

Deceleration of Electron Rings by Ions

U.Schumacher, I.Hofmann, P.Merkel  
and M.Reisert†

IPP O/31

Febr.1976



**MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK**

**8046 GARCHING BEI MÜNCHEN**



**MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK**  
**GARCHING BEI MÜNCHEN**

Deceleration of Electron Rings by Ions

U.Schumacher, I.Hofmann, P.Merkel  
and M.Reiser†

IPP O/31

Febr. 1976

†Visiting from University of Maryland, College Park/USA

*Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem  
Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die  
Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.*

Summary

The deceleration of intense relativistic electron rings by ions, that are created by impact ionization in the electron rings, is calculated analytically and numerically. It turns out that the stopping of rings with initial axial velocities of about  $\beta_0 = 0.1$  is possible under conditions, that easily can be realized experimentally in the Maryland ERA. The stopping length of the electron rings, however, depends sensitively on the initial axial ring velocities  $\beta_0$  (as  $\beta_0^3$ ). If the method of ring deceleration by ions is used, the reproducibility of the initial ring parameters (especially that of  $\beta_0$ ) should be very good to get reliable electron ring loading and acceleration.

## I. Introduction

In one of the devices for electron ring acceleration (ERA), the Maryland scheme <sup>1</sup>, an intense relativistic electron ring is formed by the passage of a hollow electron beam through a cusped magnetic field, which transforms axial into rotational velocity. The axial velocity of the electrons after the passage of the cusped field, however, is still in the range of  $dz_e/dt = \beta_0 c = 0.1c$ , where  $c$  is the speed of light. To further slow down the electrons to axial velocities, where the trapping of ions in the electron ring is possible, different methods may be applied, such as the trapping of the ring in a fast pulsed mirror magnetic field <sup>2</sup>, the stopping by resistive wires <sup>3</sup> or cylinders <sup>4,5</sup>, or the deceleration by ions, that are produced by impact ionization in the electron ring and that are partly accelerated before escaping from the ring potential well.

In this report the method of ring deceleration by ions is treated analytically and numerically, and computational results are presented for several examples with parameter values close to those of the Maryland ERA device.

## II. Analytical Considerations

In an electron ring of  $N_e$  electrons the ions are created at a rate of

$$(1) \quad dN_i/dt = \alpha N_e,$$

where  $\alpha$  is given by the pressure  $p$  of the background gas. If the electron ring is moving with an axial velocity  $v_e$ , which is so high, that the ions cannot be trapped in the ring, the average ion number  $\tilde{N}_i$  in the ring is

$$\tilde{N}_i/N_e = \alpha \frac{b}{v_e},$$

where  $b$  is the minor half axis of the electron ring. The pressure is assumed to be constant.



The ions, created in the potential well  $\Delta U$  of the electron ring, are accelerated to the velocity  $v_i$ , which is approximately given by

$$(2) \quad v_i = \frac{v_e}{1 + \alpha b / v_e \cdot M_i / m_e \gamma} \cdot \left( 1 - \sqrt{1 - \frac{e\Delta U}{M_i v_e^2 / 2} \cdot \left( 1 + \alpha \frac{b}{v_e} \cdot \frac{M_i}{m_e \gamma} \right)} \right),$$

where  $M_i$  is the ion mass and  $m_e \gamma$  the (relativistic) electron mass.

Each ion that is created with a single charge near the electron ring center takes the energy

$$W_i = \frac{M_i}{2} v_i^2 + e\Delta U$$

from the electron ring. The rate of energy loss by the electron ring is then given by

$$(3) \quad \frac{dW}{dt} = N_e \gamma \frac{m_e}{2} \frac{dv_e^2}{dt} = -\frac{dN_i}{dt} \left\{ e\Delta U + \frac{M_i}{2} v_i^2 \right\}.$$

The energy loss due to the secondary electrons created in each collision is neglected because of the nearly isotropic emission under our conditions.

$$\text{As initially } v_e^2 \gg \frac{2e\Delta U}{M_i} \left( 1 + \alpha \frac{b}{v_e} \frac{M_i}{m_e \gamma} \right)$$

in all cases under consideration,  $M_i v_i^2 / 2$  can be neglected compared to  $e\Delta U$  in equ.(3). This results in estimates for the length, the time, and the ion number needed for the stopping of electron rings with an initial  $\beta_0$ .

The square root in equ.(2) vanishes, if the trapping velocity  $v_{tr}$  of the electron ring is reached.

The equs.(1) and (3) then give

$$\gamma \frac{m_e}{2} \frac{dv_e^2}{dt} = \gamma \frac{m_e}{2} v_e \frac{dv_e^2}{dz} = -\alpha e\Delta U,$$

and the stopping length  $z_{tr}$  is about

$$(4) \quad z_{tr} \approx - \int_{v_{eo}}^{v_{tr}} \frac{\gamma m_e}{2\alpha e \Delta U} v_e dv_e^2 \approx \frac{\gamma m_e}{3\alpha e \Delta U} v_{eo}^3 = \frac{\gamma m_e}{3\alpha e \Delta U} c^3 \beta_o^3$$

The stopping time  $t_{tr}$  is given by

$$(5) \quad t_{tr} \approx \frac{\gamma m_e}{2\alpha e \Delta U} \int_{v_{eo}}^{v_{tr}} dv_e^2 \approx \frac{\gamma m_e}{2\alpha e \Delta U} v_{eo}^2 = \frac{\gamma m_e}{2\alpha e \Delta U} c^2 \beta_o^2,$$

and the ion number  $N_i$  (normalized to  $N_e$ ), needed for stopping is approximately

$$(6) \quad \frac{N_i}{N_e} \approx - \frac{\gamma m_e}{2e \Delta U} \int_{v_{eo}}^{v_{tr}} dv_e^2 \approx \frac{\gamma m_e}{2e \Delta U} v_{eo}^2 = \frac{\gamma m_e}{2e \Delta U} c^2 \beta_o^2$$

with the relations

$$(7) \quad z_{tr} \approx \frac{2}{3} \frac{c \beta_o}{\alpha} \cdot \frac{N_i}{N_e} \text{ and}$$

$$(8) \quad t_{tr} \approx \frac{1}{\alpha} \frac{N_i}{N_e} = \frac{3}{2c \beta_o} \cdot z_{tr}.$$

To get a feeling whether the stopping of electron rings with initial axial velocities in the order of  $c \beta_o = 0.1 \cdot c$  can be realized experimentally, we choose the following example:

$$\gamma = 5, \quad e \Delta U = 0.1 \text{ MeV}, \quad \beta_o = 0.1 \text{ and } z_{tr} = 30 \text{ cm}$$

and we obtain from equs.(6) to (8)

$$\frac{N_i}{N_e} = 0.127; \quad t_{tr} \approx 15 \text{ ns and } \alpha \approx 8.46 \cdot 10^6 \text{ s}^{-1}.$$

The ionization coefficient  $\alpha$  is roughly given by  $\alpha(\text{s}^{-1}) \approx 1.064 \cdot 10^8 \cdot Z_M \cdot p(\text{torr})$ , where  $Z_M$  is the number of electrons per molecule. To obtain the desired ion number one needs in the case of hydrogen ( $Z_M = 2$ ) a pressure of  $p \approx 4 \times 10^{-2}$  torr.



