably.

In order to achieve high ion energies on a certain short acceleration length in an ERA one ought to accelerate the rings as near to the acceleration limit as possible. In an electron ring experiment using magnetic expansion acceleration the accelerating field component increases monotonically from zero at the "spill-out" point (where its magnitude and that of its first derivative just vanish) to its maximum value. One would like to know the influence of the amount of the acceleration field increase on the number and the transverse energy distribution of the ions in the electron ring, which question is of special interest for the interpretation of the measurements on the electron ring acceleration [3,4].

In the Garching ERA device the acceleration increases to values above the limit given by the holding power of present electron rings. The ring acceleration is measured (by magnetic probes [4]) along the acceleration section for unloaded rings and those loaded with ions. The electron ring acceleration at a certain axial position is measured to be smaller for rings initially being loaded with ions. But one does not know whether this reduction in the acceleration value is determined by the electrostatic forces of the ions that are already lost or by the higher inertia of the electron ring that still keeps the ions. Another open question is the behaviour of the electron ring acceleration prior to and after the loss of the ions (in a fixed acceleration

structure) and the distribution of the transverse ion energy. Furthermore one would like to solve the question, what fraction of energy the ions gain from the electron ring, after they are lost (i.e. after they have crossed the point of maximum electric field strength), which question is of importance for the inter- pretation of the measured values of the ion energy [3,4].

There are some calculations [5-9] on the electron and ion motion in an accelerating field. Mostly assuming constant magnetic ex- pansion acceleration, they give the result of higher values of the radial magnetic field to be applicable in the case of in- creasing heavy ion loading of the rings [5,7]. The influence of a disturbance of an electric acceleration field - that otherwise is assumed to be constant - is calculated for an electron ring interacting with the ions as a whole. Nearly all the calcula- tions [5-8] neglect the ion distribution in phase space, so that partial ion loss cannot be taken into account. In recent calcu- lations PEREL'SHTEIN et al. [9] treat the problem of acceleration of an electron ion ring in a linearly rising accelerating field by using superparticles. Since they do not take image fields for axial ring focussing into account, they have to use magne- tic focussing first, which they suddenly switch off, if the condition for self-focussing is reached.

In this report, we discuss some simple calculations (performed already two years ago) of the ion motion in accelerated elec- tron rings that are mainly focussed in axial direction by image focussing. This type of focussing has turned out to be the domi- nant one for electron rings of present-day quality [3,10]. Under this condition the electron ring dimensions can be regarded to be independent of the (small) ion loading; i.e. independent during the acceleration process, as recently has been varified experimentally [11]. The numerical calculations are performed for the initial phase of the ion acceleration in an expanding mag- netic field with monotonically increasing radial magnetic field component B_r (from spill-out to its plateau). The shape of the axial dependence of B_r is aimed to approach to the actual depen- dence as near as possible with the objective to interpret the results of the measurement [3,4].

II. Analytical Considerations

In the following we assume an electron ring of major radius R and of a homogeneous electron density distribution inside a circular minor cross section πr_o^2 of minor radius r_o. The aspect ratio R/r_o may be very large ($R/r_o \gg 1$), such that curvature effects can be neglected. The potential of the electron ring is assumed to be constant in time, which is nearly fulfilled, if the focussing of the ring is predominantly determined by image focussing [2]. Under the conditions of the Garching ERA experiment [3] the image focussing by a structure of the "squirrel cage" type is always greater than the ion focussing, as far as the ion loading f is limited by $f = Z_i N_i / N_e < 0.05$.

We further assume the interaction of the ions with each other being negligible compared to that with the electron ring. The ring velocity dz_e/dt in the axial direction (into which the acceleration occurs) is assumed to be non-relativistic.

For an estimate of the electron ring and the ion motion during the first stage of the collective acceleration (i.e. just behind the spill-out point) we solve the motion of the electron ring center (at its axial position z_e) and that of the ions (at z_i) in a somewhat non-realistic linearly rising expansion acceleration structure, characterized by

$$B_r(z_e) = \partial B_r/\partial z \cdot z_e \quad \text{with} \quad \partial B_r/\partial z = \text{const.}$$

(In a realistic device $\partial B_r/\partial z$ has to be zero at the spill-out point and increases thereafter.)

For $|z_e - z_i| \leq r_o$ the equations of motion (using MKSA units) are

$$\ddot{z}_e = \frac{c\beta_o e B_r(z_e)}{m_e \gamma} - \frac{e^2 N_i Z}{m_e \gamma 4\pi^2 R\varepsilon_o} \cdot \frac{z_e - z_i}{r_o^2}$$

$$\ddot{z}_i = \frac{e^2 N_e Z}{M_i 4\pi^2 R\varepsilon_o} \cdot \frac{z_e - z_i}{r_o^2} ,$$

where - besides the usual constants - $c\beta_o$ is the azimuthal electron velocity, $\gamma = (1-\beta_o^2)^{-1/2}$ is the relativistic factor of the electrons, Z is the ion charge state, and N_e and N_i are the electron and ion number, respectively.

