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Technical Limitations
on Conceptual Tokamak Reactors
(Part II)

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Abstract

In a previous paper [1] some technical constraints on possible Tokamak machines have been derived for a circular and a strongly elongated plasma cross section at $q(a) = \text{const.}$ These calculations have been extended for a moderate plasma elongation ($a/a' = 2$) under the less conservative assumption of $q(a)$ varying with the plasma aspect ratio. The tendency of the results is favourable for the elliptic cross section compared the circular one in that the feasible magnetic field level is about 8 T instead of 12 T, the toroidal β_t being larger by a factor of 2, whereas the trapped ion mode $n\tau$ -value remains about the same. Interestingly enough taking account of an aspect dependent $q(a)$ would suggest that for a circular plasma cross section low aspect ratio Tokamaks at 8 T are not reactor relevant. If the applied relationships hold true, the elliptic plasma cross section may open up the way towards a feasible reactor concept.

Introduction

The evaluation of consistent possible parameters ranges for conceptual Tokamak reactors is a necessary prerequisite for a timely development of non standard components of future Tokamak machines, e.g. the superconducting magnet system. Clearly enough it is only possible in a rather rough approximation since many important scaling laws in plasma physics are not firmly established or even not known yet. On the other hand, technical constraints already known now impose restrictions which further diminish the range of configurations possible at all. In a previous study [1] already characteristic Tokamak reactor parameters have been derived for a circular and a strongly elongated plasma cross section with an ellipticity of 4. Since the moderate ellipticity values appear more likely, a comparison for an elongation of 2 with the circular cross section case is given here. Also the influence of a safety factor q depending on the plasma aspect ratio has been calculated. Such a dependency increases strongly the overall influence of the plasma aspect ratio and affects the β_t attainable. The relevance of a simplified consistent scaling also shows up, when evaluating $n\tau$ -formulae, in which the insertion of inconsistent parameters can yield rather misleading results.

As stated previously [1], it should be noted that the resulting relationships and tendencies have to be considered much more relevant than the detailed numbers.

Parameters

The most important independent scaling parameter is the rated thermal power of a Tokamak fusion reactor when the basic assumptions on the reactor plasma are kept constant. A simplified geometrical model yields the variation of the important depending parameters over a broad range of thermal power.

An important item in capital cost of a Tokamak machine is the magnetic field system for which the stored energy is a rough indication. For the purpose of comparison it is especially interesting to consider only the magnetic energy in the active reactor volume comprising the plasma and the blanket because this yields an absolute minimum of magnetic field energy required under the assumptions used.

Besides of the magnet energy a selected number of other reactor parameters have been calculated and plotted vs. thermal power and aspect ratios.

In both cases considered - circular and elliptical plasma cross section - the same basic definitions and relationships are used as listed below:

1. The plasma density and temperature ($T_e = T_i = T$) are constant respectively over the entire plasma cross section (rectangular profiles). Thus the plasma profile is characterized by the ration $y = a / r_w = a' / r'_w$. Peaked profiles would mean very low y-values. Here $y = 0.9$ is used.
2. The q-value at the plasma boundary is taken as 2.5 or alternatively as $q = 2 + 1 / A^2 + 20 / A^3$, which approximates the more pessimistic of two formulae given elsewhere [2].
3. β_{pol} is defined as \sqrt{A} .
4. For the evaluation of the plasma density the temperature is assumed to be 20 keV.
5. The total fusion energy per event is taken to be $Q_{th} = 20$ MeV.
6. A total constant blanket thickness of $t = 200$ cm is assumed. This will be true for high reactor power levels, i.e. high neutron wall loading. The possible smaller blanket thickness in smaller machines may be offset by a smaller y-value leading to roughly the same overall dimensions.

7. The radial thickness of the idealized transformer winding is calculated from an overall tensile stress of $\bar{\sigma} = 40 \text{ MN/m}^2$ taking into account that a pulsating stress is applied. With this overall tensile stress value the question of overall and local current density in the superconducting transformer winding should not present any problems. $\bar{\sigma} = 40 \text{ MN/m}^2$ limits the maximum flux density swing in the transformer core to about 20 T which could be obtained by a deviation from condition 8.

8. The maximum flux density swing in the transformer core is set equal to the maximum flux density in the toroidal magnet. In order to show the flux density dependence B_{tc} is chosen at 8, 12 and 16 T for the circular case and at 6, 8 and 12 T for the elliptical case. The bidirectional flux swing has a safety margin of 2.

9. The radial thickness of the toroidal magnet winding s is calculated from three components. A first part takes the mechanical stress, mainly in tension, a second part carries the current under the constraints of full stabilization and safety discharge at a total given voltage, the third part is constant and represents the cryogenic insulation. Practically the functions will be at least partially interconnected, and only a rough picture is to be expected from this subdivision.

The tensile stress in the mechanical part is set at 400 MN/m^2 and the cryogenic insulation is set to be $d_c = 30 \text{ cm}$ in radial thickness.

10. The torus coils closely touch at the innermost circumference which requires a common cryostat there. The effects of a discrete number of coils are not taken into account. They lead to even stronger forces on the innermost part of the coils.

11. As has been derived elsewhere [3] the current density and the current in the superconducting toroidal magnet are calculated to meet the requirements of both the full stabilization and the safety discharge. The sum voltage over all the series connected coils at safety discharge

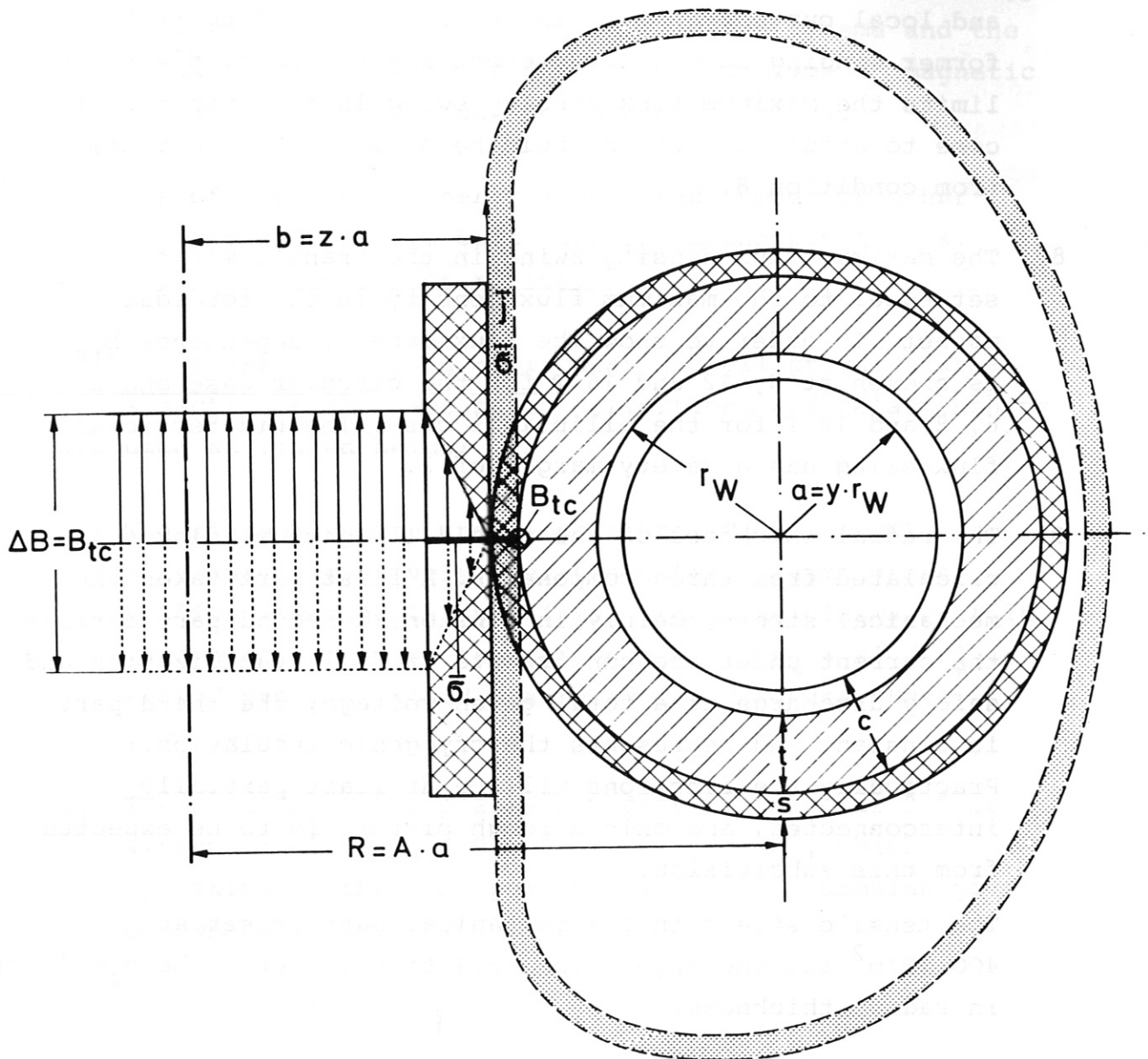


Fig. 1 Tokamak reactor - scaling model
Cylindrical plasma cross section (CCS)

