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Anomalous Heating by Ion Sound Turbulence

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Abstract

The kinetic equation for electrons including Coulomb collisions and scattering by ion sound and related spectra is reduced to a system of equations for the energy distribution and the anisotropic part. The energy distribution is obtained for the cases where Coulomb collisions, runaway or turbulent heating dominates. Resistivity, heating rate and the dispersion relation are significantly modified for the self consistent non-Maxwellian distribution. Applications to turbulent heating by ion sound are made and other transport effects will be considered in a companion paper.[9]

1. Introduction

The primary motivation for the study of plasma turbulence is its effect on transport properties. Just as in classical transport theory one would like to arrive at a closed set of equations relating the macroscopic variables of the plasma. In classical theory the possibility of such a description arises from the existence of an universal relaxation process, causing any distribution function to relax to a Maxwellian which is specified by the local fluid parameters density, mean velocity and temperature. The fluctuation spectrum is a known functional of the distribution functions. In a turbulent plasma on the other hand distribution functions and wave spectrum have to be determined from a coupled set of kinetic equations.

From particle simulation or direct numerical solution of the kinetic equations it is known that distribution functions as a rule deviate strongly from a Maxwellian as particles respond to turbulence and external forces. A lot of information has recently also become available from laboratory and space plasmas. Wave growth or decay frequently may be connected, to a large measure, to changing particle distributions rather than any fancy nonlinear saturation mechanism. For fully developed turbulence there is usually no good reason other than simplicity for the customary assumption of Maxwellian distributions. A principal aim of this paper is to demonstrate the need for a selfconsistent determination of the distribution functions and to show how this can be done for a class of instabilities. One can hardly expect to obtain a general theory of anomalous transport, and we consider one of the most researched instabilities in this context, ion acoustic turbulence and related spectra, which are characterized by small phase velocity $\omega/kv_e \ll 1$ and short wavelength $kv_e/\Omega_e \gg 1$, are sufficiently broad in the angle of $\underline{k}$, and have not too large fluctuation levels.

Such spectra can be generated by electron-ion drifts, ion-ion drifts, beams, gradients and parametric effects, and thus play an important role in laboratory and space plasmas. In the early days of quasilinear theory it was shown that the electron distribution changes considerably from an initial Maxwellian not only by quasilinear flattening but also by the development of runaway tails.[1, 2] Turbulent heating experiments showed also the formation of high energy ion tails and quenching of the instability.[3] Particle simulation of ion acoustic waves allows a detailed study of the evolution of spectrum and distribution functions. In previous communications we[4, 5] briefly reported simulation experiments specifically designed to test the numerous nonlinear theories of stabilization which had been developed over the years. The case of a current perpendicular to a weak magnetic field, which corresponds to perpendicular shocks, was considered. The magnetic field has little effect on wave dispersion but prevents electron runaway. It was demonstrated that flattening of the electron distribution and ion tail formation rather than any nonlinear effects determine the evolution of the instability. Heating and anomalous resistivity were found to be in excellent agreement with the quasi-linear prediction if one accounts for the changes in the distribution function from the initial Maxwellian and the anisotropy of the wave spectrum. The electron distribution relaxes to a distribution of the form

$$F(w) = n(C_s/v_o^3) \exp \{ - (w/v_o)^s \} \qquad (1)$$

with $s = 3.6-4$, corresponding to a reduction in the electron growth rate, resistivity and heating rate by a factor 0.33 and an increase in the effective temperature for wave dispersion $T_{eff} \gtrsim 1.6T_e$ as compared to a Maxwellian of the same energy, $T_e = m \langle w^2/3 \rangle$. Such flat topped distribution functions have also been observed downstream from the earth's bow shock[6]. The Druyvesteyn distribution of a weakly ionized plasma in a strong electric field corresponds to $s = 4$ and quasilinear theory predicts an asymptotic solution $s = 5$ for ion sound turbulence. That unmagnetized quasilinear theory describes the wave electron interaction was most accurately confirmed by a stochastic acceleration model[5] with a prescribed turbulent spectrum in which electrons relax from an initial Maxwellian to the selfsimilar distribution (1) with $4.7 \lesssim s \lesssim 5$. The relaxation process was found to be independent of the magnetic field $(\Omega_e/kv_e) = 0 - 0.1$ and the heating rate agrees with the quasilinear prediction even for fluctuation levels much larger than they occur in the full simulation,

$(e\,\Phi/T_e)^2 = 2\text{-}6.\ 10^{-2}$. (For the model, density is not a relevant parameter but the parameters are easily related to the selfconsistent problem by noting that in the latter $kv_e/\omega_e \approx 1$). The discrepancy between the quasilinear prediction $s = 5$ and the observed values of s in the selfconsistent problem is attributed to electron-electron collisions. Although the electron distribution is essentially isotropic a small anisotropy manifests itself already in the homogeneous plasma if an electric field is applied. Simulation and shock wave experiments show an asymmetry of the spectrum with respect to the current which has been connected with a small distortion of the distribution (1) and the particle orbits by the electric field and speed dependent turbulent scattering[5]. The anisotropy plays a more important role if the electric field is along the magnetic field[7] and must of course be considered for the transport connected with gradients such as heat conduction, as is well known from classical transport theory[8].

The ion distribution on the other hand always reflects the anisotropy of the ion sound spectrum. The interaction is essentially with a high energy tail in accordance with quasilinear theory. Initial tail formation is a very complicated dynamical process however. Thus at this stage we restrict ourselves to an important part of the complete problem, namely to a theory of the wave electron interaction irrespective of the generation mechanism for the spectrum. We shall not only discuss the theory underlying the simulation of current driven ion sound turbulence in more detail but guided by these results extend the theory to transport phenomena in inhomogeneous plasmas. There is already good experimental evidence that anomalous electron heat conduction plays an important role in shocks, the laser pellet interaction, the solar wind etc.

In Section II we discuss the kinetic equation for the electrons including Coulomb collisions. Coulomb collisions are included not only because they are important in certain cases but also in order to contrast classical and anomalous transport theory. We examine the conditions under which the energy distribution remains approximately Maxwellian and anomalous transport coefficients can be obtained by appropriate substitutions for Z_{eff} (ν_{eff}) and $T_{i,eff}$ in the classical transport equations. Scattering of electrons by the turbulent spectra we consider is rather similar to electron-ion collisions in that it has essentially the same speed dependence w^{-3} and is predominantly elastic. Our assumption $(\omega/kv_e)^2 \ll 1$ replaces $(v_i/v_e)^2 \sim (m/M) \ll 1$. To lowest order in these parameters the electron-ion collision term is always the isotropic Lorentz term whereas the electron-wave collision term is generally anisotropic. Nevertheless also for anisotropic spectra the electron distribution relaxes to an isotropic distribution to lowest order. We examine this isotropization process which for the turbulent plasma plays the same role as the relaxation to a local Maxwellian in classical transport.

The dominance of isotropization allows us to reduce the kinetic equations to much simpler equations for the energy distribution $F(w)$ and the small anisotropic part $\hat{f}(\underline{w})$.

In Section III we examine the relaxation of the energy distribution F(w) under the condition that e-e collisions, turbulent heating or runaway dominates. For other situations a numerical solution of the equation appears feasible. Heating rate, resistivity and electron dielectric constant are determined for a drifting isotropic distribution corresponding to a current across a sufficiently strong magnetic field. From the conservation laws and the observed wave growth and ion distribution some information about the wave ion interaction is obtained. These applications to ion sound turbulence are considered in Section IV. Conclusions from this part of the investigation are summarized in Section V. In a companion paper we present the solution for the anisotropic part of the distribution and a complete set of anomalous electron transport equations. Wave growth connected with the anisotropy of the electron distribution and application to experimental manifestations of anomalous transport will also be discussed.[9]

II. Kinetic Equation for the Electrons

Transport equations are obtained from an appropriate solution of the kinetic equation for the distribution function

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \frac{\partial f}{\partial \underline{x}} + \frac{e}{m} (\underline{E} + \frac{\underline{v}}{c} \times \underline{B}) \cdot \frac{\partial f}{\partial \underline{v}} = - \frac{\partial}{\partial \underline{v}} < \frac{e}{m} \delta\underline{E}\delta f > = Cf \quad (2)$$

where the fluctuations δf , $\delta \underline{E}$ are due to particle discreteness and collective modes. For our purposes we may separate the collision term in the form $Cf = C_{ee}f + C_{ei}f + C_{ew}f$, describing short wavelength stable fluctuations $1 \lesssim k\lambda_D < \Lambda = \lambda_D/b_o \sim 12\pi n\lambda_D^3$ (b_o impact parameter, $\lambda_D = v_e/\omega_e$) by the Landau collision integral and the collective modes $k\lambda_D \lesssim 1$ by the quasilinear collision term. Each term takes the form

$$Cf = \frac{\partial}{\partial \underline{v}} \cdot \left[- \underline{A} + \underline{\underline{D}} \cdot \frac{\partial}{\partial \underline{v}} \right] f \qquad , \tag{3}$$

where the drag force $\underline{A}$ is due to polarization of the plasma and Cerenkov emission by a test charge. For the Landau collision integral the diffusion tensor becomes[8]

$$\underline{\underline{D}}_{ab} = \frac{\Gamma_{ab}}{m_a} \int d\underline{v}' \; \underline{\underline{T}} \; (\underline{v}-\underline{v}') \; f_b \; (\underline{v}') \qquad , \tag{4}$$

where $\underline{\underline{T}} = (1/t) \left[\underline{\underline{I}} - \underline{t}\,\underline{t}/t^2 \right]$, $\underline{t} = \underline{v}-\underline{v}'$

and $\Gamma_{ab} = (2\pi e_a^2 \, e_b^2/m_a) \ln\Lambda$. The polarization force is related to the diffusion tensor by

