

A Zero-Dimensional Tokamak Transport Model

Part I:  
General Description

K. Borraß

IPP 4/146

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**MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK**

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Abstract

In this paper we describe a time-dependent zero-dimensional tokamak transport model involving besides electrons, fuel ions and  $\alpha$ -particles five types of impurities. In the model arbitrary profiles of the type  $(1-\xi) (1 - (r/a)^{2\alpha})^\beta + \xi$  can be prescribed independently for each density and temperature.

The global equations are derived from general space-dependent ones, and the relation between global models and space-dependent ones is discussed in detail.

## INTRODUCTION

In this paper we describe a zero-dimensional tokamak transport model developed for use in the IPP system study group.

In the context of system studies zero-dimensional models are needed for several reasons. Owing to their relatively simple structure and the low computing time requirements they are indispensable in all situations where a large number of calculations are required, e.g. for parameter studies and optimization problems. Furthermore, they can easily be subjected to modifications and can give at least qualitative results where space dependent calculations are not yet available.

These advantages of global models result from additional assumptions that are made in comparison with space dependent calculations, and some care is required in establishing the validity of these assumptions in order to put the model on a sound basis.

In establishing global models usually specific profiles for densities and temperatures are assumed, such as constant or parabolic ones. This implicitly means that the results should essentially be independent of the profile shapes. It is easy to verify that this is far from being true. On the other hand, as one would expect for a boundary value problem, once the boundary conditions are chosen, the profiles only weakly change in time and depend only weakly on parameters such as the plasma size and others for a large class of situations.

In our model a rather general set of profiles can therefore be prescribed independently for the densities and temperatures of each component. These profiles are found by comparison with one-dimensional codes.

In discussing the validity of global models relative to space dependent models, one must, however, be aware of the fact that the treatment of tokamak transport quite generally involves numerous, far-reaching assumptions, partly owing to lack of physical knowledge, partly owing



INTRODUCTION

to the complexity of the resulting equations. Thus, only rough estimates of diffusion, heat conduction and radiation losses in a reactor plasma are available. Furthermore, the diffusion of impurities is largely unknown and, last but not least, even the currently used set of basic transport equations is a simple heuristic modification of those of a stable plasma. Unfortunately, results are very sensitive to all these ambiguities.

Transport and radiation loss terms are dominant ones in the particle and energy balance equations respectively. The burn time of a tokamak is largely determined by the question of impurity accumulation and even slight modifications in the basic equations, e.g. Ohm's law, may impose drastic consequences on the equilibria obtainable [3].

In solving the system of transport equations further approximations are applied to simplify their solution, such as the restriction of cylindrical geometry.

Thus, the one-dimensional transport codes are only the most accurate procedure for handling our reduced knowledge, and so, despite their numerical accuracy, they may possibly be qualitative, too, or even completely wrong owing to their ambiguous physical basis. Nevertheless, we found it reasonable to discuss in detail the additional assumptions which are made in establishing global models in comparison with space dependent ones in order to have a clear distinction between those more basic ambiguities and those resulting from ad hoc assumptions. Consequently, emphasis is laid on the derivation of the zero-dimensional equations from general space dependent transport equations but our model is conventional as regards the physical assumptions.

Apart from these more basic aims, the purpose of this paper is to serve as a basis for the work within the group and as a reference for future applications. The presentation is therefore rather detailed.

The whole paper consists of two parts. The first contains the derivation and general description of the model, while the second will contain examples of applications, including a comparison with one-dimensional calculations.

In Section 1 of the first part the set of space dependent energy and particle transport equations in the cylinder approximation is derived for an impurity contaminated plasma and the respective source terms are specified.

In Section 2 the global equations are obtained by volume averaging the space dependent equations.

In Section 3 the way the particle and heat fluxes are computed and the role of neutral particles are discussed. Some conditions for the applicability of the procedure are considered.

In Section 4 we briefly discuss equilibrium and stability constraints and several operation schemes which are relevant for practical applications.



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Introduction

1-Dimer

General

Special

NOTE

In this paper all quantities are in cgs units except in formulae with decimal coefficients where temperatures are in keV, magnetic fields in kG, injection powers in MW, beam particle energies in keV and currents in MA.



# 1. 1-dimensional tokamak transport equations

## 1.1 General form of transport equations

In this paper we consider a tokamak fusion plasma which is based on the D-T reaction. Such a plasma consists of, besides electrons and fuel ions (deuterium and tritium),  $\alpha$ -particles (reaction product) and impurity ions which are chiefly a result of the plasma wall interaction.

For each component of such multicomponent plasma one has a particle and an energy equation the general forms of which are [1]

$$(1.1) \quad \frac{\partial n}{\partial t} + \nabla \cdot (n \vec{V}) = S$$

$$(1.2) \quad \frac{3}{2} \frac{\partial}{\partial t} (nT) + \nabla \cdot \left( \frac{3}{2} nT \vec{V} \right) + nT \nabla \cdot \vec{V} \\ + \Pi_{\alpha\beta} \frac{\partial V_{\alpha}}{\partial X_{\beta}} + \nabla \cdot \vec{q} = Q$$

for all species.

The right-hand sides of eqs. (1.1) and (1.2) describe sources or sinks of particles and heat respectively.

The second term in eq. (1.1) describes the streaming of particles. The second term in eq. (1.2) describes transport of heat by convection. The third term describes work done against the pressure. The last term describes heat conduction. The term  $\Pi_{\alpha\beta} \frac{\partial V_{\alpha}}{\partial X_{\beta}}$ , which results from pressure anisotropies, is usually small and can be neglected in the energy balance equation.

The plasma is assumed to be in a stable MHD equilibrium during its evolution on the relatively slow transport time scale. Such a quasistationary plasma is locally Maxwellian so that each component is characterized by a local density  $n(\vec{r}, t)$  and temperature  $T(\vec{r}, t)$ .

In what follows we shall specialize to a circular tokamak system. For such a system if described by usual toroidal coordinates  $(r, \vartheta, \varphi)$  (see fig. 1) the  $\varphi$  dependence disappears from equations (1.1) and (1.2) owing to axisymmetry.

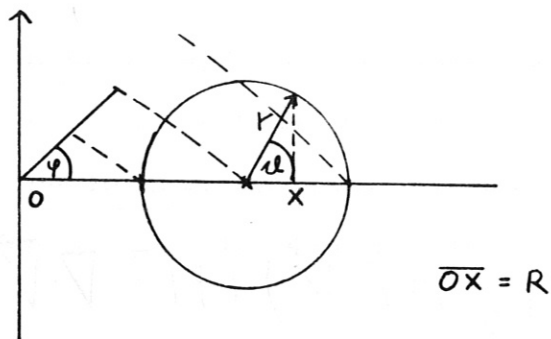


Figure 1

Further simplifications result if only lowest order terms in  $\epsilon = r/R$  are considered and one gets the so called cylinder approximation. Then the  $\vartheta$  dependence drops out too and the resulting equations contain the variable  $r$  only (1-dimensional transport equations).

This approximation is applied in practical calculations for reasons of numerical simplicity. With these simplifications equation (1.1) and (1.2) now read

$$(1.3) \quad \frac{\partial n}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r n V^r) = S$$



$$(1.4) \quad \frac{3}{2} \frac{\partial}{\partial t} (nT) + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{3}{2} r nT V^r \right) + nT \frac{1}{r} \frac{\partial}{\partial r} (r V^r) + \frac{1}{r} \frac{\partial}{\partial r} (r \cdot q^r) = Q$$

or

$$(1.5) \quad \frac{3}{2} \frac{\partial}{\partial t} (nT) + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{5}{2} nT V^r \right) - V^r \frac{\partial}{\partial r} (nT) + \frac{1}{r} \frac{\partial}{\partial r} (r \cdot q^r) = Q.$$

Within this model the magnetic surfaces are the surfaces  $r = \text{const}$ . Hence  $V^r$  is the velocity normal to the magnetic surface. A system in MHD equilibrium can move normally to the magnetic surfaces by diffusion only. Hence  $V^r$  is the respective diffusion velocity.

In what follows it is convenient to introduce the fuel ion density  $n_f$  as  $n_f = n_d + n_t$  and to use the indices  $e, f, \alpha, \zeta$  ( $= 1, 2, 3, 4, 5$ ) for electrons, fuel ions,  $\alpha$ -particles and five impurity ions respectively.

As we shall see below, the temperatures of all ions can be assumed to be equal. In this case it is possible to treat the ions as one plasma component with density

$n_i = n_f + n_\alpha + \sum_{\zeta} n_\zeta$ . The ion part can then be described by one energy balance equation which results as the sum of the energy balance equations of all constituents. That is, with eq. (1.5) one has

$$\begin{aligned}
 (1.6) \quad & \frac{3}{2} \frac{\partial}{\partial t} (n_i T_i) + \frac{1}{T} \frac{\partial}{\partial t} \left( \frac{5}{2} T_i \left[ n_f V_f^r + n_\alpha V_\alpha^r \right. \right. \\
 & \left. \left. + \sum n_\sigma V_\sigma^r \right] + T \left[ q_f^r + q_\alpha^r + \sum q_\sigma^r \right] \right) \\
 & - V_f^r \frac{\partial}{\partial t} (n_f T_i) - V_\alpha^r \frac{\partial}{\partial t} (n_\alpha T_i) \\
 & - \sum_\sigma V_\sigma^r \frac{\partial}{\partial t} (n_\sigma T_i) = Q_i
 \end{aligned}$$

In the following we shall assume that heat is essentially carried by the fuel ions and the electrons. Furthermore, we assume the impurity particle flux to be small compared with the fuel flux. Then all terms containing  $q_\alpha^r$ ,  $q_\sigma^r$ ,  $V_\alpha^r$ ,  $V_\sigma^r$  can be suppressed in eq. (1.6) and the condition of ambipolarity reduces to

$$(1.7) \quad n_f V_f^r \approx n_e V_e^r = \Gamma$$

We now have instead of eq. (1.6)

$$\begin{aligned}
 (1.9) \quad & \frac{3}{2} \frac{\partial}{\partial t} (n_i T_i) + \frac{1}{T} \frac{\partial}{\partial t} \left( \frac{5}{2} T_i \Gamma + T q_f^r \right) \\
 & - \frac{\Gamma}{n_f} \frac{\partial}{\partial t} (n_f T_i) = Q_i
 \end{aligned}$$

this again being of a type similar to eq. (1.5).

Before writing down the remaining equations (1.4) and (1.5) explicitly, we shall specify the respective source terms.

## 1.2 Specification of source terms

The physical effects that lead to source terms in the energy and particle balance equations are energy exchange, ohmic heating, nuclear heating, additional heating, radiation and refuelling. We now discuss these effects in greater detail. The resulting source terms will be characterized by a lower particle index and an upper index which characterizes the corresponding physical mechanism.

### 1.2.1 Energy exchange

The plasma particles exchange energy with each other by Coulomb interaction. The energy exchange between the heavy ions is much stronger than between the ions and the light electrons. For typical reactor parameters the characteristic time for the former process ( $\sim 10^{-2}$  sec) [2] is much shorter than the characteristic times of the other mechanisms discussed ( $\sim 1$  sec). Hence the ion temperatures remain nearly equal during the relatively slow processes under consideration. As noted above the ions can then be taken as one component the energy exchange within which need not be considered explicitly.

The energy exchange of the electrons with one ion species of charge  $Z'e$ , mass  $m'$ , temperature  $T'$  and density  $n'$  is given by [1]

$$\begin{aligned} Q^{\Delta} &= 3 \frac{n_e}{\tau_e'} \frac{m_e}{m'} (T_e - T') \\ &= 4 \sqrt{2\pi} \lambda e^4 \sqrt{m_e} \frac{T_e - T'}{T_e^{3/2}} \frac{n_e Z'^2 n'}{m'} \end{aligned}$$

where  $\lambda$  is the Coulomb logarithm.

In our case of several ion species the energy exchange to all ions is given by summation as

$$(1.10) \quad Q_e^\Delta = 4,81 \cdot 10^{-24} \frac{T_e - \bar{T}_i}{T_e^{3/2}} n_e \left( n_f \left( \frac{\chi}{M_{oi}} + \frac{1-\chi}{M_t} \right) \right. \\ \left. + 4 \frac{n_\alpha}{M_\alpha} + \sum_{\zeta} Z_\zeta^2 \frac{n_\zeta}{M_\zeta} \right)$$

( $\chi = n_d/n_d + n_t$ , and M are the mass numbers).

We shall simplify eq. (1.10) by the assumption  $Z_\zeta / M_\zeta \approx 1/2$   
 $\zeta = 1..5$  in future applications.