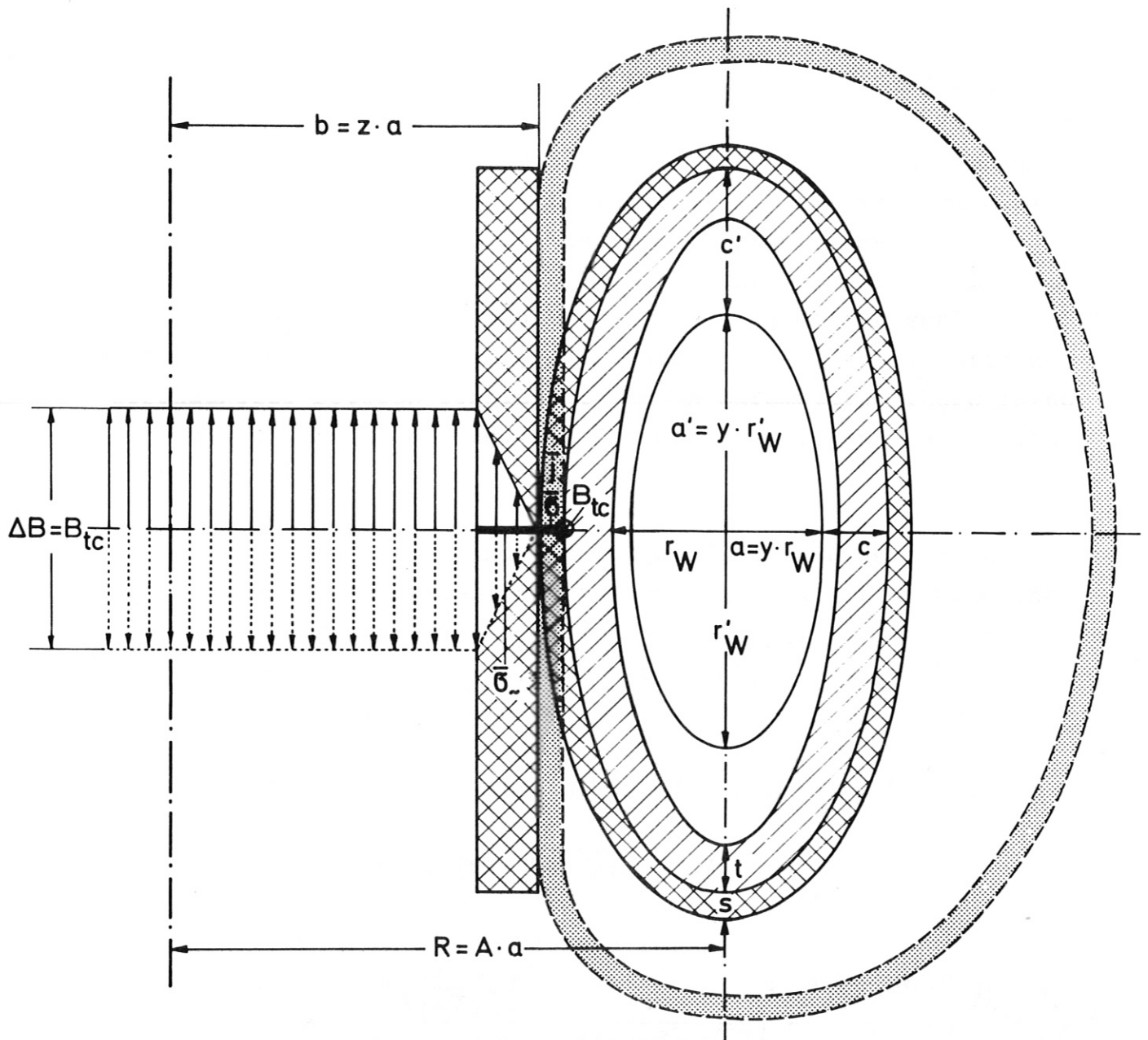


Fig. 2 Tokamak reactor - scaling model
Elliptical plasma cross section (ECS)

is set at 200 kV. The temperature rise of the winding during safety discharge is set to be 50 K, thus $f(\theta)$ is $5.10^8 \text{ (A/cm}^2\text{)}^2 \cdot \text{s}$. Further data are $q' = 0.3 \text{ W/cm}^2$ and $k = 2$, $\delta = 3 \cdot 10^{-8} \text{ cm}$.

12. The topology of Fig. 1 and 2 is put into the scaling.

Except for the above 12 items the scaling does not assume any dimension or other characteristic figure at the beginning of the calculation. Especially the reactor power, the wall loading, the aspect ratio, the plasma pressure and the plasma current are not preselected but are results of the scaling which yields under the many constraints imposed well defined dependancies between consistent design parameters. There is no kind of optimization involved. Optimum data if any may be found in the results.

Using constraints 1, 2 and 6 through 12 one can derive an equation which in an implicit form provides $a = f(A)$ with the numbers for $y, q, t, \bar{\sigma}, B_{tc}, \sigma', d_c, \Sigma V_{\max}, a'/a$ set as listed above. The equation, which can only be solved by iteration, is given elsewhere [1].

Equation (5) in the previous study [1] is changed here to

$$(\cdot/\cdot) = \ln (8A/\sqrt{a'/a}) - 1.75,$$

equation (10) *ibid.* now takes the form

$$\frac{p_w^3}{P_{th}} = \frac{[Q_{th} \frac{\langle \sigma v \rangle}{(kT)^2}]^2}{(16\mu_0)^4 \pi^2} \left(\frac{2}{1.5(1+\frac{a'}{a}) - \sqrt{\frac{a'}{a}}} \right)^3 \frac{(\frac{a'}{a})^2 [1+(\frac{a'}{a})^2]^4}{2^4} \left(\frac{B_{tc}}{q} \right)^8 \frac{y^3}{A^7}$$

All other relationships remain unchanged as already used [1].

Results

Fig. 1 and 2 show the geometrical models.

Fig. 3a and 3b represent the toroidal magnet inner radii and the ratio of axial to maximum toroidal flux density vs. power.

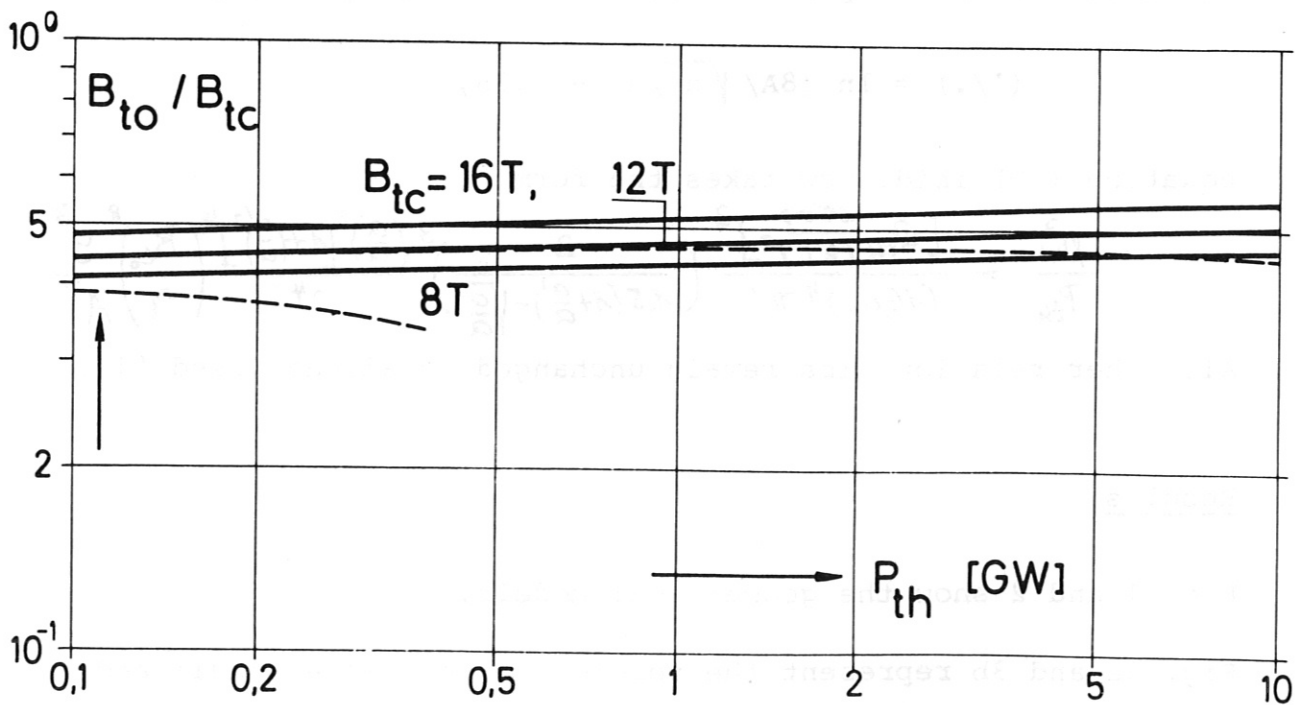
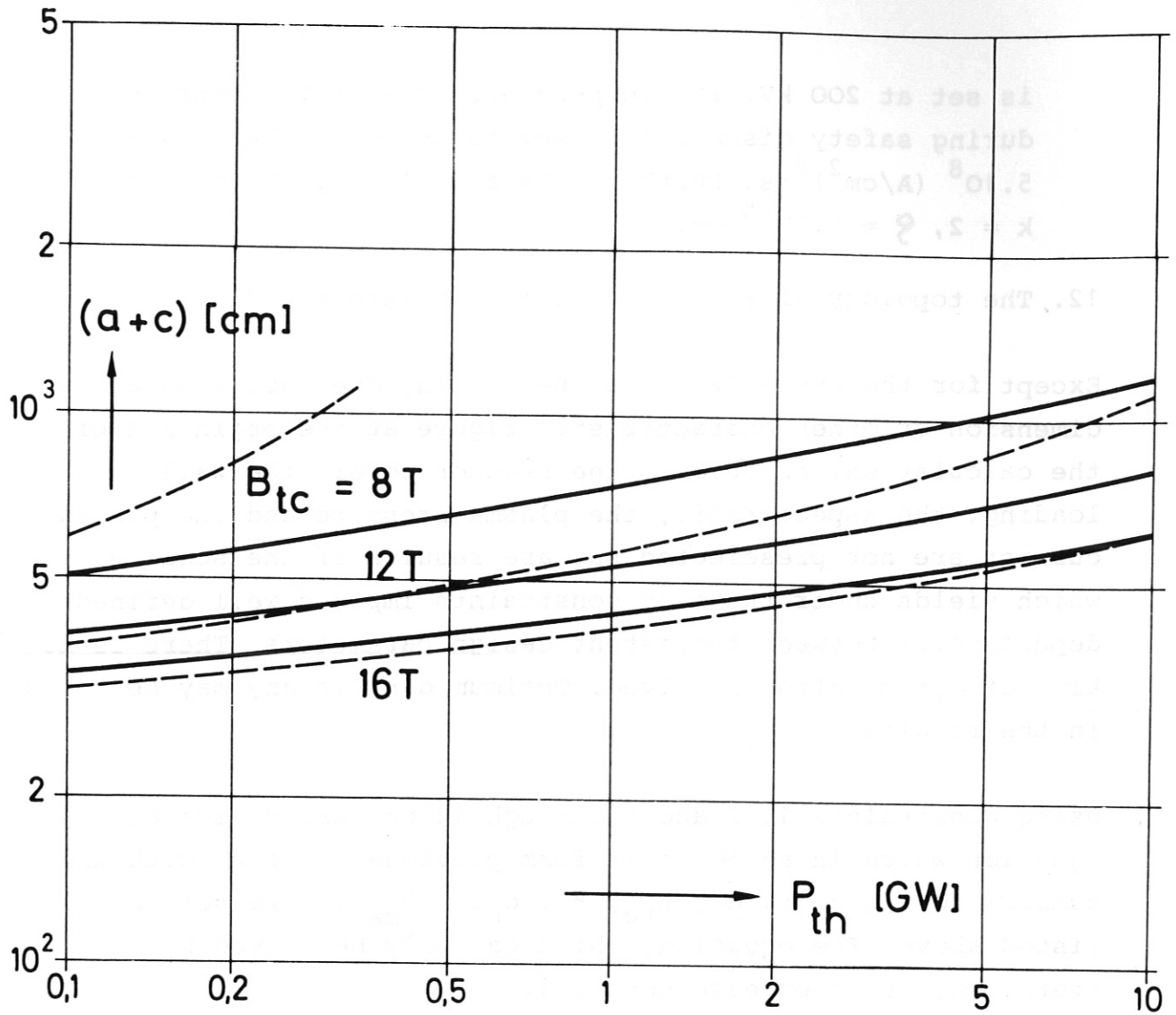