With the ion oscillation frequency ω in the potential well of the electron ring

$$\omega = \left(\frac{e^2 N_e Z}{M_i 4\pi^2 \varepsilon_o Rr_o^2} \right)^{1/2} = \left(\frac{eZE_{max}}{M_i r_o} \right)^{1/2},$$

using the maximum electric field strength

$$E_{max} = \frac{eN_e}{4\pi^2 \varepsilon_o Rr_o}$$

at the edge of the electron ring, and with

$$\alpha = \left(\frac{c\beta_o e \partial B_r/\partial z}{m_e \gamma} \right)^{1/2}$$

as well as the mass ratio

$$g = N_i M_i / (N_e m_e \gamma)$$

these equations read

$$\ddot{z}_e + (g\omega^2 - \alpha^2) z_e = g\omega^2 z_i$$

$$\ddot{z}_i + \omega^2 z_i = \omega^2 z_e.$$

In the case of very weak ion loading (g << 1) these equations are solved for the initial conditions

$$\dot{z}_e(o) = \dot{z}_i(o) = v_o \quad (\text{at } t = 0)$$

by

$$z_e = \frac{v_o}{\alpha} \text{sh}\alpha t$$

$$z_i = \frac{v_o \alpha^2}{\omega(\alpha^2 + \omega^2)} \sin\omega t + \frac{v_o}{\alpha} \cdot \frac{\omega^2}{\alpha^2 + \omega^2} \cdot \text{sh}\alpha t$$

Since under usual experimental conditions [10,11] (E_{max} = 10 MV/m, r_o = 0.3 cm, M_i = M_{proton} = 1.67·10^{-24} g, Z = 1, $\beta_o \approx$ 1, γ = 27 and $\partial B_r/\partial z$ = 2·10^{-9} Vs cm^{-3}) we have

$$\alpha^2 \ll \omega^2,$$

the ion motion z_i, which consists of a part (second term) monotonically rising in time and an oscillatory part (first term), can be approximated by

$$z_i \approx v_o/\alpha \left[(\alpha/\omega)^3 \sin\omega t + \frac{1}{1+(\alpha/\omega)^2} \, \text{sh}\alpha t \right].$$

For the difference $z_e - z_i$ of ion and electron centers we then obtain

$$z_e - z_i \approx \frac{1}{1+(\omega/\alpha)^2} \, z_e - v_o/\alpha(\alpha/\omega)^3 \sin\omega t.$$

We now ask for the axial position z_{em} (and the corresponding radial magnetic field component $B_{r\,max}$), where the acceleration limit is reached, defined by

$$z_e - z_i = r_o$$

This limiting situation is reached at

$$z_{em} \approx r_o \left[1+(\omega/\alpha)^2 \right] \approx r_o(\omega/\alpha)^2,$$

where

$$B_r(z_{em}) = B_{r\,max} = \partial B_r/\partial z \cdot z_{em} \approx \partial B_r/\partial z \cdot r_o(\omega/\alpha)^2$$

i.e.

$$B_{r\,max} = m_e\gamma/M_i \, Z/c\beta_o \cdot E_{max},$$

which is just the value corresponding to the maximum electric field strength in the electron ring.

III. Numerical Calculations

Since we are interested in the transverse energy distribution
of the ions in the electron ring during its acceleration and
especially in the process of ion loss in the vicinity of the
maximum allowable acceleration rate, we treat the problem by
solving the equations of motion of the electron ring and of
several ion subrings. The corresponding computer program has
also been used to solve the problem of deceleration of electron
rings by ions [12]. In this program it is assumed, that the ions
are accelerated by the electric field of the electron ring. If
the acceleration is too high, the ion subrings escape from the
electron ring, which is defined by their passage over the point
of maximum electric field strength. After this the ions still
interact further with the electron ring, but they normally can-
not catch up with it again. The electron ring as well as the
ion subrings are regarded to be rigid. The assumption of un-
changed electron ring shape appears to be reasonable, if the
axial focussing - as already been assumed in the introduction -
is done by an image field cylinder (of the "squirrel cage" type)
and is nearly independent of the ion loading. Hofmann [2] has
found out theoretically in fact, that image focussing of the
electron ring results in considerably higher collective ion
acceleration (higher holding power) than the focussing by ions.

The coupling of the ion subrings to each other is neglected.
The density distribution $n_e(r)$ of the electrons is assumed to
be a Gaussian one, i.e.

$$n_e(r) = n_e(o)\exp(-r^2/r_o^2),$$

where r is the distance measured from the center of the electron
ring minor cross section, and $n_e(o)$ is the electron density in
this center.