The corresponding ion energy exchange term is obviously given by

$$(1.11) \quad Q_i^\Delta = - Q_e^\Delta.$$

### 1.2.2 Ohmic heating

Ohmic heat arises owing to friction when ions and electrons move relative to each other. Because of the large mass ratio of ions and electrons the ohmic heat goes almost completely into the electrons [1]. The ohmic heating term, in general, is of the form

$$Q_e^J = \eta_{||} j_{||}^2 + \eta_{\perp} j_{\perp}^2$$



where  $\vec{j}_{\parallel}$  and  $\vec{j}_{\perp}$  are the current densities parallel and perpendicular to the magnetic field respectively, and  $\eta_{\parallel}$  and  $\eta_{\perp}$  the parallel and perpendicular resistivities respectively.

As is shown in appendix 1,  $Q_e^J$  has approximately the form

$$(1.12) \quad Q_e^J = \eta_{\parallel} j_T^2 - \frac{\Gamma_c}{n_e} \frac{\partial p}{\partial r} .$$

where  $j_T$  is the toroidal current density and

$$\Gamma_c = - \eta_{\perp} n_e c^2 \frac{\partial p}{\partial r} \frac{1}{B^2}$$

is the classical diffusion velocity. The second term in (1.12) can be neglected in comparison with the term

$$\frac{\Gamma}{n_e} \frac{\partial}{\partial r} (n_e T_e) \quad \text{if } \Gamma \gg \Gamma_c \quad . \text{ (Keep in mind that } p = p_e + p_i$$

and  $p_e \approx p_i$ )

The latter inequality holds if the anomalous diffusion prevailing in the reactor regime is not purely caused by an anomalous perpendicular resistivity. This is implicitly assumed in transport calculations and we follow this line here too.

In the low electron drift case  $\eta_{||}$  can be assumed to have its classical value which reads

$$\eta_{||} = \frac{m_e v_e}{2 e^2 n_e} ,$$

where 
$$v_e = \frac{4 \sqrt{2\pi} \lambda e^4 Z'^2 n'}{3 \sqrt{m_e} T_e^{3/2}}$$

is the electron-ion collision frequency for one ion species of charge  $Z'$  [1]. In our special case we have the more general expression

$$v_e = \frac{4 \sqrt{2\pi} \lambda e^4}{3 \sqrt{m_e} T_e^{3/2}} \left( n_f + 4 n_\alpha + \sum_G Z_G^2 n_G \right) ,$$

where  $\lambda$  is the Coulomb logarithm which can be taken as 20 in the range of interest [1]. This leads in our units to the expression

$$(1.14) \quad Q_e^J = 3,63 \cdot 10^{-18} j_T^2 T_e^{-\frac{3}{2}} n_f n_e^{-1} Z ,$$

with the abbreviation

$$Z = 1 + 4 \frac{n_\alpha}{n_f} + \sum_{\sigma} Z_{\sigma}^2 \frac{n_{\sigma}}{n_f}$$

We shall find that the ohmic heating plays no role in the reactor regime. Therefore, we need not consider neoclassical and anomaly effects.

### 1.2.3 Nuclear heating

The production rate of  $\alpha$ -particles due to the D-T reaction is given by  $n_d n_t \langle \sigma v \rangle$  where  $\langle \sigma v \rangle$  is the Maxwellian averaged reaction rate. With  $n_d/(n_d + n_t) = n_d/n_f = \alpha$  we then have

$$(1.15) \quad \dot{N}_{\alpha} = \alpha (1 - \alpha) n_f^2 \langle \sigma v \rangle$$

For each  $\alpha$ -particle two fuel ions disappear. This yields

$$(1.16) \quad \dot{N}_f = -2 \alpha (1 - \alpha) n_f^2 \langle \sigma v \rangle$$

The rate of power production by the fast  $\alpha$ 's is given by

$$\alpha (1 - \alpha) n_f^2 \langle \sigma v \rangle E_{\alpha} \quad (E_{\alpha} = 3540 \text{ KeV})$$

This energy is distributed among the other components during the slowing-down process.

The slowing-down time is relatively small compared with the characteristic times within which the plasma parameters change. Thus the slowing-down process can be assumed to happen instantaneously, and one can write

$$Q_e^N = E_\alpha \kappa (1-\kappa) n_f^2 \langle \sigma v \rangle U_e(E_\alpha)$$

and

$$Q_i^N = E_\alpha \kappa (1-\kappa) n_f^2 \langle \sigma v \rangle U_i(E_\alpha)$$

where the fractions  $U_e$  and  $U_i$  of the  $\alpha$  energy that go to electrons and ions respectively are given by the model in Appendix 2.

It is convenient to use the approximate expression [4]

$$\langle \sigma v \rangle = \frac{A}{T_i^{2/3}} e^{-\frac{B}{T_i^{1/3}}} \quad (A=3,68 \cdot 10^{-12}; B=19,94 \text{ in our units}),$$

with which we get

$$(1.17) \quad S_\alpha^N = 3,68 \cdot 10^{-12} \kappa (1-\kappa) \frac{n_f^2 e^{-\frac{19,94}{T_i^{1/3}}}}{T_i^{2/3}},$$

$$(1.18) \quad S_f^N = -7,36 \cdot 10^{-12} \kappa (1-\kappa) \frac{n_f^2 e^{-\frac{19,94}{T_i^{1/3}}}}{T_i^{2/3}},$$

$$(1.19) \quad Q_e^N = 2,07 \cdot 10^{-17} \kappa (1-\kappa) \frac{n_f^2 e^{-\frac{19,94}{T_i^{1/3}}}}{T_i^{2/3}}$$

$$\cdot (1 - 0,0137 (\kappa+2)^{2/3} T_e + 0,000849 (\kappa+2) T_e^{3/2}),$$



$$(1.20) \quad Q_i^N = 2,07 \cdot 10^{-17} \alpha(1-\alpha) \frac{n_f^2}{T_i^{2/3}} e^{-\frac{19,94}{T_i^{1/3}}} \\ (0,0137 (\alpha + 2)^{2/3} T_e - 0,000849 (\alpha + 2) \cdot T_e^{3/2})$$

Here in addition the expression(A2.5) and(A2.9) for  $U_e$  and  $U_i$  respectively in Appendix 2 are used.

#### 1.2.4 Additional heating

In the following we shall only consider heating by neutral deuterium beam injection.

A neutral particle penetrating into the plasma will, in principle, be ionized by charge exchange and electron or ion impact. For energies considered here ( $E_0 \gtrsim 200$  keV) ion impact is the dominant effect [5].

The penetration depth depends on, besides the ionization cross-section and density profile, the specific mode of injection to a large degree. We do not study the ionization process here but prescribe a certain production rate  $N_H(r)$  of energetic deuterium ions with energy  $E_0$  (beam energy). The fast ions are then assumed to give up their energy to the plasma by Coulomb interaction. As to the deposition of energy of the fast ions we shall use the test particle model described in Appendix 2.

Apart from the direct transfer of energy from the beam particles to the plasma some deuterium ions undergo a fusion reaction. Though very few beam particles react, both effects are of comparable magnitude owing to the high energy of the fusion  $\alpha$ 's (3540 keV) compared with the beam particles ( $\sim 200$  keV).

We shall write  $Q^{ZH} = Q^{ZHO}$  (direct transfer) +  $Q^{ZHF}$  (fusion).

We then get

$$Q_e^{ZHO} = N_H E_0 U_e \quad , \quad Q_i^{ZHO} = N_H E_0 U_i$$

where  $U_e$  and  $U_i$  are the fractions of energy that go to the electrons and ions respectively during the slowing down process and are given by eq. (A2.4) and eq. (A2.5) together with eq. (A2.7) of Appendix 2.

We find it convenient to express  $N_H$  by means of the total injected power  $P_{inj}$ . With

$$\bar{N}_H = \frac{1}{V} \int d\tau N_H \quad , \quad \text{where } V \text{ is the plasma volume, it follows that}$$

$$N_H = \frac{N_H}{\bar{N}_H} \frac{P_{inj}}{E_0 2\pi^2 \alpha^2 R} .$$

With the explicit form of  $U_e$  and  $U_i$  as given by eq. (A2.4) and (A2.5) we now get

$$(1.21) \quad Q_i^{ZHO} = 1,69 \cdot 10^{11} \frac{P_{inj}}{R \alpha^2} \frac{N_H}{\bar{N}_H} \eta$$

$$\cdot \left\{ \ln \left( \frac{1 - \eta^{1/2} + \eta}{1 + 2\eta^{1/2} + \eta} \right) + 2\sqrt{3} \right.$$

$$\cdot \left. \arctg \left( \frac{2 - \eta^{1/2}}{\sqrt{3} \eta^{1/2}} \right) + \frac{\pi}{\sqrt{3}} \right\}$$

and

$$(1.22) \quad Q_e^{ZH0} = 5,07 \cdot 10^{11} \frac{P_{inj}}{R \alpha^2} \frac{N_H}{N_H} \left[ 1 - \frac{\eta}{3} \right. \\ \left. \cdot \left\{ \ln \left( \frac{1 - \eta^{1/2} + \eta}{1 + 2\eta^{1/2} + \eta} \right) + 2\sqrt{3} \right. \right. \\ \left. \left. \cdot \operatorname{arctg} \left( \frac{2 - \eta^{1/2}}{\sqrt{3} \eta^{1/2}} \right) + \frac{\pi}{\sqrt{3}} \right\} \right],$$

where  $\eta = \frac{10(\kappa+2)^{2/3} T_e}{E_0}$ .

The injected particles represent a source of electrons and ions. The corresponding source terms are obviously given by

$$(1.23) \quad S_e^{ZH} = \frac{N_H}{N_H} \frac{P_{inj}}{2\pi \alpha^2 R E_0} \quad \text{and} \quad S_i^{ZH} = S_e^{ZH}$$

which in practical units yields

$$(1.24) \quad S_e^{ZH} = 3,16 \cdot 10^{20} \frac{N_H}{N_H} \frac{P_{inj}}{E_0 \alpha^2 R}$$

In the expression for  $S_i^{ZH}$  we have neglected the small number of beam particles that undergo fusion reactions. In addition, we neglect the change of  $\kappa$  due to the injection of deuterium. This can be done since  $n_f$  changes by at most a few per cent owing to injection during a particle confinement time.

The heating power generated by fusion is given by

$$Q_i^{ZHF} = N_H E_\alpha V_\alpha U_i(E_\alpha),$$

$$Q_e^{ZHF} = N_H E_\alpha V_\alpha U_e(E_\alpha)$$

$$= N_H E_\alpha V_\alpha - Q_i^{ZHF}.$$

Here  $E_\alpha$  is the energy of a fusion  $\alpha$ -particle.  $V_\alpha$  is the probability that a beam ion undergoes fusion during its slowing-down. With  $V_\alpha$ ,  $U_e$  and  $U_i$  as given by Appendix 2 we get

$$(1.25) \quad Q_i^{ZHF} = 2,58 \cdot 10^{15} \frac{P_{inj} \eta_\alpha}{R \alpha^2} \frac{N_H}{N_H} \frac{(1-\alpha)}{E_0^{5/3}} T_e^{3/2} \frac{n_f}{n_e}$$

$$\left\{ \ln \left( \frac{1 - \eta_\alpha^{1/2} + \eta_\alpha}{1 + 2\eta_\alpha^{1/2} + \eta_\alpha} \right) + 2\sqrt{3} \right.$$

$$\left. \arctg \left( \frac{2 - \eta_\alpha^{1/2}}{\sqrt{3} \eta_\alpha^{1/2}} \right) + \frac{\pi}{\sqrt{3}} \right\}$$

$$\int_0^1 dx e^{-\frac{19,94}{E_0^{1/3} X^{1/3}}} X^{-\frac{1}{6}} (X^{3/2} + \eta^{3/2})^{-1}$$

$$(1.26) \quad Q_e^{ZHF} = 7,74 \cdot 10^{15} \frac{P_{inj}}{R \alpha^2} \frac{n_f T_e^{3/2}}{E_0^{5/3} n_e} \frac{N_H}{N_H} (1-\alpha)$$

$$\int_0^1 dx e^{-\frac{19,94}{E_0^{1/3} X^{1/3}}} X^{-\frac{1}{6}} (X^{3/2} + \eta^{3/2})^{-1} - Q_i^{ZHF}$$



where  $\eta_\alpha = 5,65 \cdot 10^{-3} (\alpha + 2)^{2/3} T_e,$

$$\eta = \frac{10 (\alpha + 2)^{2/3} T_e}{E_0}$$

The fusion reactions give an additional source of  $\alpha$ 's.  
With the above notations one gets

$$(1.27) \quad S_\alpha^{ZH} = N_H V_\alpha \\ = 6,83 \cdot 10^{20} \frac{N_H}{\bar{N}_H} \frac{P_{inj} (1 - \alpha)}{R \alpha^2 E_0^{5/3}} \frac{n_f T_e^{3/2}}{n_e}$$

$$\int_0^1 dx e^{-\frac{19,94}{E_0^{1/3} x^{1/3}}} x^{-\frac{1}{6}} (x^{3/2} + \eta^{3/2})^{-1}$$

### 1.2.5 Bremsstrahlung

Bremsstrahlung results from the deflection of electrons in the ion fields. In the presence of  $\alpha$ -particles and impurity ions with charge  $Z_c$  and density  $n_\alpha$  and  $n_c$  respectively Karzas and Salter [6] give the expression

$$(1.28) \quad Q_e^{Bt} = -c \sqrt{T_e} n_e (n_f g_f + 4n_\alpha g_\alpha + \sum_c \frac{g_c Z_c^2 n_c}{6})$$

( $c = 4,86 \cdot 10^{-24}$  in our units)

for the electron energy loss term. For our purpose we may neglect the small temperature dependence of the gaunt factors  $g_{f,\alpha,\sigma}(Z^2/T_e)$  and set  $g_{f,\alpha,\sigma} \approx 1,3$ .

