Optics of Laser Plasmas
in Spherical Geometry

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Abstract :

Analytical formulae for classical reflection from pellets and plane targets under normal incidence are presented and the maximum electric field increase in smooth density profiles is determined. Density profile distortions due to light pressure and their influence on pellet compression are discussed in a steady state model.

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A. Introduction

This report treats the classical optics of laser plasmas in spherical and plane geometry. It has mainly been written for the experimentalist. In projecting or evaluating an experiment often basic formulae for simple geometries are needed. And not so seldom one discovers that such formulae do not yet exist. This is true for reflection coefficients and the increase of the electric field near the critical point. Therefore, in section D, analytical reflection coefficients will be presented for four different electron density profiles.

When the thresholds for parametric instabilities or local light pressure effects have to be estimated the knowledge of the maximum electric field amplitude is of particular interest. For this we have found a general formula which is very simple. In order to show the range of validity of these formulae in sections B and C the approximations made in the wave equation and its solution are discussed. The critical region is treated by approximating the dielectric constant by a linear function and by solving the corresponding wave equation in terms of Airy functions. With particular emphasis the limits of validity of the WKB approximation are discussed. From the numerical examples given in section C it results clearly that it has a wider range of applicability than it can be expected from the well-known criteria. Finally, in section E we discuss the influence of light pressure on density profile modifications. As a result we show that its influence on compression is much less dramatic than it has been suspected several times.

Laser plasma experiments indicate that oblique incidence of light may be of particular interest under certain angles, provocating the socalled phenomenon of resonance absorption. It would be of particular interest to have reliable analytical formulae for this situation. However, only in a later report such formulae will be presented.

B. Laser Radiation Focused on Spherically Symetric Targets

a) The wave equation

Let us consider the Fourier components of $\vec{E}, \vec{B}, \vec{j}$ for a given frequency ω, i.e. the time variation of such a component is of the form

$$\vec{E},\ \vec{B},\ \vec{j} \sim e^{-i\omega t} \tag{1}$$

Assuming $\vec{j} = \sigma \vec{E}$ for the current density with the electrical conductivity σ depending only on the frequency ω, Maxwell´s equations reduce to

$$\nabla \times \vec{B} = -i \frac{k}{c} n^2 \vec{E} , \tag{2}$$

$$\nabla \times \vec{E} = ick \vec{B} , \tag{3}$$

where k stands for $\frac{\omega}{c}$ and the square of the refractive index n^2 is related to σ by

$$n^2 = 1 + \frac{i\sigma}{\varepsilon_0 \omega} \, . \tag{4}$$

Substitution of Eq.(3) in Eq.(2) yields the familiar wave equation

$$\nabla \times \nabla \times \vec{E} - k^2 n^2 \vec{E} = 0. \tag{5}$$

This equation admits such a variety of solutions that a general discussion is impossible : nearly all classical optics in vacuum, gases and condensed matter is governed by this equation. We limit ourselves to the very special case of focused light beams normally incident on pellets the refractive index of which depends only on the radius :

$$n^2 = 1 - \frac{n_e(r)}{n_c} \cdot \frac{1}{1 + iA\, n_e(r)/n_c} \, , \tag{6}$$

where n_c is the critical electron density at which the plasma frequency $\omega_p = (n_e e^2/\varepsilon_0 m_e)^{1/2}$ equals the incident light frequency ω and A is the electron-ion collision frequency normalized to ω at the critical point r_c $(n_e(r_c) = n_c)$:

$$A = \frac{\nu_{ei}}{\omega}\bigg|_{r = r_c} \, .$$

It is thereby tacitly assumed that ν_{ei} does not depend on the temperature or that the electron temperature is constant over the whole radius owing to thermal conduction. If A were taken as equal to zero, no absorption in the pellet would take place.

By focused beams we understand electromagnetic waves where the normals to the surfaces of equal phase, i.e. the wave vectors $\vec{k}$ are all directed towards one point. However, it must be pointed out that this is an approximate concept the validity of which is within the framework of geometrical optics. Diffraction phenomena at the edges or in the focus of a beam, or strong variations of the refractive index n over one local wavelength λ , though still described by Eq.(5), invalidate the concept of focused beams. The aim in laser fusion is to illuminate a pellet uniformly from all directions so that the Poynting vector in the radial direction does not depend on the angles φ and ϑ . In the time average this goal may be reached to any desired accuracy, however, it cannot be achieved for any given time instant. In fact, there exists a topological theorem which states that on a sphere it is not possible to assign to each point a continuously varying direction. [1] In other words :

Do not try to comb a guinea pig continuously.

There will remain at least one point with undefined direction. If the electric and magnetic fields are decomposed into their tangential and radial components $\vec{E} = \vec{E}_r + \vec{E}_t$, $\vec{B} = \vec{B}_r + \vec{B}_t$, $\vec{E}_t$ and $\vec{B}_t$ represent such direction fields. Therefore, $\vec{E}_t$ must be zero at one point at least. The Poynting vector

$\vec{S}$ is determined by the sum of a tangential and a radial component, $\vec{S} = \vec{S}_t + \vec{S}_r$:

$\vec{S} \sim \vec{E} \times \vec{B} = (\vec{E}_r + \vec{E}_t) \times (\vec{B}_r + \vec{B}_t) = (\vec{E}_r \times \vec{B}_t + \vec{E}_t \times \vec{B}_r) + \vec{E}_t \times \vec{B}_t = \vec{S}_t + \vec{S}_r$,

and if, for example, $\vec{E}_t$ is zero at one point, $\vec{S}_r$ is also zero (since infinite magnetic fields have to be excluded).

Although a constant electric field in the tangential direction cannot be achieved, its variation along the surface of a sphere of radius r can be kept of the order of $|\vec{E}|r^{-1}$. The variation of $\vec{E}$ along the radius is of the order $k|\vec{E}|$, k being the local wave vector. The curl of a vector $\vec{A} = A_r\vec{e}_r + A_\varphi\vec{e}_\varphi + A_\vartheta\vec{e}_\vartheta$ in polar coordinates r, φ, ϑ is given by

$$\nabla \times \vec{A} = \frac{1}{r^2 \sin\vartheta} \begin{vmatrix} \vec{e}_r & r\sin\vartheta\,\vec{e}_\varphi & r\vec{e}_\vartheta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial \vartheta} \\ A_r & r\sin\vartheta A_\varphi & rA_\vartheta \end{vmatrix} . \tag{7}$$

Applying formula (7) twice to the wave equation (5) it follows that for such regular $\vec{E}$-fields the radial derivatives dominate those in the angular directions by the factor $kr = 2\pi r/\lambda$. For realistic pellets kr is of the order of 10^3 or larger. Therefore, all but the radial derivatives may be negected, as long as one considers "local" properties such as absorption, reflection or field maxima. But then the wave equation reduces to

$$\frac{d^2}{dr^2}(rE) + k^2n^2rE = 0$$

or, introducing $y = rE$, it becomes identical with the one-dimensional equation for a plane wave y :

$$\frac{d^2}{dr^2}y + k^2n^2y = 0 \tag{8}$$

Since the error is of the order of $\lambda/2\pi r$ it is immediately clear that in the focal region $\lambda \simeq r$ Eq.(8) becomes wrong, but in the case of pellets no light should reach shells behind the critical radius r_c. It could be further argued that λ, which is the local wavelength, could become very large at $r = r_c$ since λ drops to zero there. But our further investigations show that in realistic cases λ remains of the order of the vacuum wavelength (see, for example, Fig.3).

b) An alternative point of view.

The discussion of section a) is unsatisfactory in so far as it does not tell us quantitatively how the derivatives with respect to the angles φ and ϑ, if taken into account, would modify Eq.(8). Following Ref. (2) or preferentially (3) we observe that an arbitrary electromagnetic field can be regarded as composed of two components, one for which the electric field in the r-direction, $\vec{E}_r$, is zero and the other one for which $\vec{B}_r = 0$ holds. These are two clearly distinct groups because, according to expression (7), the radial magnetic field cannot be zero if $\vec{E}_r = 0$ and vice versa. For the $\vec{E}$-field with vanishing radial component it follows from Eq.(2) that

$$(\nabla \times \vec{B})_r = 0$$

and therefore the vector field $\{0, B_\varphi, B_\vartheta\}$ can be written as the gradient of a potential V which we assume as given in the form $V = \frac{1}{ick}\frac{\partial}{\partial r}(r\psi)$. The radial component $\vec{B}_r$ is determined from Eq.(3). However, by means of Eq.(2) the electric field components are determined from the magnetic field components, i.e. they can be expressed as derivatives of $r\psi$. Comparing the different expressions for the $\vec{E}$ and $\vec{B}$-field components and observing that n^2 depends only on r the following scalar equation for the so-called Debye potential ψ is obtained :

$$\Delta\psi + k^2 n^2 \psi = 0. \tag{9}$$

If ψ is known the five components of the electric and magnetic fields, $E_\varphi, E_\vartheta, B_r, B_\varphi, B_\vartheta$ are determined by differentiation alone. Exactly the same procedure is applied to the case for which the radial component of the magnetic field B_r is zero. The electric field $\vec{E} = \{0, E_\varphi, E_\vartheta\}$ is then determined as the gradient of a potential $U = \frac{c}{ikn^2}\frac{\partial}{\partial r} r\chi$ and $\vec{E}_r$ follows from Maxwell´s equation (2). After some tedious algebra (see again Ref.(2) or better, Ref.(3)) a scalar equation for the other Debye potential χ is obtained :

$$\Delta\chi - \frac{1}{n^2 r}\frac{\partial n^2}{\partial r}\frac{\partial}{\partial r} r\chi + k^2 n^2 \chi = 0, \tag{10}$$

from which the five field components $E_r, E_\varphi, E_\vartheta, B_\varphi, B_\vartheta$ are determined by differentiation. Equation (9) is the same as the wave equation for $\vec{E}$ and normal incidence or for the magnetic field of s-polarized light at oblique incidence in plane geometry, whereas Eq. (10) is the same as the wave equation of the magnetic field for p-polarized light at oblique incidence[4]. In fact, Eq. (10) may lead to resonance absorption (see Ref.3, pp. 53 and 54).
Let us now look for solutions of Eqs. (9) and (10) which satisfy the product ansatz

$$\psi(r, \varphi, \vartheta) = \sum_{l,m} g_l(r)\, Y_{lm}(\varphi, \vartheta),$$

$$\chi(r, \varphi, \vartheta) = \sum_{l,m} h_l(r)\, Y_{lm}(\varphi, \vartheta).$$