If the ions are divided into K subrings and if the toroidicity
is neglected (i.e. if the problem is simplified by solving the

straight beam problem), the equations of motion in axial direction are (for $z_{i,j} \neq z_e$)

$$m_e \gamma \ddot{z}_e = c\beta_o eB_r(z) - \sum_{j=1}^{K} \frac{e^2 N_{i,j} \cdot z_j}{4\pi^2 R\epsilon_o} \cdot \frac{1}{z_e - z_{i,j}} \cdot \left[1 - \exp\left\{-\left(\frac{z_e - z_{i,j}}{r_o}\right)^2\right\}\right]$$

(1)
$$\cdot \left\{1 - \frac{1}{1 + (d/(z_e - z_{i,j}))^2}\right\}$$

$$M_i \ddot{z}_{i,j} = \frac{e^2 N_e z_j}{4\pi^2 R\epsilon_o} \frac{1}{z_e - z_{i,j}} \cdot \left[1 - \exp\left\{-\left(\frac{z_e - z_{i,j}}{r_o}\right)^2\right\}\right]$$
$$\cdot \left\{1 - \frac{1}{1 + (d/(z_e - z_{i,j}))^2}\right\}$$

$j = 1, K.$

z_e and $z_{i,j}$ are the axial locations of the electron ring and the j-th ion subring, respectively. N_e is the total number of electrons, and $N_{i,j}$ is the number of ions in the j-th ion subring. β_o is the azimuthal electron velocity normalized to the speed of light c. Z_j is the charge per ion in the j-th ring, and d is the distance from the electron ring to an image field cylinder ("squirrel cage"). Its influence is included approximately in the above equations through an additional term in the force equations involving the distance between the real charge and the image charge distributions. These image fields reduce the coupling of the ion rings to the electron ring, especially at long distances between the ions and the electron ring center. The effect is demonstrated in fig.1, where the axial electric field strength is plotted in relative units versus the distance between the ion subring and the electron ring centers. With increasing distance the electric field strength in the case of an image cylinder present (solid curve) goes very much earlier down to small values than in the case without image field inclusion (dotted curve).

The electron density profile is assumed to be constant throughout the acceleration, since under the parameters of interest the squirrel cage focussing is dominant over the ion focussing. We take as an example (SCHUKO parameters [3,10]) the major and minor electron ring radii as R = 2.3 cm, a = r_o = 0.3 cm, and find the contribution
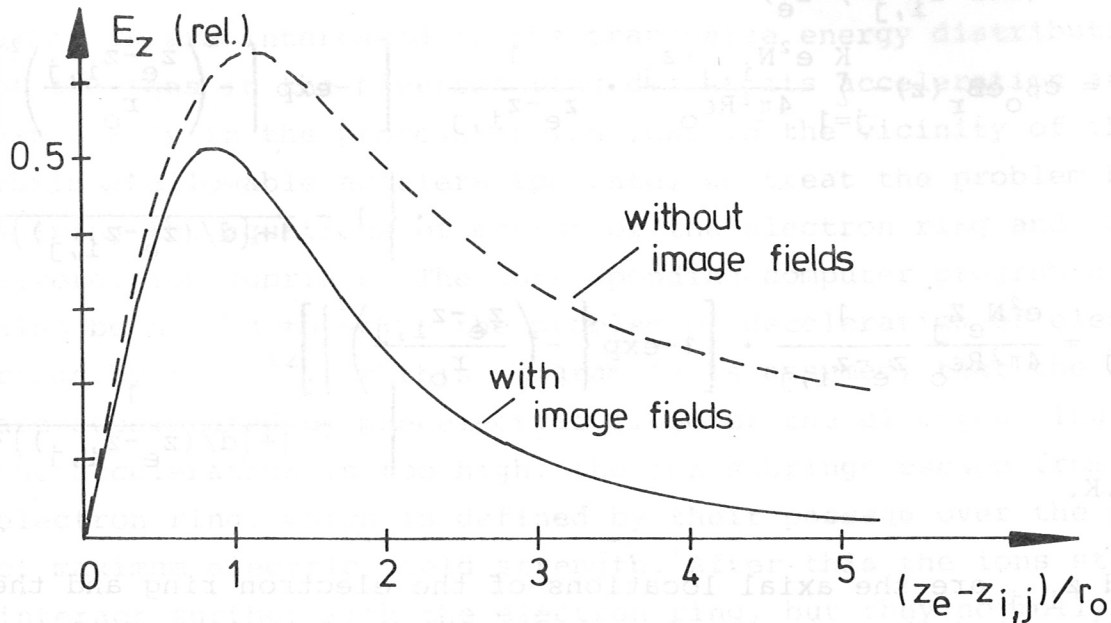
Fig. 1 The electric field strength (in relative units) versus
the axial distance of the ion to the electron ring center
with and without the inclusion of image fields

of the ion focussing to the axial betatron tune ν_z^2 to be [13]

$$\mu \cdot f \cdot 2R^2/a^2 \approx \mu \cdot f \cdot 118$$

with μ defined by

$$\mu = N_e r_e/(\gamma 2\pi R),$$

where

$$r_e = e^2/(4\pi\varepsilon_o m_e c^2)$$

is the classical electron radius. And

$$f = Z_i N_i/N_e$$

is the ion loading fraction.