Fig. 3a Toroidal magnet winding inner radii and B_{to}/B_{tc} vs. reactor power (CCS)

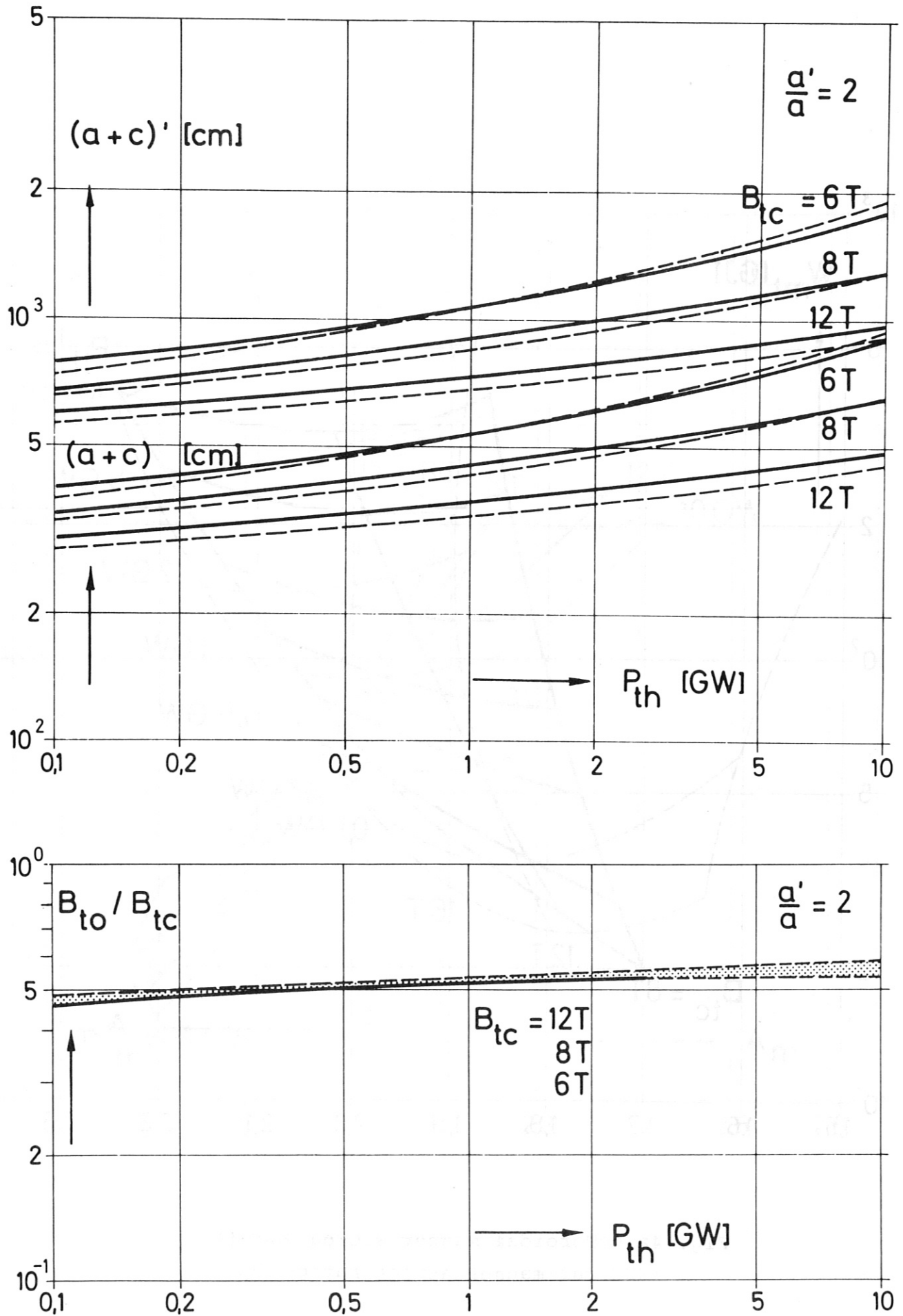


Fig. 3b Toroidal magnet winding inner radii and B_{to}/B_{tc} vs. reactor power (ECS)

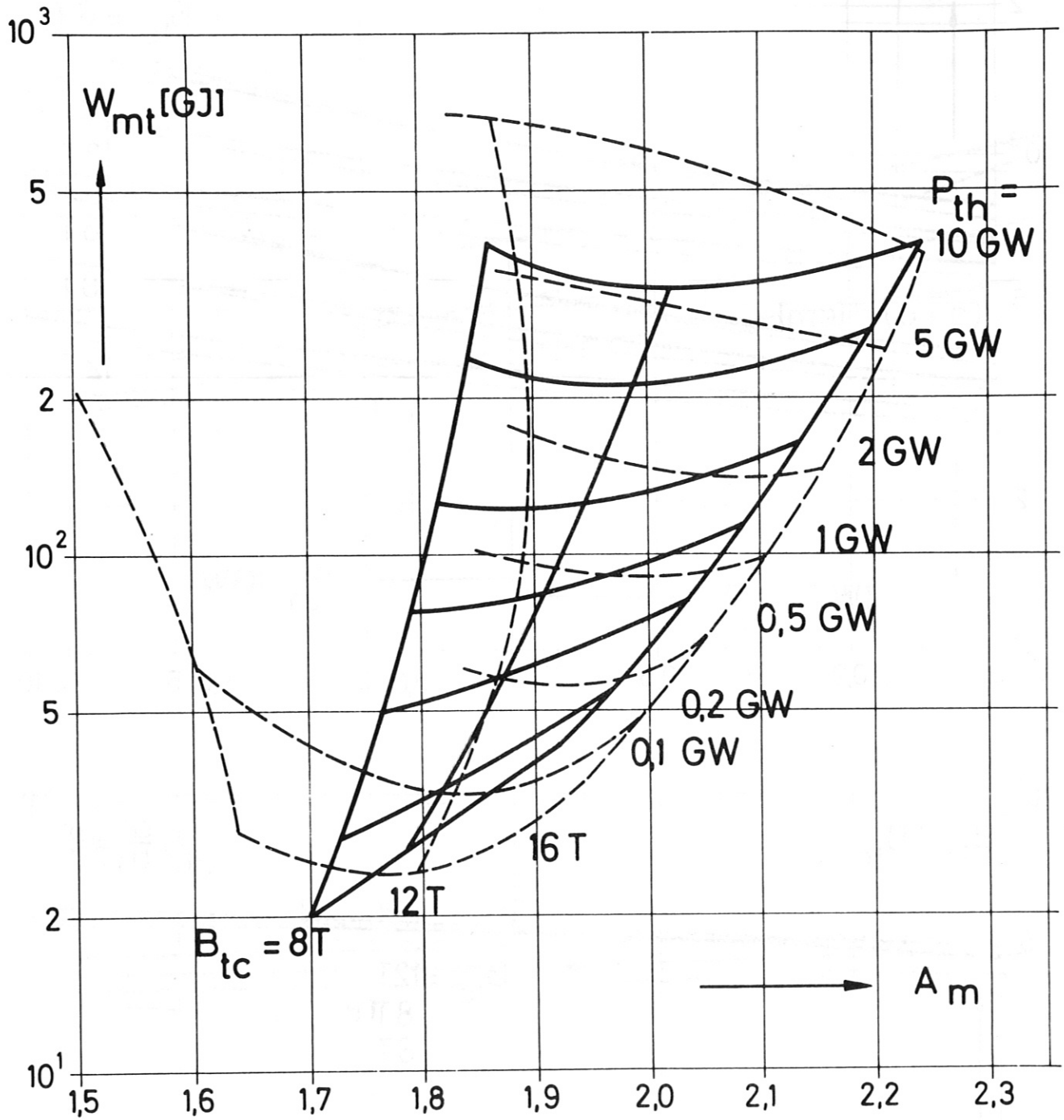


Fig. 4a Toroidal magnet stored energy
vs. magnet aspect ratio (CCS)