Y_{lm} are the Legendre polynominals (spherical harmonics) representing the eigenfunctions of the angular part $\mathcal{L}$ of the Laplace operator Δ:

$$\mathcal{L} Y_{lm} + l(l+1) Y_{lm} = 0.$$

Substituting for ψ and χ Eqs. (9) and (10) lead to the equations for the radial parts g and h :

$$\frac{d^2}{dr^2} r g_l + k^2\left[n^2 - \frac{l(l+1)}{r^2 k^2}\right] r g_l = 0, \qquad \text{for } \vec{E}_r = 0, \tag{11}$$

$$\frac{d^2}{dr^2} r h_l - \frac{1}{n^2}\frac{dn^2}{dr}\cdot\frac{d}{dr} r h_l + k^2\left[n^2 - \frac{l(l+1)}{r^2 k^2}\right] r h_l = 0, \qquad \text{for } \vec{B}_r = 0. \tag{12}$$

The focused beam through a lens can be expanded in spherical harmonics. It has been shown in Ref. (3) that for high-quality beams and low f-number lenses only the first few polynomials $l \leq 10$ contribute appreciably to the intensity of the total beam. Furthermore we observe that by multiplying Eq.(11) by $-\frac{\partial}{\partial\vartheta} Y_{lm}(\varphi,\vartheta)$ or $\frac{1}{\sin\vartheta}\frac{\partial}{\partial\varphi} Y_{lm}(\varphi,\vartheta)$ it transforms into the corresponding wave equation for E_φ or E_ϑ $(E_r = 0)$. This shows that, if y of the approximate wave equation (8) is identified with one of the spherical harmonics times $g(r)$, the error in Eq.(8) is $l(l+1)/k^2r^2$. This has to be compared with n^2. For pellets $kr_c \gtrsim 10^3$ and good beams one obtains

$$l(l+1)/k^2r^2 \lesssim 10^{-2},$$

which means that for $|n^2| \gtrsim 10^{-2}$ the field distribution is determined by the refractive index. The term $l(l+1)/k^2r^2$ is the contribution of the angular derivatives and it shifts the critical point (reflection point) from r_c to the very near value $r^2 = r_c^2 - l(l+1)/k^2\,\mathrm{Re}\,(n^2)$. Multiplying Eq. (12) by $\frac{\partial}{\partial\vartheta} Y_{lm}$ and $-\frac{1}{\sin\vartheta}\frac{\partial}{\partial\varphi} Y_{lm}$ it transforms into the corresponding wave equation for the spherical harmonic component of the magnetic field B_φ and B_ϑ respectively, which, when summed, yield

$$\frac{d^2}{dr^2} r\vec{B}_{lm} - \frac{1}{n^2}\frac{dn^2}{dr}\cdot\frac{d}{dr} r\vec{B}_{lm} + k^2\left\{n^2 - \frac{l(l+1)}{k^2r^2}\right\} r\vec{B}_{lm} = 0. \tag{13}$$

It has been numerically shown in Ref. (3) that for high-quality beams, i.e. low l-values, Eq. (13) yields the same absorption as Eq. (11). With the help of Maxwell´s equation (2) the following equation for $R_{lm} = r^2 n^2 E_{r,lm}$ is easily derived from Eq. (13) :

$$\frac{d^2}{dr^2} R_{lm} - \frac{1}{n^2}\frac{dn^2}{dr}\frac{dR_{lm}}{dr} + k^2\left(n^2 - \frac{l(l+1)}{r^2}\right) R_{lm} = 0. \tag{14}$$

This states that the radial components of the electric field behave like $\vec{E}_{r,lm} \sim r^{-2}$ for r tending to infinity, in contrast to the tangential fields $\vec{E}_t, \vec{B}_t$ which vary as r^{-1} only.
From the treatment presented here it can be seen again that uniform illumination is not possible because none among the Y_{lm} would yield a constant radial flux intensity. Y_{00} is the only constant spherical harmonic, but its amplitude must be zero because the total charge of the pellet vanishes.

c) Light pulses

In the foregoing sections only one Fourier component of frequency ω has been considered. The amplitude of a light pulse itself varies in time :

$$\vec{E}(\vec{r},t) = \hat{E}(\vec{r},t)\, e^{-i\omega t}, \tag{15}$$

which gives rise to a frequency spread $\Delta\omega \simeq 2\pi/\tau$, τ pulse duration).
Such an ansatz makes sense, of course, only if $\tau \gg 1/\omega$ is fulfilled. One can now ask how the wave equation (5) is changed in the case of a quasistationary pulse of the form (15). To answer this question, we observe that

$$\dot{\vec{x}} = -\frac{e}{m_e(\nu - i\omega)}\hat{E}(t)\,e^{-i\omega t} + \frac{e}{m_e(\nu - i\omega)}e^{-\nu t}\int^{t}\dot{\hat{E}}(t)\,e^{(\nu - i\omega)t}\,dt \tag{16}$$

is an exact solution of the equation of motion of a single electron with collision frequency ν:

$$\ddot{\vec{x}} + \nu\dot{\vec{x}} = -\frac{e}{m_e}\hat{E}(t)\,e^{-i\omega t}.$$

For clarity we indicate the amplitudes which vary slowly in time by the sign $\wedge$, i.e.

$$\vec{E}, \vec{B}, \vec{j} = \hat{E}(t), \hat{B}(t), \hat{j}(t)\,e^{-i\omega t}.$$

The current density $\vec{j}$ is then deduced from expression (16)

$$\hat{j} = -n_e e\dot{\vec{x}}\,e^{i\omega t} = \sigma\hat{E} - \sigma\,e^{-(\nu - i\omega)t}\int^{t}\dot{\hat{E}}\,e^{(\nu - i\omega)t}dt. \tag{17}$$

The electrical conductivity has the usual meaning for a pure Fourier component

$$\sigma = \frac{n_e e^2}{m_e(\nu - i\omega)}.$$

The time-varying Maxwell´s equations are

$$\nabla\times\hat{B} = \frac{1}{\varepsilon_0 c^2}\hat{j} - i\frac{\omega}{c^2}\hat{E} + \frac{1}{c^2}\dot{\hat{E}}, \tag{18}$$

$$\nabla\times\hat{E} = i\omega\hat{B} - \dot{\hat{B}}. \tag{19}$$

Combining Eqs. (17), (18), (19), we get the modified wave equation

$$\nabla\times\nabla\times\hat{E} - k^2n^2\hat{E} = 2i\frac{k}{c}\dot{\hat{E}} - \frac{\nu\sigma}{\varepsilon_0 c^2}e^{-(\nu - i\omega)t}\int^{t}\dot{\hat{E}}\,e^{(\nu - i\omega)t}dt - \frac{1}{c^2}\ddot{\hat{E}}. \tag{20}$$

The integral in this formula is best evaluated by integrating by parts, which essentially yields the term

$$i\frac{\nu}{c^2}(1 - n^2)\frac{\omega}{\nu - i\omega}\dot{\hat{E}}.$$

In most cases this equals $-(1-n^2)\frac{\nu}{c^2}\dot{\hat{E}}$ because of $\nu \ll \omega$. Therefore, Eq.(20) reduces to

$$\nabla\times\nabla\times\hat{E} - k^2n^2\hat{E} \simeq 2i\frac{k}{c}\dot{\hat{E}}, \tag{20a}$$

showing that the stationary wave equation is also a very good approximaition for light pulses since the correction is of the order of λ/l, where $l = c\tau$ is the pulse length. Only in the case of a refractive index which is nearly zero over a wide range does the quasi-stationary state be taken into account. Physically, this means that in such a situation the fields can grow to considerable intensity in the region of $n \simeq 0$ and they can store so much energy that the time τ is not sufficient to yield the energy input calculated from the stationary wave equation (5).

