

A Neutral-Particle-Based Divertor Model
for Tokamak Reactors

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Abstract

A zero-dimensional divertor model is derived from one-dimensional "fluid" equations for the plasma and neutral particles. In this model the important process determining wall bombardment is charge exchange between the background neutral gas and the ions. Simple formulas are presented for the rate of wall bombardment, for the mean temperature and for the shielding efficiency of the plasma in the scrape-off zone. Only the latter parameter is strongly dependent on the parallel transport process in the scrape-off zone. None of the parameters are dependent on the perpendicular transport process. The model shows reasonable agreement with the one-dimensional computer simulation calculations of Mense.

I. INTRODUCTION

Theoretical predictions of the performance of poloidal divertors on tokamak plasmas have been made from a variety of points of view. Hinton and Hazeltine [1] have given a neoclassical treatment based on the assumption of hot ions and cold electrons in the scrape-off zone. Boozer [2] has started from the two-fluid Braginskii equations and assumed warm electrons and cold ions. This seems appropriate for the FM-1 [3] and Wisconsin [4] divertor experiments, but is questionable for large tokamaks. A different and somewhat more phenomenological approach has been adopted by other workers [5 - 9]. In these analyses, the two-dimensional effect of particles flowing along the magnetic field lines to the collectors is simulated in a one-dimensional model by an "absorption" term in the particle continuity equation:

$$\frac{\partial n}{\partial t} + \vec{\nabla}_{\perp} \cdot \vec{\Gamma}_{\perp} = - \frac{n}{\tau_{\parallel}} \quad (1)$$

Here Γ_{\perp} is the charged particle flux perpendicular to the magnetic surface and τ_{\parallel} is an absorption time constant. According to one's prejudices about the relevant physics in the scrape-off layer, various choices for the perpendicular diffusion coefficient D_{\perp} , implicit in Γ_{\perp} , and for τ_{\parallel} are made. This phenomenological approach has been useful in developing an understanding of the consequences of various physics assumptions and their relative importance in determining divertor operation.

As a consequence of particle flow to the collectors, the particles carry energy away from the scrape-off zone. One anticipates, therefore, that the electron and ion temperatures will drop through the scrape-off zone. To consider this effect, one needs, in addition to (1), analogous one-dimensional equations for the electron and ion energy transport. In these equations the energy flow to the collectors is then modeled by an "energy absorption" term. An additional physical process that may be important is the possibility that ions may charge-exchange with the background neutral gas and thereby

produce hot neutral particles that strike the wall. Considering all these effects simultaneously leads to a very complicated and highly nonlinear set of equations. Furthermore the boundary conditions to be imposed at the separatrix are very much dependent on the plasma conditions inside the separatrix surface. Consequently one must resort to numerical techniques and solve the equations for the scrape-off zone simultaneously with the equations for the interior plasma.

The above approach has been adopted by Mense [9]. He has taken a tokamak simulation code [10], which is based on the Oak Ridge Code, and assigned some of the mesh points to the scrape-off zone. In this zone the finite-differenced transport equations are modified to include particle and energy absorption terms. The perpendicular transport is assumed to be Bohm-like but with a variable coefficient. The remaining mesh points are assigned to the interior plasma and treated in the usual way (pseudoclassical or trapped-particle transport coefficients depending on the collisionality of the plasma). Ionization and charge exchange of neutral particles is also treated, but impurities are neglected. The code determines the temporal evolution of the plasma density, temperatures, and neutral density profiles in the interior zone and in the scrape-off zone, starting from a given set of initial conditions. The resulting profiles for the UWMAK-II reactor design [11] have been given in Reference 10. These results are typical of calculations done for other designs.