The example chosen demonstrates that the stopping of electron rings by ions is possible under conditions that can be realized in the Maryland ERA experiment. The needed gas pressure, however, seems to be relatively high compared to the vacuum requirements in the acceleration section.

Furthermore the cubic dependence of the stopping length  $z_{tr}$  on the initial ring axial speed  $\beta_0$  (equ.(4)) imposes strong limitations on the reproducibility of the initial ring parameters.

In the following the stopping of the electron ring by ions is treated numerically.

### III. Numerical Calculations

For the numerical treatment of the problem of deceleration of electron rings by ions the equations of motion of the electron ring and several ion subrings are solved with a computer program that was developed for similar problems<sup>6</sup>. It is assumed, that ions are created by impact ionization in the electron rings, that they are accelerated by the electric field of the electron ring and - if the electron ring speed is too high - escape from the ring. The electron ring as well as the ion subrings are regarded to be rigid. This assumption of unchanged electron ring shape, if ions are created and lost, might be too restrictive and will be modified in further calculations. The electron density distribution  $n_e$  is assumed to be a Gaussian one, i.e.

$$n_e(r) = n_e(0)\exp(-r^2/r_0^2),$$

where  $r$  is the distance from the center of the ring minor cross section.

Dividing the ions into  $K$  subrings and neglecting the toroidicity (i.e. solving the straight beam problem) the equations for the motion in axial direction are

$$(9a) \quad m_e \gamma \ddot{z}_e = c \beta_0 B_r(z) e^{-\sum_{j=1}^K \frac{e^2 N_{i,j} \cdot z_j}{4\pi^2 R \epsilon_0} \cdot \frac{1}{z_e - z_{i,j}}} \cdot \left( 1 - e^{-((z_e - z_{i,j})/r_0)^2} \right) \left( 1 - \frac{1}{1 + (d/(z_e - z_{i,j}))^2} \right)$$

$$(9b) \quad M_i \ddot{z}_{i,j} = \frac{e^2 N_e \cdot z_j}{4\pi^2 R \epsilon_0} \cdot \frac{1}{z_e - z_{i,j}} \left( 1 - e^{-((z_e - z_{i,j})/r_0)^2} \right) \left( 1 - \frac{1}{1 + (d/(z_e - z_{i,j}))^2} \right)$$

$j = 1, K$

$z_e$  and  $z_{i,j}$  are the axial locations of the electron ring and the  $j$ -th ion subring, respectively.  $N_e$  is the total number of electrons and  $N_{i,j}$  the number of ions in the  $j$ -th ion subring.  $Z_j$  is the charge per ion in the  $j$ -th ring and  $d$  is the distance from the electron ring to an image field cylinder ("squirrel cage").  $\beta_0$  is the azimuthal electron velocity normalized to the speed of light  $c$ .

The image field cylinder turns out to be necessary for additional axial focussing of the ring, so that its influence is included in the above equations. These image fields reduce the coupling of the ion rings to the electron ring, especially at long distances between the ions and the electron ring center.

The coupling of the ion subrings to each other is neglected, as well as that of the secondary electrons, that are created at every ionization process.

In the following some examples of the numerical calculations are given for electron rings of  $r_0 = 0.5$  cm minor radius,  $R = 6$  cm major radius and  $d = 0.7$  cm distance to the image cylinder and maximum electric field strength of  $E_{\max} = 0.44 \cdot e N_e / 4\pi^2 \epsilon_0 R r_0 = 4.4$  MV/m (the factor 0.44 takes the Gaussian distribution and the image fields into account). The potential depth is  $\Delta U = 0.04$  MV. In fig.1 protons ( $M_i = m_p$ ) are created each 2 ns in the electron ring, that initially has an axial speed of  $c\beta_0 = 0.02 \cdot c$ . The total number of ions produced  $N_i$  to that of the electrons is  $N_i/N_e = 0.02$ .



The axial positions of the electron ring  $z_e$  and those of the ions  $z_{i,j}$  (dotted lines) are plotted versus the time. As can be seen from these plots the ions gain energy from the electrons; as a result, the electron ring quickly decelerates and actually stops at a distance of about 5 cm. But the ions, that had escaped earlier, catch up and interact with the electron ring. This leads to axial oscillations so that only a small fraction of the ions remains trapped in the electron ring. In this case the ions are assumed to be produced at the center of the electron ring, where the potential minimum is located. The creation of ions all over the electron ring corresponding to the electron density gives similar results.

The stopping length of the electron ring depends on the gas pressure used (or the total number of ions produced) and on the initial axial ring velocity  $\beta_0 c$ . Fig.2 shows (for an ionization interval of 10 ns) the deceleration of the electron ring for different applied gas pressures, expressed in the total number of produced ions to the electron number. Under the condition of electron rings with initial  $\beta_0 = 0.1$ , total ionization fractions of  $N_i/N_e = 0.3$  to 0.4 produced in 10 ns are necessary for ring stopping at a length of about 15 to 20 cm.

The dependence of the deceleration effect on the initial axial velocity  $\beta_0$  of the electron ring is illustrated in fig.3 for the same ionization interval of 10 ns. The stopping length depends on the initial  $\beta_0$ . A ring with  $\beta_0 = 0.08$  is stopped at  $z = 8$  cm, that with  $\beta_0 = 0.1$  at about  $z = 16$  cm, and for  $\beta_0 = 0.12$  the ionization interval is too short or the total number of ions produced is too small, respectively, to obtain complete stopping of the ring.