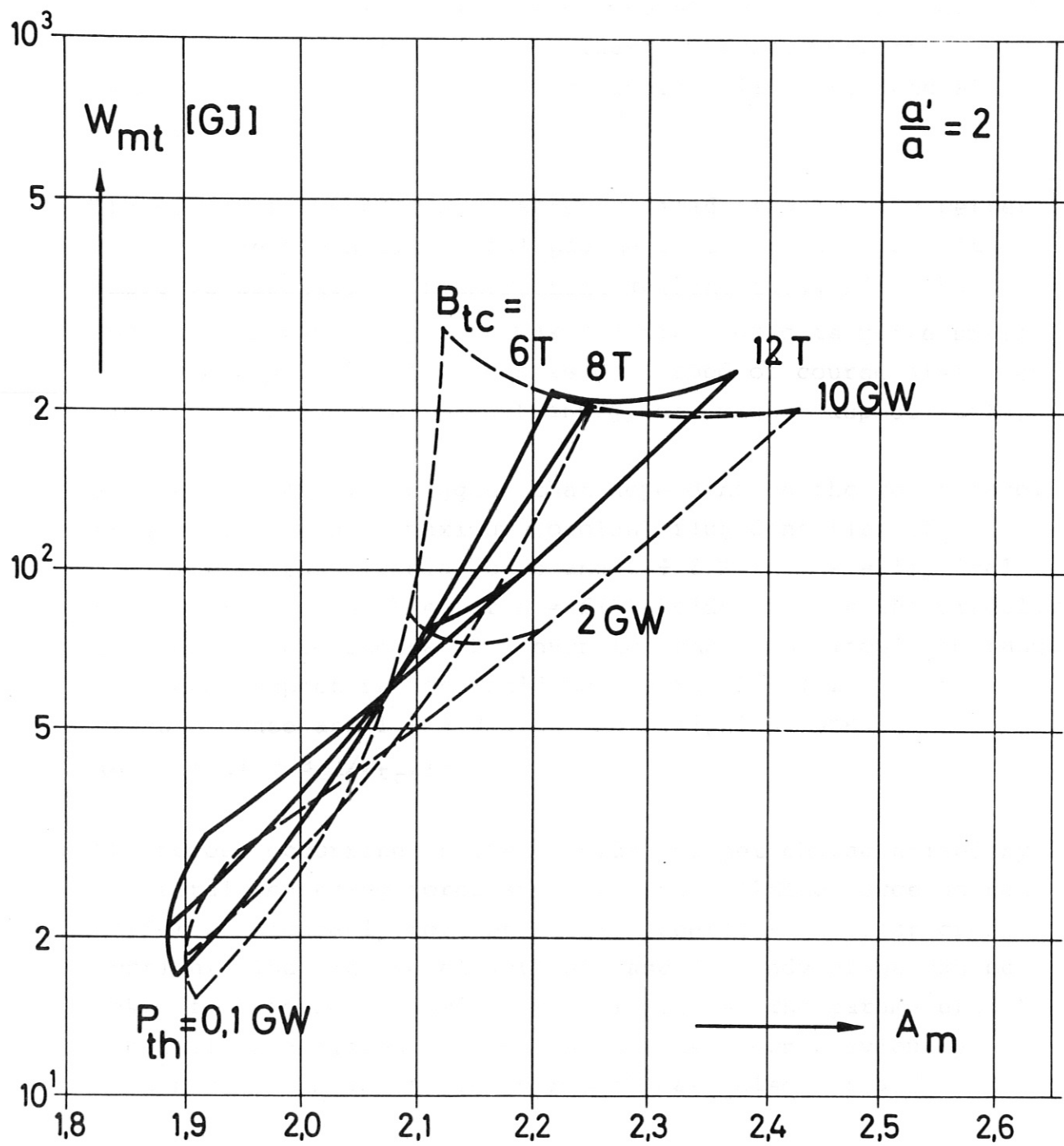


Fig. 4b Toroidal magnet stored energy vs. magnet aspect ratio (ECS)

Typically the increase of the toroidal magnet coil diameter is about 50 % per order of magnitude of thermal power. Large toroidal Tokamak magnets at 8 T maximum field are not feasible if $q = f(A)$ holds true. The strong influence of such a dependence of q (see dotted lines) especially in the circular case and at high power levels will be noticed also in the following figures.

The usage of the toroidal magnetic field is generally better in the case of an elliptical plasma cross section, and it improves slightly with increasing machine size. Also the sensitivity for changes in maximum field or q is quite small in the elliptical case. This is confirmed of course also when plotting the toroidal magnet energy vs. magnet aspect ratio.

Fig. 4a and 4b also suggest that depending on the power level at $q = 2.5 = \text{const.}$ maximum toroidal flux densities of $8 \div 12$ T in the circular case and $6 \div 8$ T in the elliptical case would be feasible. If $q = f(A)$ holds true in the circular case one should choose 12 T even for small machines! The range of magnet aspect ratios would be roughly $1.7 \div 2.2$ in the circular case and $1.9 \div 2.4$ in the elliptical one.

$$(A_m = 1/(1 - B_{tO}/B_{tC})).$$

The forces occurring in the toroidal magnet characterized by the total centering force and the total tilting force do not differ considerably for both cases (see[1] - circular cross section). Thus in the elliptical case full advantage can be taken from going to lower toroidal fields. The parameters of the fully stabilized toroidal magnet as shown previously [1] remain very similar to the circular case also for $a'/a = 2$.

Fig. 5a and 5b show plasma (minor) radius and toroidal magnet energy vs. plasma aspect ratio. At constant magnetic flux density there is a remarkable variation in plasma aspect ratio over the power range considered. This variation is necessary to meet all the technical constraints imposed. Looking for the applicability of both NbTi and Nb₃Sn from the point of view of maximum permitted field and noticing the rather flat minima of stored energy, one can conclude:

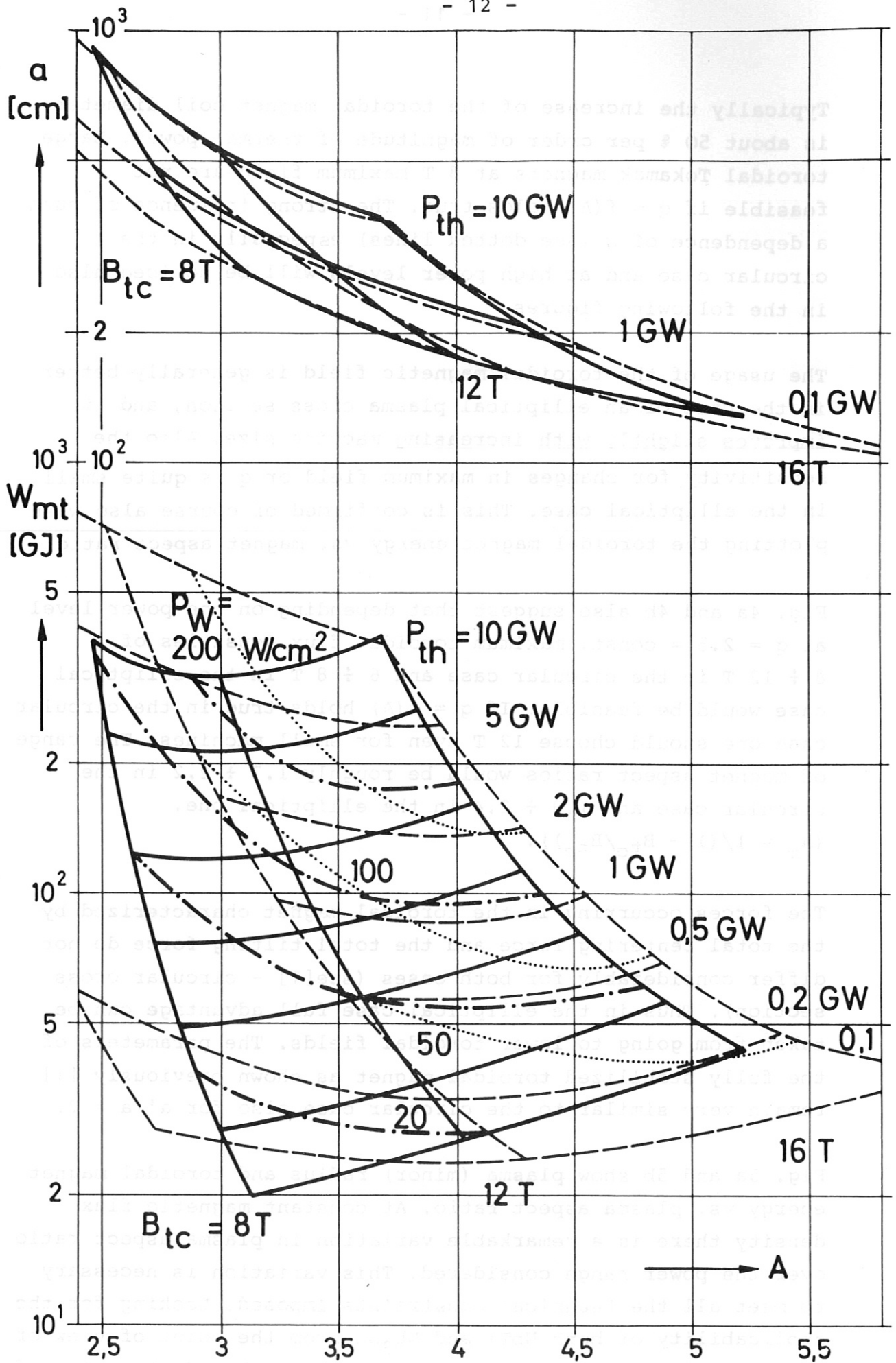


Fig. 5a Plasma radius and toroidal magnet stored energy vs. plasma aspect ratio (CCS)