The difficulty with calculations of this type is that they are computationally expensive. For the purpose of parametric studies of reactor systems, where the plasma model is only one element of the system model, one cannot afford to use such elaborate codes to determine the plasma behavior. Zero-dimensional codes for the interior plasma have been developed and compared with the more elaborate space-time codes. What is needed for systems studies is a zero-dimensional model for the divertor suitable for coupling

to a zero-dimensional plasma model and to models describing the other elements of the system. Such a model should predict reasonably well the flux and energy of particles bombarding the wall and the probability of wall-originated impurities getting through the scrape-off zone and therefore into the main plasma. All other detail is superfluous. In this context the role of the more detailed space-time model is that it is a standard against which the simpler model can be checked. In this report we present such a model.

II. DIVERTOR MODEL

The general features of the results obtained by Mense are as follows:

1. The ion flux at the wall is negligible in comparison to the ion flux crossing the separatrix.
2. The energetic particles bombarding the wall are primarily charge-exchange neutral particles.
3. The cold neutral particle density at the separatrix is negligible in comparison to the cold neutral density at the wall.
4. The energetic neutrals that reach the wall are "born" primarily in the scrape-off zone.

These four observations are not all independent. Statement 2 is a consequence of 1 and 4 is a consequence of 3. These observations seem to be general properties of reactor size divertor tokamaks and are not highly dependent on the particular assumptions about the transport processes. Therefore, we are led to consider them to be "universal" and treat them as "axioms" on which to base our model.

The observations (1-4) suggest that the scrape-off layer acts as a boundary layer for neutral particles. Backstreaming

of cold neutrals from the collection chambers produces a cold neutral population. The loss mechanism which balances the neutral source is ionization and charge exchange; this produces a hot neutral population and determines the energetic particle flux bombarding the wall.

We use a slab model for the scrape-off zone, as shown in Fig. 1. The separatrix is at $x = 0$ and the wall is at $x = d$. The neutral particles are placed in two energy groups: "cold" (with temperature T_0) and "hot" (with temperature equal to the ion temperature T_i where they are born). The "cold" temperature T_0 can be taken to be the Franck-Condon energy, although we need not specify it as long as $T_0 \ll T_i$. The plasma density n , ion temperature T_i , electron temperature T_e , and the cold neutral density n_c are assumed to satisfy the following conservation equations:

$$\frac{\partial \Gamma}{\partial x} = - \frac{n}{T_{ii}} + n n_c \langle \sigma N \rangle_I \quad (2)$$

$$\frac{\partial \Gamma_c}{\partial x} = - n n_c \langle \sigma N \rangle_{TOT} \quad (3)$$

$$\frac{\partial Q_i}{\partial x} = - \gamma_i \frac{n T_i}{T_{ii}} - 2 \alpha \alpha_E n n_c \langle \sigma N \rangle_{CX} T_i \quad (4)$$

$$\frac{\partial Q_e}{\partial x} = - \gamma_e \frac{n T_e}{T_{ii}} \quad (5)$$

Here Γ and Γ_c are the flux of ions and cold neutrals, respectively, whereas Q_i and Q_e are the ion and electron energy fluxes. The reaction rates for ionization and charge exchange are $\langle \sigma N \rangle_I$ and $\langle \sigma N \rangle_{CX}$, respectively, and $\langle \sigma N \rangle_{TOT} = \langle \sigma N \rangle_I + \langle \sigma N \rangle_{CX}$. In (2) we have neglected ionization of hot neutrals since $n(\text{hot}) \ll n_c$, at least near the boundary. The first terms on the right hand side of (4) and (5) represent energy flow to the particle collectors. The coefficients γ_i and γ_e are dependent on the parallel transport processes assumed to be present. The second term on the right hand side of (4) represents the flow to the wall

due to charge exchange in the scrape-off zone. Here $1-\alpha_E$ is the energy reflection coefficient [12] and α has the following meaning: For each first generation hot neutral produced in the plasma, α hot neutrals eventually escape and strike the wall. A two generation approximation for α is

$$\alpha = \frac{1}{2} \left[1 + \frac{1}{2} \frac{1}{1 + \frac{\langle \sigma N \rangle_I^H}{\langle \sigma N \rangle_{cx}^H}} \right] \quad (6)$$

In (6) the reaction rates are evaluated at $T(\text{neutral}) = T_i$, whereas in (2) - (4) the reaction rates are for the cold neutrals.