The bremsstrahlung can leave the plasma without reabsorption. Therefore  $Q_e^{Br}$  is the only source term due to bremsstrahlung.

### 1.2.6 Recombination and line radiation

Recombination and line radiation are caused by the interaction between electrons and partially ionized impurity atoms. Electrons lose energy owing to further ionization or excitation of these ions. This energy is radiated out of the plasma when an electron is captured (recombination radiation) or the ion returns to its ground state (excitation or line radiation).

For highly ionized ions Post [7] gives for recombination radiation losses the expression

$$(1.29) \quad Q_e^{Re} = -C \sum_{\sigma} \frac{Z_{\sigma}^4}{T_e^{1/3}} n_e n_{\sigma}$$

( $C = 1,3 \cdot 10^{-25}$  in our units).

An approximate expression for line radiation which is valid for ions with at least three rest electrons was given by Hinnov [8].

$$(1.30) \quad Q_e^{Li} = -C \sum_{\sigma} n_e n_{\sigma}$$

( $C = 2 \cdot 10^{-19}$  in our units).

For ions with one electron Post [7] gives the expression

$$(1.31) \quad Q_e^{Li} = - \left( \sum_G \frac{n_e n_G}{\sqrt{T_e}} \right) .$$

( $C = 2 \cdot 10^{-20}$  in our units).

We shall apply this formula to two-electron ions, too.

Now if  $T^1$  and  $T^3$  are the temperatures at which 0 and 1 electron and 2 and 3 electron ions respectively are in equilibrium we have

$$(1.32) \quad Q_e^{Li} = - 2 \cdot 10^{-20} \frac{n_e}{\sqrt{T_e}} \sum_G n_G \varphi(T_G^3, T_G^0, T_G^1) \\ - 2 \cdot 10^{-19} n_e \sum_G n_G \varphi(0, T_e, T_G^3),$$

$$\varphi(0, x, b) = \begin{cases} 1 & \text{if } 0 \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

The above formula for recombination radiation holds for one-electron ions and approximately for two-electron ions. The value given by the recombination radiation formula is much smaller, or at most of the same order, for very high  $Z$  values than line radiation for ions with three and more electrons. Since recombination radiation decreases rapidly for weakly ionized ions, we need not worry about recombination radiation in this range and can neglect it for ions with more than two electrons. Consequently, one has

$$(1.33) \quad Q_e^{Re} = - 1,3 \cdot 10^{-25} \frac{n_e}{\sqrt{T_e}} \sum_G Z_G^4 n_G \varphi(T_G^3, T_e, \infty).$$

### 1.2.7 Cyclotron radiation

Bremsstrahlung, recombination and line radiation are local effects inasmuch as the radiation energy which is produced at one point as a result of the interaction between electrons and ions can leave the plasma without reabsorption. This is not true of cyclotron radiation and the computation of cyclotron radiation losses is much more complicated. Therefore, only vague estimates of cyclotron radiation losses are available. Fortunately, in the temperature range expected for a tokamak reactor cyclotron radiation is relatively insignificant compared with other radiation losses.

In view of this situation we decided to make use of the relatively simple formula

$$(1.34) \quad Q_e^{zy} = -2,2 \cdot 10^{-7} B_T^{5/2} \sqrt{(1-\mu)} \frac{\sqrt{n_e} T_e^2}{\sqrt{a}}$$

proposed by Yang et al. [9]. The wall reflection coefficient  $\mu$  is taken to be 0.9 in most cases.

### 1.2.8 Refuelling

As to refuelling we only regard cold refuelling such as pellet injection. We characterize the refuelling mechanism by a given particle source  $N_F$ . The neutral fuel particles, when ionized, lead to fuel ion and electron sources of the same form:

$$(1.35) \quad S_e^F = N_F ,$$

$$(1.36) \quad S_i^F = N_F .$$



1.3 Explicit set of transport equations

We now have the following set of equations for particle and energy transport.

Electron energy balance equation:

$$(1.37) \quad \frac{3}{2} \frac{\partial}{\partial t} (n_e T_e) + \frac{1}{r} \frac{\partial}{\partial r} (r \left[ \frac{5}{2} \Gamma T_e + q_e \right]) - \frac{\Gamma}{n_e} \frac{\partial}{\partial r} (n_e T_e) = Q_e^\Delta + Q_e^J + Q_e^N + Q_e^{Br} + Q_e^{Li} + Q_e^{Re} + Q_e^{Zr} + Q_e^{ZH} ,$$

$$Q_e^\Delta = -4,81 \cdot 10^{-21} \frac{T_e - T_i}{T_i^{3/2}} n_e \left( n_f \frac{2 + \alpha}{6} + n_a + \frac{1}{2} \sum_G Z_G n_G \right) ,$$

$$Q_e^J = 3,63 \cdot 10^{-18} \frac{j_r^2}{T_e^{3/2}} \frac{n_f}{n_e} Z ,$$

$$Z = 1 + 4 \frac{n_a}{n_f} + \sum_G Z_G^2 \frac{n_G}{n_f} ,$$

$$Q_e^N = 2,07 \cdot 10^{-17} \alpha (1 - \alpha) n_f^2 \frac{e^{-\frac{19,94}{T_i^{1/3}}}}{T_i^{2/3}}$$

$$(1 - 0,0137 (\alpha + 2)^{2/3} T_e + 0,000849 (\alpha + 2) T_e^{3/2}) ,$$

$$Q_e^{Br} = -6,32 \cdot 10^{-24} \sqrt{T_e} n_e n_f Z ,$$

$$Q_e^{Li} = -2,0 \cdot 10^{-20} \frac{n_e}{\sqrt{T_e}} \sum_{\sigma} n_{\sigma} \Psi(T_{\sigma}^3, T_e, T_{\sigma}^1)$$

$$-2,0 \cdot 10^{-19} n_e \sum_{\sigma} n_{\sigma} \Psi(0, T_e, T_{\sigma}^3) ,$$

$$\Psi(a, x, b) = \begin{cases} 1 & \text{if } a \leq x \leq b \\ 0 & \text{else} \end{cases}$$

$$Q_e^{Re} = -1,3 \cdot 10^{-25} \frac{n_e}{\sqrt{T_e}} \sum_{\sigma} Z_{\sigma}^4 n_{\sigma} \Psi(T_{\sigma}^3, T_e, \infty),$$

$$Q_e^{Zr} = -2,2 \cdot 10^{-7} B_T^{2,5} \frac{\sqrt{(1-\mu) n_e} T_e^2}{\sqrt{a_1}} ,$$

$$Q_e^{ZH} = Q_e^{ZHO} + Q_e^{ZHF} ,$$

$$Q_e^{ZHO} = 5,07 \cdot 10^{11} \frac{P_i n_f}{a_1^2 \cdot R} \frac{N_H}{N_H} \left[ 1 - \frac{\eta}{3} \right.$$

$$\left. \left( \ln \left( \frac{1 - \eta^{1/2} + \eta}{1 + \eta^{1/2} + \eta} \right) + 2\sqrt{3} \arctg \left( \frac{2 - \eta^{1/2}}{\sqrt{3} \eta^{1/2}} \right) \right) \right]$$

$$+ \frac{\pi}{\sqrt{3}}) ] ,$$

$$\eta = \frac{10 (\alpha + 2)^{2/3} T_e}{E_0}$$

$$Q_e^{ZHF} = 7,74 \cdot 10^{15} \frac{P_{inj}}{R \Omega^2} \frac{n_f T_e^{3/2}}{E_0^{5/3} n_c} \frac{N_H}{N_H} (1 - \alpha)$$

$$\int_0^1 dx \frac{e^{-\frac{19,94}{E_0^{1/3} x^{1/3}}}}{x^{1/6} (x^{3/2} + \eta^{3/2})} = Q_i^{ZHF}$$

Ion energy equation:

$$(1.38) \quad \frac{3}{2} \frac{\partial}{\partial t} (n_i T_i) + \frac{1}{r} \frac{\partial}{\partial r} (r [\frac{5}{2} T_i \Gamma + q_f])$$

$$- \frac{\Gamma}{n_f} \frac{\partial}{\partial r} (n_f T_i) = Q_i^\Delta + Q_i^N + Q_i^{ZH}$$

$$Q_i^\Delta = - Q_e^\Delta ,$$

$$Q_i^N = 2,07 \cdot 10^{-17} \alpha (1 - \alpha) \frac{e^{-\frac{19,94}{T_i^{1/3}}}}{T_i^{2/3}} n_f^2$$

$$(0,0137 (\alpha+2)^{2/3} T_e - 0,000894 (\alpha+2) T_e^{3/2}),$$

$$Q_i^{ZH} = Q_i^{ZHO} + Q_i^{ZHF},$$

$$Q_i^{ZHO} = 1,69 \cdot 10^{11} \frac{P_{inj}}{R_{OI}^2} \frac{N_H}{\bar{N}_H} \eta.$$

$$\left\{ \ln \left( \frac{1 - \eta^{1/2} + \eta}{1 + 2\eta^{1/2} + \eta} \right) + 2\sqrt{3} \operatorname{arctg} \left( \frac{2 - \eta^{1/2}}{\sqrt{3} \eta^{1/2}} \right) + \frac{\pi}{\sqrt{3}} \right\}$$

$$\eta = \frac{10 (\alpha+2)^{2/3} T_e}{E_0},$$

$$Q_i^{ZHF} = 2,58 \cdot 10^{15} \frac{P_{inj}}{R_{OI}^2} \frac{N_H}{\bar{N}_H} \frac{1 - \alpha}{E_0^{5/3}} T_e^{3/2}$$

$$\eta_\alpha \frac{N_\pm}{N_e} \left\{ \ln \left( \frac{1 - \eta_\alpha^{1/2} + \eta_\alpha}{1 + 2\eta_\alpha^{1/2} + \eta_\alpha} \right) + 2\sqrt{3} \operatorname{arctg} \left( \frac{2 - \eta_\alpha^{1/2}}{\sqrt{3} \eta_\alpha^{1/2}} \right) \right.$$

$$\left. + \frac{\pi}{\sqrt{3}} \right\} \int_0^1 dx \frac{e^{-\frac{19,94}{E_0^{1/3} x^{1/3}}}}{x^{1/6} (x^{3/2} + \eta^{3/2})}.$$

Electron particle balance:

$$(1.39) \quad \frac{\partial n_e}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma) = S_e^{ZH} + S_e^F,$$

$$S_e^{ZH} = 3,16 \cdot 10^{20} \frac{N_H}{\bar{N}_H} \frac{P_{inj}}{E_0 R 01^2},$$

$$S_e^F = N_F$$

Fuel particle balance:

$$(1.40) \quad \frac{\partial n_f}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma) = S_f^N + S_f^{ZH}$$

$$+ S_f^F,$$

$$S_f^N = - 7,36 \cdot 10^{-12} \alpha (1-\alpha) n_f^2 \frac{e^{-\frac{19,94}{T_i^{1/3}}}}{T_i^{2/3}},$$

$$S_f^{ZH} = 3,16 \cdot 10^{20} \frac{N_H}{\bar{N}_H} \frac{P_{inj}}{E_0 R 01^2},$$

$$S_f^F = N_F .$$

$\alpha$ -particle balance:

$$(1.41) \quad \frac{\partial n_\alpha}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r n_\alpha V_\alpha^r) = S_\alpha^N + S_\alpha^{ZH} ,$$

$$S_\alpha^N = 3,68 \cdot 10^{-12} x(1-x) n_f^2$$

$$\cdot \frac{e^{-\frac{19,94}{T_i^{1/3}}}}{T_i^{2/3}} ,$$

$$S_\alpha^{ZH} = 6,83 \cdot 10^{20} \frac{N_H}{\bar{N}_H} \frac{P_{inj} (1-x)}{R a^2 E_0^{5/3}} \frac{n_f T_e^{3/2}}{n_e}$$

$$\int_0^1 dr x e^{-\frac{19,94}{E_0^{1/3} x^{1/3}}} x^{-\frac{1}{6}} (x^{\frac{2}{3}} + \eta^{3/2})^{-1} .$$

Impurity particle balance:

$$(1.42) \quad \frac{\partial n_G}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r n_G V_G^r) = 0 ,$$

$$G = 1 \dots 5 .$$



2. Volume averaged transport equations

2.1 General form of volume averaged transport equations

Equations (1.37) to (1.42) of the last section are a system of partial differential equations for the time and space dependent densities and temperatures which must be solved with appropriate initial and boundary conditions. To simplify the problem, one often solves the corresponding zero space moment equations with some more or less plausible assumptions as to the density and temperature profiles.