If we use an image cylinder of the squirrel cage type located at $R_{s.c.} = 1.6$ cm, its contribution to the axial focussion is given by [14]

$$4\mu(1-f)\,\varepsilon_{1,E}\bigg/\bigg(\frac{R_{s.c.}}{R}-1\bigg)^2 \approx (1-f)\,\mu \cdot 5.55,$$

such that the ratio Γ of ion to squirrel cage focussing is only depending on f:

$$\Gamma \approx 21\ f/(1-f).$$

Thus it is obvious, that the ion focussing is negligible compared to the image focussing ($\Gamma < 1$) as long as the ion loading is kept below about 5%.

We now want to discuss the results of some of the numerical calculations. Besides the acceleration d^2z_e/dt^2 of the electron ring center we are interested in the average axial ion velocity

$$\overline{dz_i/dt} = \frac{1}{K}\sum_{j=1}^{K} dz_{i,j}/dt,$$

the average ion acceleration

$$\overline{d^2z_i/dt^2} = \frac{1}{K}\sum_{j=1}^{K} d^2z_{i,j}/dt^2,$$

the mean ion energy

$$\overline{E_i}\left[keV\right] = \frac{5.218\cdot10^{-16}}{K}\cdot\sum_{j=1}^{K} A_j\,(dz_{i,j}/dt)^2,$$

the mean spread of the ion energy ("temperature")

$$kT_i\left[keV\right] = \frac{1.0436\cdot10^{-15}}{K}\cdot\sum_{j=1}^{K} A_j\left(\overline{\frac{dz_i}{dt}}\left[\frac{cm}{sec}\right]-\frac{dz_{i,j}}{dt}\left[\frac{cm}{sec}\right]\right)^2$$

(where A_j are the mass numbers of the ions), and the relative ion number in the ring

$$R_i = \frac{1}{K}\sum_{j=1}^{K} M_j,$$

with $M_j = \begin{cases} 1, & \text{if } z_e - z_{i,j} \leq r_o \\ \\ 0, & \text{if } z_e - z_{i,j} > r_o \end{cases}$,

all as a function of the axial distance z_e.

Finally, if all ions are lost ($R_i = 0$) and the elctron ring is very far in front of the ions, one is interested in the energy distribution of the ions

$$E_{i,j}\left[keV\right] = 5.218 \cdot 10^{-16} \cdot A_j (dz_{i,j}/dt \left[cm/sec\right])^2 .$$

We concentrate here on the results of numerical calculations that might roughly simulate the experimental conditions [3,4] in order to understand the collective acceleration mechanism responsible for the observed accelerated ions and furthermore on those which give information of the dependence of the acceleration process on the initial B_r field shape.

In fig.2 the measured acceleration of the electron ring along the axis z (of the experiment) for rings being loaded and unloaded with hydrogen ions is reproduced [3,4] indicating the reduced ring acceleration with ion loading (of about 1%). The proton energy obtained is in the range of 200 keV. In fig.2 the origin z = 0 is defined by the compression plane, while the acceleration starts at about z = 15 cm ("spill-out" point).
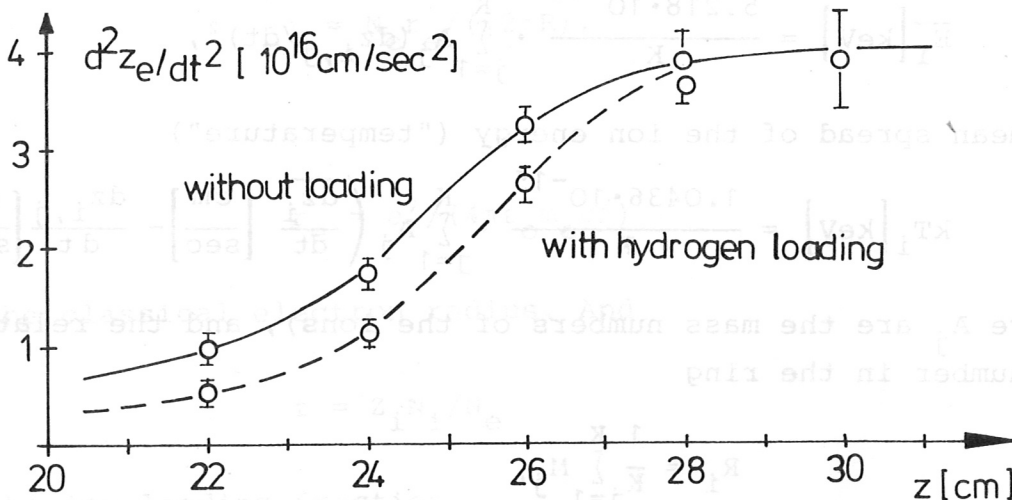

Fig.2 The measured electron ring acceleration versus z with and without hydrogen loading

For a rough simulation we take protons ($M_i = m_p$) in an electron ring of $a = r_o = 0.3$ cm minor and $R = 2.3$ cm major radius, a relativistic factor of the electrons of $\gamma = 27$ and an electric field strength given by $eZ_{max} = 4$ MeV/m (which energy gain per length corresponds roughly to the observations). The ring might be located a distance $d = 0.7$ cm from the image cylinder ("squirrel cage"). The axial dependence of the radial magnetic field component B_r might be given by

$$(2) \qquad B_r = \begin{cases} B_o \cdot (1 - \cos z_e/\zeta)/2 & \text{for } 0 \le z_e/\zeta < \pi \\ B_o & \text{for } z_e/\zeta > \pi \end{cases},$$

where B_o is chosen to be $B_o = 2$ G.