In accordance with our "axioms" 1-4, we impose the following boundary conditions on (2) - (5):

$$x=0 : \quad \Gamma = \Gamma(0), \quad Q_i = Q_i(0), \quad Q_e = Q_e(0), \quad \Gamma_c = 0. \quad (7)$$

$$x=d : \quad \Gamma = Q_i = Q_e = 0 \\ \Gamma_c(d) = -R \int_0^d \frac{n dx}{T_H} - \alpha_p \Gamma(\text{hot}) \quad (8)$$

where $\Gamma(\text{hot})$ is the hot neutral flux incident on the wall and $1 - \alpha_p$ is the particle reflection coefficient at the wall [12]. $\Gamma(\text{hot})$ is given by

$$\Gamma(\text{hot}) = \alpha \int_0^d n n_c \langle \sigma N \rangle_{cx} dx \quad (9)$$

In (8) the first term on the right describes the effect of neutral gas backstreaming from the collection chambers; we are treating it as a particle source at the boundary. One can also treat it as a distributed volume source in eq. (3). It is easy to show that there is no difference between these two approaches within the context of this analysis.

The temperature dependence in (2) - (9) occurs implicitly in $\langle \sigma N \rangle$ as well as explicitly. Fortunately the dependence

of $\langle \sigma N \rangle$ on temperature is not strong for the values of interest in reactor plasmas. Hence we replace T_i and T_e in $\langle \sigma N \rangle$ by spatial averages $\langle T_i \rangle$ and $\langle T_e \rangle$ to be defined later. With this approximation, we integrate (2) and (3) from $x = 0$ to $x = d$ and apply the boundary conditions (7) - (9) to get

$$\int_0^d n n_e dx = \frac{R \Gamma(0)}{\langle \sigma N \rangle_{TOT}} D \quad (10)$$

$$D = 1 - \alpha \alpha_p \frac{\langle \sigma N \rangle_{ex}}{\langle \sigma N \rangle_{TOT}} - R \frac{\langle \sigma N \rangle_I}{\langle \sigma N \rangle_{TOT}} \quad (11)$$

$$\Gamma(\text{hot}) = \frac{\alpha R \Gamma(0)}{D} \frac{\langle \sigma N \rangle_{ex}}{\langle \sigma N \rangle_{TOT}} \quad (12)$$

$$\int_0^d \frac{n}{T_H} dx = \Gamma(0) \left[1 + \frac{R \langle \sigma N \rangle_I}{D \langle \sigma N \rangle_{TOT}} \right] \quad (13)$$

The second term on the right side in (4) is of order R , compared with the first term. Of the recycling coefficient, R , is small (say $\sim 10^{-1}$) then the second term is not very important and we can replace T_i in it by $\langle T_i \rangle$. We can now integrate (4) from $x = 0$ to $x = d$ and use (10) to get

$$Q_i(0) = \int_0^d \gamma_i n T dx + \frac{2 \alpha \alpha_E \langle T_i \rangle}{D} \frac{R \Gamma(0)}{\langle \sigma N \rangle_{TOT}}$$

We now define

$$\langle T_i \rangle = \frac{\int_0^d \frac{n T_i}{T_H} dx}{\int_0^d \frac{n}{T_H} dx} \quad (14)$$

and use (13) to get

$$\langle T_i \rangle = \frac{Q_i(0)}{\gamma_i \Gamma(0) \left[1 + \frac{R}{D} \frac{\langle \sigma N \rangle_I + 2 \alpha \alpha_E \langle \sigma N \rangle_{ex} \alpha_i}{\langle \sigma N \rangle_{TOT}} \right]} \quad (15)$$