We shall follow this line and consider the space averages of equations (1.3) and (1.5) respectively. Quite general, one has

$$(2.1) \quad V^{-1} \int d\tau L(f(t))$$

where  $V$  is the plasma volume,  $L$  some functional,  $f$  some quantity such as  $n$  or  $T$ ,

$$= \frac{1}{2\pi^2 a^2 R} \int_0^a dt \int_0^{2\pi} d\varphi \int_0^{2\pi} d\alpha \tau \cdot R L(f(t))$$

$$= \frac{2}{a^2} \int_0^a dt \tau L(f(t)) = \overline{L(f(t))}$$

If we apply the averaging procedure to eqs. (1.3) and (1.5) we get

$$(2.2) \quad \frac{\partial \bar{n}}{\partial t} + \frac{2}{a^2} [\tau n v^r]_a = \bar{S}$$

and

$$(2.3) \quad \frac{3}{2} \frac{\partial}{\partial t} (\overline{nT}) + \frac{2}{\alpha^2} \left[ \frac{5}{2} n T V^+ r \right]_{\alpha} +$$

$$\frac{2}{\alpha^2} \left[ r \alpha \right]_{\alpha} - \frac{\partial}{\partial t} (\overline{nT}) V^+ = \overline{Q} .$$

## 2.2 Profile forms and explicit set of averaged equations

To evaluate the averages in eqs. (2.2) and (2.3), one has to know the exact solutions of eqs. (1.3) and (1.5). To get a closed system from eqs. (2.2) and (2.3) alone, one must specify the solutions of eqs. (1.3) and (1.5) in such a way that they contain at most one free parameter, the time dependence of which is then described by eqs. (2.2) and (2.3).

Usually it is assumed that the density and temperature are constant in space or have a parabolic shape. To have more flexibility in fitting density and temperature profiles we consider those of the type

$$(2.4) \quad n = n_0 \left[ (1-\varepsilon) \left( 1 - \left( \frac{r}{\alpha} \right)^{2\alpha} \right)^\beta + \varepsilon \right], \quad T = T_0 \left[ (1-\gamma) \left( 1 - \left( \frac{r}{\alpha} \right)^{2\alpha} \right)^\delta + \gamma \right]$$

The  $\alpha, \beta, \varepsilon, \gamma, \delta, \nu$  are assumed to remain constant during the time evolution of the system.

With these profiles the above averages can be explicitly evaluated. As a result, we get a set of ordinary differential equations for the time dependent peak densities  $n_0(t)$  and temperatures  $T_0(t)$ .

To put the averaged equations in a comprehensive form, it is convenient to introduce the following functions:

$$J_1(X_1, X_2, X_3, Y_1, Y_2, Y_3, Z_1, Z_2, Z_3)$$

$$= \int_0^1 d\xi \left[ (1-X_3)(1-\xi^{X_1})^{X_2} + X_3 \right] \cdot$$

$$\left[ (1-Y_3)(1-\xi^{Y_1})^{Y_2+Y_3} \right] \left[ (1-Z_3)(1-\xi^{Z_1})^{Z_2+Z_3} \right]^{1/2},$$

$$J_2(X_1, X_2, X_3, Y_1, Y_2, Y_3, Z_1, Z_2, Z_3, V_1, V_2, V_3)$$

$$= \int_0^1 d\xi \left[ (1-X_3)(1-\xi^{X_1})^{X_2+X_3} \right] \left[ (1-Y_3)(1-\xi^{Y_1})^{Y_2+Y_3} \right]$$

$$\left[ (1-Z_3)(1-\xi^{Z_1})^{Z_2+Z_3} \right]^{-\frac{3}{2}} \left[ (1-V_3)(1-\xi^{V_1})^{V_2+V_3} \right],$$

$$J_3(X_1, X_2, X_3, Y_1, Y_2, Y_3, V_1, V_2, V_3)$$

$$= \int_0^1 d\xi \left[ (1-X_3)(1-\xi^{X_1})^{X_2+X_3} \right]$$

$$\left[ (1-Y_3)(1-\xi^{Y_1})^{Y_2+Y_3} \right]^{\frac{3}{2}} \left[ (1-V_3)(1-\xi^{V_1})^{V_2+V_3} \right]^{-1},$$

$$I_X (T_0^e, X_1, X_2, X_3, Y_1, Y_2, Y_3, Z_1, Z_2, Z_3)$$

$$= \int_0^1 d\xi \ e^{-\frac{19,94}{T_0^e} \left\{ \frac{1}{[(1-Z_3)(1-\xi^{Z_1})^{Z_2+Z_3}] - 1} \right\}}$$

$$\left[ (1-X_3)(1-\xi^{X_1})^{X_2+X_3} \right]^2 \left[ (1-Y_3)(1-\xi^{Y_1})^{Y_2+Y_3} \right]^X$$

$$\left[ (1-Z_3)(1-\xi^{Z_1})^{Z_2+Z_3} \right]^{-2/3},$$

$$F_X (A, T_0^e, B, X_1, X_2, X_3, Y_1, Y_2, Y_3, Z_1, Z_2, Z_3)$$

$$= \int_0^1 d\xi \left[ (1-X_3)(1-\xi^{X_1})^{X_2+X_3} \right]$$

$$\left[ (1-Z_3)(1-\xi^{Z_1})^{Z_2+Z_3} \right] \left[ (1-Y_3)(1-\xi^{Y_1})^{Y_2+Y_3} \right]^{-X}$$

$$\Psi (A, T_0^e \left[ (1-Y_3)(1-\xi^{Y_1})^{Y_2+Y_3} \right], B),$$

$$\Psi (A, X, B) = \begin{cases} 1 & \text{if } A \leq X \leq B \\ 0 & \text{otherwise} \end{cases},$$

$$R_{x,y,z}(\eta_0, \eta'_0, E_0, X_1, X_2, X_3, Y_1, Y_2, Y_3,$$

$$Z_1, Z_2, Z_3, V_1, V_2, V_3)$$

$$= \int_0^1 d\xi \left[ (1-X_3)(1-\xi^{X_1})^{X_2+X_3} \right]$$

$$\left[ (1-Z_3)(1-\xi^{Z_1})^{Z_2+Z_3} \right]^Z \left[ (1-V_3)(1-\xi^{V_1})^{V_2+V_3} \right]^X$$

$$\left[ (1-Y_3)(1-\xi^{Y_1})^{Y_2+Y_3} \right]^{-Z}$$

$$H^Y(\eta_0 \left[ (1-V_3)(1-\xi^{V_1})^{V_2+V_3} \right])$$

$$G(\eta'_0 \left[ (1-V_3)(1-\xi^{V_1})^{V_2+V_3} \right], E_0) ,$$

$$H(\xi) = \ln \left( \frac{1-\sqrt{\xi} + \xi}{1+2\sqrt{\xi} + \xi} \right) + 2\sqrt{3} \arctg$$

$$\left( \frac{2-\sqrt{\xi}}{\sqrt{3}\sqrt{\xi}} \right) + \frac{\pi}{\sqrt{3}} ,$$

$$G(\xi, E_0) = \int_0^1 dx \frac{e^{-\frac{19.94}{E_0^{1/3} X^{1/3}}}}{X^{1/6} (X^{3/2} + \xi^{3/2})} .$$

With these auxiliary functions and eq. (A3.8) the volume averaged particle and energy balance equations take the following final form:

Electron energy balance:

$$(2.5) \quad \frac{3}{2} \frac{d}{dt} (n_0^e T_0^e) \Big]_1 (\alpha_e, \beta_e, \epsilon_e, \gamma_e, \delta_e, \nu_e, 1, 0, 1)$$

$$= \bar{Q}_e^o + \bar{Q}_e^\Delta + \bar{Q}_e^J + \bar{Q}_e^N + \bar{Q}_e^{Br} + \bar{Q}_e^{Li}$$

$$+ \bar{Q}_e^{Re} + \bar{Q}_e^{Zy} + \bar{Q}_e^{ZH} ,$$

$$\bar{Q}_e^o = - \frac{2}{\omega^2} \left[ \nu (\nu_e + \frac{5}{2} T_e \Gamma) \right]_{\omega} + \frac{\Gamma}{n_e} \frac{\partial}{\partial t} (n_e T_e) ,$$

$$\bar{Q}_e^\Delta = - 4,81 \cdot 10^{-21} \frac{n_0^e}{\sqrt{T_0^e}} \left\{ n_0^+ \frac{2 + \alpha_e}{6} \right\} .$$

$$\left[ J_2 (\alpha_e, \beta_e, \epsilon_e, \alpha_f, \beta_f, \epsilon_f, \gamma_e, \delta_e, \nu_e, \gamma_e, \delta_e, \nu_e) \right.$$

$$\left. - \frac{T_0^i}{T_0^e} J_2 (\alpha_e, \beta_e, \epsilon_e, \alpha_f, \beta_f, \epsilon_f, \gamma_e, \delta_e, \nu_e, \gamma_i, \delta_i, \nu_i) \right]$$

$$+ n_\alpha \left[ J_2 (\alpha_e, \beta_e, \epsilon_e, \alpha_\alpha, \beta_\alpha, \epsilon_\alpha, \gamma_e, \delta_e, \nu_e, \gamma_e, \delta_e, \nu_e) \right.$$

$$\left. - \frac{T_0^i}{T_0^e} J_2 (\alpha_e, \beta_e, \epsilon_e, \alpha_\alpha, \beta_\alpha, \epsilon_\alpha, \gamma_e, \delta_e, \nu_e, \gamma_i, \delta_i, \nu_i) \right]$$



$$+ \frac{1}{2} \sum_{\sigma=1}^5 Z_{\sigma} n_{\sigma}^5 \left[ J_2 (\alpha_{\sigma}, \beta_{\sigma}, \epsilon_{\sigma}, \alpha_{\sigma}, \beta_{\sigma}, \epsilon_{\sigma}, \gamma_{\sigma}, \delta_{\sigma}, \nu_{\sigma}, \gamma_{\sigma}, \delta_{\sigma}, \nu_{\sigma}) - \frac{T_0^i}{T_0^e} J_2 (\alpha_{\sigma}, \beta_{\sigma}, \epsilon_{\sigma}, \alpha_{\sigma}, \beta_{\sigma}, \epsilon_{\sigma}, \gamma_{\sigma}, \delta_{\sigma}, \nu_{\sigma}, \gamma_{\sigma}, \delta_{\sigma}, \nu_{\sigma}) \right]$$

$$\bar{Q}_e^J = 3,32 \cdot 10^{12} \frac{I^2 n_0^f \bar{Z}}{\alpha^4 n_0^e T_0^e{}^{3/2}} J_3^{-1} (\alpha_c, \beta_c, \epsilon_c, \gamma_c, \delta_c, \nu_c, \alpha_f, \beta_f, \epsilon_f)$$

$$\bar{Z} = 1 + 4 \frac{n_0^{\alpha}}{n_0^f} J_3 (\alpha_{\alpha}, \beta_{\alpha}, \epsilon_{\alpha}, 1, 0, 1, \alpha_f, \beta_f, \epsilon_f)$$