C. The Optical WKB Approximation and the Stokes Equation

An approximate solution of equation (8) is given by the WKB ansatz

$$y = \frac{C_1}{\sqrt{n}} e^{-ik\int_{r_0}^{r} n\,dr} + \frac{C_2}{\sqrt{n}} e^{ik\int_{r_0}^{r} n\,dr} . \tag{21}$$

$C_1/r\sqrt{n}$ is the amplitude of the incident wave and $C_2/r\sqrt{n}$ is that of the reflected wave. This expression would represent the exact solution if the refractive index f were of the form

$$f^2 = n^2 + \frac{1}{4k^2}\left\{2\frac{n''}{n} - 3\left(\frac{n'}{n}\right)^2\right\} =: n^2 + n_1^2 . \tag{22}$$

By differentiating y twice the reader can easily verify this result. The WKB approximation is valid as long as n_1^2 is small in comparison to n^2, which is the case when the gradient of the refractive index remains moderate and as long as n^2 differs from zero. In the region of $n^2 \simeq 0$, around the critical point, the approximation (21) breaks down because n'/n no longer remains small.

If one considers a collision free plasma with a smooth electron density, increasing from zero to a high value $n_e \gg n_c$, n^2 starts from unity, passes through zero and becomes finally strongly negative. In both regions, where (I) n is nearly unity as well as where (II) $n^2 \ll 0$ holds, the condition for the validity of formula (21) is fulfilled. If only the wave with amplitude $C_1/r\sqrt{n}$ is incident onto the plasma C_2 must vanish in region II because it would represent there an indefinitely growing wave $(n = i\sqrt{-n^2})$. In region I it follows from the energy conservation that $|C_2| = |C_1|$, with a phase factor between C_1 and C_2. When no absorption is present the incident wave is totally reflected in the critical region and an evanescent wave tunnels into the overdense region. As long as absorption is not strong (e.g. collisional absorption present only) the situation does not alter substantially. This consideration reveals clearly that expression (21) cannot be valid in the critical region for the constants change in passing from region I to region II. The WKB approximation represents the superposition of an incident and a reflected wave which do not interact. It is interesting to note that one possible system of differential equations equivalent to the wave equation is the following

$$\begin{aligned} \frac{dy_1}{dr} + ikn\left(1 - i\frac{n'}{2kn^2}\right)y_1 &= \frac{n'}{2n}y_2 \\ \frac{dy_2}{dr} - ikn\left(1 + i\frac{n'}{2kn^2}\right)y_2 &= \frac{n'}{2n}y_1 . \end{aligned} \tag{23}$$

Thereby y_1 and y_2 represent the incident and the reflected wave, the sum of them, $y = y_1 + y_2$, giving the total field of equation (8). If, for a first approximation, in the upper equation the reflected wave y_2 is ignored and in the lower equation the incident wave is suppressed the solution of this system is exactly the WKB approximation (21)*.

The WKB approximation does not hold in regions of large gradients of the refractive index or when n^2 approaches zero. Furthermore the ratio C_2/C_1 remains undetermined. One way to do both, i.e. to calculate the electric field in the critical region and to determine the constants C_1, C_2 is the following. The refractive index n^2 is approximated by a linear function around the point $\mathrm{Re}(n^2) = 0$,

$$n^2(r) = q(r - r_c)\,, \qquad q = \frac{d}{dr}\mathrm{Re}(n^2) + i\frac{d}{dr}\left|\mathrm{Im}(n^2)\right|_{r=r_c}\,,$$

and the space coordinate is conveniently substituted by the dimensionless variable

$$\xi = (k^2 q)^{1/3}(r - r_c)$$

in order to transform the wave equation (8) into the Stokes differential equation

$$\frac{d^2 y}{d\xi^2} + \xi y = 0\,. \qquad (24)$$

The solution of this equation is given by the Airy functions $Ai(\xi)$ and $Bi(\xi)$ which as power series assume the form[6]

$$Ai(\xi) = \frac{f(\xi)}{3^{2/3}\,\Gamma(2/3)} - \frac{g(\xi)}{3^{1/3}\,\Gamma(1/3)}$$

$$Bi(\xi) = \frac{f(\xi)}{3^{1/6}\,\Gamma(2/3)} + \frac{g(\xi)}{3^{-1/6}\,\Gamma(1/3)} \qquad (25)$$

with $$f(\xi) = 1 - \frac{1}{3!}\xi^3 + \frac{1\cdot 4}{6!}\xi^6 - \frac{1\cdot 4\cdot 7}{9!}\xi^9 + \frac{1\cdot 4\cdot 7\cdot 10}{12!}\xi^{12} \mp \ldots\ldots$$

and $$g(\xi) = -\xi + \frac{2}{4!}\xi^4 - \frac{2\cdot 5}{7!}\xi^7 + \frac{2\cdot 5\cdot 8}{10!}\xi^{10} \mp \ldots\ldots\ldots$$

$Ai(\xi)$ is evanescent in the overdense region $(\xi < 0)$ (see Fig.2) whereas $Bi(\xi)$ diverges there. For $|\xi|$ sufficiently large $Ai(\xi)$ and $Bi(\xi)$ must be represented by the asymptotic formulae of the WKB type (21), $Ai,\ Bi \sim \xi^{-1/4} e^{\pm\frac{2}{3} i \xi^{3/2}}$.

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*) It is not commonly known that there exists an infinite variety of systems equivalent to the equations (23), all giving the correct result y_1 and y_2 at infinity but differing considerably in the critical region, and no physical distinction can be made between "correct" and "wrong" solutions although they yield very different reflection coefficients $R = |y_2/y_1|^2$ in that region. The explanation for this fact is that in regions where y departs considerably from the WKB approximation the definition of a unique reflection coefficient is no longer possible.[5]

In contrast to the exact expressions (25), which are single valued, their asymptotic expansions are multiple-valued functions of ζ . Three different domains must be distinguished in the complex ζ-plane, as sketched in Fig. 1. The domains are separated from each other by the "anti-Stokes lines" emanating from the origin under angles of $\frac{\pi}{3}$, π and $\frac{5}{3}\pi$.

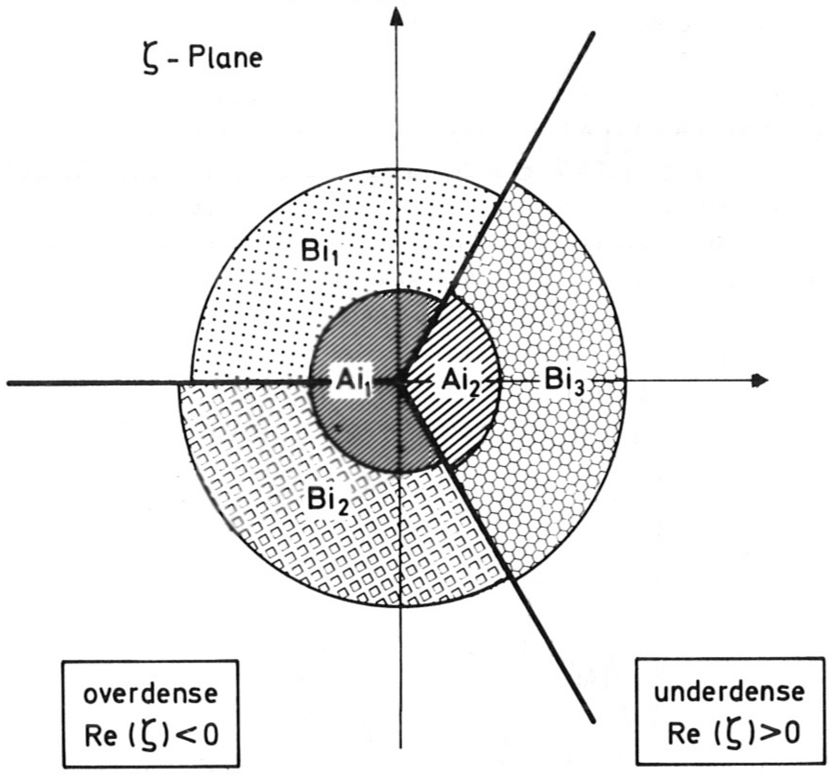

Fig. 1. Domains of validity for the different asymptotic expansions of the Airy functions $Ai(\zeta)$ and $Bi(\zeta)$. The "anti Stokes lines" emanate from the origin under angles of $\pi/3$, π and $5\pi/3$.

The asymptotic expansions of $Ai(\zeta)$ and $Bi(\zeta)$ for the domains indicated in the figure are the following, with $\eta = \frac{2}{3}\zeta^{3/2}$ and the normalization factor $g = 1/2\sqrt{\pi}$:

$$
\begin{aligned}
Ai_1 &= g\zeta^{-1/4} e^{-i\eta} & \pi/3 \leq \arg\zeta \leq 5\pi/3 \\
Ai_2 &= g\zeta^{-1/4}\{e^{i\eta} + ie^{-i\eta}\} & -\pi/3 \leq \arg\zeta \leq \pi/3 \\
Bi_1 &= g\zeta^{-1/4}\{2ie^{i\eta} + e^{-i\eta}\} & \pi/3 \leq \arg\zeta \leq \pi \\
Bi_2 &= g\zeta^{-1/4}\{ie^{-i\eta} + 2e^{i\eta}\} & \pi \leq \arg\zeta \leq 5\pi/3 \\
Bi_3 &= g\zeta^{-1/4}\{ie^{i\eta} + e^{-i\eta}\} & -\pi/3 \leq \arg\zeta \leq \pi/3
\end{aligned}
\qquad (26)
$$

The angle of $\xi^{3/2}$ has to be taken in the following manner : ⁑

$$\arg \xi^{3/2} = \frac{3}{2} \arg \xi$$

By inserting these asymptotic solutions in the Stokes equation it is directly seen that they can be expected to represent good approximations as soon as

$$|n_1^2/n^2| = \frac{5}{16|\xi|^3} \ll 1 \ ;$$

for $\xi=1.5$ the inequality yields $|n_1^2/n^2| = 0.1$ i.e. it is well fulfilled. In addition, a direct comparison shows that the approximation is even better. In Fig.2 the Airy function $Ai(\xi)$ with ξ real is compared with its asymptotic expansion $Ai_2(\xi)$ which is valid in the underdense region. As soon as $\xi \geqslant 1.3$ the relative error $|Ai - Ai_2|/|Ai|$ becomes less than 1.5%. Even in the maximum of Ai which lies at $\xi = 1.045$ the deviation is only 4%. The deviation starts becoming exponentially large at ξ as low as 0.5. It is typical for the WKB approximation in general that even the maximum E-field and its location are approximately reproduced, as in the special case of Fig.2.

