On the Use of Langmuir Probes
in a Laser-produced Plasma

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Abstract

It was found that in a plasma produced by irradiating a solid target with a laser beam Langmuir probes, besides their application for time-of-flight measurements, can be used to determine both the electron temperature T_e and the number density n_e in the asymptotic expansion phase. In the immediate vicinity of the target successful application of the probe is doubtful. In the intermediate region, although the determination of T_e might be difficult, the electron number density n_e can still be determined from the ion saturation current.

Based on a repeated guess of the plasma potential, an iterative method was found for the determination of T_e to a better accuracy in the asymptotic expansion phase.

Finally, a brief discussion of the use of probes in a magnetized plasma is given.

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1. Introduction

Owing to the simplicity of construction, ease of handling and low cost, Langmuir (or electrostatic) probes [1] have long been used as fundamental plasma diagnostic tools. In their original application, Langmuir used them in gas-discharge tubes where the plasma was stationary and of low density and temperature. Typical electron number densities n_e and temperatures were below 10^{11} and 3 eV respectively, so that both collisional and emissional effects can be neglected.

The plasma produced by irradiating a solid target with a laser beam differs in many respects from such a stationary collisionless low-temperature plasma. In the immediate vicinity of the target (e.g. in a region of a few mm) the plasma is most likely very dense and hot, so that successfull application of the Langmuir probe is doubtful. As one proceeds further away from the target, rapid conversion of the thermal energy of the plasma into directed kinetic energy of the ions takes place through the expansion process, the plasma soon reaches a collisionless state but possibly is still reasonably hot. Apart from the obvious use for time-of-flight measurements, Langmuir probes might be used to good advantage for determining the local properties of the plasma and for supplementing other diagnostic techniques, e.g. optical interferometry.

As a typical example, when a plane stainless-steel target is irradiated with a radiation power density $\phi \approx 10^{10}$ W cm^{-2} at the focal spot at a distance of 10 cm from the target, the electron temperature of the plasma is $T_e \approx 10$ eV, while the expansion speed is $u \approx 10^7$ cm/sec. The corresponding kinetic energy due to directed motion of the ions is in the keV range. Secondary emission due to thermal electrons might be negligible, but possible emissional effects due to the ion bombardment for a negatively biased probe should be borne in mind.

With due caution regarding the possible presence of secondary electron emission and with overheating of the probe taken into consideration by a suitable design of the probe (2), we may identify the application of a Langmuir probe in a laser-target-produced plasma into three regimes:

i) In the immediate vicinity of the target (e.g. a few mm), it might be impossible to use the probe

ii) At a distance greater than 10 cm, it should be possible to measure both the electron temperature T_e and the number density n_e with the probe

iii) In the intermediate region, when both n_e and T_e are high, it might be difficult to measure T_e directly, but as long as the directed flow velocity v is greater than the ion thermal velocity v_i, it should be possible to determine n_e from the ion saturation part of the probe characteristics (3).

While the above classification is based mainly on a power density of 10^{10} W cm^{-2} irradiating a plane metallic surface, the presence of the above three regimes should hold in general provided the demarcation distance is adjusted according to the laser power density, the material and the geometry of the target.

In the following, we shall present more detailed discussions on subjects regardint the design of the probe, the associated circuit, and the interpretation of the current-voltage characteristic curve. Theoretical considerations will be limited to those subjects related directly to their applications. We should like to emphasize here that for a correct interpretation of probe data a good understanding of probe theory is essential. As these subjects have appeared in abundance in various journals and specially written monographs [4, 5, 6, 7] it would be advisable for readers to refer to them directly.

Finally, it should be mentioned here that although most of the remarks made in this report were drawn mainly from the author's limited experience of disc type probes, it is easy to see that most of them should be equally applicable to cylindrical probes [8].

2. Design consideration of the probe and the accompanying circuit

The probe proper: One recalls that when the probe is negatively biased to attract ions and when the collected current is limited by the orbital motion of the ions, the saturation current is independent of the applied voltage for a plane (disc-type) probe but varies with the applied voltage for a cylindrical probe [9], thus, in principle, it should be simpler to use the plane probe. However, as recent experimental work by other investigators show [10, 11], as long as the ratio $\frac{1}{2} M_i v^2/kT_e$, i.e. the kinetic energy of the ions corresponding to the expansion velocity v with respect to the electron thermal energy kT_e, is greater than unity, the ion saturation current for a cylindrical probe is also voltage independent. In practice, it appears that either a cylindrical probe or a plane probe can be used.

In order to disturb the plasma as little as possible, it would be advisable to use plane probes at a great distance from the target since they can be conveniently mounted flush with the wall of the vacuum vessel. In regions closes to the target, especially when one is interested in studying the variation of the plasma state in space, cylindrical probes would be preferable.

For the same reasoning, whenever feasible, it is advisable to use smaller probes. Two precautions, however, should be made: firstly, the probe should be large enough without impairing its sensitivity, i.e. the signal-to-noise ratio as detected by the probe should be high enough; secondly, undesirable effects, such as the edge effect for a plane probe (12) and the end effect (13) for a cylindrical probe, should either be absent or be kept to a minimum.

As an example, if it is assumed that at a distance of 10 cm from the target for a radiation power density of $\phi \approx 10^{11}$ W/cm^2, the electron temperature T_e and number density n_e of the plasma produced are 25 eV and 10^{11} respectively, and the corresponding Debye sheath λ_D is around 35 μ. It would not be advisable to make a cylindrical probe with a radius less than 50 μ.