$$\underline{A}_{ab} = \frac{m_a}{m_b} \frac{\partial}{\partial \underline{v}} \cdot \underline{\underline{D}}_{ab} = \frac{m_a}{m_b} \frac{\partial}{\partial \underline{v}} \cdot D_{ab} \qquad , \tag{5}$$

where D is the trace of $\underline{\underline{D}}$. For a set of plane waves $(\omega_{\underline{k}}, \underline{k})$, $\omega_{\underline{k}} > 0$, the unmagnetized quasilinear diffusion tensor becomes

$$\underline{\underline{D}}_{ew} = \frac{8\pi^2 e^2}{m^2} \int d\underline{k} \; W(\underline{k}) \delta(\omega_{\underline{k}} - \underline{k} \cdot \underline{v}) \; \hat{\underline{k}} \, \hat{\underline{k}} \qquad , \tag{6}$$

where $W(\underline{k})$ is the energy spectrum $W = \langle(\delta E)^2/8\pi\rangle = \int d\underline{k} W(\underline{k})$ and $\hat{\underline{k}} = \underline{k}/k$. The drag force due to Cerenkov emission is given by[10]

$$\underline{A}_{ew} = - \int d\underline{k} \left(\frac{\partial \varepsilon}{\partial \omega_{\underline{k}}}\right)^{-1} \frac{e^2}{\pi m k^2} \, \delta(\omega_{\underline{k}} - \underline{k} \cdot \underline{v}) \; \underline{k} \qquad , \tag{7}$$

where $\varepsilon(\underline{k}, \omega_{\underline{k}}) = 0$ is the dielectric constant for wave $\underline{k}$.

We make use of our assumptions $v_i/v_e \ll 1$ and $\omega/kv_e \ll 1$ to simplify the e-i and e-w collision terms. The diffusion tensor (4) may be expanded in $v'/v < 1$ or $v'/v > 1$ from which one obtains the high and low speed limits as well as an expansion in spherical harmonics.[11] In the ion rest frame $\underline{w} = \underline{v} - \underline{u}_i$ the e-i collision term is to lowest order in m/M and v_i/v the isotropic Lorentz collison term

$$\nu(w)C_L f = \frac{\nu(w)}{2}\left[\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu} + \frac{\partial^2}{\partial\phi^2}\right]f \tag{8}$$

with the collision frequency

$$\nu_{ei}(w) = \frac{D_{ei}}{w^2} = \frac{2\Gamma_{ei}\, n_i}{m} w^{-3} \quad , \tag{9}$$

where spherical coordinates w, $\cos\theta = \mu, \phi$ have been introduced. Inelastic scattering and anisotropies appear to second order in the ion thermal velocity. Averaging over the angle of $\underline{w}$ we obtain

$$\langle \hat{\underline{w}} \cdot \underline{\underline{D}}_{ei} \cdot \hat{\underline{w}} \rangle^i = \nu_{ei}(w) v_i^2 = D_{ei}^{ww} \quad , \tag{10}$$

where $\hat{\underline{w}} = \underline{w}/w$ and the superscript indicates that the spherical average is taken in the ion rest frame. For non Maxwellian anisotropic distributions we have defined $T_b = m_b v_b^2 = m_b \langle w^2/3\rangle$, $\underline{w} = \underline{v} - \underline{u}_b$, $\underline{u}_b = \langle \underline{v} \rangle_b$ where the subscript indicates an average over $f_b(\underline{v})$.

In the high speed limit the polarization force is given by

$$\underline{A}_{ei}(w) = -\frac{m}{M}\nu_{ei}(w)\,\underline{w} \quad . \tag{11}$$

In the low speed limit $w < v_e$ we obtain from (4) and (5)

$$\langle D_{ee}\rangle^e = 3\langle D_{ee}^{ww}\rangle^e = \frac{2\Gamma_{ee}}{m}\int_0^\infty dv'\, 4\pi v' F_e(v') = 3\nu_{ee}^{*} v_e^2 \frac{a_{-1}}{a_{-3}} \tag{12}$$

$$\langle A_{ee}\hat{\underline{w}}\rangle^e = -\nu_{ee}^{*} w \quad , \tag{13}$$

where

$$\nu_{ee}^{*} = \frac{2\Gamma_{ee} n}{m v_e^3}\left(\frac{2}{\pi}\right)^{1/2}\frac{a_{-3}}{3} \tag{14}$$

and $F_e(w) = \langle f_e(\underline{w})\rangle$ is the isotropic part of the distribution function. The form factors

$$a_{-1} = \left(\frac{\pi}{2}\right)^{1/2} \left\langle\frac{v_e}{w}\right\rangle_e \tag{15}$$

$$a_{-3} = (2\pi v_e^{\,2})^{3/2} \frac{F_e(0)}{n} \tag{16}$$

are normalized to unity for a Maxwellian and are summarized in Appendix A for distributions of the form (1). Actually the exact speed dependence of the isotropic diffusion tensor is obtained by using the high and low speed limit for a field particle distribution $F_b(w)$ which is truncated at $w' \lessgtr w$ respectively. For instance

$$\langle D_{ab}(w)\rangle = \frac{2\Gamma_{ab}}{m_a}\left[N_b(w)w^{-1} + \int_w^\infty dw' 4\pi F_b(w')\right] , \tag{17}$$

where $N_b(w)$ is the number of particles of speed $w' < w$. It follows that the diffusion coefficients are monotonically decreasing with speed.

For speeds above the phase velocity range the e-w collision term has the same speed dependence as the e-i collision term in the high speed limit $v_i/w \ll 1$ but is generally anisotropic even to lowest order. For speeds below the phase velocity range, C_{ew} vanishes. If we assume that the phase velocities satisfy the condition $\omega/kv_e \ll 1$ and the spectrum is sufficiently broad in the angle of $\underline{k}$ then the resonant interaction (6) is possible except for negligibly small regions of velocity space. The extent of these regions as well as the speed dependence for $w \sim \omega/k$ depends on details of the spectrum. The drag force $\underline{A}_{ew}$ for thermal electrons is due to spontaneous emission of ion sound waves and is only a small correction to $\underline{A}_{ei}$. For the momentum transfer from the waves to the electrons we obtain from (3) and (6) for a drifting isotropic distribution $f(\underline{v}) = F(|\underline{v}-\underline{u}|)$ (indicated by the superscript)

$$\underline{R}_{ew}^{0} = - \int d\underline{v}\; m\, \underline{\underline{D}}_{ew} \cdot \frac{\partial f}{\partial \underline{v}} = - \int d\underline{w}\, \frac{mw^2}{3}\, \underline{r}_{ew}\, \frac{1}{w}\frac{\partial F}{\partial w} = n\, m\, \underline{r}_{ew}^{*} \quad , \qquad (18)$$

where

$$\underline{r}_{ew}(w) = \frac{3}{w} \langle D_{ew}^{ww} \rangle^{e} = \frac{3}{w^3} \int d\underline{k}\; W(\underline{k})\; H(\hat{\omega}^{e})\, \frac{4\pi^2 e^2}{m^2 k^3}\, (\omega_{\underline{k}} - \underline{k} \cdot \underline{u})\, \underline{k}$$

$$= -\underline{\underline{\nu}}_{ew} \cdot \left[\underline{u} - \underline{u}_{w} \right] \quad . \qquad (19)$$

The last relation in (19) defines the collision frequency $\underline{\underline{\nu}}_{ew}$ and the wave rest frame $\underline{u}_w$. With the assumption $\hat{\omega} = (\omega_{\underline{k}} - \underline{k} \cdot \underline{u})/kw \ll 1$ the resonance condition $H(\hat{\omega}) = 1$, $|\hat{\omega}| < 1$ may be dropped. For the resulting w^{-3} dependence $\underline{\underline{\nu}}_{ew}^{*}$, $\underline{r}_{ew}^{*}$ are related to $\underline{\underline{\nu}}_{ew}$, $\underline{r}_{ew}$ by

$$\underline{\underline{\nu}}_{ew}^{*} = \underline{\underline{\nu}}_{ew}\, (v_e)\, \left(\frac{2}{\pi}\right)^{1/2} \frac{a_{-3}}{3} \quad . \qquad (20)$$

The electron heating rate is given by

$$Q_{ew}^{o} = -3 \int d\underline{w}\, \frac{mw^2}{3}\, \langle D_{ew}^{ww} \rangle^{e}\, \frac{1}{w}\frac{\partial F}{\partial w} = 3\, nm\, \langle D_{ew}^{ww} \rangle^{e\,*} \quad , \qquad (21)$$

where

$$\langle D_{ew}^{ww} \rangle^{e} = \frac{1}{w^3} \int d\underline{k}\; W(\underline{k})\; H(\hat{\omega}^{e})\, \frac{4\pi^2 e^2}{m^2 k^3}\, (\omega_{\underline{k}} - \underline{k} \cdot \underline{u})^2$$

$$= \langle D_{ew}^{ww} \rangle^{w} + \frac{1}{3}\, (\underline{u} - \underline{u}_w) \cdot \underline{\underline{\nu}}_{ew} \cdot (\underline{u} - \underline{u}_w) \qquad (22)$$

is the inelastic diffusion coefficient in frame $\underline{u}$. The corresponding e-i terms are from (8-11)

$$\underline{r}_{ei} = - \nu_{ei}(w)\ (\underline{u}-\underline{u}_i) \tag{23}$$

$$\langle D^{ww}_{ei}\rangle^{e} = \nu_{ei}\left[v_i^2 + \frac{1}{3}\ (\underline{u}-\underline{u}_i)^2 \right] \tag{24}$$

$$\underline{R}^{o}_{ei} = nm\underline{r}_{ei}^{*} \tag{25}$$

$$Q^{o}_{ei} = - Q^{o}_{ie} + nm\nu^{*}_{ei}\ (\underline{u}-\underline{u}_i)^2 \tag{26}$$

where

$$Q^{o}_{ie} = 3n\,\frac{m}{M}\,\nu_{ei}\left[T_e\,\frac{a_{-1}}{a_{-3}} - T_i \right] \tag{27}$$

is the rate of heat transfer to the ions and ν^{*}_{ei} is related to $\nu_{ei}(w)$ as in (20). Comparing (9) and (14) it follows that for ions of charge Z

$$\nu_{ei}^{*} = Z\,\nu_{ee}^{*} = Z\,\omega_e\,\left(\frac{2}{\pi}\right)^{1/2} a_{-3}\,\frac{\ln\Lambda}{\Lambda} \tag{28}$$

The same e-i transfer rates are obtained from C_{ie} in the low speed limit, using momentum and energy conservation. Particle conservation by the collision term implies that generally the rate of momentum transfer $\underline{R}_e$ is independent of the reference frame while energy transfer in different frames is related by

$$Q_e = K_{e,o} - \underline{R}_e \cdot (\underline{u}-\underline{u}_o) \tag{29}$$

If, as usual for current driven instabilities $u \gg v_i,\ \omega/k$ the drift connected term (Joule heating) dominates the heating rate Q_e.