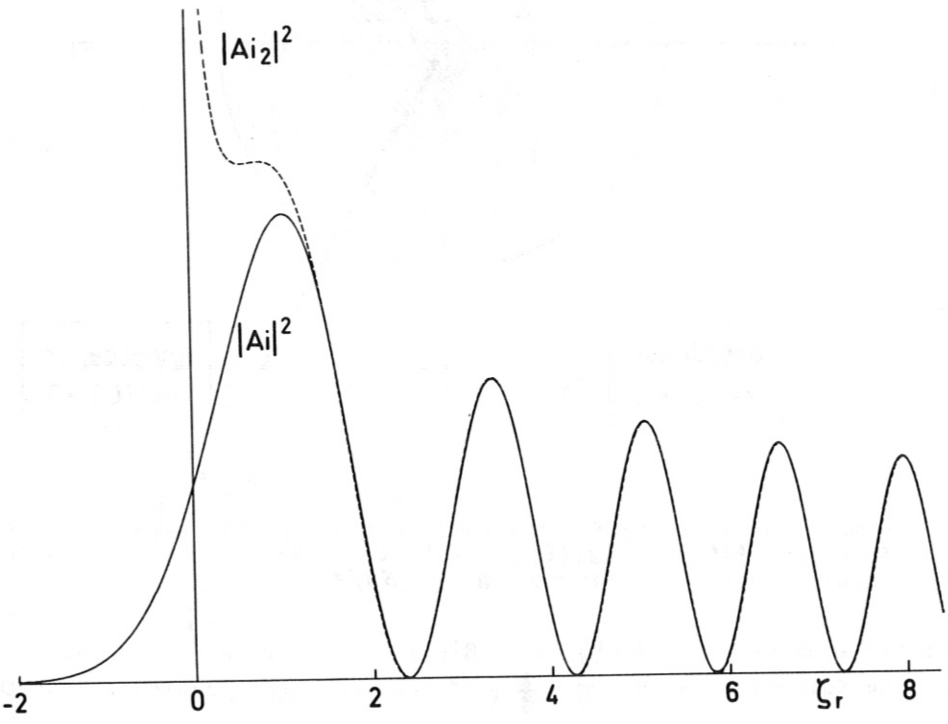

Fig. 2. Comparison of the asymptotic expansion $Ai_2 \cdot Ai_2^*$ (dashed line) with the Airy function $Ai \cdot Ai^*$. ξ is taken real (no absorption). For $|\xi| \geqslant 1.5$ the relative error $||Ai|^2 - |Ai_2|^2|/|Ai|^2$ is less than 1%. The standing wave pattern is generated by total reflection in the critical region around $\xi = 0$.

⁑ The asymptotic formulae are derived for example in Ref.7. In order to get our representation it must be taken into account that between Budden's variable ξ_B and ours the relations hold $\arg \xi = \arg \xi_B + \pi$ and $\xi_B^{3/2} = i\, \xi^{3/2}$. By these rules the Stokes lines transform into our anti-Stokes lines.

If there is only one wave incident from the underdense region a standing wave pattern is formed due to reflection in the critical region, and in the overdense region the wave becomes purely evanescent if this region is large enough. Since $Bi(\xi)$ is not bounded for $|\xi|<0$ the solution of the Stokes equation for our boundary conditions is given by $Ai(\xi)$ alone.
Consequently the right asymptotic solutions of Eq.(24) are Ai_1 for the overdense region and Ai_2 for the underdense region. Their accuracy becomes acceptable as soon as $|\xi|$ is larger than unity. At $|\xi|=1$ the relative error is nearly 5%.

Let us now assume a wave of unity amplitude incident from a remote point r_0 and let the amplitude of the reflected wave be S. Then from the requirement that the electric field must be continuous we get at a position r with the corresponding $|\xi|$ sufficiently large

$$\frac{1}{\sqrt{n}}e^{-ik\int_{r_0}^{r}ndr}+\frac{S}{\sqrt{n}}e^{ik\int_{r_0}^{r}ndr}=C\frac{q}{\xi^{1/4}}\{ie^{-i\eta}+e^{i\eta}\}. \tag{27}$$

In order to satisfy this relation the incident wave at the RHS must be equal to the incident wave at the LHS, and the same must be true for the reflected wave since, within the limit of the WKB approximation the two waves do not interact. By eliminating the factor C we obtain for the amplitude ratio S

$$S=-i\,e^{2i\{\eta+k\int_{r}^{r_0}ndr\}}. \tag{28}$$

The reflection coefficient R for the intensity is given by

$$R=SS^*=e^{-4\{\eta_i+k\int_{r}^{r_0}n_idr\}} \tag{29}$$

$(\eta_i=\mathrm{Im}(\eta),\ n_i=|\mathrm{Im}(n)|)$. The position r is somewhat arbitrary. On one hand $r-r$ must not be too small for the validity of the WKB approximation; on the other hand it should not be too large in order to keep the difference between n^2 and its linear approximation small. This latter deviation can be evaluated by taking into account the quadratic term in the expansion of n^2. The coefficient S would be independent of r if expression (21) were an exact solution; if the approximation is good the quantity $\left|\frac{1}{S}\frac{dS}{dr}\right|\Delta r$ has to be small with respect to unity. A straightforward calculation yields

$$\left|\frac{1}{S}\frac{dS}{dr}\right|\Delta r<\left|\frac{1}{S}\frac{dS}{dr}\right|(r-r_c)=\frac{1}{2}\left|q'/(kq^2)^{2/3}\xi^{5/2}\right|\ll 1\ ,\quad q'=\left.\frac{dq}{dr}\right|_{r_c}. \tag{30}$$

The inequality (30) is certainly fulfilled if

$$|q'/(kq^2)^{2/3}|\ll 1.$$

holds

<u>Critical remark on the reflection formula (29)</u>

If n_1^2 from the expression (22) is added to a given n^2 the resulting refractive index profile becomes non-reflecting because the incident wave of expression (21) alone represents an exact solution in the whole intervál. In Ref. (8) it has been shown for an Epstein layer of 10λ thickness that n_1^2 can be as small as 2×10^{-3}. Such considerations show that in calculating reflection one has to be careful in general. On the other hand one could deduce from this example that small corrections of the refractive index profile could alter completely the value of R which would invalidate formula (29). However, that is not the case. In the foregoing considerations we have tacitely assumed that n^2 can be approximated by a linear function in the critical region. In our special case this means that $|dn/dr|$ is very large (or infinite) and that the quadratic term in the Taylor series of n^2 is small (condition(30)).Therefore a very large $|n_1^2/n^2|$ results ; consequently the situation of Ref. (8) is possible only in a region with n^2 clearly different from zero. But there the WKB approximation holds (smooth n^2 profile) and practically no local reflection is generated.

Numerical solution of the wave equation

The numerical solution of Eq. (8) is straightforward if at one point r the values of y and its derivative y' are given. However, this is not the usual case. Normally, the amplitude of the incident wave is known with the additional limitation that no light is incident from the opposite side or the center. Neither y nor y' are known in this case.