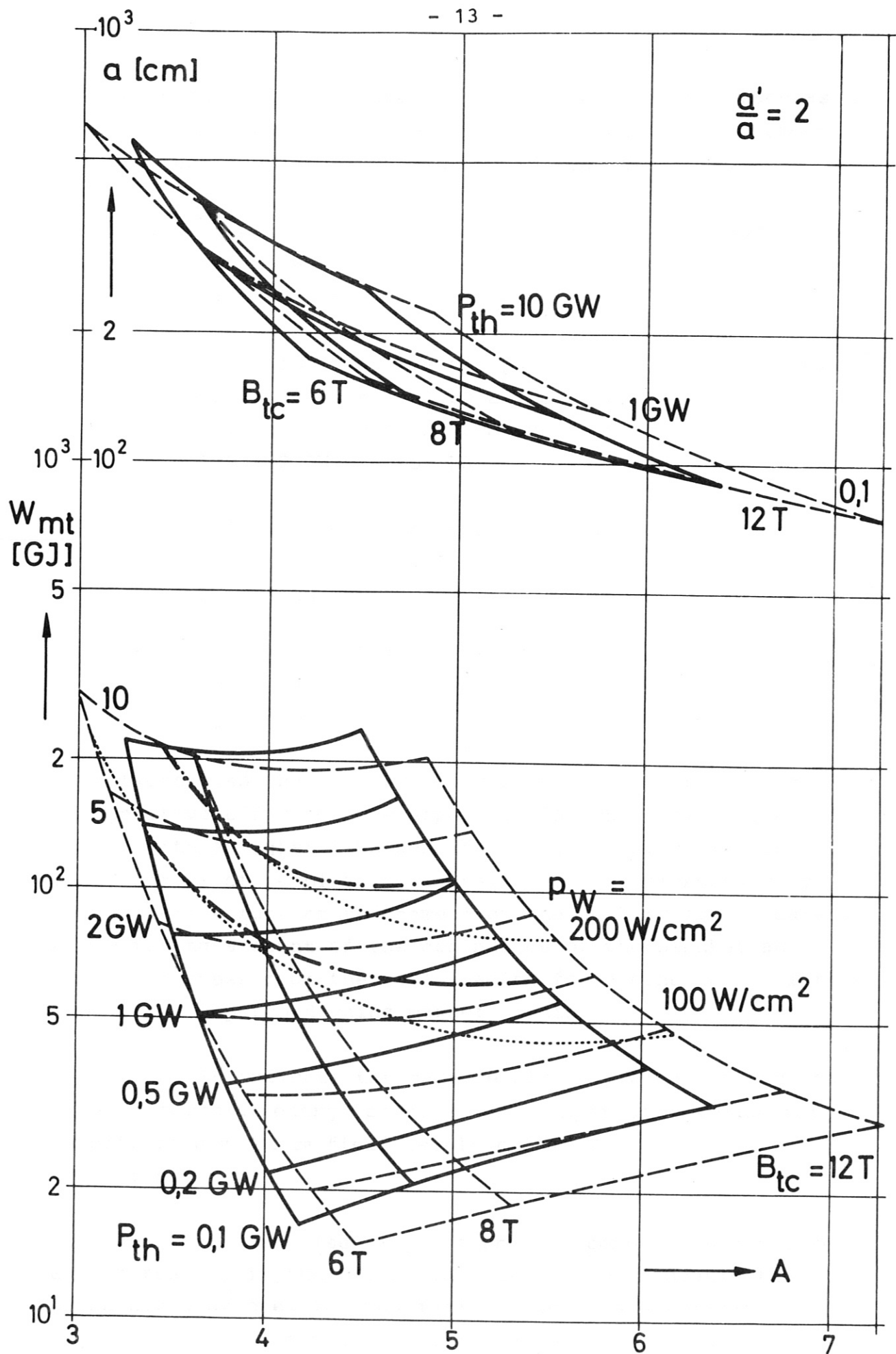


Fig. 5b Plasma radius and toroidal magnet stored energy vs. plasma aspect ratio (ECS)

- that in the circular case only low power reactors such as EPR may be feasible with NbTi at an aspect ratio of about 3 and that any larger machine would require Nb₃Sn,
- that in the elliptical case ($a'/a = 2$) the whole power range up to 10 GW would be accessible with NbTi at aspect ratios between 3.5 and 4.5,
- that in the elliptical case the power wall loading can be about the same as in the circular case, but at a lower field level. It can be seen from the figures that e.g. with $q = 2.5$ a 2 GW reactor has

with circular cross section at 12 T

$$\begin{aligned} A &= 3.3, W_{mt} = 130 \text{ GJ}, \\ p_w &= 1 \text{ MW/m}^2 \end{aligned}$$

with elliptical cross section at 8 T

$$\begin{aligned} A &= 3.9, W_{mt} = 80 \text{ GJ}, \\ p_w &= 1 \text{ MW/m}^2. \end{aligned}$$

One notices also that $q = f(A)$ strongly disturbs the otherwise positive effect of low aspect ratio. This may prevent designs with a higher flux density swing and also diminish the advantage of keeping the overall blanket thickness very small. (Here a further refinement of the scaling is indicated, namely taking account of the dependence of the blanket and shield thickness from the wall loading for a constant radiation dose at the superconducting magnet winding).

Fig. 6a and 6b indicate the same levels of plasma current and poloidal magnetic energy provided the comparison is made for the differing maximum field levels $8 \div 16$ T and $6 \div 12$ T respectively.

The plasma densities (see Fig. 7a and 7b) compare in the same way, whereas β_t in the elliptical case is about double that in the circular one. Interestingly β_t shows a saturation slightly above 2 % for the circular case and $q = f(A)$.

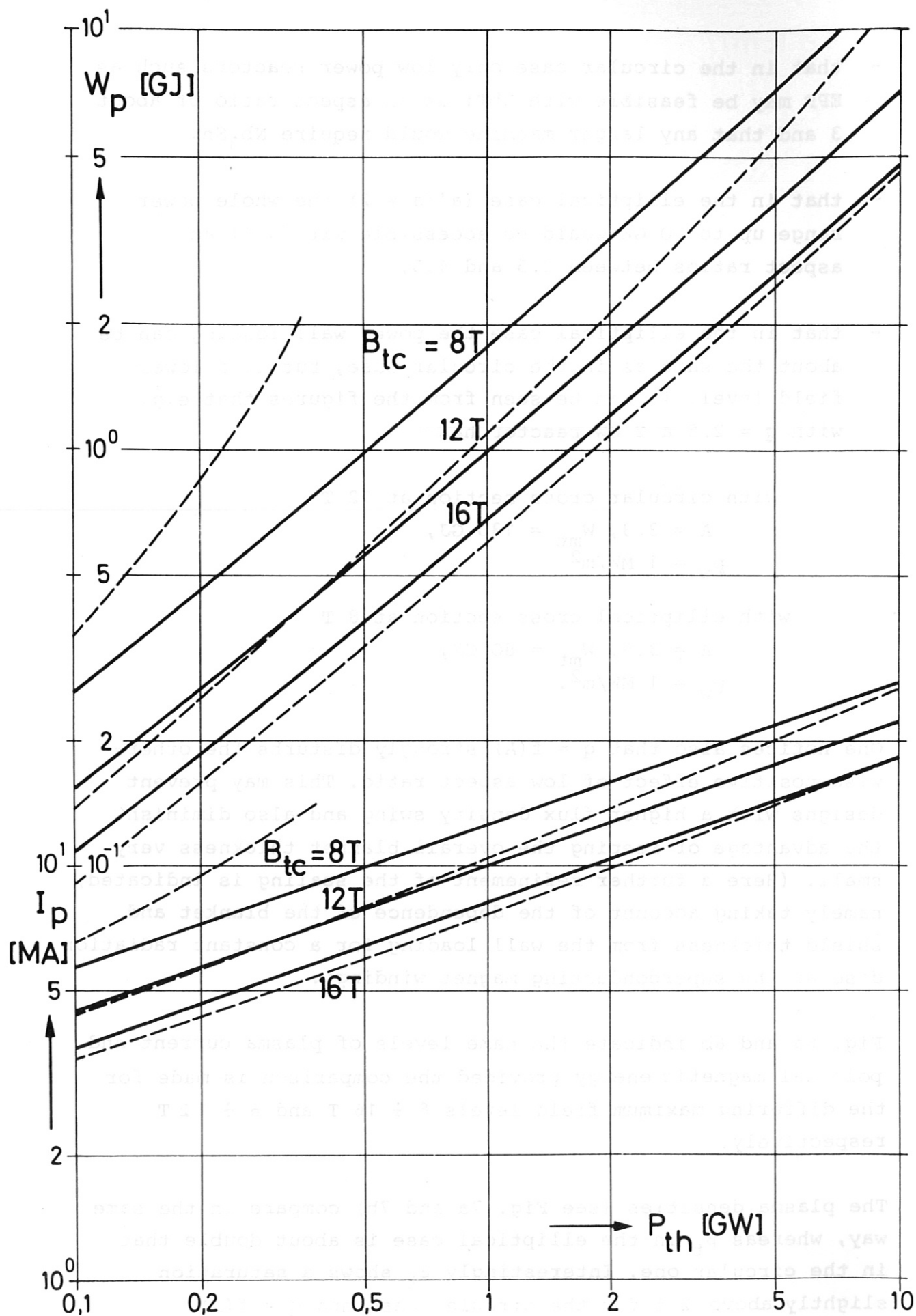


Fig. 6a Plasma current and poloidal magnetic energy vs. reactor power (CCS)