A word might be said concerning the probe material. So far molybydum has been used in our experiments and has given reasonably good results. However, as the ion attracting part of the characteristic shows a positive slope towards the voltage axis in several of our experiments when high ion current is drawn by the probe, the possibility of secondary electron emission caused by high-energy ions ($\gtrsim$ 1 keV) cannot be ruled out. As the work function of platinum is higher than that of molybydum (6 eV as compared with 4 eV), the secondary electron emission effect could be kept to a minimum if platinum is chosen as the probe material.

The circuit: The circuit used by most investigators for probe measurements in laser-produced plasmas can be represented by the diagram shown in Fig.1.

Since the capacitance C_s of the probe is usually around 100 pf for a load capacitance C in the μf range, the time response for the probe to form a sheath will be given by C_sR. For $R = 50\,\Omega$, the time response of the probe will be about 5 ns.

During the time when a current is flowing through the probe, if we assume that within 0.1 μs the plasma parameter does not alter rapidly, the voltage of the probe will change from its initial value V(o) to

$$V(t = 0.1\,\mu s) = V(0) - \int_0^t \frac{I\,dt}{C} - IR \tag{1}$$

As the voltage drop across the capacitance for $C \approx 1\,\mu$f and $t \approx 0.1\,\mu$s is much smaller than the drop across the resistive load, we have

$$\frac{V(0) - V(t)}{V(0)} \cong \frac{IR}{V(0)} . \tag{2}$$

For a current $I \approx 50$ mA (as when the probe is operating in the ion attracting part of the characteristic), a load resistance $R = 50\,\Omega$ and a bias voltage V(o) = 50 V, the change is about 5 per cent.

From the above consideration, it appears that as far as the time response and the constancy of the voltage of the probe are concerned, a small load resistance should be better. However, two points must be borne in mind:

the reflection of the signal in the cable, and the signal-to-noise ratio, especially when little current is drawn by the probe, as in the case when it is operating near the floating potential.

From our past experience, we find the following circuit parameters to be rather satisfactory

$$C \geq 2\ \mu f,$$
$$R = 50\ \Omega,$$
$$R_b \approx 10\ k\Omega\ .$$

3. Interpretation of the current-voltage characteristics

In order to utilize fully the probe for determining the plasma parameters T_e and n_e, a current-voltage characteristic (or I-V curve) must be drawn.

Fig.2 shows such an example obtained from a 1 mm dia. disc probe placed at a distance of 15 cm from a stainless-steel target. The target was placed at the focal region of a 2 J. - 30 ns Nd-glass laser. It should be mentioned that the curve was deduced from data corresponding to the maximum currents of the main plasma recorded by the oscillograms. It was also obtained under the further assumption that the experimental conditions, e.g. focal spot area, laser power density,etc.,were reasonably constant from shot to shot.

Obviously, a fast swept device capable of drawing the I-V curve automatically from - 80 to + 80 V in a time interval of 0.1 μsec will be much more desirable.

The characteristic curve in Fig.2 is seen to differ from the ideal one expected from a plane probe in two respects: firstly, the ion attracting part of the curve is not independent of the voltage but has a positive slope; secondly, no appreciable bending and flattening of the curve seems to be present even at a positive bias voltage as high as 60 eV, i.e. the value of the plasma (or space) potential V_p is very much uncertain.

As it is well known that, in order to determine the electron temperature T_e, it is necessary to make a semi-logarithm plot of the electron current I_e drawn by the probe against the bias voltage V, i.e. an ion saturation current I_{is} extrapolated to the plasma potential V_p must be substracted from the observed current drawn by the probe. A correct determination of the electron temperature therefore is much effected by this uncertainty of the plasma potential V_p and the ion saturation current I_{is}. Clearly, this complication does not arise if the probe dimension is suitably chosen so that the ion attraction part of the I-V curve is independent of the bias voltage used. This situation, however, cannot always be attained in practice, especially when the laser power density is high or when the probe is placed nearer the target. With this view in mind, in the following we shall present an iterative method which enables us to determine T_e more accurately.

3a. Iterative method for determining T_e when the ion saturation current I_{is} is uncertain

The method is based on the assumption that in the later expansion phase of a laser-produced plasma usually the following condition is satisfied:

$$v > \left(\frac{8kT_e}{\pi M_i} \right)^{1/2} > \left(\frac{8kT_i}{\pi M_i} \right)^{1/2} , \qquad (3)$$

where v is the expansion speed, M_i is the average ion mass corresponding to the target material, and the rest of the symbols are self-explanatory. Consequently, as shown in the Appendix, the ion saturation current I_{is} is given to a good degree of approximation by

$$I_{is} \approx eAv \sum_s (n_i Z_i)_s \quad ; \tag{4}$$

where A is the effective collecting area of the probe, and s refers to different species of ions corresponding to the target material. Using the condition of charge neutrality of the plasma outside the sheath, eq. (4) can be written

$$I_{is} = A e n_e v \tag{5}$$

At the floating potential V_f the electron is subject to a repelling field, and the electron current is given by

$$I_e = \frac{A}{4} n_e v_e \, e^{V_f/kT_e} \tag{6}$$

where $v_e = (8kT_e/\pi m_e)^{1/2}$, i.e. the average electron thermal velocity.

At the floating potential V_f, no net current is drawn by the probe. Using the condition $I_e = I_{is}$, i.e. by equating eqs. (5) and (6), one obtains

$$\ln 2\sqrt{\pi}\, \xi_e^{1/2} = V_f / kT_e \quad , \tag{7}$$

where $$\xi_e = \left(\frac{m_e}{2kT_e}\right) v^2 \quad ,$$

i.e. the electron temperature T_e is uniquely related to the streaming velocity v and the floating potential V_f.