For the solution of the wave equation with the boundary conditions from above it is convenient to split the second order equation into two first order equations of the following form

$$\begin{aligned} dy/dr + ikz &= 0 \\ dz/dr + ikn^2 y &= 0. \end{aligned} \qquad (31)$$

It can easily be proved that system (31) follows directly from the equations (23) by the substitution

$$z = n(y_1 - y_2).$$

The calculation proceeds in the following manner. At the back side of the irradiated slab or sufficient far away from the critical point the solution of Eqs.(31) is started with $z = ny = 1$ and is solved in the desired interval. This boundary condition means zero amplitude of the reflected wave y_2 at the starting point. After the complete solution $y = y_1 + y_2$ is normalized to the incident wave. This procedure is only applicable, of course, as long as the wave equation is linear. If, e.g. the influence of light pressure on a plasma is studied this is no longer true, for n^2 becomes a function of the electric field (see section E). In Fig. 3. the numerical solution of the equation

$$y'' + (\xi_r + 0.2i)y = 0$$

is compared with the analytical result. The equation has been splitted according (31) and integrated by the Runge-Kutta procedure. Excellent agreement is found, although seven points only have been used per unit ξ_r

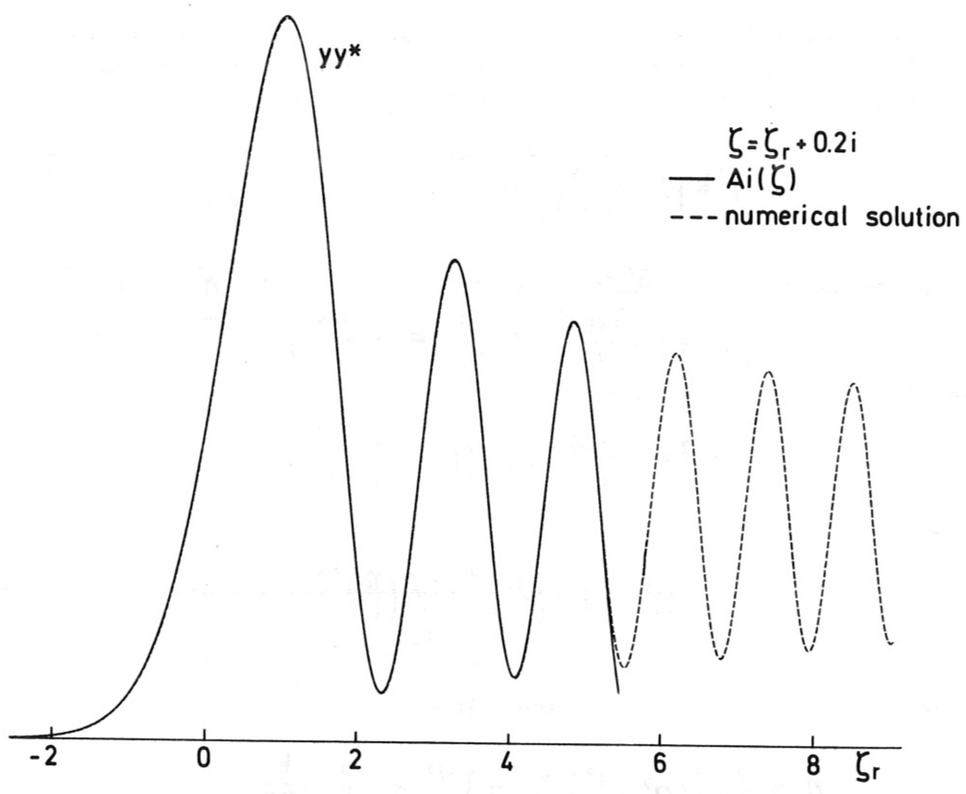

Fig. 3. Comparison of the numerical solution of the equation $y'' + (\zeta_r + 0.2i)y = 0$ by the Runge-Kutta procedure (dashed line) with its exact solution. Per unit ζ_r seven points have been taken only. The Airy function has been calculated according formula (25), which, with the number of terms used, did not converge beyond $\zeta_r = 5$.

D. Special Electron Density Profiles

a) Reflection coefficients

We shall now apply formula (29) to special profiles of the electron density n_e. We calculate R for a linear profiles of n_e

$$n_e = n_c\left(1-\frac{r-r_c}{L}\right),$$

L being the length of the profile, and for an electron density distribution

$$n_e = n_c\left(\frac{r_c}{r}\right)^{\alpha}$$

with $\alpha=1,2,3$. Among these exponents the value $\alpha=2$ is of particular interest because an isothermal steady-state rarefaction wave shows nearby such a dependence[9]. In the latter case the characteristic scale length L is given by

$$L = \left|\frac{1}{n_e}\frac{dn_e}{dr}\right|_{r_c}^{-1} = \frac{r_c}{\alpha}.$$

It can always be assumed that $A^2 \ll 1$ holds ; consequently, n^2 is given by

$$n^2 = 1-\frac{n_e}{n_c} + iA\frac{n_e^2}{n_c^2} =: a+ib,$$

which gives for the linear profile

$$n^2 = \frac{r-r_c}{L} + iA\left(1-\frac{r-r_c}{L}\right)^2$$

and for the second type

$$n^2 = 1-\left(\frac{r_c}{r}\right)^{\alpha} + iA\left(\frac{r_c}{r}\right)^{2\alpha}.$$

The imaginary part of the refractive index n is determined from

$$n_i = \frac{1}{\sqrt{2}}\left\{(a^2+b^2)^{1/2}-a\right\}^{1/2} \simeq \frac{1}{2}\frac{b}{\sqrt{a}}. \qquad (32)$$

For the correction $e^{-4\eta_i}$ around the critical point the expression for ξ from section C is needed. It is calculated to be

$$\xi = (kL)^{2/3}\frac{r-r_c}{L}(1+2iA)^{1/3} \simeq \xi_r\left(1+\frac{2}{3}iA\right) \qquad (33)$$

for the linear profile. For the electron distribution $n_e \sim r^{-\alpha}$ formula (33) still holds if the scale length is replaced by $L=r_c/\alpha$. The WKB approximation is well fulfilled for $\xi_r \gtrsim 1.3$. From expression (33) it follows that the point $\xi=1.3$ is sufficiently far away from the critical point to ensure that $b^2/a^2 \ll 1$, and so the Taylor expansion on the RHS of Eq.(32) for n_i can be used. In fact, one has

$$b^2/a^2 \simeq A^2(kL)^{4/3}/\xi_r^2$$

for both kinds of profiles. Generally, b^2/a^2 is much smaller than unity. If this condition is not fulfilled, ξ_r has to be increased. It is evident that condition (30) is well satisfied for linear electron density profiles ; for the second type of profile condition (30) becomes identical with $(kL)^{-2/3} \ll 1$ which holds for not too small pellets.

The imaginary parts of the refractive indices are given by

$$\eta_i = \frac{A}{2} \frac{[1-\frac{r_c}{L}(x-1)]^2}{(\frac{r_c}{L})^{1/2}(x-1)^{1/2}}, \qquad x = \frac{r}{r_c}, \tag{34}$$

and

$$\eta_i = \frac{A}{2} \frac{x^{2\alpha}}{(1-x^{\alpha})^{1/2}}, \qquad x = \frac{r_c}{r} \tag{35}$$

respectively .

1. The linear profile

The integral over η_i can be expressed in closed form to give

$$\int_{r>r_c}^{r_c+L} \eta_i dr = \frac{AL}{2} \int_{u>0}^{1} u^{-1/2}(1-u)^2 du = \frac{AL}{2}\left\{\frac{16}{15} - 2u^{1/2} + \frac{4}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right\}, \quad u = \frac{r_c}{L}\left(\frac{r}{r_c}-1\right) = \frac{\xi}{(kL)^{2/3}}.$$

The reflection coefficient is then

$$R_L = e^{-2kLA\left\{\frac{16}{15} - 2\frac{\xi_r^{1/2}}{(kL)^{1/3}} + \frac{8}{3}\frac{\xi_r^{3/2}}{kL}\right\}}. \tag{36}$$

The $u^{5/2}$ term can be disregarded. In the following, we choose $\mathrm{Re}(\eta) = 1$; the corresponding ξ_r is then

$$\xi_r = 1.31$$

and the reflection coefficient reduces to

$$R_L = e^{-2kLA\left\{\frac{16}{15} - \frac{2.3}{(kL)^{1/3}} + \frac{4}{kL}\right\}}. \tag{36'}$$

2. The $n_e = n_c(r_c/r)$ profile

The integral over η_i is given by

$$\int_{r>r_c}^{\infty} \eta_i dr = \frac{Ar_c}{2}\int_0^{x<1} \frac{dx}{(1-x)^{1/2}} = Ar_c\left\{1-(1-x)^{1/2}\right\},$$

which, by substitution, leads to the reflection coefficient for the intensity

$$R_1 = e^{-4kr_cA\left\{1 - \frac{\xi_r^{1/2}}{(kr_c)^{1/3}} + \frac{\frac{2}{3}\xi_r^{3/2}}{kr_c}\right\}}, \tag{37}$$

or, with ξ_r as fixed above,

$$R_1 = e^{-4kr_cA\left\{1 - \frac{1.14}{(kr_c)^{1/3}} + \frac{1}{kr_c}\right\}}. \tag{37'}$$

3. The $n_e = n_c(r_c/r)^2$ profile (spherical rarefaction wave)

For the case $\alpha = 2$ one has

$$\int_{r>r_c}^{\infty} n_i\,dr = \frac{Ar_c}{2}\int_0^{x<1}\frac{x^2dx}{(1-x^2)^{1/2}} = \frac{Ar_c}{4}\{\arcsin x - x(1-x^2)^{1/2}\} \simeq \frac{Ar_c}{4}\{\frac{\pi}{2} - 2\cdot\sqrt{2}(1-x)^{1/2}\}$$

and the reflection coefficient R_2 is

$$R_2 = e^{-kAr_c\{\frac{\pi}{2} - 2^{4/3}\frac{\xi_r^{1/2}}{(kr_c)^{1/3}} + \frac{8}{3}\frac{\xi_r^{3/2}}{kr_c}\}} \tag{38}$$

or

$$R_2 = e^{-kr_cA\{\frac{\pi}{2} - \frac{2.9}{(kr_c)^{1/3}} + \frac{4}{kr_c}\}}. \tag{38'}$$