Because of the relation

$$B_r [G] = 5.12141 \cdot 10^{-17} \, d^2z/dt^2 \, [\text{cm sec}^{-2}]$$

(for $\beta = v_\phi/c \simeq 1$) the axial acceleration d^2z_e/dt^2 of the electron ring without ion loading as a function of the axial position z_e is given by

$$d^2z_e/dt^2 \, [\text{cm sec}^{-2}] = \begin{cases} 3.905 \cdot 10^{16} (1 - \cos z_e/\zeta) & \text{for } 0 \le z_e/\zeta < \pi \\ 3.905 \cdot 10^{16} & \text{for } z_e/\zeta > \pi \end{cases}$$

This dependence is plotted in fig.3 together with the other quantities, as obtained for nonzero ion loading by solving equations (1).

The graphs of fig.3 contain three cases of different initial ion loading N_i/N_e. The case $N_i/N_e \simeq 0$ corresponds to such a negligible ion content, that the inertia of the electron-ion ring is not affected by the ions. The initial conditions of the ions are chosen to be spread over the available phase space roughly according to the initial conditions in [2]. A comparison of the upper part of fig.3, giving the electron ring acceleration,
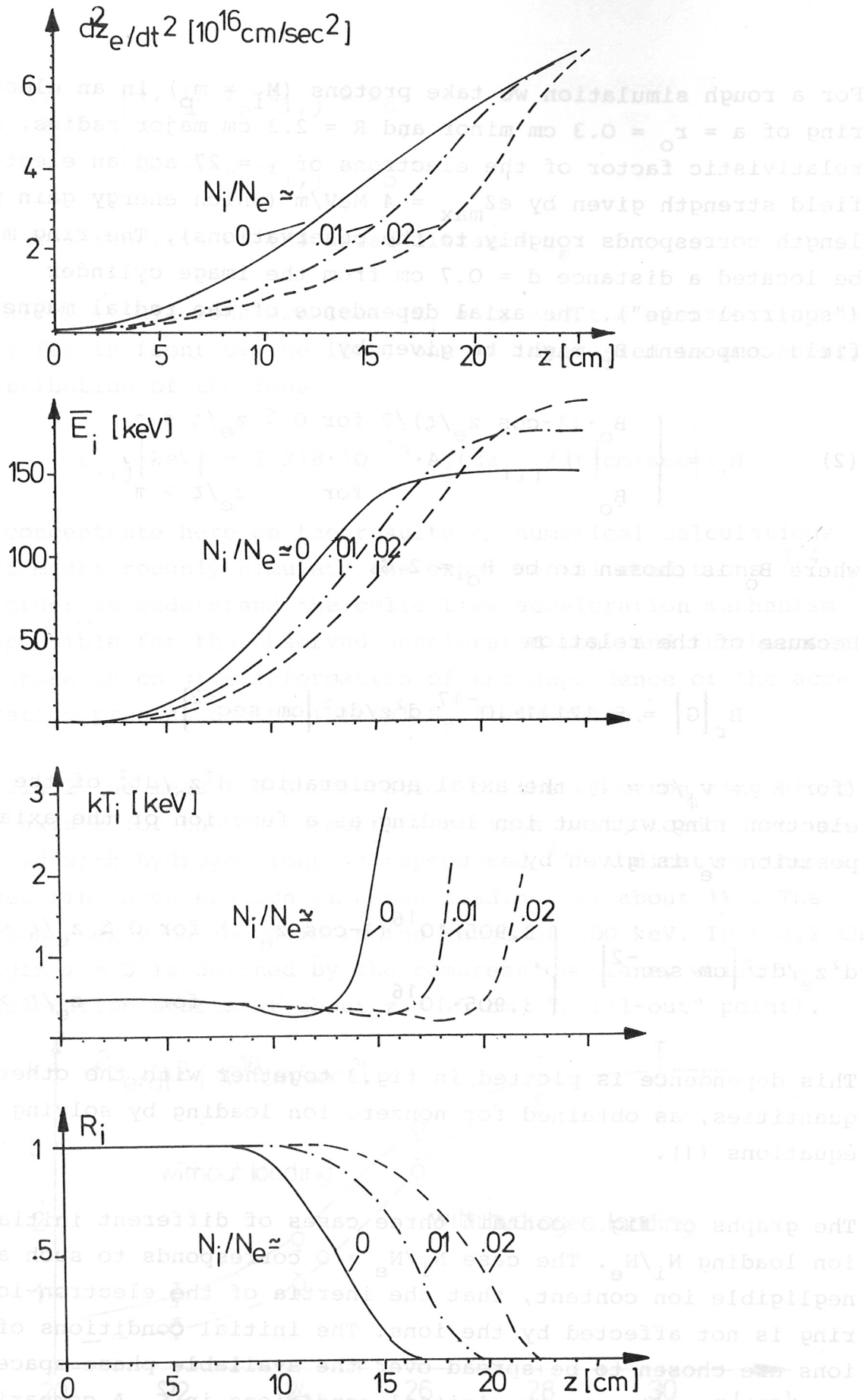
Fig.3 The electron ring acceleration, the mean ion energy, ion
"temperature" and relative ion number versus the axial
distance at the first stage of electron ring acceleration