$$+ \sum_{\sigma=1}^5 Z_{\sigma}^2 \frac{n_0^{\sigma}}{n_0^f} J_3 (\alpha_{\sigma}, \beta_{\sigma}, \epsilon_{\sigma}, 1, 0, 1, \alpha_f, \beta_f, \epsilon_f)$$

$$\bar{Q}_e^N = 2,07 \cdot 10^{-17} \alpha (1-\alpha) n_0^2$$

$$\frac{e^{-\frac{19,94}{T_0^{1/3}}}}{T_0^{2/3}} \left\{ I_0 (T_0^i, \alpha_f, \beta_f, \epsilon_f, \gamma_c, \delta_c, \nu_c, \gamma_i, \delta_i, \nu_i) \right.$$

$$- 0,0137 T_0^e (\alpha+2)^{2/3} I_1 (T_0^i, \alpha_f, \beta_f, \epsilon_f, \gamma_c, \delta_c, \nu_c,$$

$$\gamma_i, \delta_i, \nu_i) + 0,000849 (\alpha+2) T_0^e{}^{3/2}$$

$$I_{\frac{3}{2}} (T_0^i, \alpha_f, \beta_f, \epsilon_f, \gamma_e, \delta_e, \nu_e, \gamma_i, \delta_i, \nu_i) \},$$

$$\bar{Q}_e^{Br} = -6,32 \cdot 10^{-24} T_0^e^{-\frac{1}{2}} n_0^e n_0^f$$

$$\{ J_1(\alpha_e, \beta_e, \epsilon_e, \alpha_f, \beta_f, \epsilon_f, \gamma_e, \delta_e, \nu_e)$$

$$+ 4 \frac{n_0^\alpha}{n_0^f} J_1(\alpha_e, \beta_e, \epsilon_e, \alpha_\alpha, \beta_\alpha, \epsilon_\alpha, \gamma_e, \delta_e, \nu_e)$$

$$+ \sum_{\sigma=1}^5 Z_\sigma^2 \frac{n_0^\sigma}{n_0^f} J_1(\alpha_e, \beta_e, \epsilon_e, \alpha_\sigma, \beta_\sigma, \epsilon_\sigma, \gamma_e, \delta_e, \nu_e) \},$$

$$\bar{Q}_e^{Li} = -2 \cdot 10^{-20} \frac{n_0^e}{\sqrt{T_0^e}} \sum_{\sigma=1}^5 n_0^\sigma$$

$$F_{1/2}(T_\sigma^3, T_0^e, T_\sigma^1, \alpha_e, \beta_e, \epsilon_e, \gamma_e, \delta_e, \nu_e, \alpha_\sigma, \beta_\sigma, \epsilon_\sigma)$$

$$- 2 \cdot 10^{-19} n_0^e \sum_{\sigma=1}^5 n_0^\sigma$$

$$F_0(0, T_0^e, T_\sigma^3, \alpha_e, \beta_e, \epsilon_e, 1, 0, 1, \alpha_\sigma, \beta_\sigma, \epsilon_\sigma),$$

$$\bar{Q}_e^{Re} = -1,3 \cdot 10^{-25} n_0^e T_0^e^{-\frac{1}{2}} \sum_{\sigma=1}^5 Z_{\sigma}^4$$

$$n_0^e F_{1/2}(T_{\sigma}^3, T_0^e, \infty, \alpha_e, \beta_e, \epsilon_e, \gamma_e, \delta_e, \nu_e, \alpha_{\sigma}, \beta_{\sigma}, \epsilon_{\sigma}),$$

$$\bar{Q}_e^{zy} = -2,02 \cdot 10^{-7} B_T^{5/2} T_0^e{}^2 \sqrt{\frac{n_0^e (1-\mu)}{01}}$$

$$J_1(\gamma_e, \delta_e, \nu_e, \gamma_e, \delta_e, \nu_e, \alpha_e, \beta_e, \epsilon_e),$$

$$\bar{Q}_e^{ZH} = \bar{Q}_e^{ZHO} + \bar{Q}_e^{ZHF},$$

$$\bar{Q}_e^{ZHO} = 5,07 \cdot 10^{11} \frac{P_{inj}}{R \cdot 01^2} \Theta(t_H - t) - \bar{Q}_i^{ZHO},$$

$$\Theta(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$\bar{Q}_e^{ZHF} = 7,74 \cdot 10^{15} \frac{P_{inj}}{R \cdot 01^2} \frac{n_0^+ T_0^e{}^{3/2}}{E_0^{5/3} n_0^e}$$

$$\Theta(t_H - t) J_1^{-1}(\alpha_H, \beta_H, \epsilon_H, 1, 0, 1, 1, 0, 1)$$

$$R_{\frac{3}{2}, 0, 1} (0, \eta_0, E_0, \alpha_H, \beta_H, \varepsilon_H, \alpha_e, \beta_e, \varepsilon_e,$$

$$\alpha_f, \beta_f, \varepsilon_f, \gamma_e, \delta_e, \nu_e) - \bar{Q}_i^{\text{ZHF}},$$

$$\eta_0 = \frac{10(\alpha+2)^{2/3} T_0^e}{E_0}.$$

Ion energy balance:

$$(2.6) \quad \frac{3}{2} \frac{d}{dt} (T_0^i [n_0^f] J_1(\alpha_f, \beta_f, \varepsilon_f, \gamma_i, \delta_i, \nu_i, 1, 0, 1))$$

$$+ n_0^\alpha J_1(\alpha_\alpha, \beta_\alpha, \varepsilon_\alpha, \gamma_i, \delta_i, \nu_i, 1, 0, 1)$$

$$+ \sum_{\sigma=1}^5 n_0^\sigma J_1(\alpha_\sigma, \beta_\sigma, \varepsilon_\sigma, \gamma_i, \delta_i, \nu_i, 1, 0, 1)]$$

$$= \bar{Q}_i^0 + \bar{Q}_i^\Delta + \bar{Q}_i^N + \bar{Q}_i^{\text{ZH}}$$

$$\bar{Q}_i^0 = -\frac{2}{\alpha^2} \left[ \Gamma(\alpha_i + \frac{5}{2} T_i \Gamma) \right]_{01} + \frac{\Gamma}{n_f} \frac{\partial}{\partial t} (n_f T_i),$$

$$\bar{Q}_i^\Delta = -\bar{Q}_e^\Delta,$$

$$\bar{Q}_i^N = 2.07 \cdot 10^{-17} n_0^{f^2} \alpha(1-\alpha) \frac{e^{-\frac{19.94}{T_0^{i/3}}}}{T_0^{i^{2/3}}}$$

$$\left\{ 0, 0.137 (\alpha+2)^{2/3} T_0^e I_1(T_0^i, \alpha_f, \beta_f, \varepsilon_f, \gamma_e, \delta_e, \nu_e, \gamma_i, \delta_i, \nu_i) \right\}$$

$$- 0,000894 (\chi + 2) T_0^c \frac{3}{2} I_{\frac{3}{2}}(T_0^c, \alpha_f, \beta_f, \epsilon_f, \gamma_e, \delta_e, \nu_e, \gamma_i, \delta_i, \nu_i) \}$$

$$\bar{Q}_i^{ZH} = \bar{Q}_i^{ZHO} + \bar{Q}_i^{ZHF}$$

$$\bar{Q}_i^{ZHO} = 1,69 \cdot 10^{11} \frac{P_{inj} \eta_0 \theta(t_H - t)}{R \Omega^2 J_1(\alpha_H, \beta_H, \epsilon_H, 1, 0, 1, 1, 0, 1)}$$

$$R_{1,1,0}(\eta_0, 0, E_0, \alpha_H, \beta_H, \epsilon_H, 1, 0, 1, 1, 0, 1, \gamma_e, \delta_e, \nu_e),$$

$$\bar{Q}_i^{ZHF} = 2,58 \cdot 10^{15} \frac{P_{inj} \eta_0^{\alpha} \theta(t_H - t)}{R \Omega^2 J_1(\alpha_H, \beta_H, \epsilon_H, 1, 0, 1, 1, 0, 1)}$$

$$\frac{1 - \chi}{E_0^{5/3}} T_0^c \frac{3}{2} \frac{n_0^f}{n_0^c}$$

$$R_{5/2,1,1}(\eta_0^{\alpha}, \eta_0, E_0, \alpha_H, \beta_H, \epsilon_H, \alpha_e, \beta_e, \epsilon_e,$$

$$\alpha_f, \beta_f, \epsilon_f, \gamma_e, \delta_e, \nu_e) ,$$

$$\eta_0^{\alpha} = 5,65 \cdot 10^{-3} (\chi + 2)^{2/3} T_0^c .$$

Electron particle balance:

$$(2.7) \quad \frac{d}{dt} n_0^c J_1(\alpha_e, \beta_e, \epsilon_e, 1, 0, 1, 1, 0, 1) = \bar{S}_e^0 + \bar{S}_e^{ZH} + \bar{S}_e^F,$$

$$\bar{S}_e^0 = -\frac{2}{\omega^2} [\Gamma \Gamma]_{01} ,$$

$$\bar{S}_e^{ZH} = 3,16 \cdot 10^{20} \Theta(t_H - t) \frac{P_{inj}}{R \omega^2 E_0} ,$$

$$\bar{S}_e^F = \bar{N}_F .$$

Fuel particle balance:

$$(2.8) \frac{d}{dt} n_0^f \mathcal{J}_1(\alpha_f, \beta_f, \varepsilon_f, 1, 0, 1, 1, 0, 1) =$$

$$\bar{S}_f^0 + \bar{S}_f^N + \bar{S}_f^{ZH} + \bar{S}_f^F ,$$

$$\bar{S}_f^0 = -\frac{2}{\omega^2} [\Gamma \Gamma]_{01} ,$$

$$\bar{S}_f^N = -7,36 \cdot 10^{-12} \alpha(1-\alpha) n_0^f{}^2 \frac{e^{-\frac{19,94}{T_0^i{}^{1/3}}}}{T_0^i{}^{2/3}}$$

$$I_0(T_0^i, \alpha_f, \beta_f, \varepsilon_f, \gamma_e, \delta_e, \gamma_i, \delta_i, \gamma_i) ,$$

$$\bar{S}_f^{ZH} = 3,16 \cdot 10^{20} \frac{P_{inj}}{R \omega^2 E_0} \Theta(t_H - t) ,$$

$$\bar{S}_f^F = N_F .$$



$\alpha$ -particle balance:

$$(2.9) \quad \frac{d}{dt} n_{\alpha}^{\alpha} \cdot J_1(\alpha_{\alpha}, \beta_{\alpha}, \epsilon_{\alpha}, 1, 0, 1, 1, 0, 1)$$

$$= \bar{S}_{\alpha}^0 + \bar{S}_{\alpha}^N + \bar{S}_{\alpha}^{ZH},$$

$$\bar{S}_{\alpha}^0 = -\frac{2}{\alpha^2} [n_{\alpha} V_{\alpha}^{\alpha} t]_{01},$$

$$\bar{S}_{\alpha}^N = 3,68 \cdot 10^{-12} \alpha(1-\alpha) n_{\alpha}^{\alpha 2} \frac{e^{-\frac{19,94}{T_0^i 1/3}}}{T_0^i 2/3}$$

$$I_0(T_0^i, \alpha_f, \beta_f, \epsilon_f, \gamma_e, \delta_e, \gamma_e, \gamma_i, \delta_i, \gamma_i),$$

$$\bar{S}_{\alpha}^{ZH} = 6,83 \cdot 10^{20} \frac{P_{inj} (1-\alpha) n_{\alpha}^{\alpha} T_0^c \frac{3}{2} \Theta(t_H - t)}{R \alpha^2 E_0^{5/3} n_0^c J_1(\alpha_H, \beta_H, \epsilon_H, 1, 0, 1, 1, 0, 1)}$$

$$R_{3/2, 0, 1}(0, n_0, E_0, \alpha_H, \beta_H, \epsilon_H, \alpha_e, \beta_e, \epsilon_e, \alpha_f, \beta_f, \epsilon_f$$

$$\gamma_e, \delta_e, \gamma_e).$$

Impurity particle balance:

$$(2.10) \quad \frac{d}{dt} n_{\sigma}^{\sigma} \cdot J_1(\alpha_{\sigma}, \beta_{\sigma}, \epsilon_{\sigma}, 1, 0, 1, 1, 0, 1) = \bar{S}_{\sigma}^0,$$

$$\bar{S}_{\sigma}^0 = -\frac{2}{\alpha^2} [n_{\sigma} V_{\sigma}^{\sigma} t]_{01}, \quad \sigma = 1 \dots 5.$$

The first terms on the right-hand side of eqs. (2.5) to (2.10) describe the flux of energy and particles respectively through the plasma boundary and work that is done by the diffusing plasma against the pressure. These terms will be specified in the next section.