In the case of  $\beta_0 = 0.11$  the energy balance can easily be checked at  $t \approx 11$  ns, where the electron ring axial velocity has come to zero and its initial kinetic energy

$$E_{e0} = \frac{1}{2} m_e \gamma \beta_0^2 c^2$$

has been transferred to the kinetic and potential energy of the ions:

$$E_{\text{kin}} = 3.77 \cdot 10^{-9} \frac{\text{gcm}^2}{\text{s}^2} \quad \text{and} \quad E_{\text{pot}} = 2.18 \cdot 10^{-8} \frac{\text{gcm}^2}{\text{s}^2}$$

The corresponding  $\beta$ , calculated from

$$\beta^2 = \frac{2(E_{\text{kin}} + E_{\text{pot}})}{m_e \gamma c^2}$$

is  $\beta = 0.112$ , which is in relative good agreement with the initial  $\beta_0 = 0.11$  of the electron ring.

The decrease of the electron ring velocity  $dz_e/dt$  for the same conditions as in fig.3 is plotted in fig.4 versus the time; as expected, the strongest decrease is at smaller velocities. This is even more obvious in the plot of the electron ring deceleration  $(-d^2z_e/dt^2)$  in fig.5 versus the axial distance. These curves, which show a Bragg-curve-like structure, demonstrate the sensitive dependence of the stopping length  $z_{\text{tr}}$  (where the ions are trapped) from the initial ring velocity  $\beta_0$ .

The stopping length  $z_{\text{tr}}$  is proportional to the third power of  $\beta_0$

$$z_{\text{tr}} \sim \beta_0^3$$

in agreement with the analytic result in equ.(4), and the integral of the deceleration over  $z$  from zero to the stopping point

$$-\int_0^{z_{\text{tr}}} \frac{d^2z_e}{dt^2} dz \sim \beta_0^2$$

is proportional to  $\beta_0^2$ , i.e. proportional to the initial electron ring kinetic energy, as expected from the analytical estimate.

Fig.6 summarizes the dependence of the stopping length  $z_{\text{tr}}$  and the ion loading  $\tilde{N}_i/N_e$  of the rings (at  $t = 50$  ns) under the conditions of fig.3. Both the stopping length and the ion loading



fraction depend very sensitively on the initial electron velocity  $\beta_0$ .

While for the calculations mentioned so far the radial magnetic field component  $B_r$  in equ.(9a) has been chosen to vanish, fig.7 gives an example of the application of  $B_r \approx 4G$  for axial positions  $z \gtrsim 15$  cm. Again, as the figure indicates, the initial  $\beta_0$  has to be chosen very carefully to get the desired result of stopping, ion loading and acceleration of the electron rings.

Because of the sensitive dependence of the stopping length and ion loading fraction on the initial electron ring velocity  $\beta_0$  the practical application of this method of ring stopping demands a very good reproducibility of the electron ring initial properties.

#### IV. Application

In the calculations the pressure always was assumed to be constant. In the actual experiment at Maryland, however, the pressure varies with distance ( $p = p(z)$ ), as it is produced by opening of a puff valve. A typical curve (according to the measurements of Misra <sup>7</sup>) is plotted in fig.8. Both peak pressure  $p_0$  as well as the "width"  $\Delta z$  of this  $p$ -versus  $z$ -curve can be varied experimentally by increasing the amount of gas in the "puff" or by injecting the electron ring at a different time after opening the puff valve. At later times the  $p(z)$  curve gets flatter and broader, for instance.

The spatial variation of the pressure  $p$  is taken into account by the relation

$$dN_i = N_e \alpha(z) dz / v_e.$$

The pressure pulse shape should have a certain axial distribution for a successful experiment. Since the electron ring axial velocity decreases due to the ion stopping, there is a speed, where some of the ions may catch up with the ring, before it enters the acce-

leration region. To avoid overloading of the ring, it appears better to work with a broad pressure pulse shape rather than the sharp curve shown in the figure.

Let  $z_{tr}$  be the position downstream from the cusp where the electron ring is slowed down to the trapping velocity  $v_{tr}$ . The number of ions that are loaded into the ring is then determined by the integral

$$\int_{z_{tr}}^{z_{acc}} \frac{\alpha(z)}{v_e(z)} dz,$$

where  $z_{acc}$  is the beginning of the expansion-acceleration region. An important problem for the experiment is how to control this integral to avoid overloading, which results in very low final ion energy, or underloading, so that the ring accelerates too fast and all ions are lost. As in the case of ring stopping by ions, this problem also requires good reproducibility of the operating parameters, especially the initial axial velocity  $\beta_0 c$  of the electron ring. In view of the strong dependence on  $\beta_0$ , it would be desirable to slow down the electron ring by some other method, such as resistive wall interaction, to a value close to the trapping velocity before it enters the gas cloud.

Acknowledgements

The authors are grateful to E. Springmann who carried out the numerical calculations.

## Figure Captions

- Fig.1 The axial positions of the electron ring and of protons versus time for  $\beta_0 = 0.02$
- Fig.2 The electron ring motion for different total ion numbers
- Fig.3 The electron ring motion for different initial axial velocities  $\beta_0 \cdot c$
- Fig.4 The electron velocity for different  $\beta_0$
- Fig.5 The electron ring deceleration for different  $\beta_0$
- Fig.6 The dependence of  $z_{tr}$  and  $\tilde{N}_i/N_e$  on  $\beta_0$
- Fig.7 The ring motion for different  $\beta_0$  with a superimposed  $B_r$ -field
- Fig.8 The axial pressure distribution as obtained with a puff valve (from Ref.7, Fig.33)

## Acknowledgements

The authors are grateful to H. Springmann who carried out the numerical calculations.