especially for zero and 1% ion loading, with fig.2 gives rela-
tive good agreement with the measurement, although the axial
dependence of B_r has not properly been simulated. The numerical
calculations demonstrate that an axial behaviour of the elec-
tron ring acceleration during the initial phase with monotoni-
cally increasing B_r can have the shape as observed in the ex-
periment. The electron acceleration is appreciably reduced with
increasing ion loading, its amount depending on the inertia as
related to the relative ion number R_i in the ring, plotted in
the lowest part of fig.3. One observes that as soon as ions are
lost from the ring the relative difference between the values
of the ring acceleration d^2z_e/dt^2 with and without initial ion
loading goes down. Also the ion "temperature" slightly decreases
(because the ions with the highest transverse energies disappear
first) as long as about one third of the ions have escaped. The
initial value of this ion "temperature" certainly depends on
the choice of the ion initial conditions. Before about half of
the ions are lost, the "temperature" increases sharply, indica-
ting the beginning of the wide spreading of ion velocities due
to the continuous losses.

The loss of ions starts and ends later when the initial ion
loading is increased. The average ion energy increases as long
as there are still ions in the ring and levels out after the
complete ion loss. The energy of about 170 keV is in rough
agreement with the experimental findings. Looking at the indi-
vidual ions one finds that the ions gain nearly all of their
energy while still being in the electron ring, which is defined
by the ions not yet having crossed the point of maximum electric
field strength.

Since the loss occurs during a relatively wide axial period, the
energy spread of the ions at the end of the acceleration process
is expected to be wide, as the histogram in fig.4 (of the case
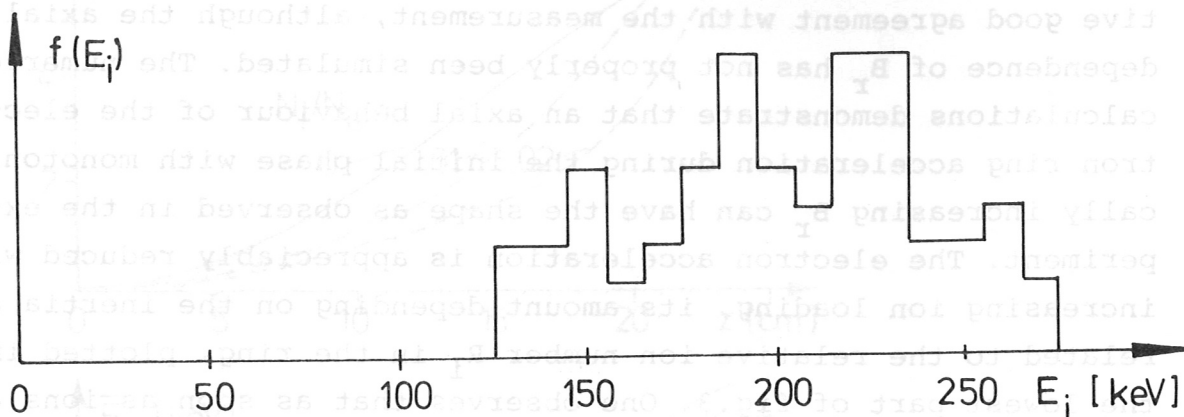with an initial ion loading of $N_i/N_e = 0.02$) clearly demonstrates.

Fig. 4 Histogram of the ion energy distribution in the case of $N_i/N_e = 0.02$.

It is obvious, however, that such a wide energy distribution function is only obtained under the specific conditions of our problem, that treats the ion motion in the initial phase of the electron ring acceleration, during which ion loss already occurs. For a ring of higher holding power or a B_r-field of lower magnitude, such that the ions are not lost from the electron ring, we result in very much smaller final ion energy distributions.