### 3. Neutral particles and the plasma boundary

Up to now nothing has been said about neutral particles. Particles (deuterium, tritium, wall material) leaving the wall are neutral owing to the low wall temperature and the low temperature in the region between wall and plasma [10]. On entering the plasma, the slow neutrals are either ionized, giving a source of electrons or ions, or undergo charge exchange reactions, giving a source of slow ions and fast neutrals some of which may fly to the wall.

Ions that reach the plasma boundary through transport (diffusion) are usually neutralized to slow neutrals before reaching the wall.

On hitting the wall, the slow and fast neutrals are reflected or absorbed. Furthermore they may cause sputtering of the wall material, giving a source of impurities.

These particles then undergo the reactions described above when they enter the plasma or hit the wall again.

In a large reactor plasma these processes happen in a boundary layer the diameter of which is small compared with the plasma radius. This layer must be taken into account when the terms in brackets in equations (2.5) to (2.10) are specified.

### 3.1 Electron and fuel ion flux through the boundary

We shall assume the refuelled particles to be deposited near the plasma centre. Furthermore the  $\alpha$ -heating may be regarded as an energy source localized at the centre owing to the strong temperature dependence of the D-T-reaction rate [11].

Energy and particles are transported to the plasma boundary by heat conduction and diffusion. At the plasma periphery additional transport is provided by energy exchange reactions with neutrals as described above.

The particle flux and heat flux of each species are in general a linear functional of the densities and temperatures of all plasma components. Plasma transport theory has developed the complete form of the diffusion and heat conduction tensor only in the classical and neoclassical domains. With respect to the uncertainties in the theory of anomalous transport we shall, as usual, make the most simple assumption

$$\Gamma = -D \frac{\partial n}{\partial r}$$

$$q = -n \chi \frac{\partial T}{\partial r}$$

for electrons and fuel ions in the domains of pseudoclassical, trapped electron and trapped ion transport, which will be relevant in future experiments and fusion plasmas. As to the specific form of the transport coefficients in these domains we shall use [12]

$$(3.2) \quad D_{PS} = 8,2 \cdot 10^{-11} \frac{Z n_f}{T_e^{1/2} B_T^2} \frac{R^2 q^2}{r^2}, \quad \chi_{ePS} = 3 D_{PS}$$

$$(3.3) \quad D_{TE} = 3,7 \cdot 10^{-9} \left(\frac{R}{r}\right)^{3/2} \frac{q^2 Z n_f}{B_T^2 T_e^{1/2}} \frac{n_e}{T_e}$$

$$\left(\frac{\partial n_e}{\partial t}\right)^{-1} \frac{\partial T_e}{\partial t}$$

$$\left(1 + 6,8 \cdot 10^{-34} \frac{Z^2 n_f^2 R^3 q^2}{\alpha^3 T_e^4} \left(\frac{1}{n_e} \frac{\partial n_e}{\partial t}\right)^{-2}\right)^{-1}$$

$$\chi_{eTE} = \frac{R}{r} D_{TE} \quad ,$$

$$(3.4) \quad D_{TI} = 5,4 \cdot 10^{24} \left(\frac{r}{R}\right)^{2,5} \frac{T_e^{3,5}}{Z n_f B_T^2} \left(\frac{1}{n_e} \frac{\partial n_e}{\partial t}\right)^2$$

$$\cdot \left(1 + \frac{T_e}{T_i}\right)^{-2} \quad , \quad \chi_{eTI} = \frac{R}{r} D_{TI}$$

As to the ion heat flux we take  $\chi_e \gg \chi_i$  , except for the trapped ion regime, where  $\chi_{eTI} = \chi_{iTI}$  .

The behaviour in the whole domain can be sufficiently approximated by

$$(3.6) \quad D = D_{PS} + D_{TE} + D_{TI} \quad , \quad \chi_e = 3 D_{PS} + \frac{R}{r} D_{TE} + D_{TI} \quad , \quad \chi_i = D_{TI}$$

All relevant source terms decrease rapidly towards the plasma boundary, apart from those including neutrals which are localized in the outer plasma periphery. Hence the particle flux (of electrons and fuel ions) is nearly constant, apart from the central and outer regions, and is

determined by diffusion and heat conduction. Therefore, we compute the electron and D-T particle and heat flux at some point  $r = \lambda a$  (typically  $\lambda = 0,7$ ) instead of  $r = a$  and do not consider the effect of charge exchange reactions on transport in the peripheral region. That is, we assume that the particle and heat fluxes through the plasma boundary are determined by anomalous transport in the region between the centre and periphery. We then get the following expressions for the fluxes of heat and particles through the surface in equs. (2.5), (2.6), (2.7) and (2.8) respectively

$$(3.7) \quad \frac{2}{a^2} \left[ \Gamma \left( \frac{0}{e} + \frac{5}{2} \Gamma T_e \right) \right]_{a_1} = \frac{2}{a^2} \lambda n_0^e T_0^e \left( \chi_e(\lambda a) \frac{\xi_n \xi_T}{l_T} + \frac{5}{2} \frac{\xi_T \xi_n}{l_n} D(\lambda a) \right),$$

$$(3.8) \quad \frac{2}{a^2} \left[ \Gamma \left( \frac{0}{i} + \frac{5}{2} \Gamma T_i \right) \right]_{a_1} = \frac{2}{a^2} \lambda n_0^e T_0^i \left( \chi_i(\lambda a) \frac{\xi_n \xi_T}{l_T} + \frac{5}{2} \frac{\xi_T \xi_n}{l_n} D(\lambda a) \right).$$

$$(3.9) \quad \frac{2}{a^2} [\Gamma T]_{a_1} = \frac{2 \lambda}{a^2} \xi_n \frac{n_0^e}{l_n} D(\lambda a)$$

Here we have used the abbreviations

$$\xi_n = \frac{n_f(\lambda a)}{n_0^f} \approx \frac{n_e(\lambda a)}{n_0^e}, \quad \xi_T = \frac{T_i(\lambda a)}{T_0^i} \approx \frac{T_e(\lambda a)}{T_0^e},$$

$$\frac{1}{a l_n} = \frac{1}{n_e(\lambda a)} \left| \frac{\partial n_e(\lambda a)}{\partial r} \right| \approx \frac{1}{n_i(\lambda a)} \left| \frac{\partial n_i(\lambda a)}{\partial r} \right|$$

( $l_T$  in analogous fashion)

(that is we assume that  $n_f$  and  $n_e$  as well as  $T_i$  and  $T_e$  are similar in shape)

and

$$D(\lambda a) = D_{PS}(\lambda a) + D_{TE}(\lambda a) + D_{TI}(\lambda a),$$

$$\chi_e(\lambda a) = 3 D_{PS}(\lambda a) + R/\lambda a D_{TE}(\lambda a) + D_{TI}(\lambda a),$$

$$\chi_i(\lambda a) = D_{TI}(\lambda a),$$

$$D_{PS}(\lambda a) = 8,2 \cdot 10^{-11} \frac{\bar{Z} n_o^+ R^2 a^2(a)}{T_o^{e 1/2} a^2 B_T^2} \frac{\xi_n}{\xi_T^{1/2} \lambda^2},$$

$$D_{TE} = 3,7 \cdot 10^{-9} \left(\frac{R}{a}\right)^{1,5} \frac{\bar{Z} n_o^+ a^2(a)}{B_T^2 T_o^{e 1/2}} \frac{\xi_n \ell_n}{\lambda^{1,5} \xi_T^{1/2} \ell_T}$$

$$\left(1 + 6,8 \cdot 10^{-34} \frac{\bar{Z}^2 n_o^{+2} R^3 a^2(a)}{a T_o^{e 4}} \frac{\ell_n^2 \xi_n^2}{\lambda^3 \xi_T^4}\right)^{-1}$$

$$D_{TI} = 5,4 \cdot 10^{24} \left(\frac{a}{R}\right)^{2,5} \frac{T_o^{e 3,5}}{\bar{Z} a^2 n_o^+ B_T^2} \left(1 + \frac{T_o^e}{T_o^i}\right)^{-2} \frac{\xi_T^{3,5} \lambda^{2,5}}{\ell_n^2 \xi_n}.$$

The assumption of similar shape of  $n_f$  and  $n_e$  is critical. From quasineutrality one has

$$n_e = n_f + 2 n_\alpha + \sum_{\epsilon} Z_{\epsilon} n_{\epsilon}$$

Now in cases where one has impurity accumulation it is still true that  $n_f \gg n_\alpha, n_{\epsilon}$ , but it is only roughly true that  $n_f \gg 2 n_\alpha, \sum n_{\epsilon}$ . In such a case, if the impurity



distribution differs from that of the fuel ions,  $n_e$  and  $n_f$  can no longer be similar in shape. On the other hand at the beginning of the burn, when no impurities are present,  $n_e \approx n_f$ . Hence the concept of constant profiles becomes questionable in the case of impurity accumulation if the impurities are much more or much less peaked than the fuel.

Within this limitation  $Z$  depends only weakly on  $r$  and hence we set  $Z(ra) = \bar{Z}$  in the expression for  $D(ra)$ .

Finally, we have to evaluate the terms  $\overline{\Gamma/n_e \frac{\partial}{\partial r} (n_e T_e)}$  and  $\overline{\Gamma/n_f \frac{\partial}{\partial r} (n_f T_i)}$  in eqs. (2.5) and (2.6). By definition we have

$$\overline{\Gamma/n_e \frac{\partial}{\partial r} (n_e T_e)} = \frac{2}{a^2} \int_0^{a'} dr r \frac{\Gamma}{n_e} \frac{\partial}{\partial r} (n_e T_e).$$

Keeping in mind what was said about the sources we have as a reasonable approximation  $r\Gamma \approx \text{const}$ . It thus follows that

$$\overline{\frac{\Gamma}{n_e} \frac{\partial}{\partial r} (n_e T_e)} \approx \frac{2}{a^2} [\Gamma r]_{a'} \int_0^{a'} dr \left( \frac{\partial T_e}{\partial r} + \frac{T_e}{n_e} \frac{\partial n_e}{\partial r} \right).$$

Within the parentheses on the right-hand side the contribution of the second term relative to the first one depends much on the shape of  $T_e$  and  $n_e$ . One has

$$\int_0^{a'} dr \left( \frac{\partial T_e}{\partial r} + \frac{T_e}{n_e} \frac{\partial n_e}{\partial r} \right) \approx$$

$$-T_e(0) + \begin{cases} 0 & \text{if } n_e \text{ is very flat compared with } T_e. \\ -T_e(0) \ln(n_e(0)/n_e(a)) & \text{if } T_e \text{ is very flat compared with } n_e. \end{cases}$$

It thus follows that

$$(3.10) \quad \overline{\Gamma_{n_e} \frac{\partial}{\partial r} (n_e T_e)} = -\frac{2}{a^2} [\Gamma_T]_{a1} T_0^e \phi, \\ 1 \leq \phi \leq \ln(n_e(0)/n_e(a)).$$

$\phi$  depends purely on the profiles of densities and temperature and hence will be, like all other profile parameters, prescribed as an input data, according to the situation.

Usually the term  $n T \frac{1}{r} \frac{\partial}{\partial r} (r \cdot V_r)$  in eq. (1.4) is not considered. However, we must emphasize that this term may even be considerably larger than the diffusion and heat conduction loss term for certain situations.

By analogy one gets

$$(3.11) \quad \overline{\Gamma_{n_f} \frac{\partial}{\partial r} (n_f T_i)} \approx -\frac{2}{a^2} [\Gamma_T]_{a1} T_0^i \phi.$$

(In the case of not too high impurity concentration  $n_f$  and  $n_e$  do not differ greatly so that the same value  $\phi$  can be taken.)

Combining equations (3.7), (3.8), (3.10) and (3.11), it now follows that

$$(3.12) \quad Q_e^0 = -\frac{2\lambda}{a^2} n_0^e \xi_n \left[ \mathcal{I}_e(\lambda a) T_0^e \frac{\xi_T}{\ell_T} + \frac{5}{2} \frac{\xi_T}{\ell_n} T_0^e D(\lambda a) + \frac{1}{\ell_n} T_0^e \phi D(\lambda a) \right],$$

$$(3.13) \quad Q_i^0 = -\frac{2}{\alpha^2} \lambda n_0^e \xi_n \left[ X_i(\lambda_{01}) T_0^i \frac{n_0^i}{n_0^e} \frac{\xi_T}{\ell_T} \right. \\ \left. + \frac{5}{2} \frac{\xi_T T_0^i}{\ell_n} D(\lambda_{01}) + \frac{1}{\ell_n} T_0^i \phi D(\lambda_{01}) \right].$$

### 3.2 $\alpha$ -particle and impurity ion fluxes

As to the  $\alpha$ -particles and impurity ions we shall assume that they diffuse inwards and stay in the plasma. The effect of the boundary layer is then characterized by a surface particles source for each type of impurity.