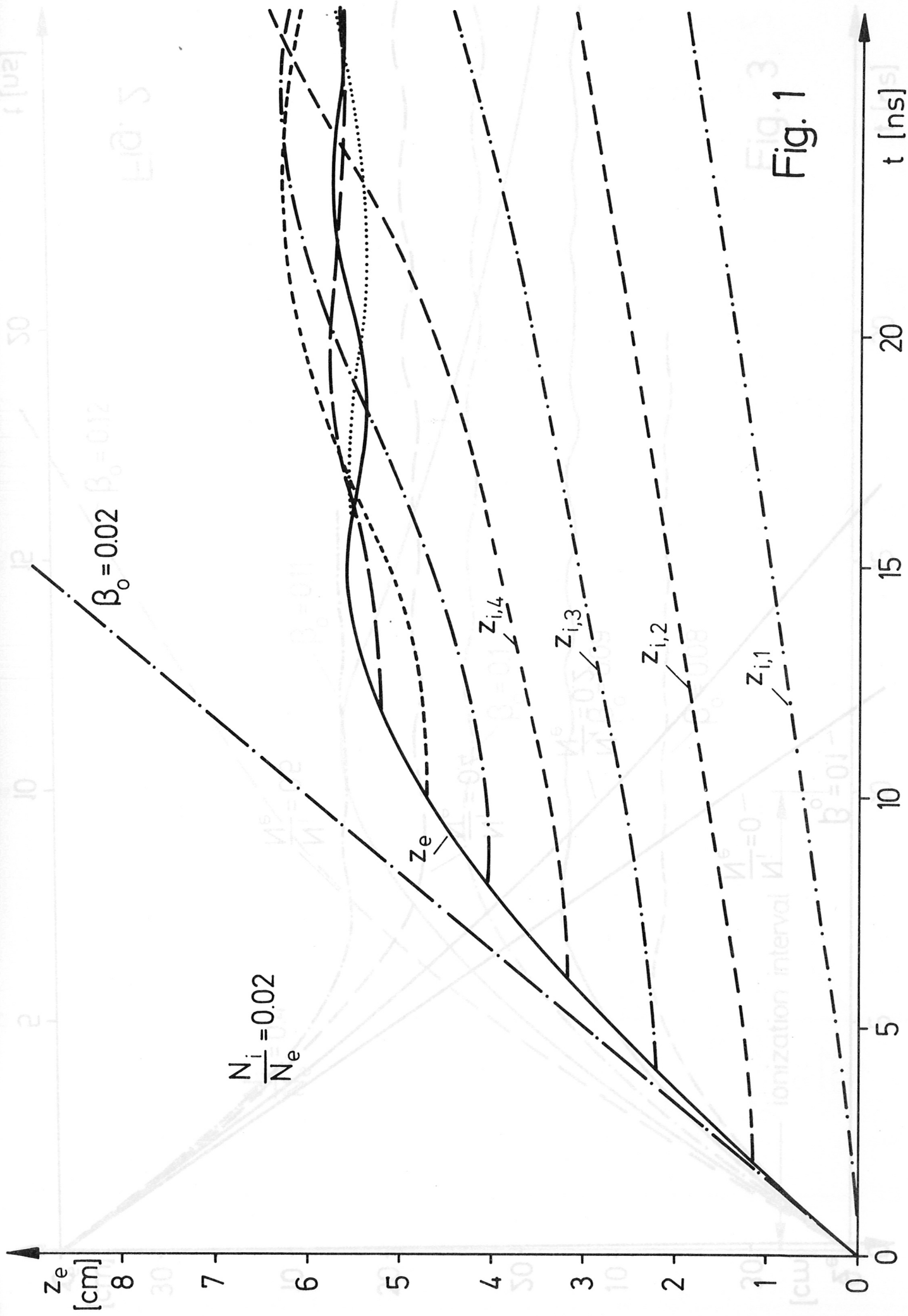


Fig. 1

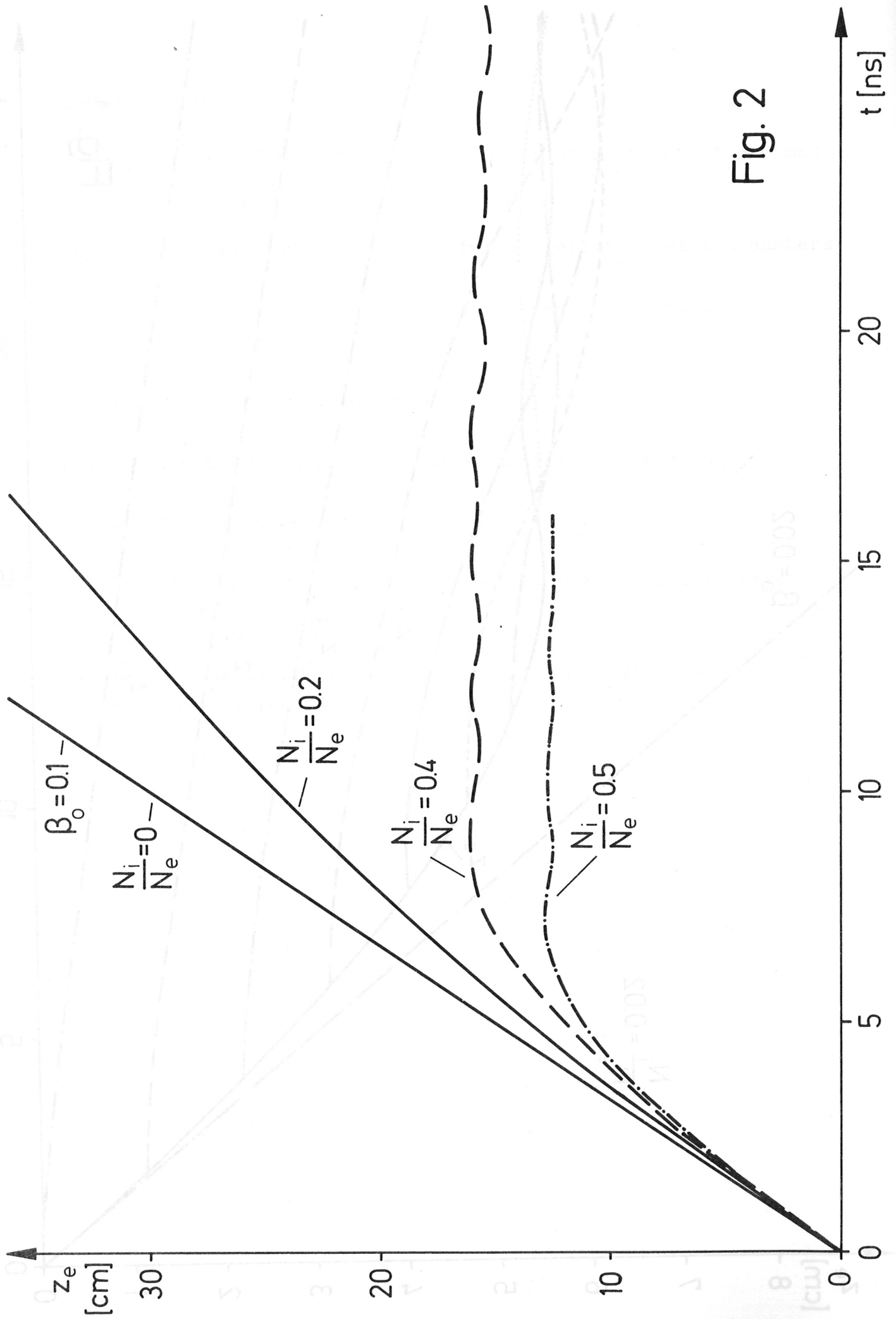


Fig. 2

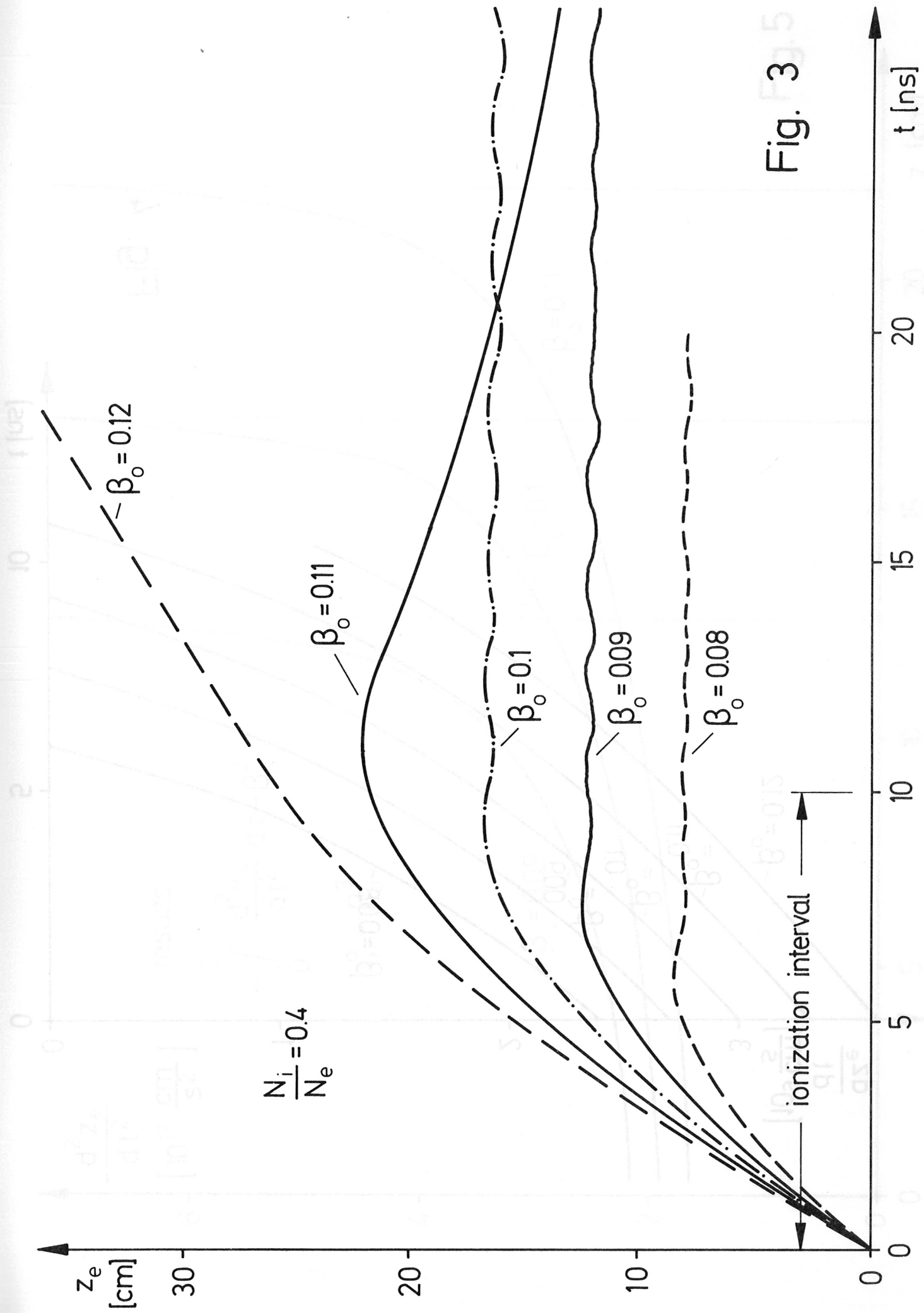


Fig. 33.5



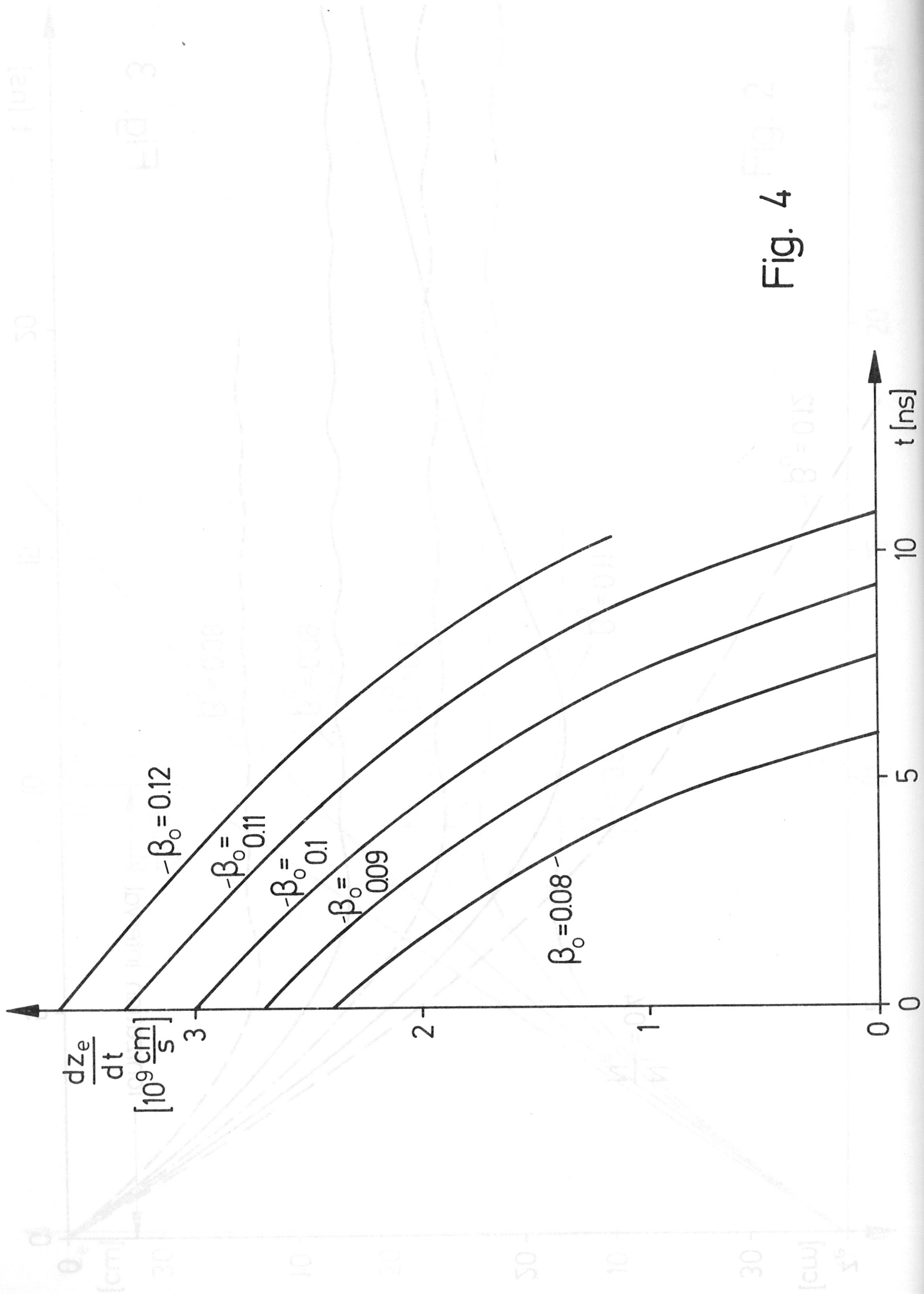


Fig. 4

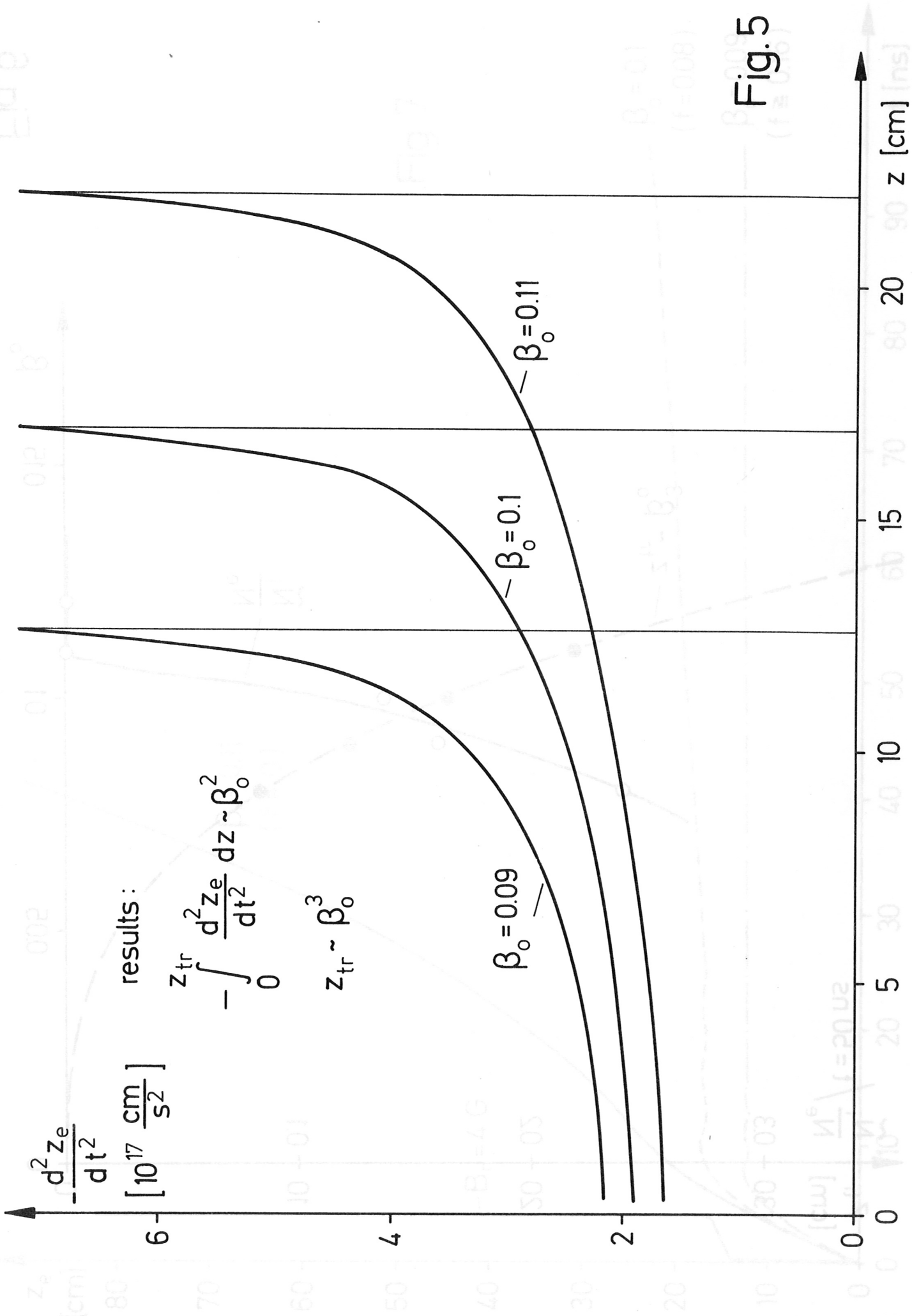


Fig.5

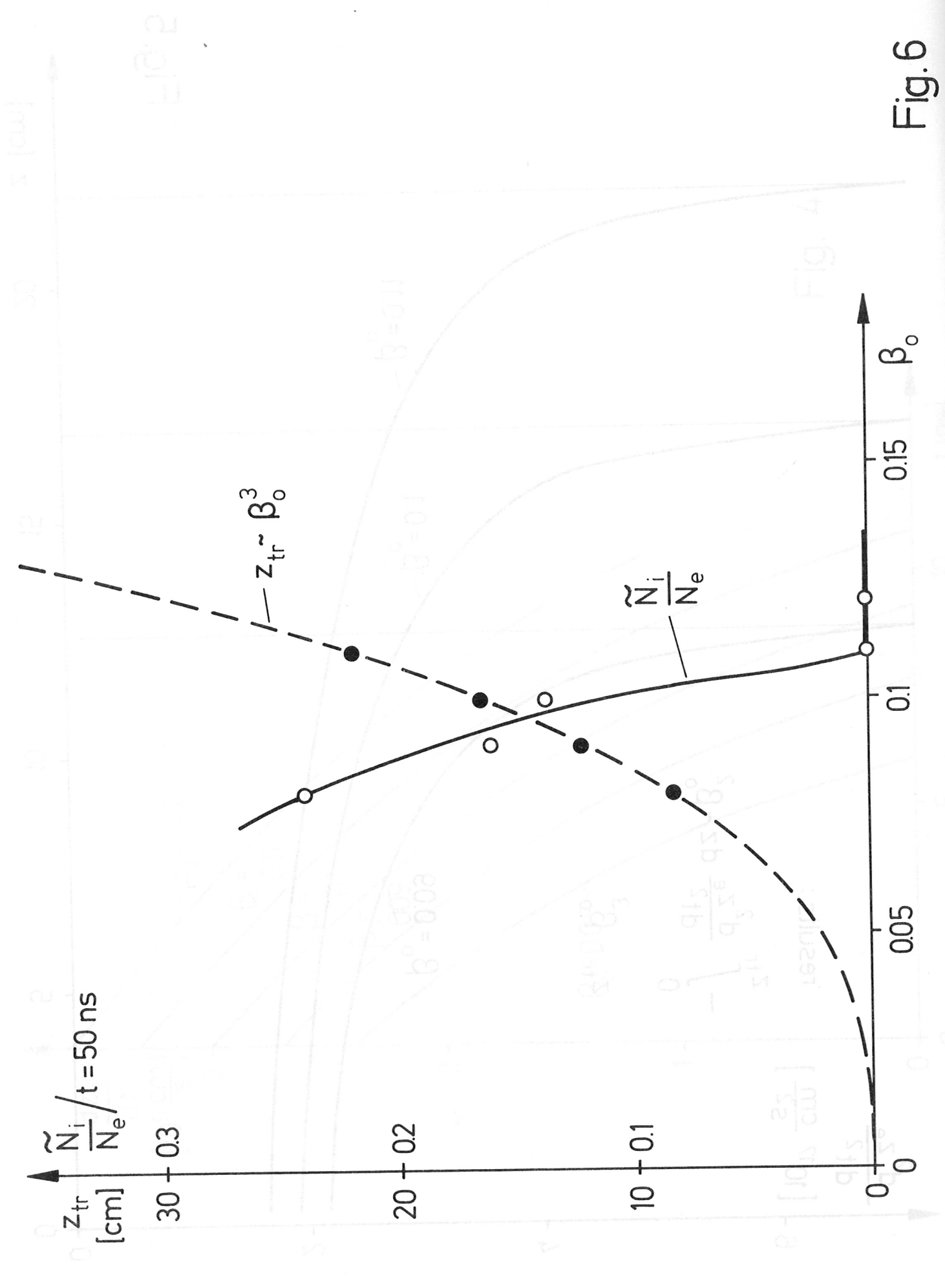


Fig. 6



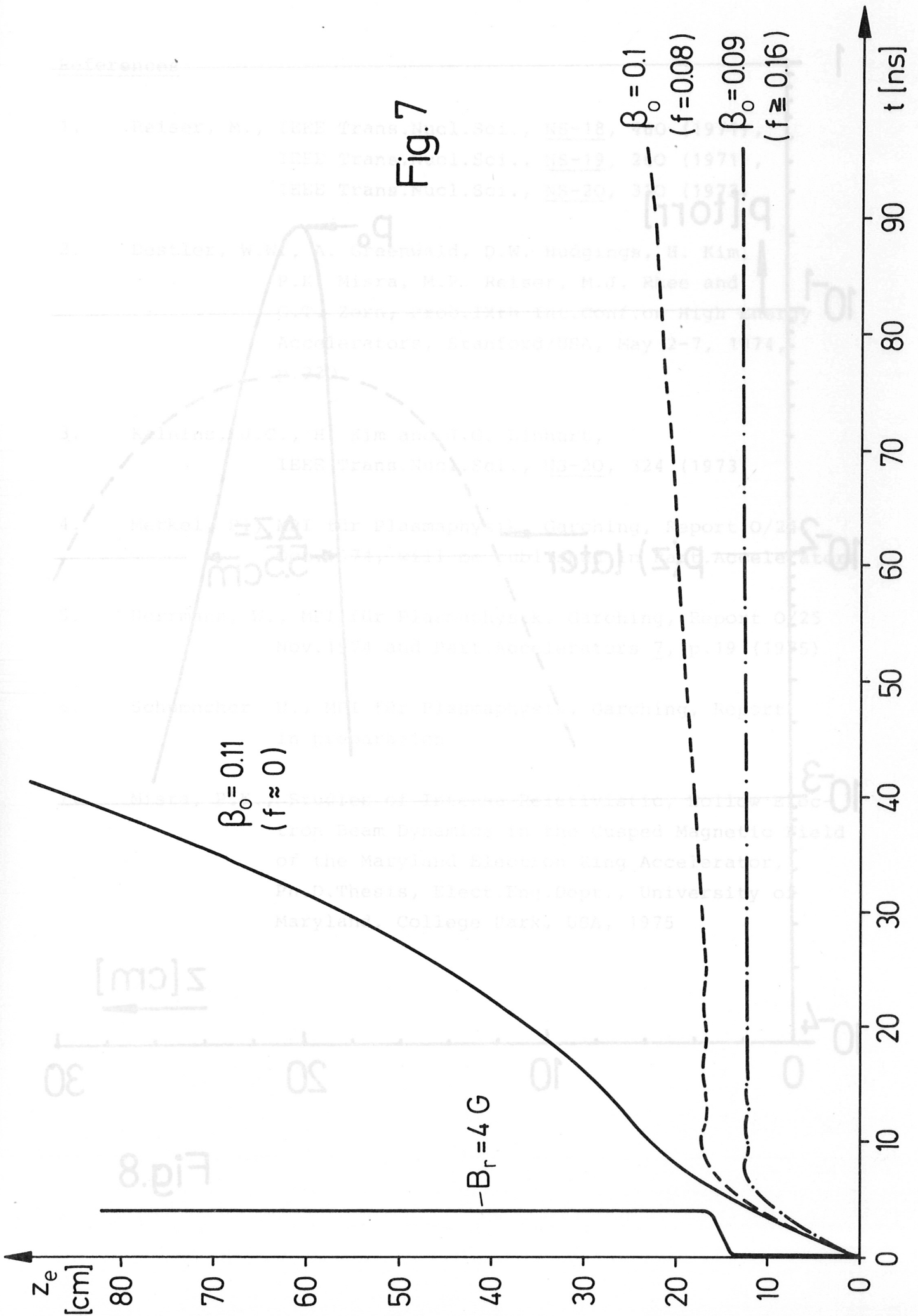


Fig. 7

References

1. H. Weiser, *IEEE Trans. Nucl. Sci.*, **18**, 45 (1971); *IEEE Trans. Nucl. Sci.*, **19**, 20 (1972); *IEEE Trans. Nucl. Sci.*, **20**, 30 (1973).