The analogy between e-w and e-i transfer rates would be complete if the drag force $\underline{A}_{ew} \ll \underline{A}_{ei}$ had been included in (21) as it was in (27). The e-i terms correspond to an isotropic collision frequency $\underline{\underline{\nu}}_{ei} = \nu_{ei}\,\underline{\underline{I}}$. If the wave spectrum were also isotropic to lowest order, the e-i and e-w collision terms could be combined to an effective e-i term with Z_{eff} and $T_{i,eff}$ defined by

$$\nu(w) = \langle D_{ei} + D_{ew} \rangle \frac{1}{w^2} = \left[3Z \frac{\ln\Lambda}{\Lambda} + \pi \frac{W}{nT_e} \left\langle \frac{1}{k\lambda_D} \right\rangle_w \right] \omega_e \left(\frac{v_e}{w}\right)^3 \tag{30}$$

$$= 3Z_{eff}\, \omega_e \left(\frac{v_e}{w}\right)^3 \frac{\ln\Lambda}{\Lambda}$$

$$\langle D^{ww}_{ei} + D^{ww}_{ew} \rangle^i = \left[3T_i \frac{\ln\Lambda}{\Lambda} + \pi \frac{W}{nT_e} \left\langle \frac{1}{k\lambda_D} \left(\frac{\omega_k^i}{kc_s}\right)^2 \right\rangle_w T_e \right] \omega_e \frac{Z}{M} \left(\frac{v_e}{w}\right)^3$$

$$= 3 \frac{T_{i,eff}}{M} Z_{eff} \left(\frac{v_e}{w}\right)^3 \omega_e \frac{\ln\Lambda}{\Lambda} \tag{31}$$

where $\omega_k^i = \omega_k - \underline{k} \cdot \underline{u}_i$ is now the frequency in the ion frame and $c_s^2 = ZT_e/M \approx (\omega/k)^2$. The effect of isotropic turbulence $\omega/kv_e \ll 1$ on electron transport is obtained by replacing Z and T_i in the classical transport equations[8] by Z_{eff} and $T_{i,eff}$, provided that the distribution function remains close to a Maxwellian. This will be the case if e-e collisions still dominate inelastic scattering that is

$$\frac{W}{nT_e} \left\langle \frac{1}{k\lambda_D} \left(\frac{\omega_k - \underline{k}\cdot\underline{u}}{kv_e}\right)^2 \right\rangle_w \ll \frac{M}{Zm} \frac{\ln\Lambda}{\Lambda} \tag{32}$$

comparing (12) and (22). Usually, however, turbulence not only dominates elastic scattering ($Z_{eff} \gg 1$) but inelastic scattering as well. If the inequality opposite to (32) is satisfied the lowest order distribution is no longer a quasistationary Maxwellian but evolves by turbulent heating to the selfsimilar distribution (1), $s = 5$. Classical transport equations with an enhanced collision frequency have been used to describe anomalous transport. We see that this is justified only under rather restricted circumstances and that Z and T_i must also be modified. $T_{i,eff}$ enters the electron transport equations only through the heat transfer (27). Z_{eff} determines the effective collision frequency and the relative importance of e-e collisions through (28). As is well known an increased Z_{eff} allows for a greater distortion of the distribution function by the perturbing forces and gradients.[8] The numerical coefficient in the heat conductivity $\kappa_T = 3.16\ nT_e/m\nu^*$ for example is changed from 3.16 ($Z = 1$) to 12.58 as $Z_{eff} \to \infty$(Lorentz gas).

The transfer rates (18), (21), (25) and (26) for a drifting isotropic distribution may be used if the drift is across a magnetic field which is sufficiently strong $(\nu^* /\Omega_e) << 1$ to keep the distribution isotropic. Distortion of the distribution function is significant even for $Z = 1$ if the current is along the magnetic field. Recall that $R_e^{\parallel}$ is reduced by the factor 0.52 for $Z = 1$ and 0.29 for $Z \to \infty$. To obtain the total transfer rates in this case the contribution from the anisotropic part of the distribution must be added.

For the anisotropic part of the distribution function we must consider scattering in angle (elastic scattering). With our assumption $\hat{\omega} << 1$ it follows from (6) that scattering by the waves is predominantly in angle[2]

$$D^{ww} \approx \hat{\omega}^2 D^{\mu\mu}; \qquad D^{\mu w} \approx D^{\phi w} \approx \hat{\omega} D^{\mu\mu}; \qquad D^{\mu\mu} \approx D^{\mu\phi} \approx D^{\phi\phi} \tag{33}$$

just as for e-i collisions. Whereas to zero order in (v_i/v), e-i collisions are described by the isotropic Lorentz collision term (8) the e-w diffusion coefficients are generally angle dependent. Nethertheless to zero order in $\hat{\omega}$ the electron distribution should relax to an isotropic distribution F(w) even for an anisotropic spectrum. The isotropization rate depends on speed and the spectrum in the directions $\underline{k} \cdot \underline{w} \approx 0$. Defining the quasilinear H function[12] $H = \int d\underline{w}\, f^2(\underline{w})/2$ it follows from (6) that

$$\left.\frac{\delta H}{\delta t}\right|_{ew} = - \int d\underline{w}\, \frac{\partial f}{\partial \underline{w}} \cdot \underline{\underline{D}}_{ew} \cdot \frac{\partial f}{\partial \underline{w}} \leq 0 \tag{34}$$

Although there is no "plateau", that is a steady state $\delta H/\delta t = 0$ coexistent with a broad wave spectrum (in $\hat{\underline{k}}$), the isotropic distribution function F(w) represents a quasiplateau for speeds which satisfy $\hat{\omega} << 1$. If there are waves $\underline{k} \cdot \underline{w} \approx 0$ the condition $\delta H/\delta t = 0$ implies $\underline{k} \cdot \partial f/\partial \underline{w} = 0$ i.e. isotropy to zero order in $\hat{\omega}$. For a given energy distribution $F(w) = \langle f(\underline{w})\rangle$ the H function $H = \int d\underline{w} \left[F^2 + \hat{f}^2\right]/2$ assumes its minimum for vanishing anisotropy $\hat{f}(\underline{w}) = f(\underline{w}) - F(w)$. The anisotropy of the spectrum leads to a forced anisotropy $\hat{f}$ to first order in $\hat{\omega}$. Weak perturbing electric fields, drifts and gradients also lead to small anisotropies $\hat{f} << F$ which we can determine by perturbation theory. The forced anisotropy is established in roughly an elastic scattering time which in the turbulent case, is much faster than the relaxation rate of the isotropic part of the distribution. Relaxation of the isotropic part of the distribution F(w) is second order in $\hat{\omega}$ and the perturbing forces. Writing the kinetic equation (1) in the drift frame $\underline{u}$ and taking a spherical average we obtain

$$\frac{dF}{dt} - \frac{w}{3} \nabla \cdot \underline{u} \frac{\partial F}{\partial w} + \frac{\partial}{\partial \underline{x}} \cdot \langle \underline{w} \hat{f} \rangle + \frac{1}{w^2} \frac{\partial}{\partial w} w \langle (\underline{a} \cdot \underline{w} - \underline{\underline{U}}:\underline{\underline{W}}) \hat{f} \rangle$$

$$= \langle C \rangle F + \langle \hat{C} \hat{f} \rangle \tag{35}$$

where

$$\underline{a} = \frac{e}{m} \left[\underline{E} + \frac{\underline{u}}{c} \times \underline{B} \right] - \frac{d\underline{u}}{dt} \tag{36}$$

and $d/dt = \partial/\partial t + \underline{u} \cdot \partial/\partial \underline{x}$, also

$$U_{ik} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) - \frac{1}{3} \delta_{ik} \nabla \cdot \underline{u} \tag{37}$$

is the shear tensor and $\underline{\underline{W}} = \underline{w}\underline{w} - w^2 \underline{\underline{I}}/3$. To first order in the perturbing forces the anisotropic part of the distribution satisfies

$$\underline{w} \cdot \frac{\partial F}{\partial \underline{x}} + (\underline{a} \cdot \underline{w} - \underline{\underline{U}}:\underline{\underline{W}}) \frac{1}{w} \frac{\partial F}{\partial w} - \hat{C}^1 F = (\underline{w} \times \underline{\Omega}) \cdot \frac{\partial \hat{f}}{\partial \underline{w}} + C^o \hat{f} - \langle \hat{C}\hat{f} \rangle \tag{38}$$

using that $\hat{C}^o F = 0$ by our assumption $v_i/w \ll 1$ and $\hat{\omega} \ll 1$. We also assume that scattering is dominant $\nu L/v_e \gg 1$, $\nu T \gg 1$ where L and T are the macroscopic length and time scales. Perpendicular to the magnetic field gyration also isotropizes the electron distribution. For $\nu/\Omega \ll 1$ this takes place in a few gyro periods and it is then convenient to split $\hat{f}$ by angle averaging into $\hat{f} = \bar{f}\,(w_\perp, w_\parallel) + \tilde{f}\,(w_\perp, w_\parallel, \phi)$, $\langle \tilde{f} \rangle = 0$ and (38) into

$$\underline{w}_\parallel \cdot \frac{\partial F}{\partial \underline{x}} + (a_\parallel w_\parallel - \underline{\underline{U}}_I:\underline{\underline{W}}_I) \frac{1}{w} \frac{\partial F}{\partial w} - \bar{C}^1 F = \bar{C}^{(o)} \bar{f} + \overline{\tilde{C}^o \tilde{f}} - \langle C^o \tilde{f} \rangle \tag{39}$$

$$\underline{w}_{\perp} \cdot \frac{\partial F}{\partial \underline{x}} + (\underline{a}_{\perp} \cdot \underline{w}_{\perp} - \underline{\underline{U}}_{\perp} : \underline{\underline{W}}_{\perp}) \frac{1}{w} \frac{\partial F}{\partial w} - \tilde{C}^{1} F = - \Omega \frac{\partial f}{\partial \phi} + \tilde{C}^{o} \bar{f} + C^{o} \tilde{f} - \overline{C^{o} \tilde{f}} \quad . \tag{40}$$