Simple models of the plasma wall region which will be adopted here have been proposed by several authors [10], [13], [14], [15]. It is a common feature of these models that always

$$(3.15) \quad S_\sigma^0 = -\frac{2}{\alpha^2} [\Gamma n_\sigma V_\sigma^t] = + \gamma_\sigma \frac{2}{\alpha^2} [\Gamma t]_{01} \\ = - \gamma_\sigma S_f^0$$

with some constant coefficients  $\gamma_\sigma$ . We shall specify the  $\gamma_\sigma$  in the respective applications.

As to the  $\alpha$ -particles we shall assume inward diffusion too. Since there is no source of  $\alpha$ 's from the boundary it holds that

$$(3.16) \quad S_\alpha^0 = -\frac{2}{\alpha^2} [n_\alpha V_t^t]_{01} = 0.$$

In specifying the fluxes of energy and particles, we have now completed the equations (2.5) to (2.10) to a closed set of ordinary differential equations for the time dependent quantities  $T_0^e(t)$ ,  $T_0^i(t)$ ,  $n_0^e(t)$ ,  $n_0^i(t)$ ,  $n_0^\alpha(t)$ ,  $n_0^\sigma(t)$ .

#### 4. Numerical treatment and stability constraints

The above derived system of 10 ordinary differential equations for the time dependent quantities  $T_0^e(t)$ ,  $T_0^i(t)$ ,  $n_0^e(t)$ ,  $n_0^i(t)$ ,  $n_0^\alpha(t)$ ,  $n_0^1(t)$ .... $n_0^5(t)$  is solved numerically.

Four types of input data must be considered:

- 1) The system parameters,  $R, a, B_T, \dots$ , which characterize the plasma under consideration.
- 2) Profile indices such as  $\alpha_e, \beta_e, \epsilon_e, \gamma_e, \delta_e, \nu_e \dots$ . At the beginning of each run all auxiliary functions that depend only on profile indices are calculated. Those which depend on time dependent quantities too (such as  $I_x(T_0^i, \dots)$ ) are tabulated.
- 3) The initial values of the time dependent quantities, that is  $T_0^e(0)$ ,  $T_0^i(0)$ ,  $n_0^e(0)$ ,  $n_0^1(0)$ .... $n_0^5(0)$ .

- 4) The time dependence of arbitrary source terms.

In the above equations the heating power  $P_{inj}$  and the refuelling rate  $\bar{N}_F$  can be chosen arbitrarily.

As to additional heating three modes of operation are intended:

- a) Heating with constant power for some time  $t_H$ .
- b) Heating with constant power until  $T_0^i$  reaches a given value.
- c) Heating with constant power until  $\beta_p$  (for definition of  $\beta_p$  see eq. (4.1)) reaches a given values.

As to refuelling the following modes are intended:

a)  $\bar{N}_F = - (\bar{S}_f^O + \bar{S}_f^{ZH} + \bar{S}_f^N)$  so that  $\frac{dn_o^f}{dt} = 0$  and  $n_o^f$  remains constant.

b)  $\bar{N}_F = \left\{ \begin{array}{l} -0,8 (\bar{S}_f^O + \bar{S}_f^{ZH} + \bar{S}_f^N) \text{ if } \beta_p - \beta_o \geq 0, \\ -(\bar{S}_f^O + \bar{S}_f^{ZH} + \bar{S}_f^N) \text{ if } \beta_p - \beta_o < 0 \text{ and } \bar{Q}_i < 0, \\ -(\bar{S}_f^O + \bar{S}_f^{ZH} + \bar{S}_f^N) + 2 \cdot 10^8 \frac{\bar{Q}_i}{T_o} \\ \frac{J_1(\alpha_i, \beta_i, \epsilon_i, 1, 0, 1, 1, 0, 1)}{J_1(\alpha_i, \beta_i, \epsilon_i, \gamma_i, \delta_i, \nu_i, 1, 0, 1)} \text{ if } \beta_p - \beta_o < 0 \\ \text{and } \bar{Q}_i \geq 0. \end{array} \right.$

$$\bar{Q}_i = \bar{Q}_i^O + \bar{Q}_i^A + \bar{Q}_i^N + \bar{Q}_i^{ZH}$$

This mode controls the fuelling rate in such a way that the value of the poloidal beta (for definition see eq. (4.1)) is kept fixed near some values  $\beta_o$  as long as possible during a burn phase.

These modes are the most important ones but obviously others can easily be provided.

In solving the above derived equations two constraints are relevant which are given by tokamak stability and equilibrium theory respectively:

1)  $\beta_p \leq$  some given value.

Here  $\beta_p$  is the poloidal beta defined as

$$\begin{aligned}
 (4.1) \quad \beta_p &= \frac{8\pi}{B_p^2(0)} V^{-1} \int d\tau (n_e T_e + n_i T_i) \\
 &= 4,026 \cdot 10^{-14} \frac{A^2}{B_T^2} q^2(0) \left\{ n_0^e T_0^e \right. \\
 &\quad J_1(\alpha_e, \beta_e, \epsilon_e, \gamma_e, \delta_e, \nu_e, 1, 0, 1) + T_0^i [n_0^i J_1(\alpha_f, \beta_f, \epsilon_f, \\
 &\quad \gamma_i, \delta_i, \nu_i, 1, 0, 1) + n_0^a J_1(\alpha_a, \beta_a, \epsilon_a, \gamma_i, \delta_i, \nu_i, 1, 0, 1) \\
 &\quad \left. + \sum_{\sigma=1}^5 n_0^\sigma J_1(\alpha_\sigma, \beta_\sigma, \epsilon_\sigma, \gamma_i, \delta_i, \nu_i, 1, 0, 1) \right\}
 \end{aligned}$$

where  $A = R/a$  is the aspect ratio and  $q(r) = r B_T / R B_p$  is the safety factor.

$\beta_p$  is computed at each time step and the computation terminates if  $\beta_p$  exceeds the limit value.

$$2) \quad q(r) \geq 1 \text{ for } 0 \leq r \leq a$$

In our calculations only  $q(a)$  is needed.  $q(a)$  is connected with the plasma current  $I$  through Ampere's law, which reads in our coordinates (stationary case)

$$(4.2) \quad \frac{1}{r} \frac{\partial}{\partial r} (r B_p) = \frac{4\pi}{c} j_T$$

It thus follows that

$$B_p(0) = \frac{4\pi}{c} \frac{1}{0} \int_0^a dr \cdot r \cdot j_T \quad \text{by integration.}$$

With  $I = 2\pi \int_0^a dr \cdot r \cdot j_T$  it follows that

$$(4.3) \quad B_p(0) = \frac{2}{c} \frac{I}{0} \quad \text{and} \quad q(0) = \frac{a^2 B_T c}{2 R I}$$

or

$$(4.4) \quad B_p(0) = 2,00 \cdot 10^2 \frac{I}{0} \quad \text{and} \quad q(0) = 5,00 \cdot 10^{-3} \frac{0^2 B_T}{R I}$$

To take the condition  $q(r) \geq 1$  into account,  $q(a)$  is computed by solving eq. (4.2) so that  $q(r)$  takes its minimal value  $q(0) = 1$ . This has been described elsewhere [16]. It is found that the corresponding  $q(a)$  is given by

$$(4.5) \quad \frac{1}{q(a)} = \int_0^1 dz \left[ (1 - \gamma_e) (1 - Z^{\alpha_e})^{\beta_e} + \gamma_e \right]^{3/2} \\ \frac{[(1 - \xi_e) (1 - Z^{\alpha_e})^{\beta_e} + \xi_e]}{[(1 - \xi_f) (1 - Z^{\alpha_f})^{\beta_f} + \xi_f]} =$$

$$J_3(\alpha_e, \beta_e, \xi_e, \gamma_e, \delta_e, \gamma_e, \alpha_f, \beta_f, \xi_f)$$

The corresponding  $I$  is then computed from eq. (4.3). This procedure is reasonable for stationary situations. There is some experimental evidence for a flattening of the temperature profile within the central region giving  $q(r) = 1$  in this whole region. To cope with this situation, where the simple considerations of [16] might not apply,  $q(a)$  and  $I$  can be prescribed as arbitrary input data too.

These two modes are adequate for the  $I = \text{const}$  case. In the more general case when  $I$  is time dependent as is the case for instance during the current rise phase the coupled system of equations (2.5) to (2.10) and the transformer



equations that describe the ohmic heating transformer must be solved simultaneously. Instead of  $I$  the primary voltage and current are then given and  $I(t)$  is computed self-consistently. We shall treat this procedure in detail in a later study.

Apart from the densities and temperatures all other interesting quantities which are functions of them are computed such as all source terms, wall loadings etc. For all these quantities the integral  $\int_0^t \dots dt'$  can be computed if desired. All computed quantities can be plotted in any combination.

APPENDIX 1. Ohmic heating

In this appendix we compute  $Q_e^j = \eta_{||} j_{||}^2 + \eta_{\perp} j_{\perp}^2$ .

For convenience we consider the cylinder approximation with usual cylinder coordinates  $(r, \vartheta, z)$  and assume  $B_p \ll B_T$  ( $B_p$  = poloidal field,  $B_T$  = toroidal field), so that  $B_T \approx B$ . We then get

$$(A1.1) \quad j_{||}^2 = \left[ \frac{\vec{B}}{B^2} (\vec{B} \cdot \vec{j}) \right]^2 = \frac{(B_p^2 + B_T^2)}{B^4}$$

$$[ B_p \cdot j_p + B_T \cdot j_T ]^2$$

$$= \frac{1}{B^2} [ B_p \cdot j_p + B_T \cdot j_T ]^2$$

The system under consideration is assumed to be in MHD equilibrium. Hence the force balance equation  $c \nabla p = \vec{j} \times \vec{B}$  must be fulfilled, which in our coordinates reads

$$(A1.2) \quad c \frac{\partial p}{\partial r} = j_p \cdot B_T - j_T \cdot B_p$$

Eliminating  $j_p$  from eq. (A1.1) by means of eq. (A1.2) we get

$$j_{||}^2 = \frac{1}{B^2} \left[ j_T B_T + j_T \frac{B_p^2}{B_T} + c \frac{B_p}{B_T} \frac{\partial p}{\partial r} \right]^2$$

$$\approx \frac{1}{B^2} \left[ j_T B_T + c \frac{B_p}{B_T} \frac{\partial p}{\partial r} \right]^2$$

The last term in the bracket can be neglected compared with the first one if  $\beta \ll 1$ . It thus follows that

$$(A1.3) \quad j_{\parallel}^2 \approx j_{\perp}^2.$$

Next we consider the term  $\eta_{\perp} \cdot j_{\perp}^2$ . From the force balance equation it follows that

$$\vec{j}_{\perp} = - \frac{c \nabla p \times \vec{B}}{B^2}.$$

One thus has

$$(A1.4) \quad \eta_{\perp} \cdot \vec{j}_{\perp}^2 = \eta_{\perp} \cdot c^2 \frac{(\nabla p \times \vec{B})^2}{B^4} \\ = \eta_{\perp} c^2 \frac{(\nabla p)^2}{B^2} = - \frac{\Gamma_c}{n_c} \frac{\partial p}{\partial r},$$

where  $\Gamma_c/n_c = -\eta_{\perp} c^2 \frac{\partial p}{\partial r} \frac{1}{B^2}$  is the classical diffusion velocity (in this geometry). Combining equations (A1.3) and (A1.4) it now follows

$$(A1.5) \quad Q_e^J \approx \eta_{\parallel} j_{\perp}^2 - \frac{\Gamma_c}{n_c} \frac{\partial p}{\partial r}.$$

APPENDIX 2. Thermalization of fast ions

In the plasma considered in this paper fast  $\alpha$ -particles and fuel ions are produced by fusion reactions and ionization of fast beam neutrals. The respective distribution functions are then no longer Maxwellian and the thermalization should be treated by a kinetic equation. A much simpler method is to treat the fast ions as a test particle slowing down in a time constant, homogeneous and Maxwellian background plasma. This procedure is reasonable if the plasma does not change much during the slowing down time and over a slowing down length. The theory of a slowing down test particle has been rederived for a multicomponent plasma by W.A. Houlberg [17].