Unfortunately, one recalls $V_f = V_o - V_p$, where V_o is the observed potential at which there is no net current drawn by the probe. V_p is the plasma (or space) potential and is unknown. The iterative method is essentially based on a repeated guess of V_p and can be summarized by the following procedure:

i) Assuming a reasonable plasma potential, (e.g. $V_p \approx 10$ or 20 V), read from the experimentally determined characteristic curve the extrapolated I_{is}.

ii) Using the I_{is} thus obtained, calculate $I_e = I_{measured} - I_{is}$.

iii) From the semilogarithm plot $\ln I_e$ vs V_o, determine T_e.

iv) Using the T_e obtained and eq. (7), evaluate the space potential V_p.

v) Use the determined V_p and read I_{is} from the characteristic curve again, thus repeat the process until finally the computed V_p agrees reasonably well with the previously guessed value V_p.

3b. Illustrative example

To illustrate the use of the Langmuir probe and its characteristic for the determination of plasma parameters, and to demonstrate the influence of I_{is} on the deduced value of T_e, a concrete example is given in the following (see Fig. 2 for general reference).

i) Streaming velocity and ion kinetic energy

From the oscillogram, using the photon emission signal (the first small pulse as shown in the oscillogram taken at $V \approx -21$ V) as an indicator, one observes that the main peak of the bulk of the plasma arrives 2.2 μs later. As the probe is located 153 mm from the target,

$$\upsilon = \frac{15.3}{2{,}2 \times 10^{-6}} = 6.95 \times 10^{6} \text{ cm/sec.}$$

For the stainless-steel target used (standard No 1.4301, 18 % Cr, 10 % Ni) the average ion mass is $M_i = 66.4 \times 10^{-24}$ g_m and the corresponding ion kinetic energy is

$$\frac{M_i V^2}{2} = 1.604 \times 10^{-9} \text{ erg} \approx 1 \text{ keV.}$$

ii) Estimation of n_e

From eq. (5), for the 1 mm diam. probe used, i.e. $A = 7.854 \times 10^{-3}$ cm^2, if we take $I_{is} = 26$ mA, we have

$$n_e = \frac{I_{is}}{Ae\upsilon} = \frac{26 \times 10^{-3}}{1.258 \times 10^{-21} \times 6.95 \times 10^{6}} = 2.97 \times 10^{12}.$$

As we shall soon demonstrate, a more accurate value of I_{is} is about 20 mA, the corresponding value of $n_e = 2.28 \times 10^{12}$, and so n_e is not very sensitive to the estimated value of I_{is} as T_e will be. This is why one can use the ion attracting part of the characteristics for a reasonable estimate of n_e.

iii) Determination of T_e

From the characteristic shown in Fig. 2, we observe that it crosses the I = 0 axis at $V_o = -1$ V. As a first guess, we assume the plasma potential to be $V_p = 10$ V. Using the extrapolated $I_{is} = 26$ mA, we plotted the log I_e vs. the V curve as shown in Fig. 3 (ignoring the point at V = - 6 V, because the sensitivity of the probe at such a low bias voltage is rather doubtful). It is interesting to notice that, whereas there is no breaking of the curve at high positive voltages in the I-V characteristic, a

break occurs in the semi-logarithmic plot.
The appearance of two segmental lines can be interpreted in two ways. We may interpret it as no electron saturation current, I_{es} being reached and we may consider the plasma to consist of two groups, at different electron temperatures T_e.[14] It is easy to see that the upper segmental line corresponds to higher T_e. This implies that the higher energy group of electrons occurs at higher attracting voltage. This does not seem to be physically reasonable. Alternatively, we may interpret that the electron saturation current I_{es} does occur somewhere near the intersection of the two lines, and we may then consider the appearance of the upper segmental line to be caused by the abnormal space charge effect [15].

Taking the latter interpretation, from the slope of the lower segmental line, where most experimental point lies, we attain $kT_e = 15.7$ eV.

This, in turn, gives

$$\left(\frac{2kT_e}{M_i}\right)^{1/2} = 8.709 \times 10^5 \text{ cm/sec}$$

and together with the streaming velocity $v = 6.95 \times 10^6$ cm/sec gives

$$\varrho_e^{1/2} = \left(\frac{m_e}{M_i}\right)^{1/2} \left(\frac{M_i}{2kT_e}\right)^{1/2} = 2.955 \times 10^{-2}$$

and $V_f = kT_e \cdot \ln[2\,(\pi\,\varrho_e)^{1/2}] = -35.42$.

Consequently, the calculated plasma potential is $V_p = V_o - V_f \approx 34$ V.

This indicates that the guessed value of V_p is too low.

As a second guess, we take $V_p = 30$ V . From the characteristic shown in Fig. 2, we obtain $I_{is} = 20$ mA. From the semi-logarithm plot using $I_{is} = 26$ mA as shown in Fig. 4, we obtain $T_e = 13.6$ eV. Following a similar calculation procedure, we obtain a calculated $V_p = 29$ V, which is very close to the estimated $V_p = 30$ V.

iV) Estimation of the electron saturation current I_{es}

On the assumption that $v_e > \upsilon$, where v_e is the average electron thermal velocity, the electron saturation current is

$$I_{es} = e\, A\, n_e\, v_e / 4$$

since the ion saturation current under the condition given by eq. (3) is

$$I_{is} = e\, A\, n_e\, \upsilon \quad ,$$

we obtain $I_{es}/I_{is} = \frac{1}{\upsilon}\left(\frac{8\,k\,T_e}{\pi\, m_e}\right)^{1/2} V = \frac{1}{2\sqrt{\pi}\,\varrho_e}$.