Moreover, it can be deduced from fig. 3 that there is a maximum applicable acceleration for the ring (about equal for all initial ion loading fractions), which might be defined through the minimum of the ion "temperature". In our specific case this is an acceleration of about $2.6 \cdot 10^{16}$ cm sec^{-2}, which corresponds to roughly $\tilde{B}_r = 1.3$ G and (for protons and for a relativistic factor of $\gamma = 27$) to an electric field of $\tilde{E}_m = 2.72 \cdot 10^4$ V/cm, which is a fraction of 68% of the maximum electric field strength $E_{max} = 4 \cdot 10^4$ V/cm assumed in the ring. If the radial magnetic field component has increased just to its maximum value of $B_r \simeq 2$ G, all ions are lost.

In order to find out whether the behaviour of the electron ring and the ion motion shown in fig. 3 are specific for the assumed magnetic field shape of equ. (2) or not, we varied the width ζ of

the magnetic field increase and its shape. Fig.5 gives an example of the variation of the width ζ for a magnetic field dependence given by equ.(2). The resulting mean ion energy $\overline{E}_i$ is a monotonically increasing function versus ζ, as one would expect.
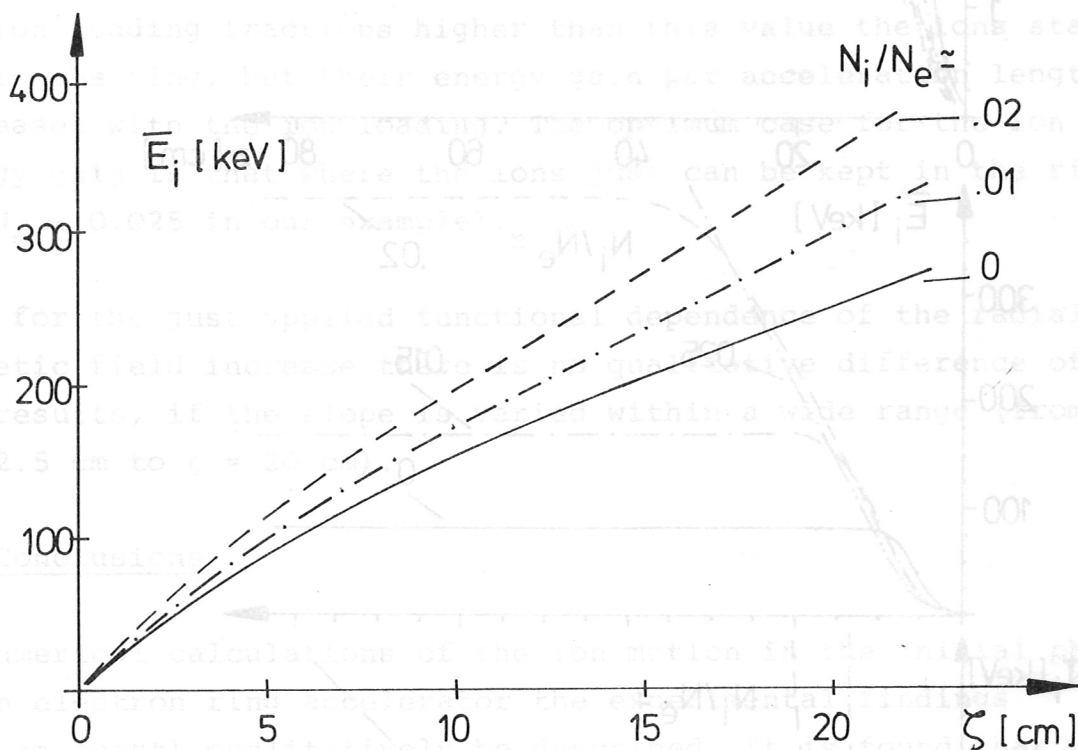

Fig.5 The mean ion energy obtained for different slopes of the magnetic field increase.

There is no difference qualitatively in the results obtained, if other axial dependences of the radial magnetic field component B_r are chosen. As such an example we take