4. The $n_e = n_c(r_c/r)^3$ profile (asymptotic spherical rarefaction wave)

In this case the integral $\int n_i dr$ is no longer expressible in closed form ; however, it can be evaluated with the help of the gamma function :

$$\frac{2}{r_cA}\int_{r>r_c}^{\infty} n_i\,dr = \int_0^{x<1}\frac{x^4dx}{(1-x^3)^{1/2}} = \int_0^1\frac{x^4dx}{(1-x^3)^{1/2}} - \int_x^1\frac{x^4dx}{(1-x^3)^{1/2}} = \frac{4}{7}\int_0^1\frac{xdx}{(1-x^3)^{1/2}} - \{\frac{2}{7}x^2(1-x^3)^{1/2} +$$

$$\frac{4}{7}\int_x^1\frac{xdx}{(1-x^3)^{1/2}}\} \simeq \frac{4^{2/3}\cdot 3^{1/2}}{7\pi}\Gamma^2(2/3) - \{\frac{2}{\sqrt{3}}(1-x)^{1/2} - \frac{8}{21\sqrt{3}}(1-x)^{3/2}\}.$$

The integral from x to 1 was approximated by $\frac{8}{7\sqrt{3}}(1-x)^{1/2}[1+\frac{1}{3}(1-x)]$. Finally, the following reflection coefficient results :

$$R_3 = e^{-kr_cA\{2\frac{4^{2/3}\cdot 3^{1/2}}{7\pi}\Gamma^2(2/3) - 4\frac{1}{3^{1/3}}\frac{\xi_r^{1/2}}{(kr_c)^{1/3}} + \frac{152}{63}\frac{\xi_r^{3/2}}{kr_c}\}}, \tag{39}$$

or

$$R_3 = e^{-kr_cA\{0.73 - \frac{2.2}{(kr_c)^{1/3}} + \frac{2.4}{kr_c}\}}. \tag{39'}$$

The reflection coefficients calculated here are equally valid, of course, for plane targets with the same electron density profiles. This is a consequence of the wave equation (8), which, in our approximation, has the same form for plane and spherical geometries. The corrections in the reflection formulae are of the form $\left(\xi_r/(kL)^{2/3}\right)^{\nu}$ with $\nu = 1/2$, 1, $3/2$ etc. They show that, although local absorption is highest in the critical region, the WKB approximation is an acceptable method for calculating R of not too small pellets. For larger pellets it holds that $(kr_c)^{-1/3} \ll 1$ and the reflection coefficients reduce to

$$
\begin{aligned}
R_L &= e^{-\frac{32}{15}kLA}, && n_e \text{ linear} \\
R_1 &= e^{-4kr_cA} = e^{-4kLA}, && n_e = n_c\left(\frac{r_c}{r}\right) \\
R_2 &= e^{-\frac{\pi}{2}kr_cA} = e^{-\pi kLA}, && n_e = n_c\left(\frac{r_c}{r}\right)^2 \\
R_3 &= e^{-0.73kr_cA} = e^{-2.19kLA}, && n_e = n_c\left(\frac{r_c}{r}\right)^3.
\end{aligned}
\tag{40}
$$

The critical radius has thereby been replaced by the characteristic length $L = r_c/\alpha$. If the absorption is low, i.e. kr_cA is small, all four formulae give results within an acceptable range. However, in the case of strong absorption the formulae (40) show that the form of the density profile very much influences the result.
The parameter characterizing absorption is the product kr_cA. From this expression the wavelength dependence can easily be studied under various conditions. If, for example, the critical radius is kept constant the absorption is independent of λ for a given density profile. In fact k varies as λ^{-1} and $A = \nu_{coll}/\omega$ is proportional to λ. On the other hand, from formulae (40) a very strong dependence of R on the ion charge number Z_{eff} should be expected. Since $A \sim Z_{eff}$ holds, under identical conditions for the rest, the reflection coefficients for deuterium and a high Z material are related by the power law

$$R_{Z_{eff}} = R_{D_2}^{Z_{eff}}.$$

If heat conduction plays a role this relation is drastically modified.[9]

b) Electric field increase

As discussed in section C, the electromagnetic field may increase considerably near the critical point. We describe the field maximum by the factor β, defined as

$$\beta = \frac{|\hat{E}_{max}|_{Plasma}}{|\hat{E}|_{Vacuum}} = \frac{|y_{max}|_{Pl}}{|y|_{Vac}},$$

i.e. as the ratio between the maximum E field in the plasma and the vacuum field at the same position. Let the incident wave have an amplitude of unity. Then at a suitable position r Eq. (27) holds, from which the scaling factor C is determined :

$$|C| = \frac{1}{g}\left|\frac{\xi^{1/4}}{\sqrt{n}}\right| e^{-(k\int_r^\infty n_i dr + \eta_i)} = 2\sqrt{\pi}\, k^{1/6} \left|\frac{dn^2}{dr}\right|_{r_c}^{-1/6} \cdot R^{1/4}.$$

Keeping in mind that $\beta = |\hat{E}_{max}|_{Plasma} = |C|\cdot|Ai|_{max} = 0.54\,|C|$, for the field increase β the following expression results:

$$\beta = 2\times 0.54\sqrt{\pi}\; k^{1/6} |q|^{-1/6} \cdot R^{1/4} \tag{41}$$

This formula is valid for an arbitrary electron density distribution that is smooth. Its limitation consists only in the approximation of Eq. (8) by the Stokes equation (24) in the critical region, the validity of which is guaranteed by condition (30). In the case of considerable density profile distortions formula (41) fails (see the next section). For our special density profiles we obtain for β

$$\beta = 1.90\,(kL)^{1/6} R^{1/4}, \qquad L \text{ linear or } L = r_c/\alpha. \tag{42}$$

The dependence of β on R is very weak. It reflects the author´s experience that in numerical calculations, even with considerable absorption, the field still increased towards the critical point. In fact, when, for example, 90% of light is absorbed (R = 0.1), β reduces by a factor of nearly 2 only with respect to the non-absorbing case. The knowledge of the β factor may be useful for calculating threshold intensities of parametric instabilities or local light pressure effects.

E. Light Pressure Effects

By light pressure we mean the integral of Lorentz force density κ, averaged over the fast oscillation factor $e^{-i\omega t}$, i.e.

$$\kappa = \langle (n_i - n_e) e\vec{E} \rangle + \langle \vec{j} \times \vec{B} \rangle . \tag{43}$$

By substituting the charge and current densities in this expression from Maxwell´s equations and taking into account that $\nabla\vec{B} = 0$ one obtains

$$\kappa + \langle \varepsilon_0 \frac{\partial}{\partial t}(\vec{E} \times \vec{B}) \rangle = \langle \varepsilon_0 \{ \vec{E}(\nabla\vec{E}) - \vec{E} \times (\nabla \times \vec{E}) + c^2\vec{B}(\nabla\vec{B}) - c^2\vec{B} \times (\nabla \times \vec{B}) \} \rangle \tag{44}$$

By suitable transformations the RHS term of Eq.(44) can be written as the divergence of the stress tensor $\mathbf{T}$ with the components

$$T_{ij} = \varepsilon_0 [E_i E_j + c^2 B_i B_j - \frac{1}{2} \delta_{ij} (\vec{E}^2 + c^2 \vec{B}^2)]$$

and therefore the term $\varepsilon_0 \vec{E} \times \vec{B} = \frac{1}{c^2} \vec{S}$ can be interpreted as the electromagnetic momentum density. In vaccum it is responsible for the fact that the ponderomotive force κ vanishes, in agreement with Eq. (43). When the time averaged expressions are used, the $\frac{1}{c^2}\vec{S}$ term can nearly always be neglected. From Poynting´s theorem it follows that this term is $(\Delta x / c\tau)^2$ times less than the RHS expression of eq. (44), where τ is the pulse lenght and Δx is the characteristic scale length of the spatial structure; owing to interference of waves Δx can become as small as the local wavelength.

In most cases the ionic component of $\vec{j}$ can be omitted, which means that light pressure acts on the electrons and is then transmitted to the ions by an electrostatic field $\vec{E}_s$. Generally, the $\vec{E}$ field of Eq. (44) is the sum of the electromagnetic field as well as $\vec{E}_s$, $\vec{E} = \hat{E} + \vec{E}_s$, which makes calculation of the correct light pressure a difficult task. However, when $\vec{E}_s$ is not resonant, i.e. ω_{pe} differs sufficiently from ω, $\vec{E}_s$ can be neglected in the expression for κ. In fact, it holds that

$$m_e n_e \frac{d\vec{v}_e}{dt} = -\nabla p_e - n_e e \vec{E} + \vec{j}_e \times \vec{B} , \tag{45}$$

$$m_i n_i \frac{d\vec{v}_i}{dt} = -\nabla p_i + n_i e \langle \vec{E}_s \rangle . \tag{46}$$

Averaging in time the first equation over the fast oscillations,

$$m_e n_e \frac{d\langle \vec{v}_e \rangle}{dt} = -\nabla p_e - n_e e \langle \vec{E}_s \rangle + \langle \vec{j}_e \times \vec{B} \rangle , \tag{47}$$

it is seen by comparison with Eq. (46) that the LHS of Eq. (47) can be put equal to zero since $\langle \vec{v}_e \rangle \simeq \vec{v}_i$. But then the electrostatic field produced by the ponderomotive force is determined by

$$n_e e \langle E_s \rangle = |\vec{j} \times \vec{B}| = n_e e v_{osc} B \simeq n_e e v_{osc} \cdot \frac{\hat{E}}{c}$$

or

$$|E_s/\hat{E}| \simeq \frac{v_{osc}}{c} \ll 1 .$$

In the resonant case, v_{osc} can become very large. In the non-resonant case a Nd-glass laser intensity of $10^{18}\ W/cm^2$ is required to produce $v_{osc} = c$. Therefore, E_s can be neglected in (44). With the help of Eq. (2) and (3) Eq.(44) reduces to the following simple formula for the time averaged force density produced by the light wave :

$$\kappa = \frac{\varepsilon_0}{4}\left[\mathrm{Re}(n^2) - 1\right] \nabla |\hat{E}|^2, \tag{48}$$

where $\hat{E}$ is the complex amplitude of the field. The same result is obtained from single particle motion in the oscillation center approximation[10]. Since κ is proportional to the particle density, only in a homogeneous medium can κ be written as the gradient of a light pressure $p_L = \frac{\varepsilon_0}{4}\left[1 - \mathrm{Re}(n^2)\right] |\hat{E}|^2$. In an inhomogeneous medium such as a laserproduced plasma, the force density is given by

$$\kappa = -\nabla p_L - \frac{\varepsilon_0}{4} |\hat{E}|^2 \nabla \mathrm{Re}(n^2).$$

The influence of light pressure on laser plasma acceleration and plasma profile modifications has been studied by several authors[11-14]. Here we summarize only the main aspects and point out a few consequences.