Integrating (5) and using (13) gives

$$\langle T_e \rangle = \frac{Q_e(0)}{\gamma_e \Gamma(0) \left[1 + \frac{R}{D} \frac{\langle \sigma N \rangle_I}{\langle \sigma N \rangle_{TOT}} \right]} \quad (16)$$

Recall that the reaction rates are evaluated with $T_{i,e}$ replaced by $\langle T_{i,e} \rangle$. Hence (15) and (16) are to be solved simultaneously for $\langle T_i \rangle$ and $\langle T_e \rangle$. Since R is presumably small, this can be done rather easily in an iterative fashion. Given the solutions for $\langle T_i \rangle$ and $\langle T_e \rangle$, we can compute the hot charge exchange neutral particle flux at the wall from (12); we also assign the temperature $\langle T_i \rangle$ to these neutrals. It is interesting to note that these results are independent of the mechanism for cross-field ion transport in the scrape-off zone (as long as "axiom 1" is not violated) and only weakly dependent on the parallel transport mechanism through the coefficients γ_i and γ_e .

The remaining parameter of interest is the effectiveness of the scrape-off layer plasma in ionizing wall-originated impurities. The probability P of an impurity atom traversing the scrape-off layer without ionization depends on the parameter β ,

$$\beta = \frac{\langle \sigma N \rangle_{imp}}{v_0} \int_0^d n dx$$

where $\langle \sigma N \rangle_{imp}$ is the ionization rate and v_0 is the velocity of the impurity atom. The probability P also depends on the angular velocity distribution of the incident atoms. For normal incidence $P = e^{-\beta}$, for a cosine distribution $P = 2E_3(\beta)$, and for an isotropic distribution $P = E_2(\beta)$. Here E_2 and E_3 are the exponential integrals. Hence we need $\int_0^d n dx$ to evaluate P . Unfortunately our analysis only gives us $\int_0^d n/\tau_{||} dx$. We need to specify a model for $\tau_{||}$ and then estimate a mean value for $\tau_{||}$. Hence

$$\int_0^d n dx \approx \bar{\tau}_{||} \int_0^d \frac{n}{\tau_{||}} dx = \bar{\tau}_{||} \Gamma(0) \left[1 + \frac{R \langle \sigma N \rangle_I}{D \langle \sigma N \rangle_{TOT}} \right] \quad (17)$$

For the ion-sound model with $T_i > T_e$,

$$\bar{\tau}_{\parallel} \approx L \sqrt{\frac{2\pi M_i}{\langle \tau_i \rangle}} \quad (18)$$

where L is the field line length and M_i is the ion-mass. The dominant contribution to $\int_0^d n dx$ is $\tau_{\parallel} \Gamma(0)$ and has been obtained earlier by Keilhacker [5]. The second term on the right side of (12) represents the increase in $\int_0^d n dx$ due to ionization of neutral atoms. This is an effect of order R and hence is small. It is small even if $R \sim 1$ since $D \sim .5$ and $\langle n \rangle_I \ll \langle n \rangle_{TOT}$ for temperatures anticipated in the scrape-off zone.

It is worthwhile to repeat here some of the assumptions on which this model is based. We have used some observations from numerical calculations as a basis for our model. If these, the assumption most questionable for divertor Tokamaks of the ASDEX/PDX size is that the scrape-off layer plasma is opaque to cold neutrals. If this assumption is not true, we can define a distance λ (measured from the separatrix) beyond which few cold neutrals penetrate. If λ is not too large, we can apply our analysis to the zone $-\lambda < x < d$ and define $\tau_{\parallel} = \infty$ for $-\lambda < x < 0$. The results are formally identical except that $\Gamma(0) \rightarrow \Gamma(-\lambda) \approx \Gamma(0)$, $Q(0) \rightarrow Q(-\lambda) \approx Q(0)$. In this case, the weakest element is probably the ion energy equation. Our definition of $\langle T_i \rangle$ gives us a value near the separatrix temperature, whereas the charge-exchange neutrals are now coming from deeper into the plasma and therefore have $T > \langle T_i \rangle$. This model then underestimates the temperature of the charge-exchange neutrals striking the wall. This, however, has only a minor effect on the flux, $\Gamma(\text{hot})$, since the cross-sections are relatively insensitive to T_i .