If the reasonable assumptions  $V \ll v_{th}^e$  and  $V \gg v_{th}^j$ ,  $j = f, \alpha, \delta$  ( $V$  the (ensemble) average test particle velocity,  $v_{th}$  the thermal velocities of the respective particles) are made the mean test particle energy  $E$  changes according to

$$(A2.1) \quad \frac{dE}{dt} E^{3/2} = \frac{1}{\tau} (E_c^{3/2} + E^{3/2})$$

$$\frac{1}{\tau} = 1,5 \cdot 10^{-13} \frac{Z_{te}^2}{M_{te}} \frac{n_e \ln \Lambda_e}{T_e^{3/2}}$$

$$E_c = 14,8 T_e M_{te} \left[ \frac{1}{n_e \ln \Lambda_e} \right]$$

$$\left[ \sum_{j=f,\alpha,\delta} \frac{Z_j^2 n_j \ln \Lambda_j}{M_j} \right]^{2/3}$$

( $t_e$  indicates the test particle).

$E_c$  is the test particle energy at which the rate of energy loss to the electrons is equal to the rate of energy loss to all ions.

Equation (A2.1) can immediately be solved:

$$(A2.2) \quad E^{3/2}(t) = E_0^{3/2} e^{-\frac{t}{\tau}} - E_c^{3/2} (1 - e^{-\frac{t}{\tau}})$$

$E_0$  is the initial energy of the test particle.

From eq. (A2.2) the slowing down time at which  $E = T_e$  is derived as

$$(A2.3) \quad \tau_s = \tau \ln \left( \frac{E_c^{3/2} + E_0^{3/2}}{T_e^{3/2} + E_c^{3/2}} \right) \\ \approx \tau \ln \left( 1 + \left( \frac{E_0}{E_c} \right)^{3/2} \right) \quad (\text{since } T_e \ll E_c)$$

In deriving the energy balance for electrons and ions, we need the fraction  $U_e$  and  $U_i$  of the test particle energy that goes to electrons and ions respectively. From [17] we get

$$(A2.4) \quad U_i = \frac{\eta}{3} \left[ \ln \left( \frac{1 - \sqrt{\eta} + \eta}{1 + 2\sqrt{\eta} + \eta} \right) + 2\sqrt{3} \cdot \operatorname{arctg} \left( \frac{2 - \sqrt{\eta}}{\sqrt{3} \sqrt{\eta}} \right) + \frac{\pi}{\sqrt{3}} \right], \quad \eta = \frac{E_c}{E_0}$$

$$(A2.5) \quad U_e = 1 - U_i$$

In deriving eq. (A2.4)  $T_e \ll E_c$  and  $T_e \ll E_0$  are used.

With equation (A2.1) one gets

$$(A2.6) \quad \eta = \frac{E_c}{E_0} = \frac{14,8 T_e M_{te}}{E_0} \left[ \frac{1}{n_e \ln \Delta_c} \left( \frac{n_f \alpha \ln \Delta_{ol}}{2} + \frac{n_f (1-\alpha) \ln \Delta_+}{3} + 4 \frac{n_\alpha \ln \Delta_\alpha}{4} + \sum_{\sigma} \frac{Z_\sigma^2 n_\sigma \ln \Delta_\sigma}{M_\sigma} \right) \right]^{2/3}$$

In the range of interest one has approximately

$$\ln \Delta_\sigma \approx \ln \Delta_\alpha \approx \ln \Delta_{ol} \approx \ln \Delta_+ \approx 20$$

and  $\ln \Delta_c \approx 17$ . Furthermore for  $\sigma = 1 \dots 5$  one has  $Z_\sigma/M_\sigma \approx \frac{1}{2}$  or smaller. With these approximations we get

$$\eta = 5,0 \frac{M_{te} T_e}{E_0} \left[ \frac{n_f}{n_e} \left( 2 + \alpha + 3 \frac{n_e - n_f}{n_f} \right) \right]^{2/3}$$

In typical applications  $n_f/n_e$  is not smaller than 0,3. In this range the bracket deviates only slightly from  $2 + \alpha$ . Hence we take the following final expression for  $\eta$

$$(A2.7) \quad \eta = 5,0 \frac{M_{te} (2 + \alpha)^{2/3} T_e}{E_0}$$

Equations (A2.4), (A2.5) and (A2.7) must be applied to fast beam ions and fusion  $\alpha$ -particles.

We shall only consider deuterium beams and therefore must set  $M_{te} = 2$  in eq. (A1.7) in this case.

In the case of fusion  $\alpha$ 's  $M_t = 4$  and  $E_0$  has the fixed value  $E_\alpha = 3540$  keV. Owing to the smallness of

$$(A2.8) \quad \eta_\alpha = 20,0 \frac{(\alpha + 2)^{2/3} T_e}{3540} = 5,65 \cdot 10^{-3} (\alpha + 2)^{2/3} T_e$$

it is sufficient to expand the bracket in eq. (A2.4) to first order in  $\eta_\alpha^{1/2}$ .

$$(A2.9) \quad U_i^* = 0,0137 (\chi+2)^{2/3} T_e - 0,000849 (\chi+2) T_e^{3/2}$$

The fast deuterium ions may undergo fusion reactions to an appreciable extent when slowing down. To compute the resulting energy production rate, one has to determine the probability  $V_\alpha$  that a deuterium ion undergoes a fusion reaction. Obviously, it is valid that

$$(A2.10) \quad V_\alpha = \int_0^{\tau_s} n_t \langle \sigma v \rangle (E(t)) dt$$

From eq. (A2.1) one gets  $dt = -\frac{3}{2} \tau \frac{E^{1/2} dE}{E^{3/2} + E_c^{3/2}}$  so that

$$V_\alpha = -\frac{3}{2} \tau n_t \int_{E_0}^{T_e} \frac{\langle \sigma v \rangle (E) E^{1/2} dE}{E^{3/2} + E_c^{3/2}}$$

With  $E_c = 20,0 \cdot (\chi+2)^{2/3} T_e$ ,  $n_t = (1-\chi) n_f$ ,

$\tau$  according to eq. (A2.1) and the approximation

$$\langle \sigma v \rangle = 3,68 \cdot 10^{-12} \frac{e^{-\frac{19,94}{E^{1/3}}}}{E^{2/3}}$$

we get

$$(A2.11) \quad V_\alpha = 4,33 \frac{(1-\chi) n_f T_e^{1/2}}{n_e} \int_{T_e}^{E_0} dE \frac{e^{-\frac{19,94}{E^{1/3}}}}{E^{1/6} (E^{3/2} + (20(\chi+2)^{2/3} T_e)^{3/2})}$$



Since the integrand of eq. (A2.11) is very small in the range  $E < T_e$ , one can integrate from 0 to  $E_0$ . Finally, with  $X = E/E_0$ , one gets

$$V_\alpha = 4,33 \frac{(1-\alpha) n_f T_e^{3/2}}{n_e E_0^{2/3}} \int_0^1 dx \frac{e^{-\frac{19,94}{E_0^{1/3} X^{1/3}}}}{X^{1/6} (X^{3/2} + \eta^{3/2})}$$

where  $\eta$  is as given by eq. (A2.7).

APPENDIX 3. Volume averaged ohmic heating power density

From eq. (1.14) we have

$$(A3.1) \quad Q_e^J = 3,63 \cdot 10^{-18} j_T^2 T_e^{-\frac{3}{2}} Z_{\text{eff}} \quad ,$$

$$Z_{\text{eff}} = \frac{n_f}{n_e} + 4 \frac{n_\alpha}{n_e} + \sum_{\sigma} Z_{\sigma}^2 \frac{n_{\sigma}}{n_e} \quad .$$

We shall assume the toroidal E field to be independent of r as is valid in the stationary case. One then has

$$(A3.2) \quad E = \frac{j_T}{\eta_{||}} \sim j_T T^{-\frac{3}{2}} Z_{\text{eff}} \quad \text{independently of } r.$$

It thus follows that

$$(A3.3) \quad j_T T_e^{-\frac{3}{2}} Z_{\text{eff}} = j_{T_0} T_0^{-\frac{3}{2}} Z_{\text{eff}0}$$

or  $j_T = j_{T_0} Z_{\text{eff}0} T_0^{-\frac{3}{2}} T_e^{\frac{3}{2}} Z_{\text{eff}}^{-1}$  ( $_0$  indicates the value at  $r=0$  as usual).

From eqs. (A3.1) and (A3.3) we eliminate  $j_T$ :

$$(A3.4) \quad Q_e^J = 3,63 \cdot 10^{-18} j_{T_0}^2 Z_{\text{eff}0}^2 T_0^{-3} T_e^{\frac{3}{2}} Z_{\text{eff}}^{-1} \quad .$$

By definition one has

$$(A3.5) \quad I = 2\pi \int_0^r \omega r \tau \, j_T \quad , \text{ where } I \text{ is the total current,}$$

$$= \pi \omega^2 \bar{j}_T^2 \quad , \text{ where } \bar{j}_T \text{ is defined as in}$$

eq. (2.1),

$$= \pi \omega^2 j_{T_0} Z_{eff_0} T_0^{e^{-\frac{3}{2}}} \overline{T_e^{3/2} Z_{eff}^{-1}}$$

$$\text{or } I = 1,047 \cdot 10^{-15} \omega^2 j_{T_0} Z_{eff_0} T_0^{e^{-\frac{3}{2}}} \overline{T_e^{3/2} Z_{eff}^{-1}}$$

in our units. On the other hand, from eq. (A3.4) one gets

$$(A3.6) \quad \bar{Q}_e^j = 3,63 \cdot 10^{-18} j_{T_0}^2 Z_{eff_0}^2 T_0^{e^{-3}} \overline{T_e^{3/2} Z_{eff}^{-1}}$$

From eqs. (A3.6) and (A3.5) we eliminate  $j_{T_0}$ :

$$(A3.7) \quad \bar{Q}_e^j = 3,31 \cdot 10^{12} \frac{I^2}{\omega^4} \frac{1}{\overline{T_e^{3/2} Z_{eff}^{-1}}}$$

It holds that

$$\overline{T_e^{3/2} Z_{\text{eff}}^{-1}} = \overline{T_e^{3/2} n_e n_f^{-1} Z^{-1}}$$

$$= \int_0^1 d\xi \frac{T_0^e n_0^e}{n_0^f} \frac{[(1-\gamma_e)(1-\xi\gamma_e)^{\delta_e} + \gamma_e]^{3/2}}{[(1-\varepsilon_f)(1-\xi\alpha_f)^{\beta_f} + \varepsilon_f] Z}$$

$$\cdot [(1-\varepsilon_e)(1-\xi\alpha_e)^{\beta_e} + \varepsilon_e]$$

$$\approx \frac{T_0^e n_0^e}{n_0^f \overline{Z}} \int_0^1 d\xi \frac{[(1-\gamma_e)(1-\xi\gamma_e)^{\delta_e} + \gamma_e]^{3/2}}{[(1-\varepsilon_f)(1-\xi\alpha_f)^{\beta_f} + \varepsilon_f]}$$

$$[(1-\varepsilon_e)(1-\xi\alpha_e)^{\beta_e} + \varepsilon_e]$$

$$= \frac{T_0^e n_0^e}{n_0^f \overline{Z}} J_3(\alpha_e, \beta_e, \varepsilon_e, \gamma_e, \delta_e, \gamma_e, \alpha_f, \beta_f, \varepsilon_f),$$

$$\overline{Z} = 1 + 4 \frac{n_0^{\alpha}}{n_0^f} J_3(\alpha_{\alpha}, \beta_{\alpha}, \varepsilon_{\alpha}, 1, 0, 1, \alpha_f, \beta_f, \varepsilon_f)$$

$$+ \sum_{\sigma=1}^5 Z_{\sigma}^2 \frac{n_0^{\sigma}}{n_0^f} J_3(\alpha_{\sigma}, \beta_{\sigma}, \varepsilon_{\sigma}, 1, 0, 1, \alpha_f, \beta_f, \varepsilon_f)$$

( $J_3$  is as defined in section 2.2, and  $Z$  is assumed to be only

weakly dependent on  $r$ , which is in agreement with our previous assumptions).

It follows that

$$(A3.8) \quad Q_e^J = 3,31 \cdot 10^{12} \frac{I^2 N_0^f \bar{Z}}{0,4 N_0^e T_0^{e/2}}$$

$$J_3^{-1} (\alpha_e, \beta_e, \epsilon_e, \gamma_e, \delta_e, \gamma_e, \alpha_f, \beta_f, \epsilon_f).$$

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