I. It has been suspected time and again that light pressure in connection with the steepening of the electric field in the critical region should strongly accelerate the plasma created. However, it has been shown in Ref. 15 that this predicted acceleration is ineffective as long as the electrostatic field can be neglected (non-resonant case) and the pulse duration is long in comparison to the time an ion spends in crossing the first amplitude maximum of the light wave. In many experiments these conditions may be satisfied.

II. The effect of profile modification due to light pressure is best studied in an isothermal steady state model in spherical geometry[16]. Starting from the equations of mass and momentum conservation - the latter one containing the force term of Eq. (48)- the following relation is obtained for the Mach number $M = v/s$.

$$M^2 - \ln M^2 = M_0^2 - \ln M_0^2 + 2\mu\left(|\hat{E}_0|^2 - |\hat{E}|^2\right) + 4 \ln \frac{r}{r_0} , \tag{49}$$

$$\mu = e^2/(4 m_e m_i \omega^2 s^2), \quad 2\mu |\hat{E}|^2 = 3\langle v_{osc}^2 \rangle / 2\langle v_{thermal}^2 \rangle ;$$

the index "0" refers to an arbitrary but fixed radius r_0 . The electric field amplitude has to be obtained from the wave equation (8). Absorption can thereby be disregarded as long it is collisional only and/or weak.

Without light pressure the spherical rarefaction wave is supersonic everywhere $(M \geq 1)$.

If light pressure is taken into account two cases have to be distinguished :

a) The flow in the overdense region is supersonic $(M_0 > 1)$. In this case a density plateau forms just below the critical density. Its maximum length Δr is given by

$$\Delta r / r_0 = M_0^{1/2} - 1 .$$

If one increases the laser intensity further, no steady state plateau-like solution exists[16].

b) The flow in the overdense region is subsonic $(M_0 < 1)$. In this case a step-like density solution builds up and the flow passes from $M < 1$ to $M > 1$ values very near the maximum of the electric field amplitude. Since the width of the step Δr is only of small extent, the term $4 \ln r/r_0$ in Eq.(49) can be disregarded as long as only the step structure is considered (see Figs. 4 and 5 ; $\Delta r \simeq \lambda$ and $\lambda/5$ respectively).This simplification corresponds to the assumption of a plane model as used in Ref.14.

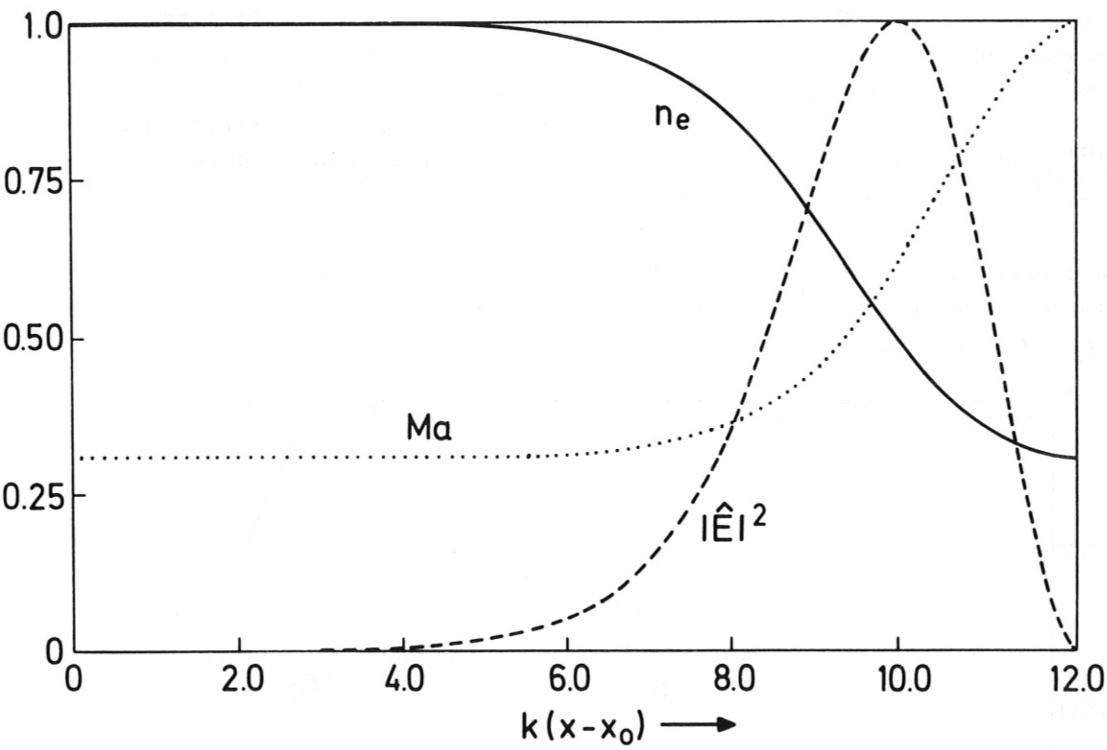

Fig. 4 Distribution of electron density n_e, Mach number $M = v/s$ and $2\mu|\hat{E}|^2$ over x. All quantities are normalized to unity. n_0, M_0 are the values of n_e and M on the LHS. The laser impinges from the RHS. The following values hold : $n_0 = 1.27\, n_c$, $M_0 = 0.5$, $2\mu|\hat{E}|^2_{max} = 0.64$. The width of the density transition is approximately λ.

The calculation of M, E and n_e as functions of the space coordinate x is then straightforward, i.e.

$$k(x-x_0) = \int_{Ma_0}^{Ma} \frac{\left|M - \frac{1}{M}\right| dM}{\left\{\ln M^2 - M^2 + M_0^2 - \ln M_0^2 + 2\mu|\hat{E}|_0^2\right\}^{1/2}\left\{M^2 - \ln M^2 - 1 - 2\frac{n_0}{n_c} M_0 \frac{(M-1)^2}{M}\right\}^{1/2}} \qquad (50)$$

$$\frac{n_0}{n_c} = \frac{M_0^2 - \ln M_0^2 - 1}{2(M_0-1)^2}, \qquad n_e = n_0\frac{M_0}{M}, \qquad n_1 = n_0\frac{M_0}{M_1},$$

and $|\hat{E}|$ is determined directly from Eq. (49). M_1, n_1 refer to a point X_1 where the electric field amplitude reduces again to zero and the flow is supersonic, $M_1^2 - \ln M_1^2 = M_0^2 - \ln M_0^2$. By differentiating expression (50) with respect to x it can be verified that the integral represents the correct solution of the wave equation (8) in plane geometry $(y = E)$ and that the boundary conditions are fulfilled. If the laser wavelength and one of the following five quantities are specified : the density or Mach number n_0, M_0 which is reached immediately behind the evanescent light wave (on the LHS in Figs. 4 and 5) n_1, M_1 or the maximum electric field $|\hat{E}|_{max} = (M_0^2 - \ln M_0^2 - 1 + 2\mu|\hat{E}|_0^2)^{1/2}/(2\mu)^{1/2}$ the distribution of all other quantities is determined. So also is the pressure uniquely determined in the overdense region. In a real target the matching in this region occurs owing to a compression (shock) wave running in the opposite direction to the outflowing plasma. In Figs. 4 and 5 two examples of step-like solutions are reported for $M_0 = 0.5$ and $M_0 = 0.005$ corresponding to maximum ratios $3\langle v_{osc}^2\rangle / 2\langle v_{th}^2\rangle$ of 0.64 and 9.6.