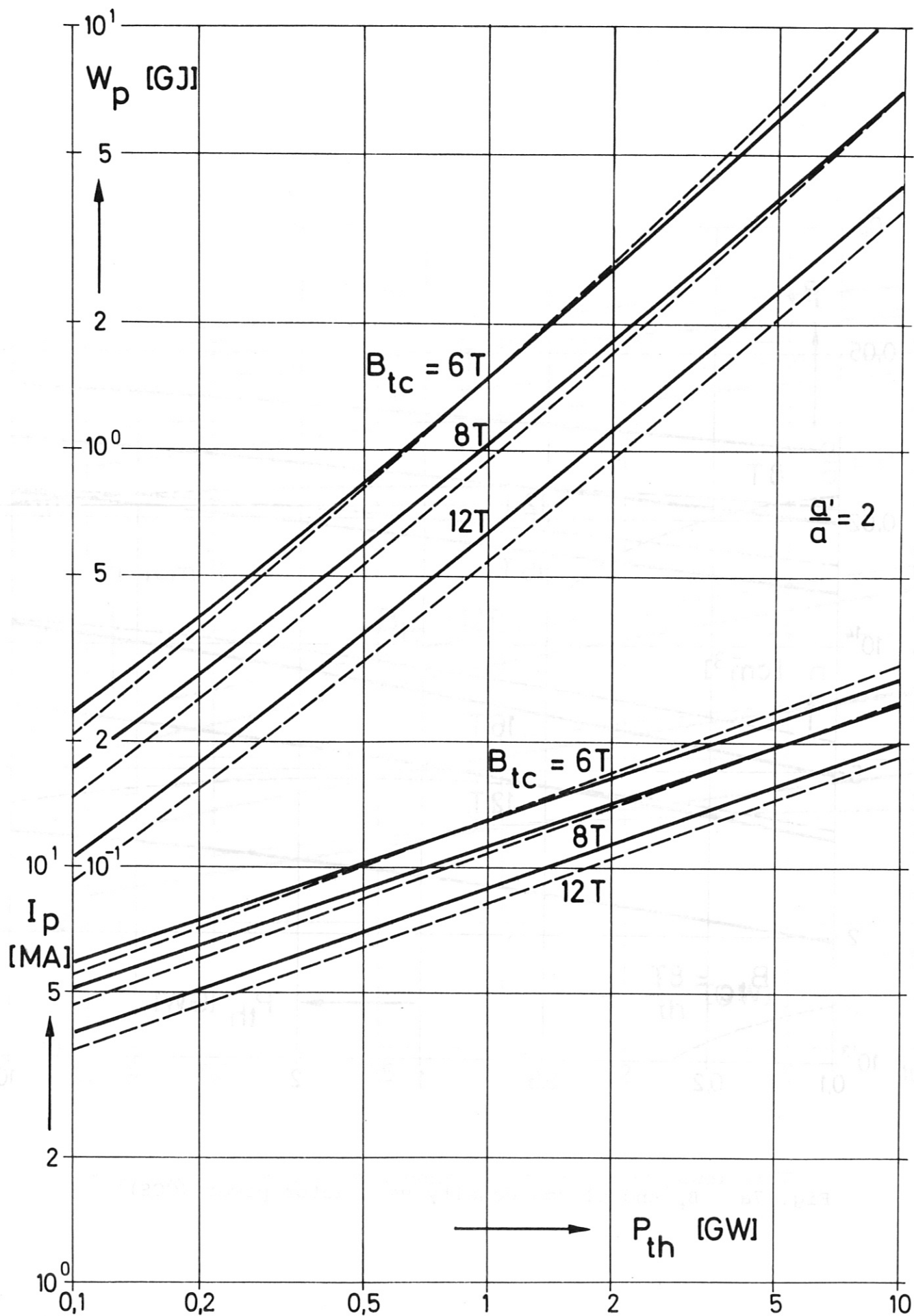


Fig. 6b Plasma current and poloidal magnetic energy vs. reactor power (ECS)

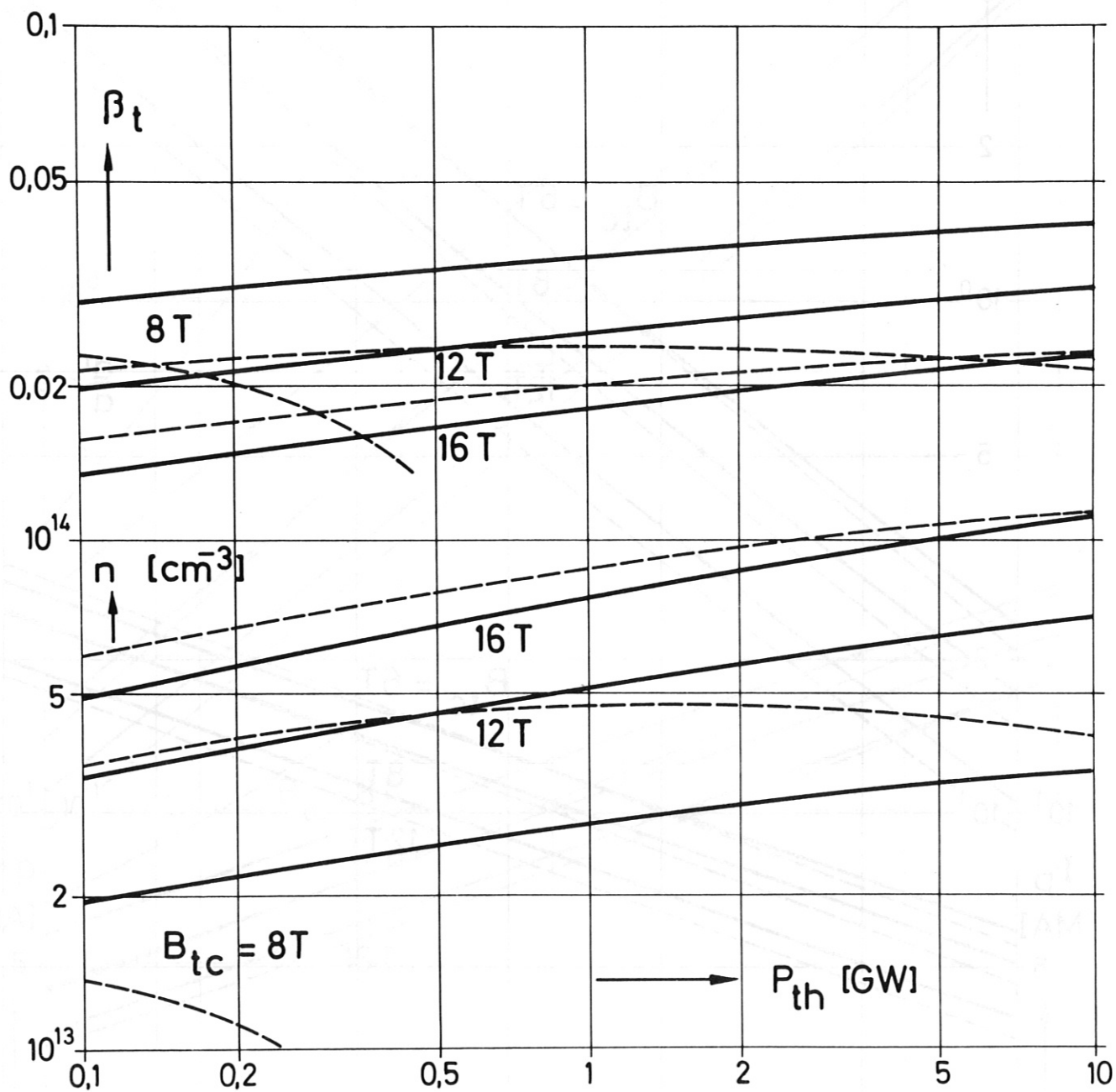


Fig. 7a β_t and plasma density vs. reactor power (CCS)

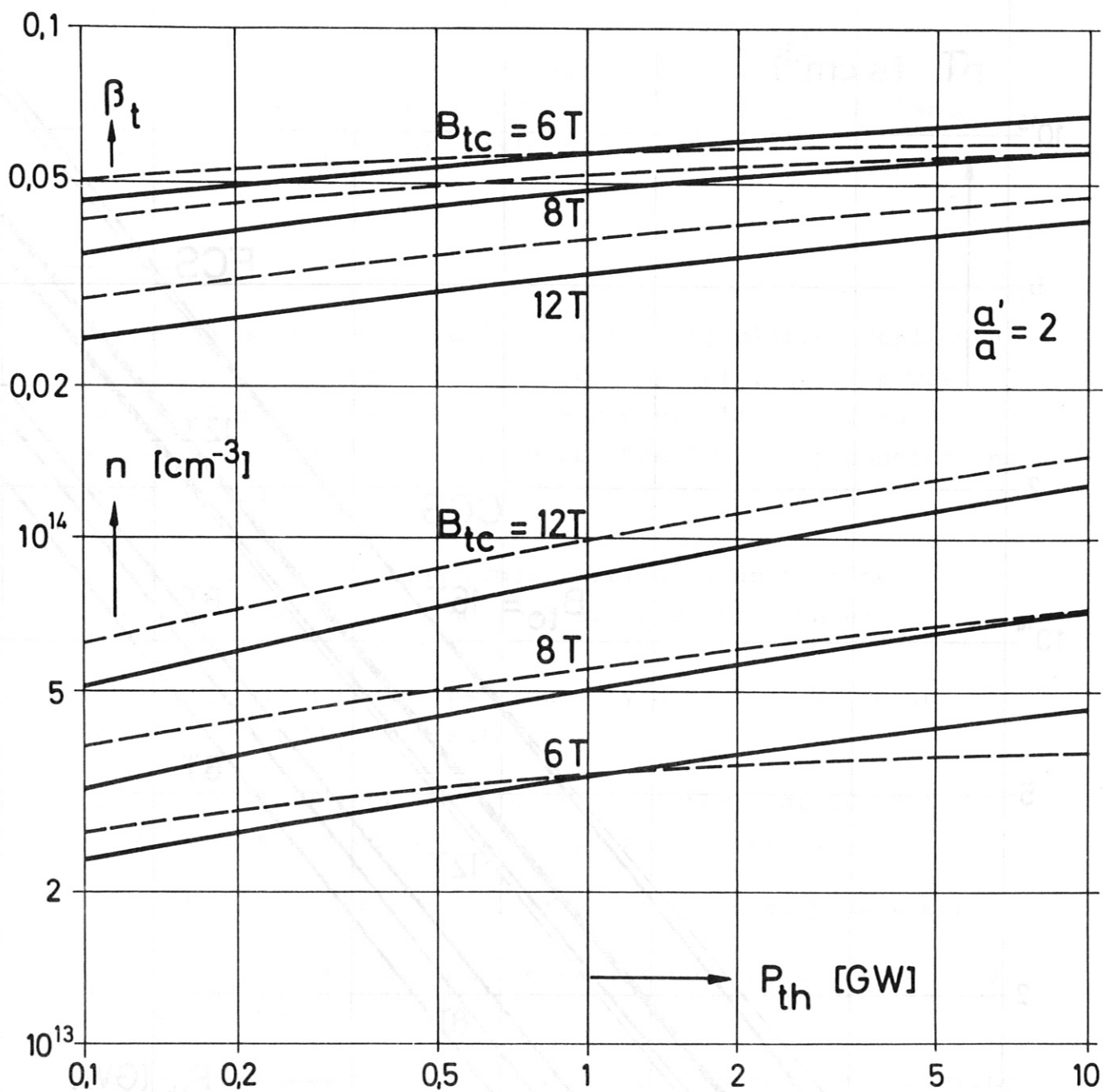


Fig. 7b β_t and plasma density vs. reactor power (ECS)