$$B_r = B_o \cdot (th\left[z_e/\zeta - 2\right] + 1),$$

again with $B_o = 2$ G,

for which fig.6 gives the same data as in fig.3 for the ion acceleration process in the first phase of the magnetic expansion acceleration structure.
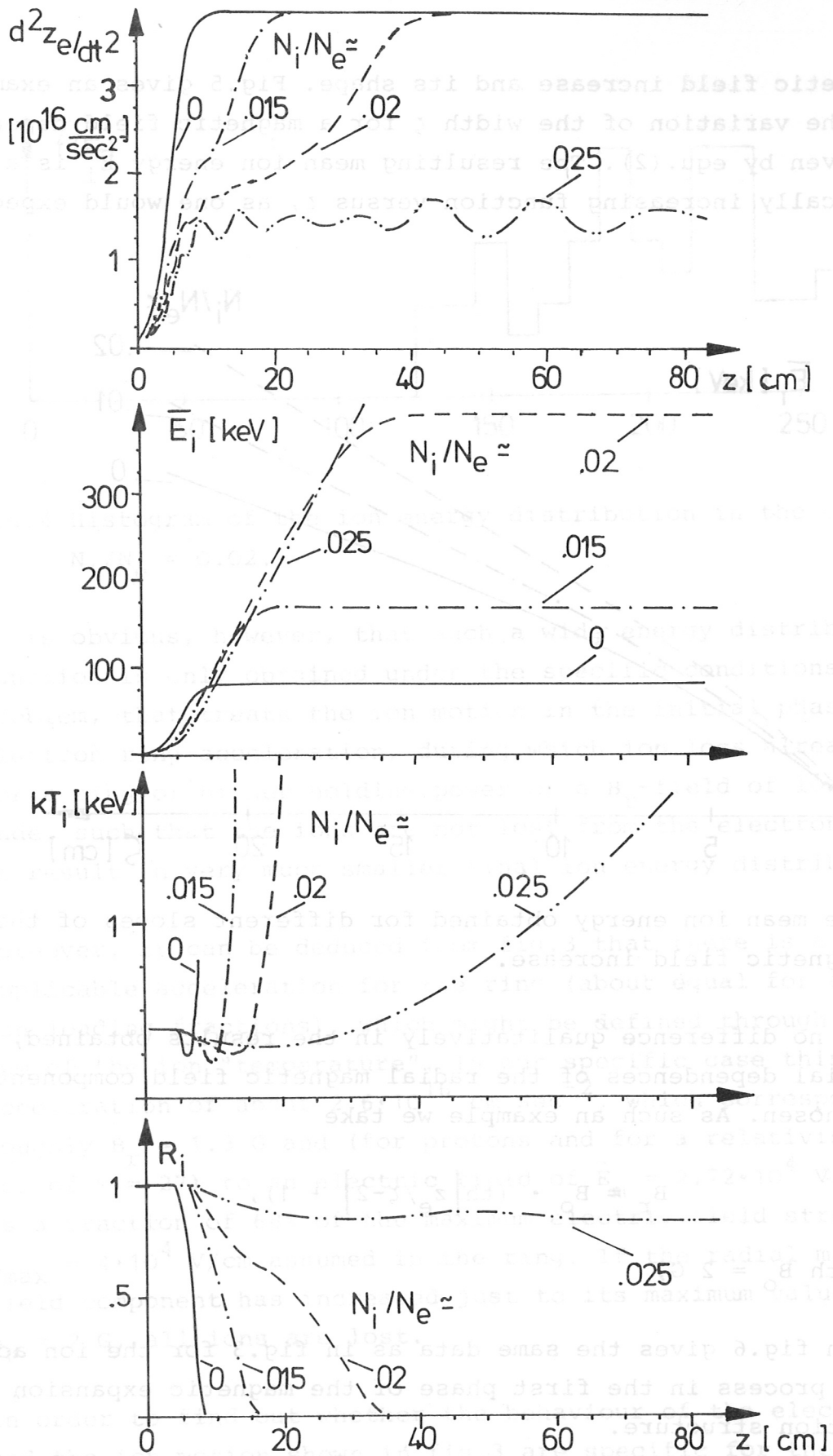
Fig.6 The electron ring acceleration, the mean ion energy, ion
"temperature" and relative ion number versus the axial dis-
tance for another type of first stage of electron ring acce-
leration

Qualitatively we get the same results for this case, for which
ζ = 2.5 cm has been chosen, although it might clearly demon-
strate the dependence of the acceleration on the ion loading
N_i/N_e, that at the amount of about $N_i/N_e \approx 0.025$ reaches a
situation, in which the ions can catch up with the electron ring.
For ion loading fractions higher than this value the ions stay
within the ring, but their energy gain per acceleration length
decreases with the ion loading. The optimum case for the ion
energy gain is that where the ions just can be kept in the ring
($N_i/N_e \approx 0.025$ in our example).

Also for the just applied functional dependence of the radial
magnetic field increase there is no qualitative difference of
the results, if the slope is varied within a wide range (from
ζ = 2.5 cm to ζ = 20 cm).

IV. Conclusions

By numerical calculations of the ion motion in the initial phase
of an electron ring accelerator the experimental findings [3,4]
can (at least) qualitatively be described. It is found that the
ions gain most of their energy while they are still kept by the
electron ring, and only a very small part originates from the
electron ring field accelerating the ion if it is already escap-
ed over the point of maximum electric field. In all treated ca-
ses, where the ions are lost during the initial phase of the
electron ring acceleration, we observe that with increasing ion
loading the mean ion energy gets up, and the mean ion and elec-
tron ring acceleration stay at smaller values for a longer time,
as well as the transverse ion "temperature" increases later,
while also the ion loss occurs later. It is furthermore found
that as soon as strong ion losses are observed, the ion "tempe-
rature" steeply goes up indicating the spreading of ion veloci-
ties during the process of continuous ion losses. It turns out
that the mean ion energy increases as long as the last ion es-
capes over the point of maximum electric field strength. Conse-
quently the ion energy distribution at the end of the accelera-
tion process is found to be very wide.

Varying the slope of the radial magnetic field increase (covering a range of about one order of magnitude) or its functional dependence does not lead to unexpected results, and no new limitations show up.

Acknowledgements

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