At $kT_e = 13.6$ eV, $\varrho_e^{1/2} = 3.177 \times 10^{-2}$, or $I_{es}/I_{is} \approx 8.88$, this gives

$$I_{es} = 8.88 \times 20 \approx 178 \text{ mA},$$

which is not far from the intersecting point ($I_e = 190$ mA) of the two straight lines, as shown in Fig. 4. This indicates that the suggested iterative procedures seems to be self-consistent.

4. Effect of magnetic field

In view of the current interest of filling magnetic traps with laser-produced plasmas [16, 17, 18], a few remarks concerning the application of Langmuir probes in magnetized plasmas might be made.

Owing to the different rates of the motion of charged particles along and across the field lines, a magnetized plasma is an anisotropic medium. Consequently, in a sufficiently strongly magnetized plasma not only might the electron temperature differ from the ion temperature; even the electron temperature itself might have different values along or across the field lines.

In spite of this complication, Langmuir probes have been used in magnetized plasmas as useful diagnostic tools in, for example, gas discharge tubes [19, 20, 21], Q-machines [22, 23], a conical gun [24] and a microwave cavity device [25].

According to the relative sizes of the electron and ion Larmor radii ρ_e and ρ_i and the probe radius R, the collection phenomena of probes in a magnetized plasma can be broadly divided into three categories: weak field case $R \ll \rho_e$, moderate field case $\rho_e < R < \rho_i$, and strong field case $\rho_i < R$.

Generally speaking, for the weak field case no great difficulty will be encountered in interpreting the probe data. For the moderate field case, the ion attracting part and the electron repelling part of the characteristic can with some precaution, e.g. the use of simple models and interpolation formulas [25], still be used to measure the electron number density n_e and temperature T_e. With regard to the strong field case, very little practical experience is available concerning the probe response.

When a laser-produced plasma expands into a magnetic field depending on the relative direction of the laser beam and the main field lines, two cases must be distinguished. When the beam is directed across the main field lines, past experiments indicate that rapid oscillations occur even when a relatively weak field is applied [26]. These oscillations tend to be damped only when a strong field is applied. The use of the probe for measuring plasma parameters (i.e. the mean quantities), at best, will be a complicated matter. Concerning the case when the laser beam is directed along the field lines, so far limited experiments indicate that such oscillations might be absent;probes can then be used as simple diagnostic tools as long as the field is not very strong.

5. Conclusions

From past experiments conducted in plasmas produced by laser-target interactions, we conclude that the use of Langmuir probes can be divided into three regions:

a) In the asymptotic expansion phase, both the electron temperature T_e and number density n_e can be measured.

b) In the vicinity of the target, it might be impossible to use the probe.

c) In the intermediate region, n_e can be determined from the ion-saturation current.

When such a plasma expands into a magnetic field, two cases must be distinguished according to the orientation of the laser beam relative to the direction of the field lines. Owing to the presence of rapid oscillations in the cross-field case, the application of the probe for the determination of plasma parameters, if not impossible, will be difficult. In the parallel-field case, probes might still be used for the determination of plasma parameters as long as the field is not so strong that the ion Larmor radius ρ_i becomes smaller than the probe radius R.

Acknowledgment

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The probes used in the earlier experiments were provided by M. Hashmi and in the later experiments by K. Büchl.

Notes and References

1) Most of the original work regarding the probe can be found in the Collected Works of Irving Langmuir vol. 4, C. Guy Smits, general editor (Pergamon Press, 1961);
in particular, the two long articles written together with H.M. Mott-Smith, appearing on pages 23 and 99, should be consulted.

2) Specifically, this means using a probe of sufficiently small size, but not so small as to cause the difficulty of the edge effect (plane probe) or of the end effect (cylindrical probe).

3) Langmuir probes have been used as close as 5 mm to a carbon plate target irradiated at a power density 7×10^{10} W/cm^2; see article by P.T. Rumby and J.W.M. Paul, Plasma Physics 16, 247 (1974).

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6) J.D. Swift and M.J.R. Schwar "Electric Probes for Plasma Diagnostics" (Iliffe Books, London, 1971)

7) P.M. Chung, L.Talbot and K.J. Touryan "Electric Probes in Stationary and Flowing Plasmas" (Springer Verlag, 1975).

8) In interpreting data obtained by a cylindrical probe in a flowing plasma, care must be taken regarding the alignment of the cylindrical axis with respect to the flow direction; see papers by

D.W. Koopman, Phys.Fluids 14, 1707 (1971)
S.B. Segall and D.Koopman, Phys.Fluids 16, 1149 (1973)

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12) Ref. 1, pp. 39-47

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14) See, for example, Fig.6, p. 136, Ref. 4

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23) J.A. Williams, J.Phys.D: Appl. Phys. 7, 2205 (1974)

24) L.Lindberg and L.Kristofferson, Plasma Physics 12, 831 (1970)

25) I.G. Brown, A.B. Compher, and W.B. Kunkel, Phys. Fluids 14, 1377 (1971)

26) C.T. Chang and M.Hashmi, Phys.Fluids (to be published, 1977)

Figure Captions

Fig. 1 Circuit diagram of the probe -
C_s is the stray capacitance of the probe itself,
V is a variable voltage source,
R_b is the charging resistance, and
C and R are the capacitance and resistance of the load respectively.