A second limitation is that we have neglected the ionization of hot neutrals in the scrape-off zone. This becomes more important in larger tokamaks, especially if we can make τ_n longer than that given by the ion-sound model. This effect produces a correction of order R in $\langle T_i \rangle$ and $\int_0^d n/\tau_{\parallel} dx$ and

of order R^2 in $\Gamma(\text{hot})$, and thus becomes more important with increased recycling. Generalizing the model to consider this is not trivial, however, and therefore is not considered here.

III. COMPARISON WITH 1-D CALCULATIONS

As a check on the above analysis we compare its predictions with the results from the more detailed one-dimensional calculations of Mense [9]. This is shown in Table I for the UWMAK-II, -III, and Oak Ridge EPR designs. For these calculations, the ion sound model for τ_{ii} has been used.

TABLE I

Comparison of 0-D and 1-D calculations

| | DESIGN | 1-D | 0-D |
|---|-----------|----------------------|----------------------|
| $\langle T_i \rangle$ (keV) | UWMAK-II | 4 | 3.1 |
| | UWMAK-III | 3.4 | 3.2 |
| | EPR | 3 | 2.7 |
| $\frac{\Gamma(\text{hot})}{\Gamma(0)}$ | UWMAK-II | .04 | .12 |
| | UWMAK-III | .14 | .12 |
| | EPR | .11 | .13 |
| $\int_0^d n dx$ (cm^{-2}) | UWMAK-II | 1.2×10^{13} | 6.3×10^{12} |
| | UWMAK-III | 2.6×10^{13} | 1.2×10^{13} |
| | EPR | 1.4×10^{13} | 1.4×10^{12} |

The values listed above for $\langle T_i \rangle$ for the 1-D calculations are actually the ion temperature at the separatrix. Because of the density weighting factor in eq. (14), $\langle T_i \rangle$ is close to, but less than, the separatrix temperature. The agreement for $\langle T_i \rangle$ and $\Gamma(\text{hot})/\Gamma(0)$ is good.

The agreement between the 1-D and O-D calculations of $\int n dx$ is good for the UWMAK-II and - III cases but not for the EPR case. In eq. (17) the term involving the ionization of neutrals is small. Hence the disagreement between the O-D and 1-D results has to be attributed to the estimation of $\bar{\tau}_{\parallel}$. The 1-D code uses a variable $L(r)$ obtained from a magnetic field code. In the O-D estimate of $\bar{\tau}_{\parallel}$ (eq.(18)) a single value of L , typical of what an ion experiences in the scrape-off zone, is used. Since $L(r)$ is singular at the separatrix and the density is also greatest there, a disagreement by a factor of 2 between the O-D and I-D results is not surprising. The factor of 10 disagreement for the EPR case is surprising and not understood.

An interesting result in Table I is the rather constant value of $\langle T_i \rangle$ even though the three designs vary considerably in their basic parameters. In each case, though, the bulk of the plasma is in the trapped-ion collisionality regime. This is probably the determining factor in setting $Q_i(o)/\Gamma(o)$, which is of principal importance in determining $\langle T_i \rangle$.