For a strong magnetic field $\tilde{f}$ can be expanded further in the small parameters $\nu/\Omega << 1$, $v_e/\Omega L_{\perp} << 1$ which considerably simplifies the problem, particularly for anisotropic turbulence. In (40) it is then required only that scattering by collisions and turbulence is sufficiently frequent to prevent magnetic trapping in an inhomogeneous magnetic field. In the longitudinal equation we must still require $\nu L_{\parallel}/v_e >> 1$, $\nu T >> 1$ for $\bar{f} << F$. Because of the speed and angle dependence of turbulent scattering Coulomb collisions may play a role in certain regions of velocity space even for elevated turbulence levels. Actually the conditions for dominance of scattering are much less severe since we require our equations to be valid only in the regions of velocity space which make the dominant contributions to the macroscopic variables we are ultimately interested in. The e-i and e-w collision terms must be transformed from their old reference frames to the drift frame $\underline{u}$. The e-e collision term is invariant under this transformation. Correct to first order in $(\underline{u} - \underline{u}_i)/v_e$ and v_i/w the new e-i term consists of the isotropic collision operator (8) and the anisotropic part

$$\hat{C}^{1}_{ei} F = - \underline{r}_{ei} (w) \cdot \underline{w} \frac{1}{w} \frac{\partial F}{\partial w} \quad , \tag{41}$$

In the e-w collision term the transformation is performed simply by Doppler shifting the frequency, $\omega \to \omega - \underline{k} \cdot \underline{u}$. Again, the drift must be restricted to $\underline{u} - \underline{u}_w/v_e << 1$ for $\hat{\omega}$ to remain small. Within this restriction the drift frame can be chosen arbitrarily. Choosing the electron drift frame $\underline{u} = \underline{u}_e$ is most convenient since we get the anisotropic part of the distribution connected with a relative drift and shear from the first order equation (38). In the frame $\underline{u} = 0$ the drift would be determined as a first orderquantity proportional to the electric field and the spatial gradients. The viscous stress would then be determined from the second order equation for $\hat{f}$. In the electron rest frame $\underline{u} = \underline{u}_e$ we obtain by taking a first moment of (38) the solubility condition

$$\underline{a}^1 = \left(\frac{\nabla p - \underline{R}}{nm}\right) \quad . \tag{42}$$

Since $\hat{f}$ depends on $\underline{R}$ through (38), $\underline{R} = \int d\underline{w}\ m\underline{w}\ Cf$ must be determined from the condition $n(\underline{u}_e - \underline{u}) = \int d\underline{w}\ \underline{w}\ \hat{f}(\underline{w}) = 0$. (43)

In earlier papers on classical transport theory which attempt a one fluid description the center of mass velocity is chosen for $\underline{u}$.

III. Relaxation of the Energy Distribution.

Our approach resembles the derivation of the quasilinear equations in that the anisotropic part of the distribution $\hat{f}$ is determined as a functional of the slowly varying isotropic distribution $F(w)$, linear in the perturbing forces.

Inserting $\hat{f}$ into (35) gives a closed kinetic equation for F which includes besides local relaxation also speed dependent transport terms. The reduction of the kinetic equation (2) to the system (35), (38) is based upon the dominance of isotropization. The equations apply also if Coulomb collisions dominate, but are more general.

For Coulomb collisions scattering in angle and speed occur at the same rate, cf. (12). Thus in a few collision times the lowest order distribution F(w) becomes a quasistationary Maxwellian and the anisotropic part also has only an implicit space and time dependence through the local fluid parameters and external forces. These facts form the basis of the Chapman - Enskog expansion scheme for a collision dominated plasma. [8] Making use of the small mass ratio $m/M \ll 1$ collisions between unlike particles may be neglected in the lowest order relaxation process and one obtains separate equations for the electrons and ions with $T_e \neq T_i$, $\underline{u}_e \neq \underline{u}_i$. The anisotropic part of the distribution is determined from (38), using for F_o a local Maxwellian. Higher order corrections are rarely considered but for (35) could be found from the expansion scheme

$$C_{ee} F_o = 0 \tag{44}$$

$$C_{ee} F_1 = C_{ee}(F_o) F_1 + C_{ee}(F_1) F_o = \frac{\partial^1}{\partial t} F_o + LF_o + C^1 F_o \tag{45}$$

etc., where L represents the convection terms and C^1 the remaining collision terms. The implicit time dependence

$$\frac{\partial^1}{\partial t} F_o = \left[\frac{1}{n} \frac{\partial^1 n}{\partial t} - \frac{1}{2T_e} \frac{\partial^1 T_e}{\partial t} \frac{1}{w^2} \frac{\partial}{\partial w} w^3 \right] F_o \tag{46}$$

is obtained form the continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot n\underline{u}_e = 0 \tag{47}$$

and the heat equation

$$\frac{3}{2} n \left[\frac{\partial}{\partial t} + \underline{u}_e \cdot \nabla \right] T_e + p_e \nabla \cdot \underline{u}_e + \nabla \cdot \underline{q}_e + \underline{\underline{\pi}}_e : \underline{\underline{U}}_e = Q_e \tag{48}$$

with the transport terms evaluated from (38) using F_o etc. Equations (47-48) are the solubility conditions for (45).

If on the other hand there is intense turbulent heating $\tau_H^{-1} = (1/T_e)\, \partial T_e/\partial t >> \nu_{ee}$, this conventional method fails. Already to lowest order there is an explicit time dependence

$$\frac{\partial F_o}{\partial t} - \frac{1}{w^2}\frac{\partial}{\partial w}\, w^2 D_o \frac{\partial F_o}{\partial w} = 0 \tag{49}$$

$$\frac{\partial F_1}{\partial t} = \frac{1}{w^2}\frac{\partial}{\partial w}\, w^2 D_o \frac{\partial F_1}{\partial w} = - LF_o + C_{ee} F_o + C^1 F_o \tag{50}$$

etc. If the drift is across the magnetic field as in perpendicular shocks the dominant term D_o has a w^{-3} dependence, cf. (22). For $t >> \tau_H$ we expect then that F_o becomes the selfsimilar distribution (1), s = 5 with the time dependence given by the zero order heating rate (21). This is immediately verified by expressing the time dependence in (49) by (46), using (48) in zero order. Introducing the selfsimilar velocity variable $\hat{w} = w/v_e$ it is not difficult to show that the general diffusion term which leads to a selfsimilar solution $F = (n/v_e^3)\, \hat{F}(w/v_e)$ must be of the form

$$D_o\, (\underline{x},w,t) = \bar{D}\, (x,t) w^{-(s-2)} \tag{51}$$

with s > o. The selfsimilar solution is just (1). The corresponding heating rate is

$$\frac{d}{dt}\, v_o^{\,s} = s^2\, \bar{D}\, (x,t) \tag{52}$$

and

$$\left(\frac{v_e}{v_o}\right)^2 = \frac{\Gamma(5/s)}{3\Gamma(3/s)} \tag{53}$$

More special selfsimilar solutions are possible if D_o has an explicit v_e dependence other than through $w = \hat{w}v_e$. The quasilinear diffusion equation has a strictly selfsimilar solution for all w not only $w > \omega/k$ if the spectrum is also selfsimilar $W(\underline{k},\underline{x},t) = \bar{W}(\underline{x},t)\,\hat{W}(kv_e/\omega_e)$. If the heating rate satisfies $(\partial/\partial t)v_e^3 = \text{const}$ a selfsimilar solution in the presence of e-e collisions is possible. Such special cases are however of no interest to us here. More generally (49-50) are solved by using the Green's function for (49).

$$F + F_1 = \int d\underline{w}'\, G(w,w',t,t_o)\, F_o(w',t_o) + \int_{t_o}^{t} dt' \int d\underline{w}'\, G(w,w',t,t')\, S^1(\underline{x},w',t') \tag{54}$$

where S^1 is the r.h.s. of (50). For diffusion coefficients (51) the Green's function is found using some Bessel function identities[13] and the appropriate boundary conditions.