Fig. 2 Typical oscillogram and current - - voltage characteristic obtained from a 1 mm dia. disc probe. Target: stainless-steel plate; laser energy 2 J., normal incidence.

Fig. 3 Plot of log I_e vs. V. I_{is} = 26 mA is used to obtain $I_e = I - I_{is}$ from Fig. 2.

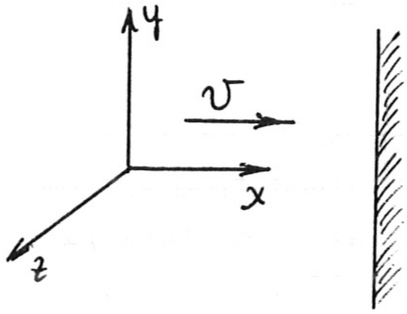
Fig. 4 Plot of log I_e vs. I_{is} = 20 mA is used to obtain $I_e = I - I_{is}$ from Fig. 2

Fig. 5 Variation of the function $G(\varsigma)$ with respect to the parameter $\varsigma^{1/2} = (m/2kT)^{1/2} v$.

Appendix

Saturation Current for a Flowing Plasma

In the absence of emission, ionization effect, etc., the ideal saturation current density drawn by a probe is given by the random current at the sheath edge.



When a stream velocity V is present in addition to the random velocity, the distribution function in a coordinate system moving with V should still be a Maxwellian, i.e.

$$f(u,v,w) = A \exp. \left\{ - \frac{(u-V)^2 + v^2 + w^2}{2kT} \right\} du\, dv\, dw \qquad (A1)$$

For an infinitely extended plane surface, the current density I, accordingly, is given by

$$I = \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u\, f(u,v,w)\, du\, dv\, dw \qquad (A2)$$

The constant A can be determined from the total number of charges present in a unit volume, i.e.

$$Zen = A \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ - \frac{(u-V)^2 + v^2 + w^2}{2\ T} \right\} du\, dv\, dw \qquad (A3)$$

where n is the number density of the particle concerned.

By putting u' = u-v and then du = du' , the integral reduces to the same form as the usual Maxwellian distribution, and consequently

$$A = Zne \left(\frac{m}{2\pi k T} \right)^{3/2} \qquad (A4)$$

Substituting the value of A given by eq. (A-4) into (A-2) and carrying out the integration, one obtains

$$I = Zne \left(\frac{kT}{2\pi m} \right)^{1/2} G(\varphi) \ ,$$

where $G(\varphi) = e^{-\varphi} + (\pi \varphi)^{1/2} \left\{ 1 + \text{erf.}(\varphi^{1/2}) \right\}$

$$\varphi = \frac{mV^2}{2kT} \qquad \text{(A 5)}$$

i.e. φ is essentially the ratio of the energy in the ordered motion comparing with that in the random motion. $\varphi^{1/2}$ is defined as the positive branch of the root of φ.

The variation of the function G (φ) with respect to the parameter $\varphi^{1/2}$ is shown in Fig. 5. Two special cases are of particular interest, namely

$$G(o) = 1 \ , \qquad \text{(A 6)}$$

$$G(\varphi \gg 1) = 2(\pi \varphi)^{1/2} \qquad \text{(A 7).}$$

Detailed computation, however, shows that the function $G(\varphi)$ approaches its asymptotic $2(\pi\varphi)^{1/2}$ very quickly, and so even at $\varphi = 1$, $G(\varphi)$ differs from $2(\pi\varphi)^{1/2}$ only by 2.5 per cent.

Comparing the random current density in the absence of the streaming velocity

$$I(\varphi = o) = \frac{Ze\ n}{4} \left(\frac{8kT}{\pi m} \right)^{1/2} = Ze\ n \left(\frac{kT}{2\pi m} \right)^{1/2} \qquad \text{(A 8)}$$

with eq. (A-5), we can rewrite eq. (A-5) as

$$I(\varphi) = I(o)\ G(\varphi). \qquad \text{(A 7)}$$

The function $G(\varphi)$ can then be interpreted as a correction factor due to the streaming effect.

In the special case where $\varphi \gg 1$, we notice that

$$I\ (\varphi \gg 1) = Ze\,n\ \left(\frac{T}{2\pi m}\right)^{1/2} 2\,(\pi\varphi)^{1/2} = Z\,e\,n\,v\,. \qquad \text{(A 8)}$$

In practice, as we mentioned previously, even at $\varphi = 1$, $I\ (\varphi = 1)$ differs from $I\ (\varphi \gg 1)$ by only 2.5 per cent, i.e. as long as the flow kinetic energy is comparable with its thermal energy, the saturation current is mainly given by the streaming motion.

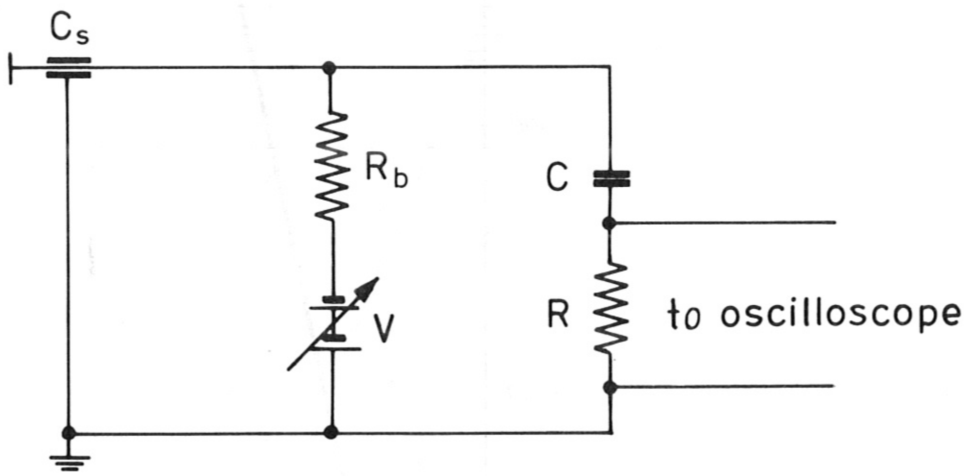


Fig. 1 Circuit diagram of the probe. C_s is the stray capacitance of the probe itself, V is a variable voltage source, R_b is the charging resistance, and C and R are the capacitance and the resistance of the load, respectively.