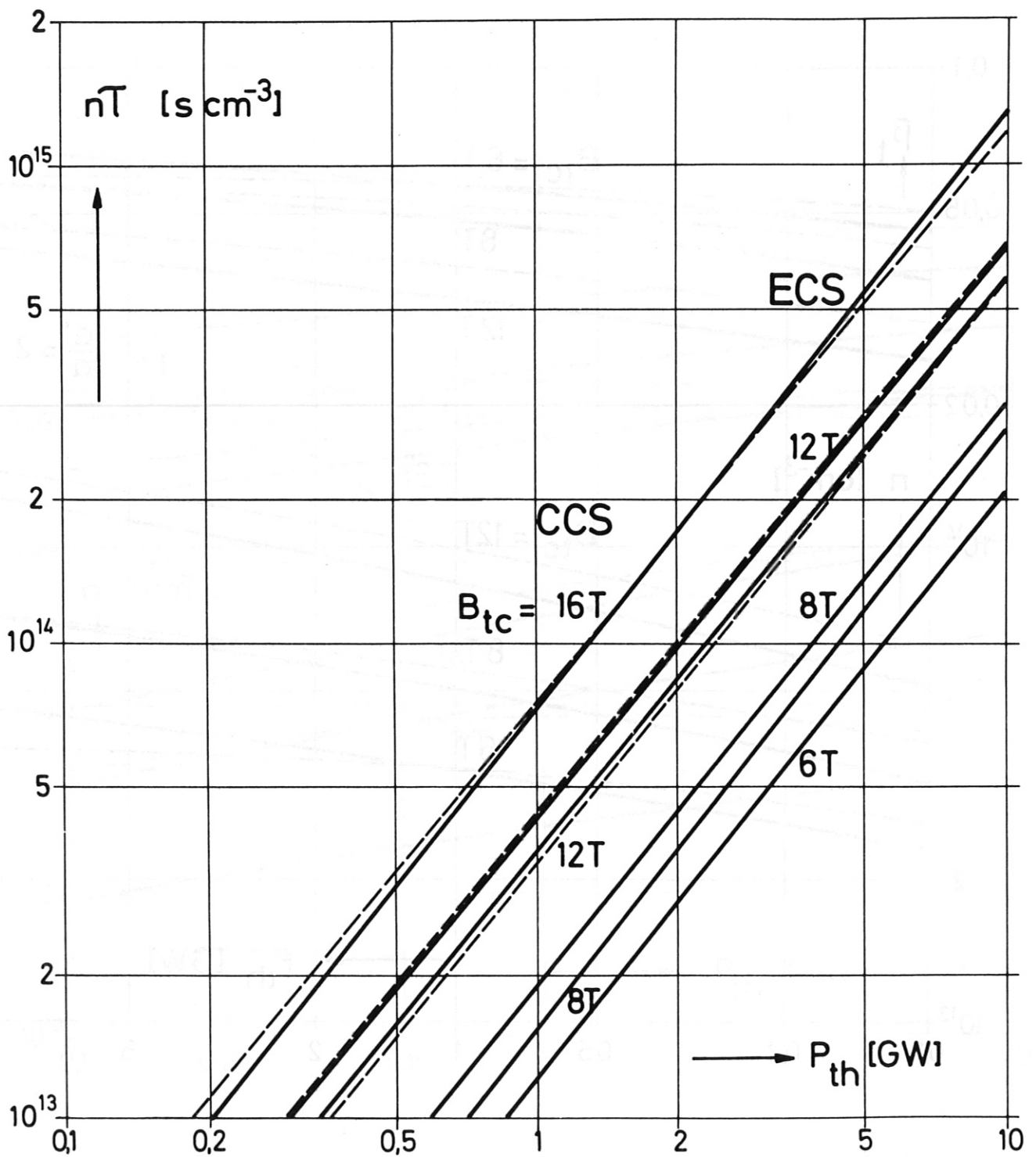


Fig. 8 $n\tau$ (trapped ion mode) vs. reactor power

Finally Fig. 8 presents $n\tau$ -data belonging to the different cases shown before. Roughly the elliptical case yields the same $n\tau$ as in the circular one at the same maximum field or a factor of 2 lower at 8 T vs. 12 T. The results are generated from the trapped ion mode formula mentioned elsewhere [4]. If this trapped ion mode scaling would turn out real, low power reactors with α -heating and without a hybrid blanket would have to apply high maximum field.

Summary

On the basis of a previously published simplified scaling a comparison of feasible data for conceptual Tokamak power reactors for a cylindrical and a modest elliptical ($a'/a = 2$) plasma cross section is presented. The following conclusions can be drawn:

Modest elliptical plasma cross section leads to some advantages over the cylindrical cross section case:

- Lower field in the torus magnet by a factor 1.5 to the extent, that NbTi may suffice.
- Lower toroidal magnet energy by a factor 2 at about the same plasma current and poloidal field energy.
- Higher β_t by a factor of 2 at about the same plasma density.
- Larger aspect ratios meaning easier design, accessibility and maintenance.
- Smaller centering and tilting forces and smaller centering pressure in the toroidal magnet.
- But: according to trapped ion mode scaling lower $n\tau$ by a factor of 2.

The situation would become even more favourable if the optimistic $q(A)$ -predictions [2] would turn out true. There

is possibly one over-optimistic figure in the scaling presented here, however, namely $y = 0.9$. If only $y < 0.9$ can be achieved this might have to be made up by a lower q (hopefully) than assumed here.

A P P E N D I X

Here are some additional curves which originate from compiling the material given in the main report.

Figs. 9 through 17 refer to the circular case and the elliptical case with $\frac{a'}{a} = 2$.

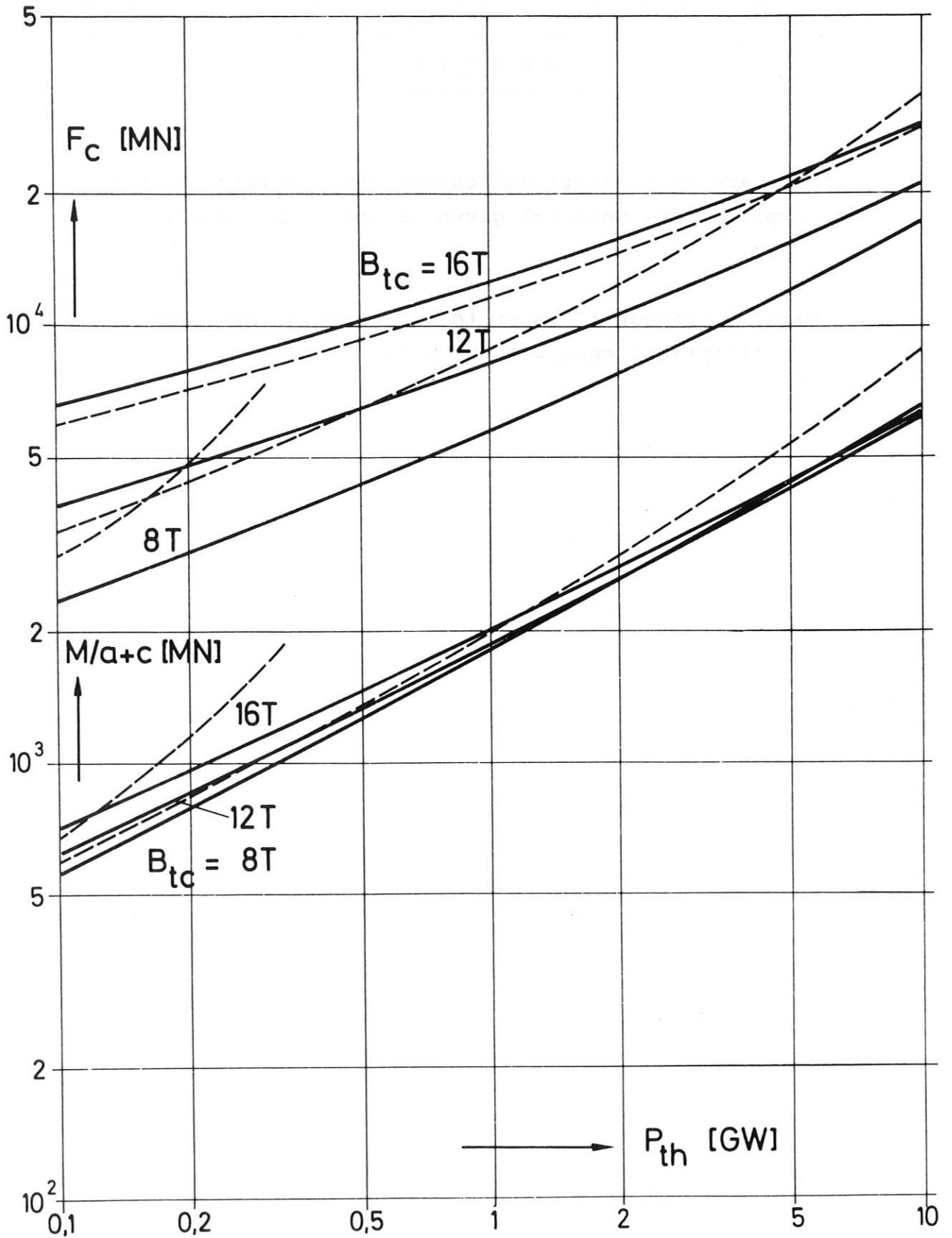


Fig. 9a Toroidal magnet centering and tilting forces vs. reactor power (CCS)