$$G(w,w',t,t') = \frac{|s|}{4\pi\tau}\,(w\,w')^{(s-3/2)}\; I_p\left[\frac{2}{\tau}\,(ww')^{5/2}\right] \exp\left\{-\frac{w^s + w'^s}{\tau}\right\} \tag{55}$$

where $\tau = \int_{t'}^{t} dt''\, s^2\, \bar{D}(x,t'')$

and $p = (3-s)/|s|$ is the order of the modified Bessel function. We have included the case $s < 0$ which corresponds to runaway, $s = -1$ for an ideal Lorentz gas.[14] For $z = (2/\tau)(ww')^{s/2} \to 0$ one obtains

$$G(w,w',t,t') \approx \frac{|s|\,(w\,w')^x}{4\pi\tau^{p+1}}\,\Gamma(p+1)\,\exp\left[-\frac{(w^s + w'^s)}{\tau}\right] \tag{56}$$

where $x = 0$ for $s > 0$ and $x = s-3$ for $s < 0$. It follows for $s > 0$ that F becomes selfsimilar for $\tau \gg v_{eo}^s$ and that this occurs most rapidly for small w. Using the asymptotic form of I_p for $z \to \infty$ one verifies that (55) becomes a delta function for $t \to t'$ and that F remains unchanged for $w \to \infty$, $s > 0$ or $w \to 0$, $s < 0$. From (49) the rate of change in the number of particles of speed less than w is

$$\frac{\partial N(w)}{\partial t} = 4\pi w^2 D \frac{\partial F}{\partial w} \tag{57}$$

For $s < o$ we obtain from (51) and (57) the runaway rate

$$\frac{\partial N}{\partial t} = -4\pi \bar{D}(x,t) (|s| + 3) C \tag{58}$$

using the asymptotic solution $F = Cw^{-(|s| + 3)}$ from (56). It is interesting to note that the heat produced in this asymptotic region is just carried away by runaway particles. One may also show that eventually all particles runaway. Runaway arises for an electric field $a_{\parallel} = \frac{e}{m} E_{\parallel}$ along the magnetic field. The anisotropic part of the distribution is from (38) $\hat{f} = (a_{\parallel} w_{\parallel}/\nu w)\, \partial F/\partial w$ giving rise to a diffusion term

$$D_a^{ww} = \frac{a_{\parallel}^2 \tau_1(w)}{3} \tag{59}$$

in (35). For $\tau_1 \sim \nu^{-1} \sim w^3$ the runaway term for an ideal Lorentz gas is obtained $s = -1$. For a plasma this term must be compared with the other terms in (35). Comparing (59) with the inelastic diffusion term (22) we see that (59) becomes important in the velocity region $w > v_e$ if the drift $u_{\parallel}$ exceeds a few times $\omega/k \sim c_s$, using $a_{\parallel} \approx \nu^* u_{\parallel}$ Such drifts are required for current driven ion sound instability. Experiments and simulation confirm that turbulent heating by current along the magnetic field is necessarily connected with runaway. Due to the w^{-3} dependence of D_{ew}^{ww} this term should dominate at lower speeds. We have already noted that D_{ew}^{ww} is increased considerably by a drift across the magnetic field without changing the speed dependence. The turbulent runaway problem is essentially non-stationary. In the classical case a nearly stationary solution is made possible by the polarization term A_{ee} balancing the diffusion terms. The diffusion terms have a similar speed dependence as in the turbulent case if for $w > v_e$ the high speed limit of C_{ee} is used, cf. (10-11). The bulk of the distribution $w < v_e$, however, remains Maxwellian[15]. If electron neutral collisons dominate, $\nu(w) \sim w$, a strong electric field does not lead to runaway but to a steady state distribution [16](1), s = 4. Finally we note that in the runaway problem the collision dominated region for which (35) and (38) are valid must be connected to the runaway region where the acceleration terms dominate and the distribution function is strongly anisotropic [14, 15]. The critical speed is determined by $(e/m)\, E \approx \nu_{eff}(w) w$ where ν_{eff} is the total effective collision frequency for elastic scattering. For large enough ν_{eff} the runaway flux is formed, however, at lower speeds where the distribution function is still nearly isotropic[15]. The flux depends strongly on the shape of F(w), cf. (57).

Turbulent heating by a current across the magnetic field is described by the diffusion term (22), s =5. The heating rate $Q = 4\pi m\bar{D}(x,t)\, F(0)$, cf. (21), is proportional to $F(w=0,\tau)$ which may be determined from (56). It follows that the heating rate rapidly approaches the asymptotic heating rate (52) for the selfsimilar distribution, even before the higher w regions of F become selfsimilar. Fig. 1 shows the heating rate

$$\frac{d}{d\hat{\tau}} \left(\frac{T_e}{T_{eo}}\right)^{5/2} = \frac{1}{15} \left(\frac{2}{\pi}\right)^{1/2} a_{-3}\, (\hat{\tau}) \rightarrow \left(\frac{v_e}{v_o}\right)^5 = 0.024 \qquad (60)$$

and the form factor $a_{-3} = \left[F(0,\tau)/F(0,0) \right] (T_e/T_{eo})^{3/2} \rightarrow 0.45$ as a function of $\hat{\tau}$

$$\hat{\tau} = \frac{\tau}{v_{eo}^{5}} = 25\pi \int_0^t dt' \omega_e \frac{W}{nT_{eo}} < \frac{\omega_e}{kv_{eo}} \left(\frac{\omega_k - \underline{k} \cdot \underline{u}}{kv_{eo}}\right)^2 > \qquad (61)$$

for a distribution which was initially Maxwellian. A qualitatively similar behavior is observed for the 2D stochastic acceleration model[5], except that in 2D, a_{-3} is determined by an integral of $F(w_\perp)$, cf. (B 5).

We must consider the convective terms in the equation (35). In the classical case dominance of elastic scattering implies also dominance of local relaxation for the bulk $w < v_e$ of the distribution $F(w)$. In the turbulent case inelastic scattering occurs at a rate slow compared to elastic scattering thus the gradients must satisfy additional restrictions for a local solution (49-50) to be valid. The nonlocal case where gradient terms are of the same order as the collision terms generally requires a numerical solution. In a recent numerical treatment of runaway [17] it has been shown that a significant reduction of the runaway rate results if the Joule heat is removed by a speed dependent loss term. This was done in an ad hoc manner. Within our model, that is no anomalies other than the short wavelength fluctuations,such terms are included selfconsistently in (35).

IV. Quasilinear Theory of the Wave-Electron Interaction.

We have described the wave-electron interaction by the unmagnetized resonant quasilinear diffusion term (6). Ample evidence from the simulation studies has been cited to justify the validity of this description. We briefly discuss the theoretical basis.

Magnetic field effects in the wave electron interaction term are not important if the correlation time τ_c between particle and wave spectrum is short compared to the gyro period. For modes propagating at an angle to the magnetic fields this can be accomplished simply by phase mixing (Doppler broadening) $\tau_c^{-1} \approx \Delta(\omega - k_{\parallel} v_{\parallel})$. If we assume $\Omega_e/kv_e \ll 1$ there remains a small wedge $k_{\parallel}/k \ll 1$ in which a different broadening mechanism is required. Particle diffusion across the magnetic field due to collisions and turbulent scattering is such a mechanism, requiring $\Omega_e{}^3/k_{\perp}^2 D \ll 1$ or a critical fluctuation level[18]

$$\frac{W}{nT_e} > \left(\frac{\Omega_e}{\omega_e}\right)^2 \frac{\Omega_e}{kv_e} \tag{62}$$

if (30) is used. For quasilinear behavior on the other hand the decrease in the interaction time from the linear unmagnetized correlation time $\tau_c^{-1} \approx \Delta\,(\omega - \underline{k}\cdot\underline{v})$ due to resonance broadening must remain small, $k^2 D \tau_c{}^3 \approx \nu/kv \ll 1$. Since $\nu \sim v^{-3}$ resonance broadening becomes important for low speed particles, perhaps extending the interaction to linearly nonresonant particles[19], typically $(v \sim v_e)$, however,

$$\frac{\nu^*}{kv_e} \approx \left(\frac{\omega_e}{kv_e}\right)^2 \frac{W}{nT_e} \ll 1 \tag{63}$$

estimating ν^* from (30) and $W/nT_e \approx 10^{-2}$, $\omega/kv_e = O(1)$. Comparing (62) and (63) we see that for $kv_e/\Omega_e \gg 1$ and typical values of $(\Omega_e/\omega_e)^2 \sim 10^{-3} - 10^{-2}$ (shocks and laser-pellet interaction) there is a wide range of fluctuation levels big enough to wipe out magnetic field effects yet small enough for other modifications of particle wave resonance to remain insignificant, thus corresponding to a regime in which unmagnetized quasilinear theory should be valid.

The Coulomb collision term undergoes only a weak modification[20] of the $\ln\Lambda$ term even for strong magnetic fields $\Omega_e > \omega_e$ since most of the fluctuations are in the short wave length range $1<k\lambda_D<\Lambda$. The magnetic field still enters into processes which occur on longer time and length scales. The magnetic field term on the left hand side of the kinetic equation (2) remains important and for modes $kv_e/\Omega_e \lesssim 1$ propagating essentially across the magnetic field, as they have been considered in the shock problem, the collision term retains its magnetized form.

The same arguments about resonance broadening and the magnetic field effects apply to the dielectric constant. The only difference is that we must now consider the interaction between a given wave packet $(\underline{k},\omega_{\underline{k}})$ with the particle distribution whereas before the interaction between particles of a given velocity and the wave spectrum was examined. It has been shown that for $kv_e/\Omega_e >> 1$ the dielectric constant reduces to the unmagnetized form

$$\varepsilon_e\ (\underline{k},\omega) = \frac{4\pi e^2}{mk^2} \int d\underline{v}\ \frac{1}{\omega - \underline{k}\cdot\underline{v}}\ \underline{k} \cdot \frac{\partial f}{\partial \underline{v}} \tag{64}$$

either linearly[21] for oblique modes or by resonance broadening for modes propagating perpendicular to the magnetic field[22]. Recently[23] a quantitative theory of resonance broadening has been given for the dielectric constant with $\Omega_e = 0$ but arbitrary ν^*/kv_e. It has been shown that for $\nu^*/kv_e \lesssim 1$ correction to the dielectric constant remain very small, the correction to the real part being even orders of magnitude smaller than the correction to the imaginary part (collisional damping). For distributions which are flat in the low speed region, such as (1), s = 5, the resonance broadening effect is reduced further.