The primary factor determining the hot charge-exchange flux at the wall is the recycling coefficient, R . In the above results, the value $R = .1$ has been used for computational purposes. We have made no attempt to estimate R for a specific design so the numerical values for $\Gamma(\text{hot})$ presented here are not to be taken literally. The scaling of $\Gamma(\text{hot})$ with R from this model is clear, however: Reducing R reduces $\Gamma(\text{hot})$. This conclusion can be modified, however, depending on the refuelling mechanism used to maintain constant density. If the refuelling mechanism introduces a large number of neutrals into the scrape-off layer then this will increase $\Gamma(\text{hot})$. This has been considered to some extent by Haas and Keilhacker [13] for ASDEX.

IV. THE INFLUENCE OF IMPURITIES

Impurity ions in the scrape-off zone have the effect of 1) increasing the electron density relative to the ion density and 2) cooling the electrons by dilution and radiation. Increasing the electron density in the scrape-off zone increases the probability of ionization relative to charge exchange and thereby reduces the rate of bombardment of the wall by hot charge-exchange neutrals. The analysis of this effect is of minor importance, at least for plasmas bounded by low-Z liners.

The electron temperature enters the analysis only through the determination of the ionization rate; this is rather insensitive to T_e for $T_e > 100$ eV. In the above calculations $\langle T_e \rangle > 1$ keV so that a drop in T_e by at least one order of magnitude is necessary in order to have a significant effect. This seems unlikely in view of the low impurity concentration predicted in the scrape-off zone.

The more important problem concerning impurities is the probability of impurity ions in the scrape-off zone diffusing into the main plasma or to the wall before reaching the divertor collection chambers. This is very much dependent on the transport processes that are operative in the scrape-off zone for both the impurity ions and for the hydrogenic ions. One also has the time-dependent processes of stripping and heating of the impurities entering into the problem. This problem is worthy of study but beyond the scope of the work presented here.

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VI. APPENDIX

Influence of Impurities on the Scrape-off Layer Plasma

The influence of impurities is treated by considering continuity equation for the neutral impurity atoms and for the impurity ions in addition to those for the plasma. Thus the set of equations to consider is

$$\frac{\partial \Gamma_i}{\partial x} = - \frac{n_i}{\tau_i} + n_e n_c \langle \sigma v \rangle_I \quad (A1)$$

$$\frac{\partial \Gamma_c}{\partial x} = - n_i n_c \langle \sigma v \rangle_{cx} - n_e n_c \langle \sigma v \rangle_I \quad (A2)$$

$$\frac{\partial \Gamma_z^{(0)}}{\partial x} = - n_e n_z^{(0)} \langle \sigma v \rangle_z \quad (A3)$$

$$\frac{\partial \Gamma_z}{\partial x} = - \frac{n_z}{\tau_z} + n_e n_z^{(0)} \langle \sigma v \rangle_z \quad (A4)$$

$$n_e = n_i + Z n_z \quad (A5)$$

Here, the subscript i denotes the plasma ions, z denotes the impurity ions (with charge Ze) and e denotes the electrons. $\Gamma_z^{(0)}$ (Γ_z) and $n_z^{(0)}$ (n_z) are the flux and density, respectively, of the impurity atoms (ions) and $\langle \sigma v \rangle_z$ is their ionization rate.

In addition to the previous boundary conditions, (7) and (8), we require that

$$\Gamma_z(d) = - S \Gamma(\text{hot}) \quad (A6)$$

where S is the sputtering coefficient.

Using (A5), we rewrite (A2) as

$$\frac{\partial \Gamma_c}{\partial x} = -n_i n_c \langle \sigma v \rangle_{TOT} - z n_z n_c \langle \sigma v \rangle_I$$

Integrating this from $X = 0$ to $x = d$ and using (7), (8), (9) we obtain

$$\Gamma(\text{hot}) = \frac{\alpha \langle \sigma v \rangle_{cx}}{\langle \sigma v \rangle_{TOT}} \frac{R \Gamma(0) - z \int_0^d n_z n_c \langle \sigma v \rangle_I dx}{D} \quad (\text{A7})$$

where D is as before. From (A7), it is apparent that one effect of impurities in the scrape-off layer is to reduce the bombardment of the wall by hot neutral particles. This is because the ionization of impurities increases the electron density and thereby increases the probability of ionization (relative to charge exchange) of cold neutral fuel atoms coming from the wall.