2. W. M. Linstler, A. Greenwald, D. W. Huggins, H. Kim, P. K. Misra, M. Weiser, M. J. Lee and J. F. Zarka, *Proceedings of the High Energy Accelerators*, Stanford/USA, May 2-7, 1974.
3. J. L. Lichten, H. Kim and J. G. Schubert, *IEEE Trans. Nucl. Sci.*, **19**-**20**, 124 (1973).
4. J. L. Lichten, *Report O/25*, Engineering Department, University of Maryland, College Park, Nov. 1974 and *High Energy Accelerators*, 3, 19 (1975).
5. J. L. Lichten, *Report O/25*, Engineering Department, University of Maryland, College Park, Nov. 1974 and *High Energy Accelerators*, 3, 19 (1975).
6. P. K. Misra, *Study of the Dynamics of a Particle in a Cusp Magnetic Field of the Maryland Electron Ring Accelerator*, M.S. Thesis, Elect. Eng. Dept., University of Maryland, College Park, USA, 1975.

Fig. 8

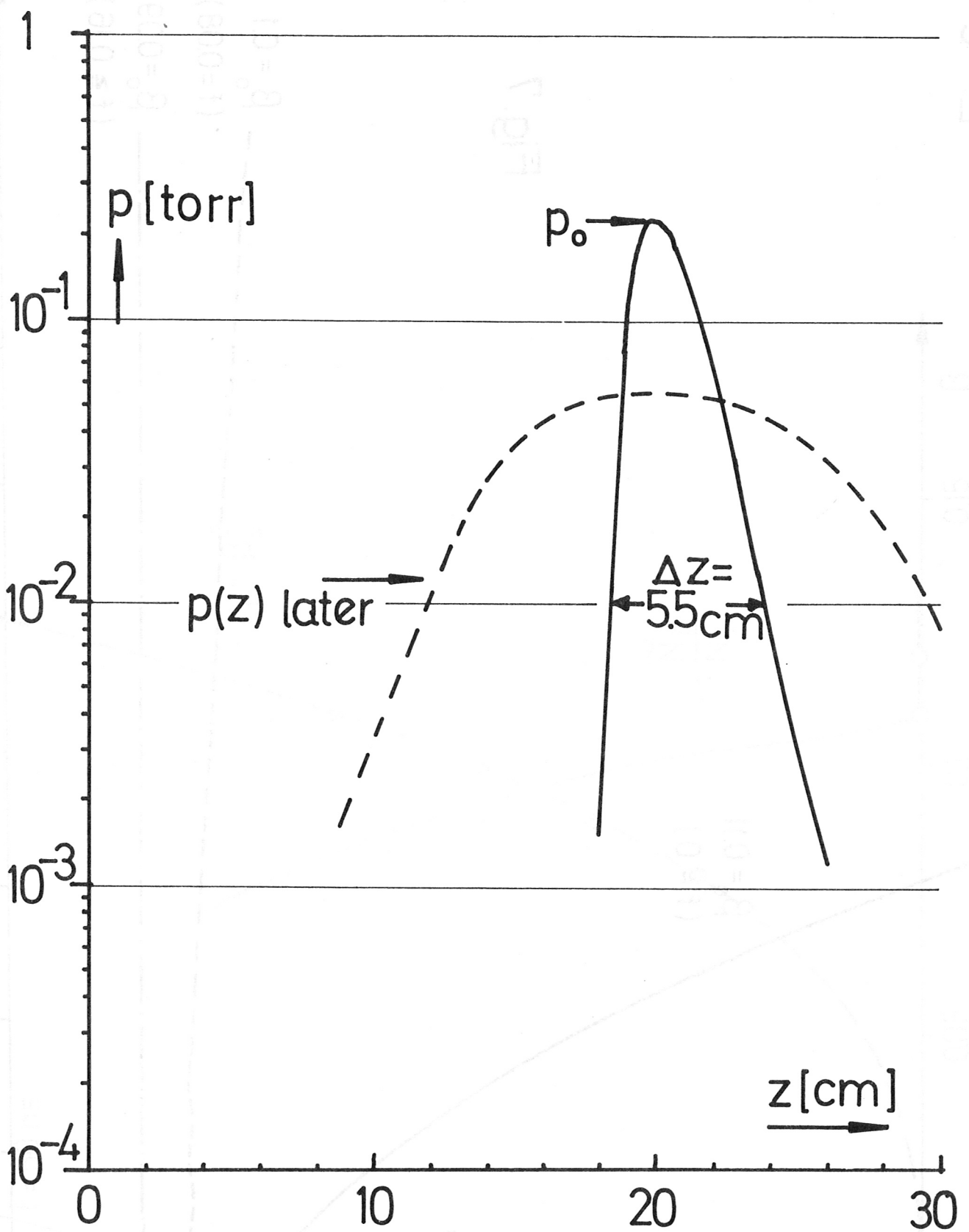


Fig.8