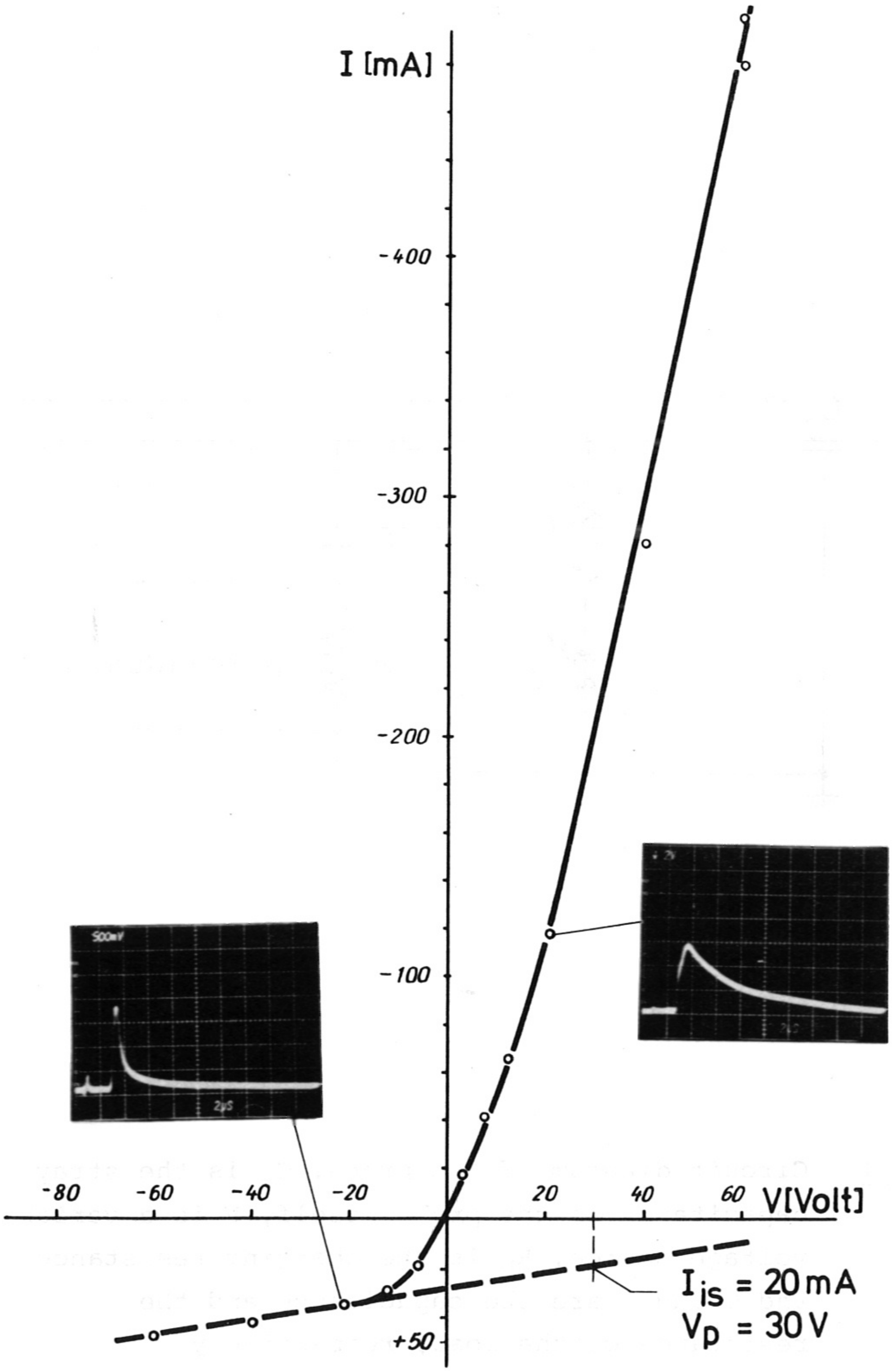


Fig. 2 Typical oscillograms and current-voltage characteristic obtained from a 1 mm dia-disc probe. Target: stainless steel plate; laser energy: 2 J, normal incidence.