For turbulent spectra which satisfy $\Omega_e/kv_e \ll 1$, $\omega/kv_e \ll 1$, $W/nT_e \ll 1$ and are sufficiently broad in $\underline{k}$ we are thus justified in using the standard unmagnetized quasilinear diffusion term and dielectric constant to describe the wave electron interaction.

In the transport problem we must naturally consider a slow space and time dependence of both spectrum and the distribution functions. The condition for the geometrical optics (eikonal) approximation $kL \gg 1, T \gg \tau_c$ are easily satisfied since collision dominated transport requires the space and time scales to satisfy $L/\lambda \gg 1$, $\nu\ T \gg 1$, $\lambda = v_e/\nu$ or $v_e/\Omega_e L \ll 1$ across the magnetic field and on the other hand for unmagnetized quasilinear behavior we must have $\nu/kv_e \ll 1$, $kv_e/\Omega_e \gg 1$. In the geometrical optics approximation the space and time dependence of the spectrum is governed by the usual kinetic equation for wave packets[24]. The ingredients of this equation are obtained from the local dielectric constant. Connected with the weak space and time dependence are also adiabatic diffusion terms not included in (6). These terms represent reversible changes associated with the linear wave motion (sloshing) whereas the resonant interaction term describes the secular motion of the oscillation centers[25] and dissipation. Since the overwhelming majority, $\omega/kv < 1$, of the electrons can take part in the resonant interaction and since the energy in the wave motion is small compared to the thermal energy for $W/nT_e \ll 1$, adiabatic terms are not important for the electrons. They may be important for the colder ions however and must be included for the exact conservation laws to be satisfied[24]. Taking moments of the collision term in (2) we obtain particle conservation $\delta n / \delta t = 0$ and the rates of momentum and energy transfer

$$\underline{R}_j = \langle \delta \underline{E} \ \delta\sigma_j \rangle + \langle \frac{1}{c} \delta \underline{J}_j \times \delta \underline{B} \rangle \quad , \tag{65}$$

$$K_j = \langle \delta \underline{E} \cdot \delta \underline{J}_j \rangle \quad . \tag{66}$$

The heating rate is defined in the restframe $\underline{u}_j$ and is related to (65-66) by (29). Maxwell's equations give the conservation laws

$$\underline{R} = \sum_j \underline{R}_j = - \frac{\partial}{\partial t} \underline{G}^{em} - \frac{\partial}{\partial \underline{x}} \cdot \underline{\underline{T}}^{em} \quad , \tag{67}$$

$$K = \sum_j K_j = - \frac{\partial}{\partial t} U^{em} - \frac{\partial}{\partial \underline{x}} \cdot \underline{S}^{em} \tag{68}$$

which are the usual momentum conservation law and Poyntings theorem applied to the fluctuations $\delta \underline{E}$, $\delta \underline{B}$, $\underline{G}^{em} = (1/c)^2 \underline{S}^{em} = (c/4\pi) \langle \delta \underline{E} \times \delta \underline{B} \rangle$, $U^{em} = \langle \delta E^2 + \delta B^2 \rangle / 8\pi$, $\underline{\underline{T}}^{em} = U^{em} \underline{\underline{I}} - \langle \delta \underline{E} \, \delta \underline{E} + \delta \underline{B} \, \delta \underline{B} \rangle / 4\pi$

For homogeneous electrostatic fluctuations $\underline{R}_e = - \underline{R}_i$ that is anomalous resistivity can only be due to fluctuations in which the ions participate. The energy extracted from the electrons by electrostatic modes is delivered to the ions and the electric field $-K_e = K_i + \partial \langle \delta E^2 / 8\pi \rangle / \partial t$.

On the basis of these general considerations it is immediately possible to exclude theories which arrive at a stationary spectrum solely by quasilinear or nonlinear modifications, such as trapping or nonlinear Landau damping, of the wave - electron interaction without wave dissipation by the ions if such spectra, in agreement with observation, are connected with anomalous resistivity and heating. Simulation of ion sound turbulence shows that stabilization is due to the resonant interaction with a hot ion tail[4].

The quasilinear diffusion term implies a quasilinear relation between the fluctuating electric field and the fluctuating current $\delta \underline{J}_j$. The transfer rates (65-67) are then expressed in terms of the conductivity tensor and the spectrum. For a set of electrostatic plane waves

$$\underline{R}_j = 2\mathrm{Im} \int d\underline{k}\ W(\underline{k})\ \varepsilon_j\ (\underline{k}, \omega_{\underline{k}})\ \underline{k} \tag{69}$$

$$\underline{K}_j = 2\mathrm{Im} \int d\underline{k}\ W(\underline{k})\ \varepsilon_j\ (\underline{k}, \omega_{\underline{k}})\ \omega_{\underline{k}} \tag{70}$$

using the definition $\sigma_j\ (\underline{k}) = k^2 \varepsilon_j\ \phi_k / 4\pi$ of the dielectric constant. Separating resonant and adiabatic terms the conservation laws may be written as

$$\underline{R}_i^r + \frac{\partial}{\partial t}\ \underline{G} + \frac{\partial}{\partial \underline{x}} \cdot \underline{\underline{T}} = - \underline{R}_e^r \tag{71}$$

$$K_i^r + \frac{\partial}{\partial t}\ U + \frac{\partial}{\partial \underline{x}} \cdot \underline{S} = - K_e^r \tag{72}$$

where $\underline{G} = \underline{G}^{em} + \underline{G}^e + \underline{G}^i$ is the total momentum and $U = U^{em} + U^e + U^i$ the total energy associated with the wave motion, etc. The simplest of the adiabatic terms are obtained by letting $\omega \to \omega + i\partial/\partial t$ in the transfer rates and expanding in the weak time dependence of the spectrum

$$\underline{R}_j = \int d\underline{k}\ W(\underline{k}) \frac{\partial \varepsilon}{\partial \omega_{\underline{k}}} \underline{k} \tag{73}$$

$$U_j = \int d\underline{k}\ W(\underline{k}) \frac{\partial \omega \varepsilon}{\partial \omega_{\underline{k}}} \tag{74}$$

and represent sloshing momentum and energy associated with the wave motion in species j. The weak time dependence of the plasma and the space dependence should also be considered however.[24,26]
For an isotropic distribution we obtain from (64)

$$\mathrm{Re}\ \varepsilon_e\ (\underline{k},\omega) = \frac{4\pi e^2}{mk^2} \int_0^\infty dw\ 4\pi \left\{ F(w) + \frac{1}{w}\frac{\partial F}{\partial w}\frac{1}{2}\hat{\omega}\ \ln\left|\frac{1+\hat{\omega}}{1-\hat{\omega}}\right| \right\} \tag{75}$$

$$\mathrm{Im}\ \varepsilon_e\ (\underline{k},\omega) = \frac{8\pi^3 e^2}{mk^2} \frac{\omega}{k} F\left(\frac{\omega}{k}\right) \quad . \tag{76}$$

Expanding the log term in $\hat{\omega}=\omega/kw<<1$ generates the asymptotic expansion for Re ε_e. To lowest order in $\omega/kv_e<<1$

$$\mathrm{Re}\ \varepsilon_e\ (\underline{k},\omega) = \left(\frac{\omega_e}{kv_e}\right)^2 \left[a_{-2} - a_{-4} \left(\frac{\omega}{kv_e}\right)^2 + \ldots \right] \tag{77}$$

$$\mathrm{Im}\ \varepsilon_e\ (\underline{k},\omega) = \left(\frac{\omega_e}{kv_e}\right)^2 \left(\frac{\pi}{2}\right)^{1/2} a_{-3} \frac{\omega}{kv_e} \quad . \tag{78}$$

The form factor a_{-3} has been defined by (16) and measures the slope of the reduced distribution function compared to a Maxwellian of the same energy. The form factor $a_{-2} = \langle (v_e/w)^2 \rangle$ determines the effective temperature $T_e^* = T_e/a_{-2} = mv_e^{*2}$ for wave dispersion. For ion sound waves

$$\left(\frac{\omega}{kv_e^*}\right)^2 = \frac{Zm}{M} \frac{1}{1+(kv_e^*/\omega_e)^2} \quad . \tag{79}$$

The form factors for distributions (1) and 2D distributions are evaluated in the appendix. The resonant transfer rates for a drifting isotropic distribution may be obtained by making the replacement $\omega \to \omega - \underline{k} \cdot \underline{u}$ in (78) and using (69-70). They agree with the transfer rates obtained in Section II from the resonant diffusion term. The energy extracted from the electrons becomes e.g.

$$-K_e^{or} = a_{-3}(2\pi)^{1/2} \left\langle \frac{\omega_{\underline{k}}}{kv_e} \frac{\underline{k}\cdot\underline{u}-\omega_{\underline{k}}}{kv_e} \frac{\omega_e}{kv_e} \right\rangle \frac{W}{nT_e} \; nT_e\,\omega_e \qquad (80)$$

The factor $\omega_{\underline{k}}$ in the spectral average is replaced by $\underline{k}\cdot\underline{u} - \omega_{\underline{k}}$ in the heating rate (21) and by $-\underline{k}$ in the momentum transfer rate (18). The superscript indicates again that these expressions are valid for an isotropic distribution.