To be more quantitative, we need to evaluate $\int_0^d n_z n_c dx$. A general expression has not been found but we can obtain an upper limit for it. This gives us, then, a lower bound on $\Gamma(\text{hot})$. An upper bound on $\Gamma(\text{hot})$ is trivial; simply put $n_z = 0$ in (A7). To evaluate $\int_0^d n_z n_c dx$ we need specific expressions for n_z and n_c in term of n_e and n_i . For a cosine distribution of the direction of the incident velocity, the solution to (A3) is an exponential integral (E_3) which can be approximated by an exponential function if we put

$$\Gamma_z^{(0)} = -n_z^{(0)} \nu_E$$

where $\nu_E = \frac{1}{2} \sqrt{2E/m}$. Here E is the energy of the particles sputtered off the wall. Then the solution to (A3) is

$$n_z^{(0)}(x) = n_z^{(0)}(d) e^{-\int_x^d n_e \sigma_z dx} \quad (\text{A8})$$

where
$$\sigma_z = \langle \sigma_N \rangle_z / N_z \tag{A9}$$

We put (A8) into (A4) and neglect $\partial \Gamma_z / \partial x$ compared with n_z / τ_z ; this gives an upper limit on n_z :

$$n_z(x) = -\tau_z \Gamma_z^{(0)}(d) \sigma_z n_e(x) e^{-\int_x^d n_e \sigma_z dx'} \tag{A10}$$

In (A2) we let $\Gamma_c = -n_c N_a$ and solve it to get

$$n_c(x) = -\frac{\Gamma_c(d)}{N_a} e^{-\int_x^d n_e \sigma_{Iz} dx'} e^{-\int_x^d n_i \sigma_{cx} dx'} \tag{A11}$$

where σ_I and σ_{cx} are defined in the same manner as (A9).

Thus

$$\begin{aligned} \langle \sigma_N \rangle_I \int_0^d n_z n_c dx &= \tau_z \Gamma_z^{(0)}(d) \Gamma_c(d) \sigma_z \sigma_I \cdot \\ &\cdot \int_0^d e^{-\int_{x'}^d n_e (\sigma_z + \sigma_I) dx''} e^{-\int_{x'}^d n_i \sigma_{cx} dx''} n_e(x') dx' \end{aligned} \tag{A12}$$

Evaluation of the integral is, in general, not possible.

It can be bounded, however, by putting $n_i = 0$ (upper bound) or $n_i = n_e$ (lower bound). We also assume $\int_0^d n_e (\sigma_z + \sigma_I) dx \gg 1$ so that

$$\langle \sigma_N \rangle_I \int_0^d n_c n_z dx = \tau_z \Gamma_c(d) \Gamma_z^{(0)}(d) \sigma_I f \tag{A13}$$

where

$$\frac{\sigma_z}{\sigma_z + \sigma_I + \sigma_{cx}} < f < \frac{\sigma_z}{\sigma_z + \sigma_I} \tag{A14}$$

We put (A13), together with (A6) and (8), into (A7) and obtain a quadratic equation for $\Gamma(\text{hot})$:

$$a_1 \Gamma^2(\text{hot}) + a_2 \Gamma(\text{hot}) + a_3 = 0 \tag{A15}$$

where

$$a_1 = S Z \tau_z \int \sigma_I \alpha_p \alpha \frac{\langle \sigma_N \rangle_{CX}}{\langle \sigma_N \rangle_{TOT}}$$

$$a_2 = D + Z \tau_z \int \sigma_I \alpha S R \Gamma(o) \frac{\langle \sigma_N \rangle_{CX}}{\langle \sigma_N \rangle_{TOT}}$$

$$a_3 = - \alpha \frac{\langle \sigma_N \rangle_{CX}}{\langle \sigma_N \rangle_{TOT}} R \Gamma(o)$$

The existence of two roots to (A15) presents no problem. The desired root is always positive; the second root is always negative and therefore unphysical. The coefficients a_1 , a_2 , a_3 can be numerically evaluated and the solution for $\Gamma(\text{hot})$ obtained.