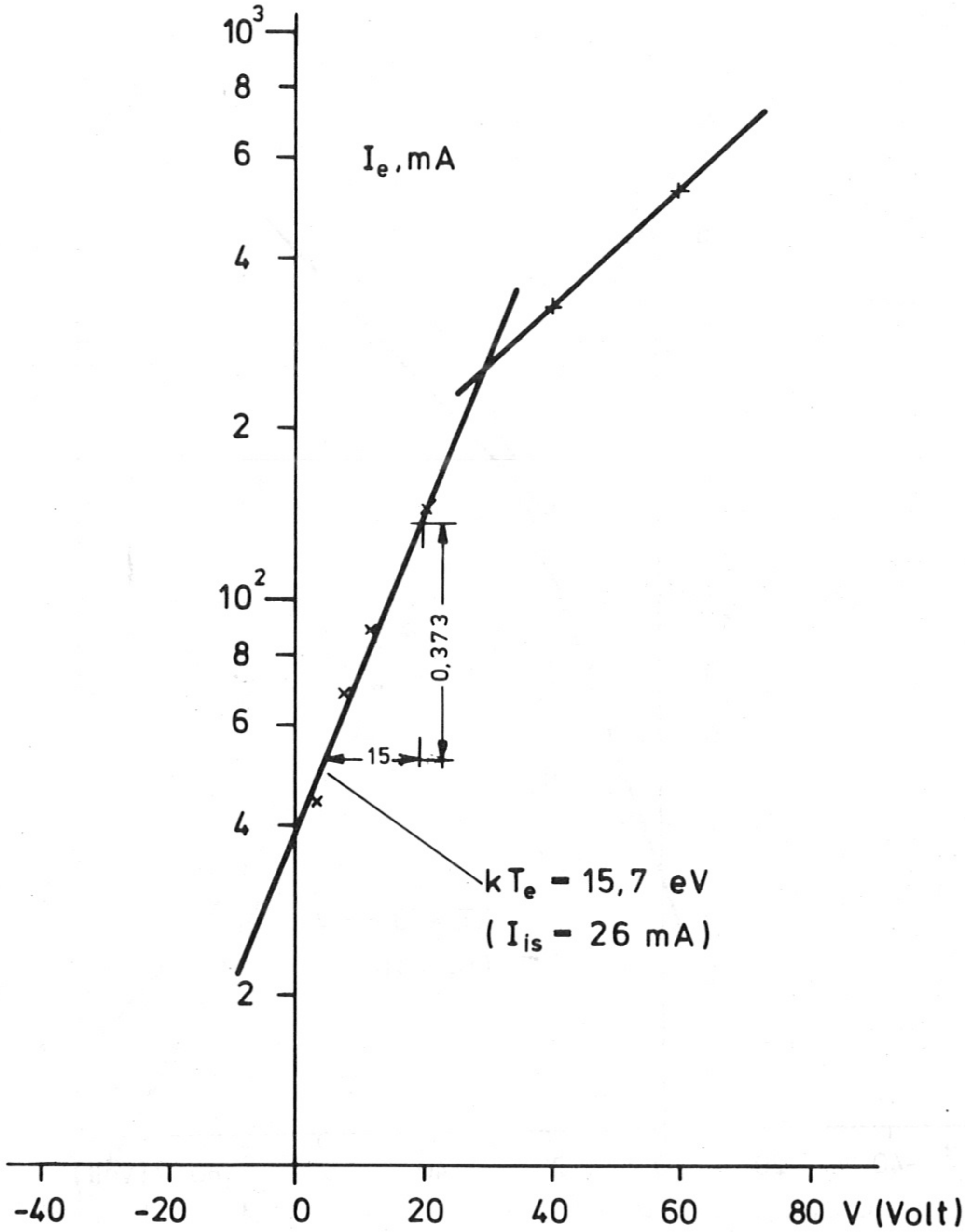
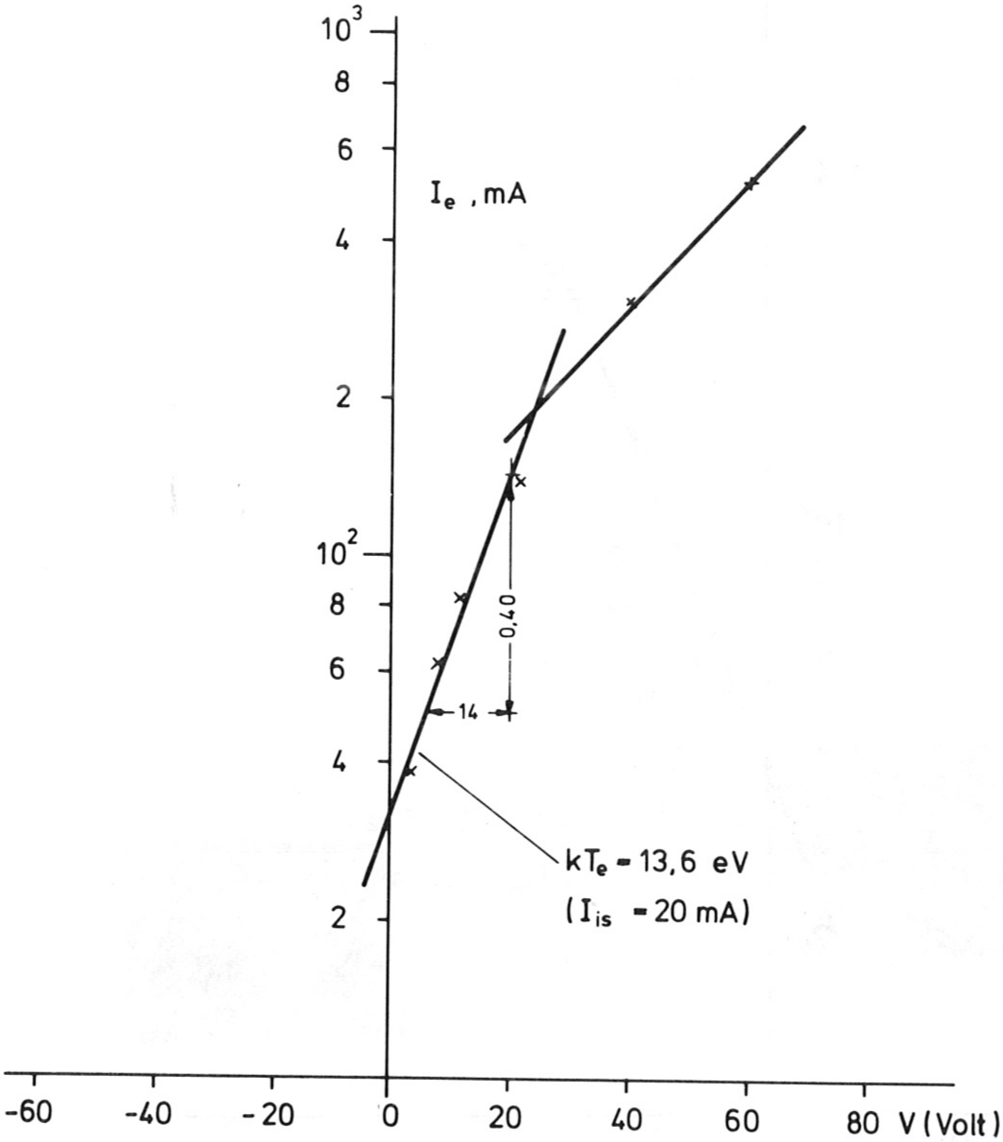