## References

1. Reiser, M., IEEE Trans.Nucl.Sci., NS-18, 460 (1971),  
IEEE Trans.Nucl.Sci., NS-19, 280 (1971),  
IEEE Trans.Nucl.Sci., NS-20, 310 (1973)
2. Destler, W.W., A. Greenwald, D.W. Hudgings, H. Kim,  
P.K. Misra, M.P. Reiser, M.J. Rhee and  
G.T. Zorn, Proc.IXth Int.Conf.on High Energy  
Accelerators, Stanford/USA, May 2-7, 1974,  
p.230
3. Kalnins, J.G., H. Kim and J.G. Linhart,  
IEEE Trans.Nucl.Sci., NS-20, 324 (1973)
4. Merkel, P., MPI für Plasmaphysik, Garching, Report O/24,  
Oct.1974, will be published in Part.Accelerators
5. Herrmann, W., MPI für Plasmaphysik, Garching, Report O/25  
Nov.1974 and Part.Accelerators 7, p.19 (1975)
6. Schumacher, U., MPI für Plasmaphysik, Garching, Report  
in preparation
7. Misra, P.K., Studies of Intense Relativistic, Hollow Elec-  
tron Beam Dynamics in the Cusped Magnetic Field  
of the Maryland Electron Ring Accelerator,  
Ph.D.Thesis, Elect.Eng.Dept., University of  
Maryland, College Park, USA, 1975