The degree of impurity accumulation in the scrape-off zone can be characterized by the parameter

$$\bar{Z} = \frac{\int_0^d (Z n_z + n_i) dx}{\int_0^d n_i dx} = \frac{\int_0^d n_e dx}{\int_0^d n_i dx} \quad (\text{A16})$$

where
$$\int_0^d n_z dx = S \tau_z \Gamma(\text{hot}) \quad (\text{A17})$$

using (A3), (A4), and (A6).

As a numerical example we apply these results to the UWMAK-III divertor. We consider a carbon impurity sputtered off the wall with an energy of 5 eV. As an upper limit on the residence time, τ_z , we neglect any effects (e.g. friction with the ions or electric fields) which might increase the parallel velocity. Then

$$\tau_z = 2.4 \times 10^{-2} \text{ sec.}$$

and

$$.11 < \frac{\Gamma(\text{hot})}{\Gamma(o)} < .12$$

Hence we find that, for this example, the effect of impurity accumulation is at most a 10 % reduction in the rate of wall bombardment. For this example

$$\bar{Z} = 1.2,$$

the scrape-off zone plasma remains relatively clean.

Applying this analysis to possible high Z impurities can give a somewhat greater reduction in $\Gamma(\text{hot})$. In this case, however, determination of the charge state, Z_r , of the impurities and their residence time, τ_Z , is more important. As a pessimistic approximation, however, one can always ignore impurities and use (12) to determine an upper limit for $\Gamma(\text{hot})$.

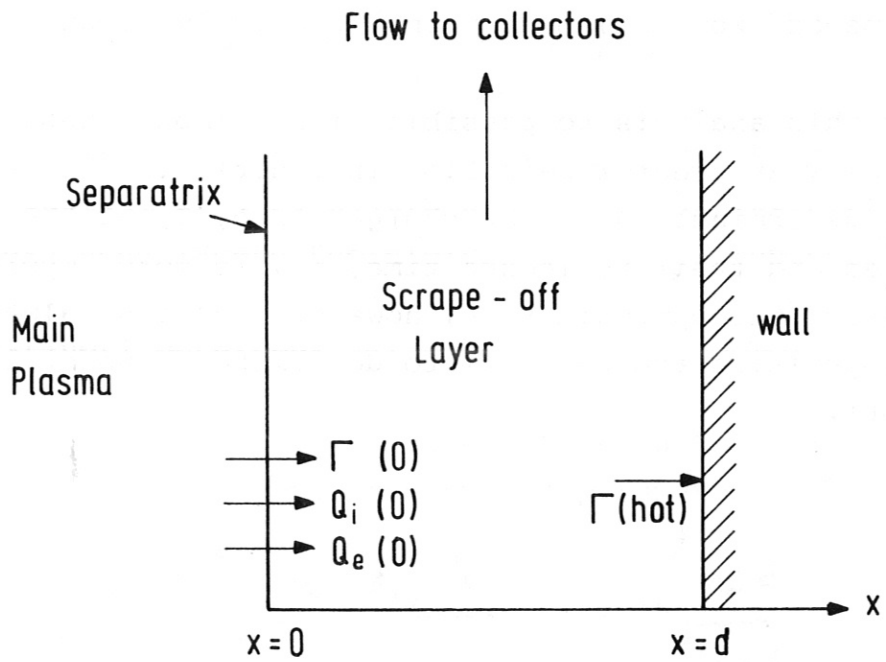


Fig. 1 SLAB MODEL FOR SCRAPE-OFF ZONE