Fig. 3   Plot of log I_e vs. V. I_{is} = 25 mA is used to obtain $I_e = I - I_{is}$ from Fig. 2.



<u>Fig. 4</u> Plot of log I_e vs. V. I_{is} = 20 mA ist used to obtain $I_e = I - I_{is}$ from Fig. 2.

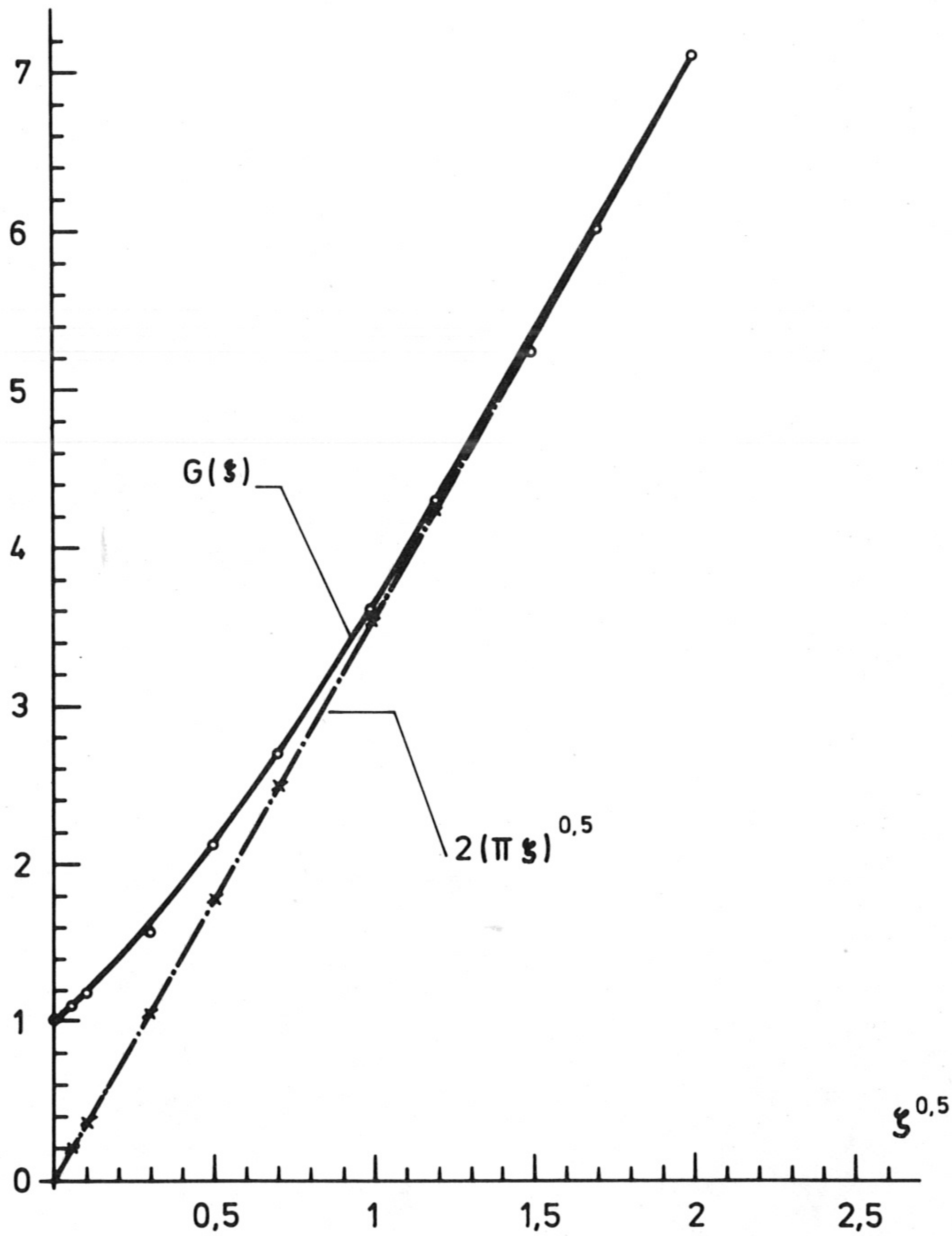


Fig. 5 Variation of the function G(ξ) with respect to the parameter $\xi^{1/2} = (\frac{m}{2kT})^{1/2} \mathbf{U}$.