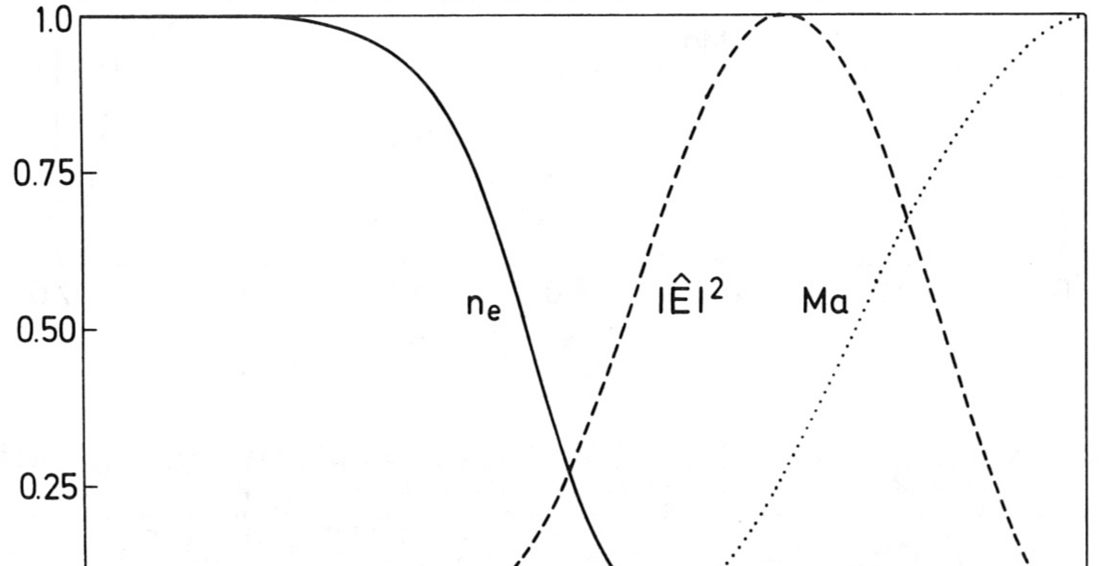

Fig. 5 The same case as in Fig.4, but with laser intensity much higher : $2\mu|\hat{E}|_{max}^2 = 9.60$. $n_0 = 4.85\, n_c$, $M_0 = 0.005$. The width of the density transition is approximately $\lambda/5$. The laser impinges from RHS.

With increasing light intensity (or decreasing plasma temperature) the density step becomes more and more pronounced and the maximum of $|\hat{E}|$ is shifted to lower and lower densities (see Fig.6). As is well known, with the density (conductivity) of the plasma going to infinity, i.e. with $2\mu|\hat{E}|^2 \to \infty$ for a given temperature, a sharp boundary builds up and the standing electric field has a node at the surface.
The total force exerted by the light in this limiting case is expected to be given then by

$$p_L = \frac{2I}{c}$$

In fact,

$$p_L = \int \kappa \, dx = \frac{\varepsilon_0}{2} \int \frac{\omega_p^2}{\omega^2} \nabla |\hat{E}_p|^2 dx ,$$

$\hat{E}_p$ being the electric field amplitude in the interior.
At a sharp boundary it is connected with the outer field $\hat{E}$ by

$$\hat{E}_p = \frac{2}{1+n} \hat{E}$$

For $\omega_p \gg \omega$ the refractive index becomes $n \simeq i \frac{\omega_p}{\omega}$ and p_L reduces to $p_L = \varepsilon_0 |\hat{E}|^2 = \frac{2I}{c}$.
It should be pointed out that the confinement of a warm plasma by HF fields is never perfect; however it improves with increasing field strength. If the plasma is in equilibrium, Eqs. (45,46) reduce to

$$\nabla(p_e + p_i) + \frac{\varepsilon_0}{4} \frac{\omega_p^2}{\omega^2} \nabla |\hat{E}|^2 = 0.$$

In the isothermal case this can be integrated to yield

$$n_e = n_0 \, e^{-\mu(|\hat{E}|^2 - |\hat{E}|_0^2)} .$$

III. For pellet or plasma compression it is interesting to know how strongly the compression might be modified by light pressure. Two distinct aspects have to be considered : firstly, compresssion may be altered owing to the density and velocity profile modifications discussed above; secondly, light presssure effects may be involved in the absorption process and may act on the corona temperature and, as a consequence, on the plasma pressure. This point will not be discussed here. The hydrodynamic aspect of light pressure can be discussed as follows. In a spherical flow the combination of momentum and mass conservation equations

$$\rho v r^2 = \rho_0 v_0 r_0^2 , \quad \rho v \frac{\partial v}{\partial r} = -\frac{\partial p}{\partial r} - \rho \mu s^2 \frac{\partial}{\partial r} |\hat{E}|^2$$

yields the relation

$$\frac{\partial}{\partial r}(p + \rho v^2) = -\rho\left(\frac{2v^2}{r} + \mu s^2 \frac{\partial}{\partial r}|\hat{E}|^2\right),$$

or, in its integrated form,

$$P_0 = p + \rho v^2 + 2\int_{r_0}^{r} \frac{\rho v^2}{r} dr + \mu \int_{r_0}^{r} \rho s^2 \frac{\partial |\hat{E}|^2}{\partial r} dr , \qquad (52)$$

where P_0 is a constant. When no light pressure is present, in plane geometry P_0 represents the hydrostatic pressure at a point where the flow velocity is zero. It can be regarded as the suitable parameter characterizing compression. In the case of $M_0 > 1$ (formation of a plateau) P_0 is not affected by light pressure for in Eq. (52) the upper limit of the integrals can be taken smaller than r_c ; in this region hydrodynamics is not changed because $\hat{E}$ is zero there. The situation is different for $M_0 < 1$ (step formation [13,14]). For the small extent of the step profile plane geometry is a good approximation. Eq.(52) then reduces (assuming $s = const$) to

$$P_0 = \rho_1 v_1^2 + p_1 - s^2 \mu \int_{x_0}^{x} |\hat{E}|^2 \frac{\partial \rho}{\partial x} dx ,$$

if x_1 is a point with zero $\hat{E}$ field. P_0 is calculated and reported as a function of $\langle v_{osc}^2 \rangle / \langle v_{thermal}^2 \rangle$ in Fig. 6. With increasing light pressure P_0 decreases first slightly

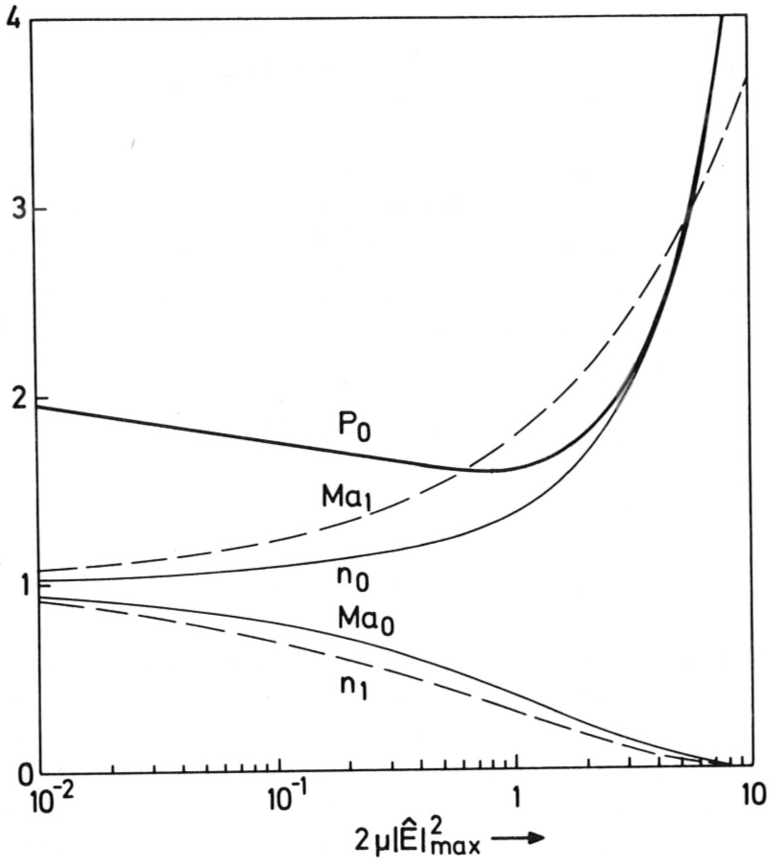

Fig. 6 Densities n_0 and n_1, normalized to the critical density n_c, Mach numbers M_0 and M_1 and the total pressure P_0 (plasma pressure + light pressure) as functions of $2\mu|\hat{E}^2|_{max} = 3\langle v_{osc}^2 \rangle / 2 \langle v_{thermal}^2 \rangle$. In plane geometry n_0 is the maximum and n_1 is the minimum density of the step at a fixed value of $2\mu|\hat{E}^2|_{max}$. M_0 and M_1 are the corresponding Mach numbers. The minimum of P_0 is reached at $2\mu|\hat{E}^2|_{max} = 0.7$. For $|\hat{E}^2|_{max} = 0$ $P_0 = 2$ is assumed and the other quantities meet at the value 1.

(by 20%) and shows a strong increase only when light pressure dominates clearly, which could be the case only if no appreciable fraction of light were absorbed and consequently the plasma temperature were kept low. The minimum of P_0 is reached approximately in the situation reported in Fig.4 with $2\mu|\hat{E}|^2_{max} = 0.64$. Within our model $2\mu|\hat{E}^2|_{max}$ values higher than unity must be considered as purely academic. Therefore, we can also conclude in this case that light pressure does not directly affect the compression of a pellet in a remarkable manner as long as the processes in the critical region can be regarded as quasistationary. However, the indirect influence of ponderomotive forces via instabilities and absorption may be important. It is further evident that a stability analysis of laser light induced hydrodynamic structures is needed.

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