The growth rate for ion acoustic waves may be written in the form

$$\gamma_{\underline{k}} = - \operatorname{Im} \varepsilon(\underline{k},\omega_{\underline{k}})/(\partial\varepsilon/\partial\omega_{\underline{k}}) = \omega_i \frac{k\lambda_D^*}{2(1+k^2\lambda_D^{*2})^{\frac{3}{2}}} \left(\frac{\pi}{2}\right)^{1/2} \frac{a_{-3}}{a_{-2}} \frac{\hat{\underline{k}}\cdot\underline{u}_{eff}-u^*}{v_e} \qquad (81)$$

where $\lambda_D^* = v_e^*/\omega_e$ and $\underline{u}_{eff}$ includes the relative e-i drift and effective drifts connected with the anisotropy of the electron distribution.[9] The critical drift velocity $u^* > \omega/k$ depends on the interaction with the ions. We obtain

$$\frac{u^*}{v_e} = \frac{\omega}{kv_e} + \frac{1}{a_{-3}} \frac{Z\,T_e}{T_i} \frac{\omega}{kv_i} \exp\left[-\frac{1}{2}\left(\frac{\omega}{kv_i}\right)^2\right] , \qquad (82)$$

assuming that the wave-electron interaction is modified according to (77-78) while the wave-ion interaction remains linear with a

Maxwellian ion distribution. According to Fig. 1 such a situation should be established shortly after the onset of the stability. The linear interaction with a high energy ion tail $(\omega/kv_h)^2 \approx 1$ which dominates at later times gives rise to an additional damping

$$\frac{u^{**}}{v_e} \approx \frac{1.2}{a_{-3}} \delta \frac{Z T_e}{T_i} \tag{83}$$

where the tail is modeled by a Maxwellian in the half space $\underline{k}\cdot\underline{v} > 0$ of wave propagation. The tail parameters $\delta = n_h/n_i$, T_h may be estimated from the conservation laws[27] if the observation is used that most of the energy and momentum extracted from the electrons is delivered by resonant interaction to a strongly anisotropic tail extending in the direction of the drift $\underline{u}$, whereas the bulk is heated only weakly and undergoes nearly free acceleration by the applied electric field[4]. From (69-70) we obtain

$$\frac{\partial}{\partial t} \alpha \delta M v_h \approx < 1/(1- \frac{\omega_{\underline{k}}}{\underline{k}\cdot\underline{u}}) > \frac{1}{u} \frac{\partial}{\partial t} Z T_e \tag{84}$$

$$\frac{\partial}{\partial t} \beta \delta T_h \approx < \frac{\omega_{\underline{k}}}{\underline{k}\cdot\underline{u}} /(1- \frac{\omega_{\underline{k}}}{\underline{k}\cdot\underline{u}})> \frac{\partial}{\partial t} Z T_e \tag{85}$$

using the appropriate spectral averages obtained from the relations between the electron transfer rates, cf. (69-70) and the comment following (80). For a Maxwellian tail in the halfspace $\underline{u}\cdot\underline{v}>0$ the form factors are $\alpha = (2/3)\ (2/\pi)^{1/2}$ and $\beta = 1$. These equations are easily solved for $\underline{u}$ = const, $\omega_{\underline{k}}/k \sim T_e^{1/2}$ or $\omega_{\underline{k}}/\underline{k}\cdot\underline{u}$ = const. For $\underline{u}$ = const and $(\omega_{\underline{k}}/\underline{k}\cdot\underline{u}) \ll 1$ the number of particles in the tail is built up as

$$\delta \approx (\frac{Zm}{M})^{1/2} \frac{v_{eo}}{\alpha u} (\frac{Z T_e}{T_h})^{1/2} \frac{\hat{T}_e - 1}{\hat{T}_e^{1/2}} \tag{86}$$

where $\hat{T}_e = T_e/T_{eo}$. The temperature of the tail varies as

$$\left(\frac{T_h}{Z\,T_e}\right)^{1/2} \approx \frac{2\alpha}{3\beta}\left(\frac{1}{a_{-2}}\right)^{1/2} \left\langle \frac{\underline{k}\,\underline{u}}{\underline{k}\cdot\underline{u}} \right\rangle \frac{\hat{T}_e^{3/2} - 1}{\hat{T}_e^{3/2} - \hat{T}_e^{1/2}} \qquad (87)$$

using (79). Similar relations are obtained for $(u/v_e) = \text{const}$. The evolution of the total ion thermal energy $3/2\, n_i T_i$ and the temperature ratio T_i/ZT_e as a function of T_e is obtained from (85) if $\beta\delta T_h$ is replaced by T_i. With the tail parameters (86-87) the growth rate is obtained from (81) and (83). Using also the heating rate (21) the evolution of the wave energy W and the fluctuation level W/nT_e may be found. The maximum of W/nT_e is given by the condition that wave growth is overtaken by heating, rather than nonlinear saturation. The important feature of tail formation is the increase in the number of tail particles[4,27-28] rather than tail heating at fixed n_h/n which has been modeled by other authors[29]. According to (87) the temperature ratio T_h/ZT_e stays relatively constant and of order unity. The tail buildup leads to self-quenching of the instability for $u \lesssim u^* \sim (m/M)^{1/4}\, v_e$ (typically in $\omega_i t \gtrsim 100$ for the simulation), as follows from (83), (86-87). If heating is significant the total temperature ratio T_i/ZT_e approaches a value quite independent of the initial temperature ratio, according to (85). These scaling laws describe the later stages of the simulation experiments[4] quite well.

The model could be refined by including adiabatic interaction terms[30] which are important in the initial phase, linear and non-linear interaction with the bulk[29, 31] and representation of the spectrum by sample wave modes. Anyway however, a more accurate description requires the use of a kinetic equation for the ion distribution. There have been several attempts. Selfsimilar solutions of the quasilinear equations at marginal stability have been studied[32], but the simulation shows a continued tail buildup until the instability is quenched, even if u/v_e is held constant, which is necessary for a selfsimilar solution. The quasilinear equations of Section II do not apply to the interaction of ion sound waves with the ions unless it is assumed that the bulk is dominated by i-i collisions and the tail by elastic scattering from the waves which requires $\omega/kv << 1$. Such assumptions have been made by Kovrizhnykh[33] when considering current driven ion sound in a homogenous plasma and $B = 0$. In an earlier paper on the same problem by Rudakov and Korablev[2] the resonant interaction with the ions was neglected altogether. In these papers it was also assumed that the quasilinear effects reduce the spectrum to a single wave number k_o, with constant growth rate $\gamma \gtrsim 0$ in a cone $|\Theta| < (\pi/2)$ about the drift direction. Simulation and experiments, however, show a broad spectrum in $\underline{k}$, strongly anisotropic ion tails extending to about 2 c_s i.e. $\omega/kv > 0.5$, and anisotropic ion bulks.

V. Conclusion

We have discussed the kinetic equation for the electrons including Coulomb collisions and turbulence. It was assumed that the turbulence is due to ion sound and related spectra which are characterized by small phase velocities $(\omega/kv_e) \ll 1$ and short wave lengths $(kv_e/\Omega_e) \gg 1$. Scattering by such fluctuations is rather similar to electron-ion collisions in that it has the same speed dependence w^{-3} and is predominently elastic. For e-e collisions which dominate in the classical case on the other hand elastic and inelastic scattering occur at the same rate.

Classical transport equations with an enhanced effective collision frequency have been used to describe turbulence effects. We have shown that this is justified only in rather restricted circumstances i.e. very low fluctuation levels (32), isotropic turbulence, and that Z_{eff} and $T_{i\ eff}$ in the classical transport equations must also be modified. Z_{eff} not only determines the effective collision frequency but also the relative importance of electron-electron collisions. Usually, however, turbulence not only dominates elastic scattering, $Z_{eff} \gg 1$, but inelastic scattering as well. Electron-electron collisions are then no longer able to maintain the distribution function close to a quasistationary Maxwellian and the energy distribution must be determined self consistently. Scattering in angle and the magnetic field keep the electron distribution essentially isotropic, but the anisotropy must be determined for applied electric fields along the magnetic field and transport connected with gradients. The reduction of the kinetic equation to the much simpler system (35), (36) for the energy distribution and the anisotropic part is based upon the dominance of isotropization. This process replaces the usual relaxation to a local Maxwellian in classical transport theory. The solution of the equation for the energy distribution is discussed for the cases where Coulomb collisions, runaway or turbulent heating dominates. Numerical solution for other cases appears feasible. Quasilinear flattening of the electron distribution leads to a significant modification of the dispersion relation, momentum and energy transfer rates,etc. by the time turbulent heating increases the temperature by a few percent,cf. Fig. 1.

Ample evidence from simulation studies of current driven ion sound turbulence has been cited to justify the use of the unmagnetized quasilinear diffusion coefficients and dielectric constant for the electrons and the theoretical basis is briefly discussed. Momentum and energy transfer rates and the conservation laws have been used to construct a dynamical model for ion tail formation which is based upon the observation that most of the energy and momentum extracted from the electrons is delivered by resonant interaction to a strongly anisotropic ion tail, whereas the bulk of the ions is heated only weakly and nearly freely accelerated by the applied electric field. Resistivity, heating rates and the dispersion relation for non Maxwellian isotropic electron distribution and model ion distributions are obtained.Turbulent heating by a current across the magnetic field is described quite well by this simple model, including self quenching of the instability.

To describe heating by a current along the magnetic field and transport connected with gradients the anisotropic part of the electron distribution will be determined self-consistently in a companion paper.[9]

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Appendix A Form Factors of the Energy Distribution.

The form factors of the distribution (1) may be expressed in terms of the Γ function $\Gamma(z) = \int_0^\infty dt\, e^{-t}\, t^{z-1}$

$$< w^a > = \frac{1}{n}\int d\underline{w}\; F(w) w^a = \frac{4\pi C_s}{s}\, \Gamma\, (\frac{a+3}{s})\, v_o^a \tag{A1}$$

$$-\frac{1}{n}\int d\underline{w}\; w^{a+1}\, \frac{\partial F}{\partial w} = \frac{4\pi C_s}{s}\, \Gamma\, (\frac{a+3}{s} + 1)\, v_o^a \tag{A2}$$

Thus for s	s = 2	s = 5	
$C_s = \frac{s}{4\pi\Gamma(3/s)}$	0.1796	0.2672	(A3)
$(\frac{v_e}{v_o})^2 = \frac{\Gamma(5/s)}{3\Gamma(3/s)}$	0.5	0.2238	(A4)

The following form factors are all normalized to unity for a Maxwellian.

	s = 5	
$a_{-1} = -\frac{1}{2n}(\frac{\pi}{2})^{1/2}\, v_e \int d\underline{w}\, \frac{\partial F}{\partial w}$	0.8832	(A5)
$a_{-2} = -\frac{1}{n}\, v_e^2 \int d\underline{w}\; w^{-1}\, \frac{\partial F}{\partial w}$	1/1.4492	(A6)
$a_{-3} = -\frac{1}{n}\, (\frac{\pi}{2})^{1/2} v_e^3 \int d\underline{w}\; w^{-2}\, \frac{\partial F}{\partial w}$	0.44563	(A7)
$a_{-4} = -\frac{1}{n}\, v_e^4 \int d\underline{w}\; w^{-3}\, \frac{\partial F}{\partial w}$	0.19585	(A8)

The dielectric constant (73-74) becomes

$$\varepsilon_e (\underline{k},\omega) = \left(\frac{\omega_e}{kv_e}\right)^2 \left[a_{-2} - a_{-4} \left(\frac{\omega}{kv_e}\right)^2 - \ldots + i a_{-3} \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega}{kv_e} \right] .$$

The effective collision frequency for resistivity and heating is reduced by a_{-3} compared to a Maxwellian $\nu^{*} = \nu(v_e)\ (2/\pi)^{1/2} a_{-3}/3$ and the effective temperature in the low speed limit of the Coulomb diffusion coefficients (12) is $T_e a_{-1}/ a_{-3}$.

Appendix B: Two Dimensional Diffusion

For modes in the plane perpendicular to $\underline{B}$ which also satisfy $(\omega/kw_\perp) \ll 1$, as is the case for some loss cone flute modes and the ion distributions or in 2D simulation of ion sound, a two dimensional theory can be carried out. The diffusion coefficient

$$\langle D^{\perp\perp} \rangle^e = \frac{8\pi e^2}{m^2} \int d\underline{k}_\perp\, W(\underline{k}_\perp) \frac{1}{(1 - \hat{\omega}^2)^{1/2}} \hat{\omega}^2 \frac{1}{k_\perp w_\perp} \tag{B1}$$

retains its $w_\perp^{-3}$ speed dependence for $\hat{\omega} = (\omega/k_\perp w_\perp) \ll 1$.
The selfsimilar solution for the 2D distribution $F(w_\perp) = \int dw_\parallel \frac{d\phi}{2\pi} f(\underline{w})$ thus has the form

$$F(w_\perp) = n(C_s/v_o^2) \exp\left[- (w_\perp/v_o)^s \right] \tag{B2}$$

with s = 5. The Green's function can be found in the same manner as (66) for the 3D case. For $D^{\perp\perp} = \bar{D}/w_\perp^3$ i.e. s = 5 we have

$$G(w_\perp, w_\perp', t, t') = \frac{5}{2\pi\tau} (w_\perp \; w_\perp')^{3/2} \, I_{-3/5} \left[\frac{2}{\tau} (w_\perp w_\perp')^{5/2}\right]$$

$$\exp\left[-\frac{w_\perp^5 + w_\perp'^5}{\tau}\right] \quad \text{(B3)}$$

where $\tau = \int_{t'}^{t} dt'' \; 25 \; \bar{D} \; (t'')$.

The form factors in the dielectric constant (B8) are modified to

$$a_{-2} = - v_e^2 \frac{1}{n} \int d\underline{w}_\perp \frac{1}{w_\perp} \frac{\partial F}{\partial w_\perp} \quad \text{(B4)}$$

$$a_{-3} = - \left(\frac{2}{\pi}\right)^{1/2} v_e^3 \frac{1}{n} \int d\underline{w}_\perp \frac{1}{w_\perp^2} \frac{\partial F}{\partial w_\perp} \quad \text{(B5)}$$

where $d\underline{w}_\perp = 2\pi w_\perp \, dw_\perp$. The collision frequency for elastic scattering also retains its $w_\perp^{-3}$ dependence

$$\underline{\underline{\nu}}_1 (w_\perp) = \frac{2}{a_{-3}} \left(\frac{2}{\pi}\right)^{1/2} \underline{\underline{\nu}}_1^* \left(\frac{v_e}{w_\perp}\right)^3 \quad \text{(B6)}$$

where in 2D and 3D, cf. (13-20) and (78)

$$\underline{\underline{\nu}}_1^* = \omega_e \frac{W}{nT_e} (2\pi)^{1/2} a_{-3} \left\langle \frac{\omega_e}{kv_e} \frac{\underline{k}\,\underline{k}}{k^2} \right\rangle \quad \text{(B7)}$$

For the distributions (B2) we obtain the form factors

s	s = 2	s = 5	
$C_s = \frac{s}{2\pi\Gamma(2/s)}$	$\frac{1}{\pi} = 0.3183$	0.3588	(B8)
$\left(\frac{v_e}{v_o}\right)^2 = \frac{s\,\Gamma(4/s)}{\Gamma(2/s)}$	0.5	0.2624	(B9)
$a_{-2} = \frac{s(v_e/v_o)^2}{\Gamma(2/s)}$	1	$\frac{1}{1.6905}$	(B10)
$a_{-3} = \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{v_e}{v_o}\right)^3 s \frac{\Gamma(1-1/s)}{\Gamma(2/s)}$	1	0.2815	(B11)

As expected, quasilinear effects are stronger in 2D.

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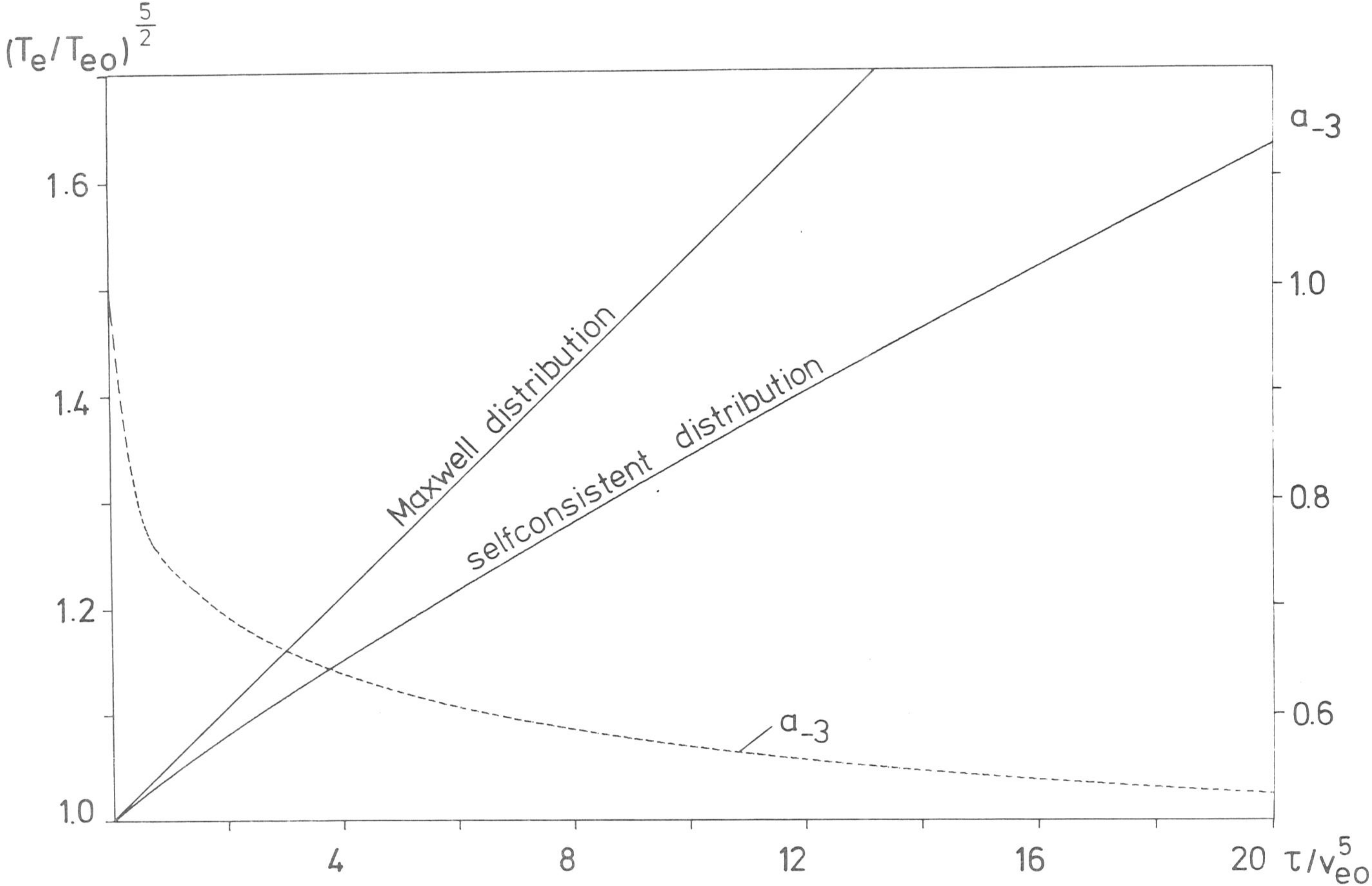
Figure Captions

Fig. 1 Electron temperature and form factor a_{-3} versus diffusion time (61), for the selfconsistent distribution that was initially Maxwellian.



$(T_e/T_{eo})^{\frac{5}{2}}$
1.6
1.4
1.2
1.0
Maxwell distribution
selfconsistent distribution
a_{-3}
1.0
0.8
0.6
4
8
12
16
20 τ/v_{eo}^5