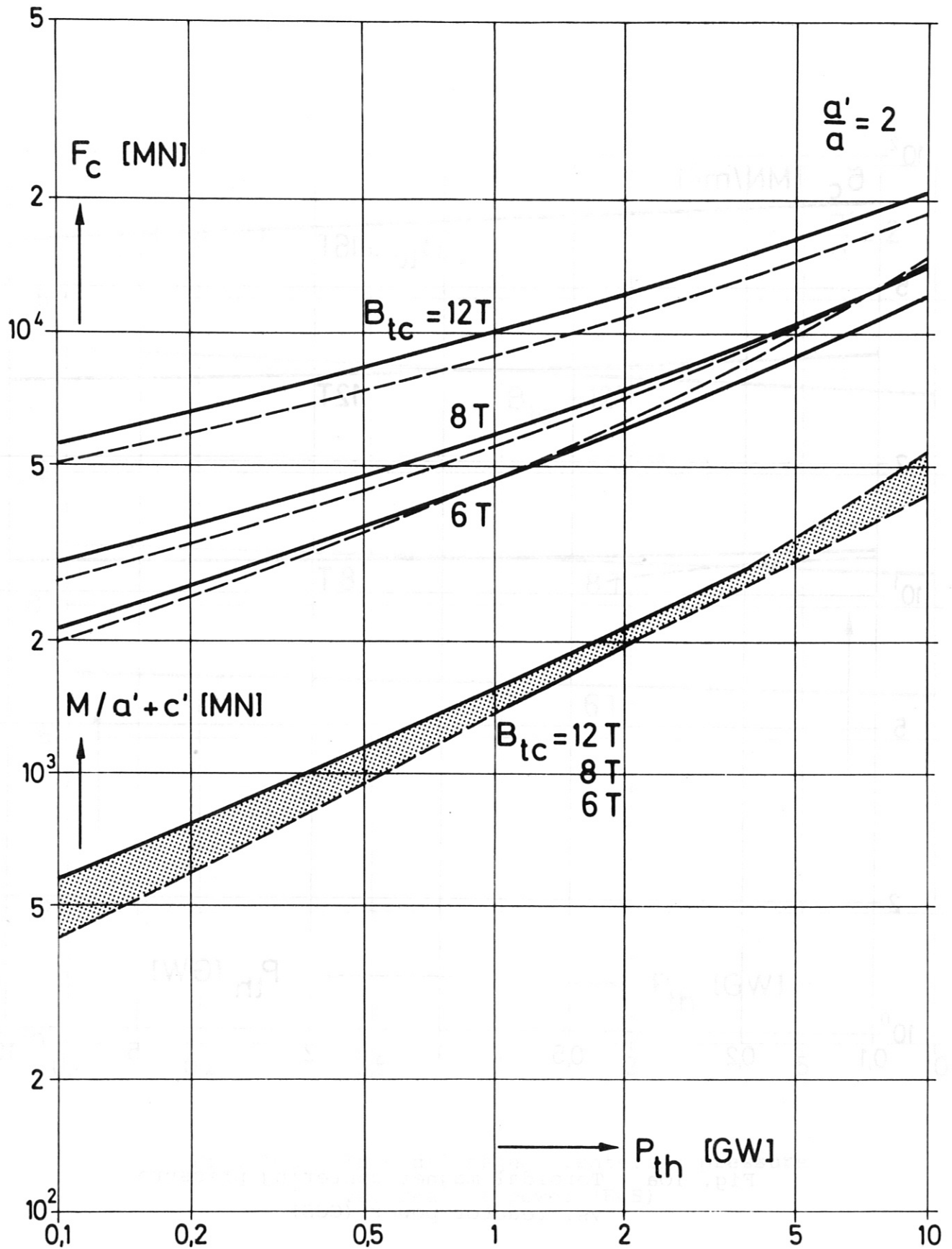


Fig. 9b Toroidal magnet centering and tilting forces vs. reactor power (ECS)

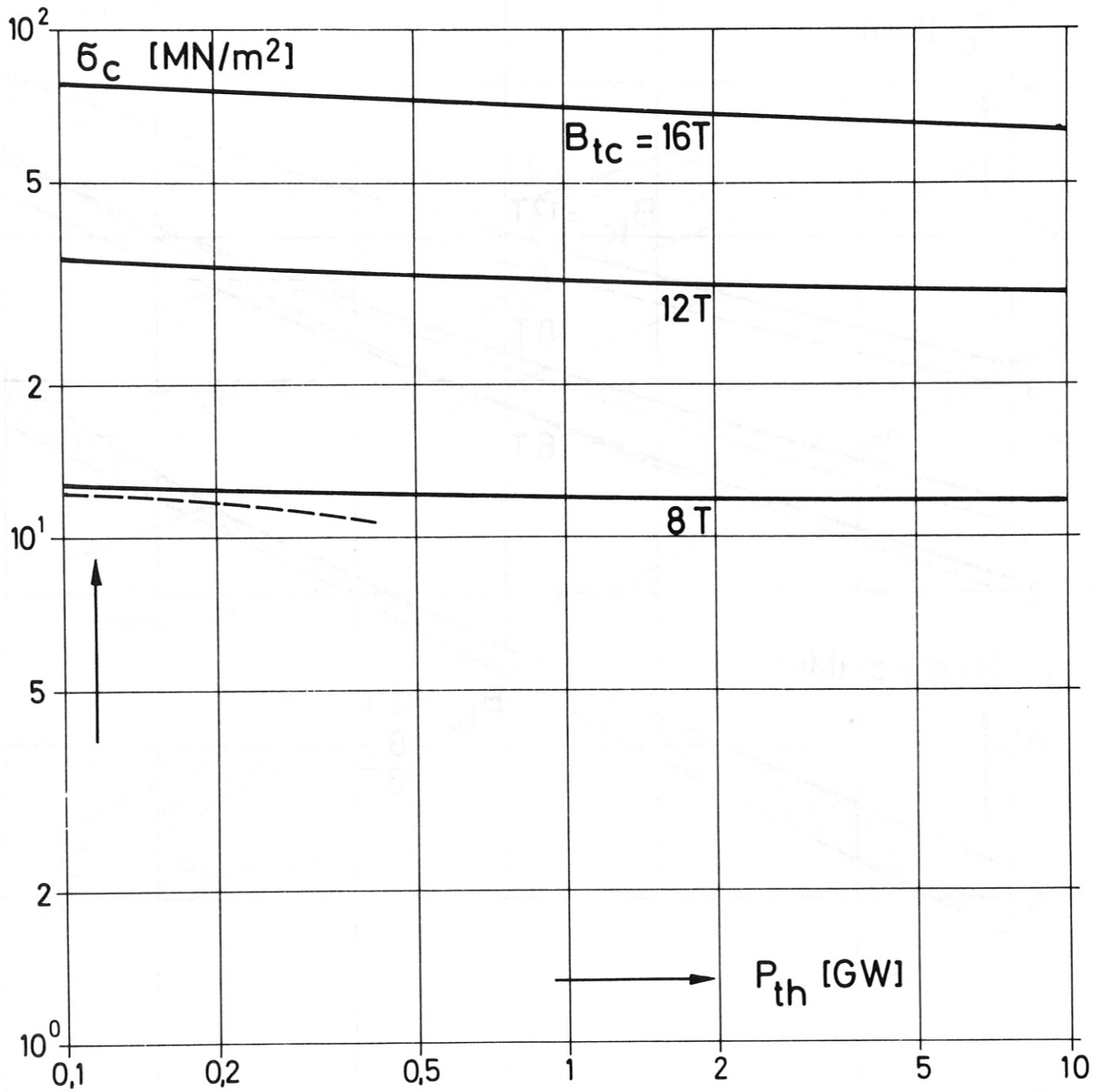


Fig. 10a Toroidal magnet centering pressure
vs. reactor power (CCS)

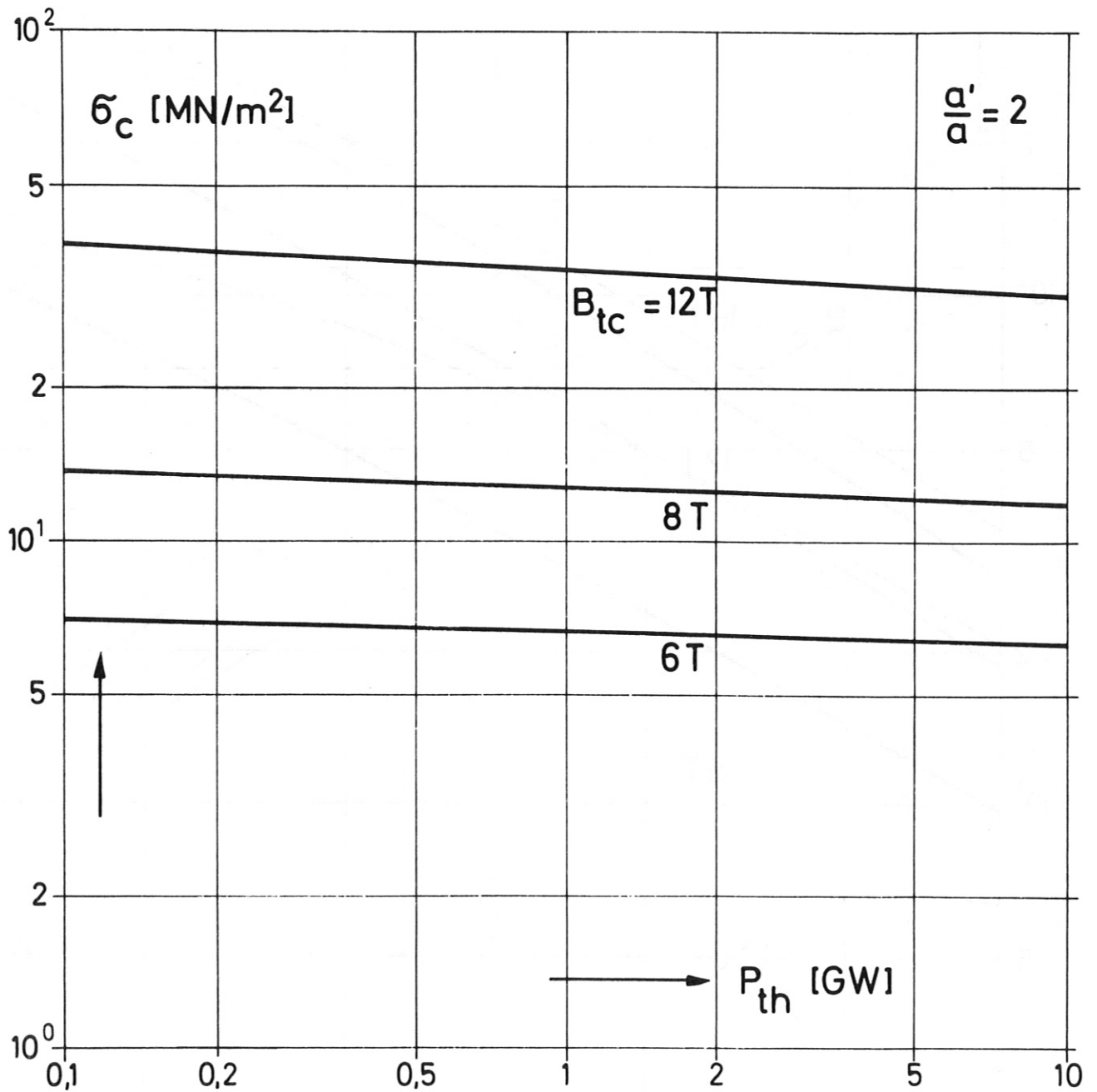


Fig. 10b Toroidal magnet centering pressure
vs. reactor power (ECS)

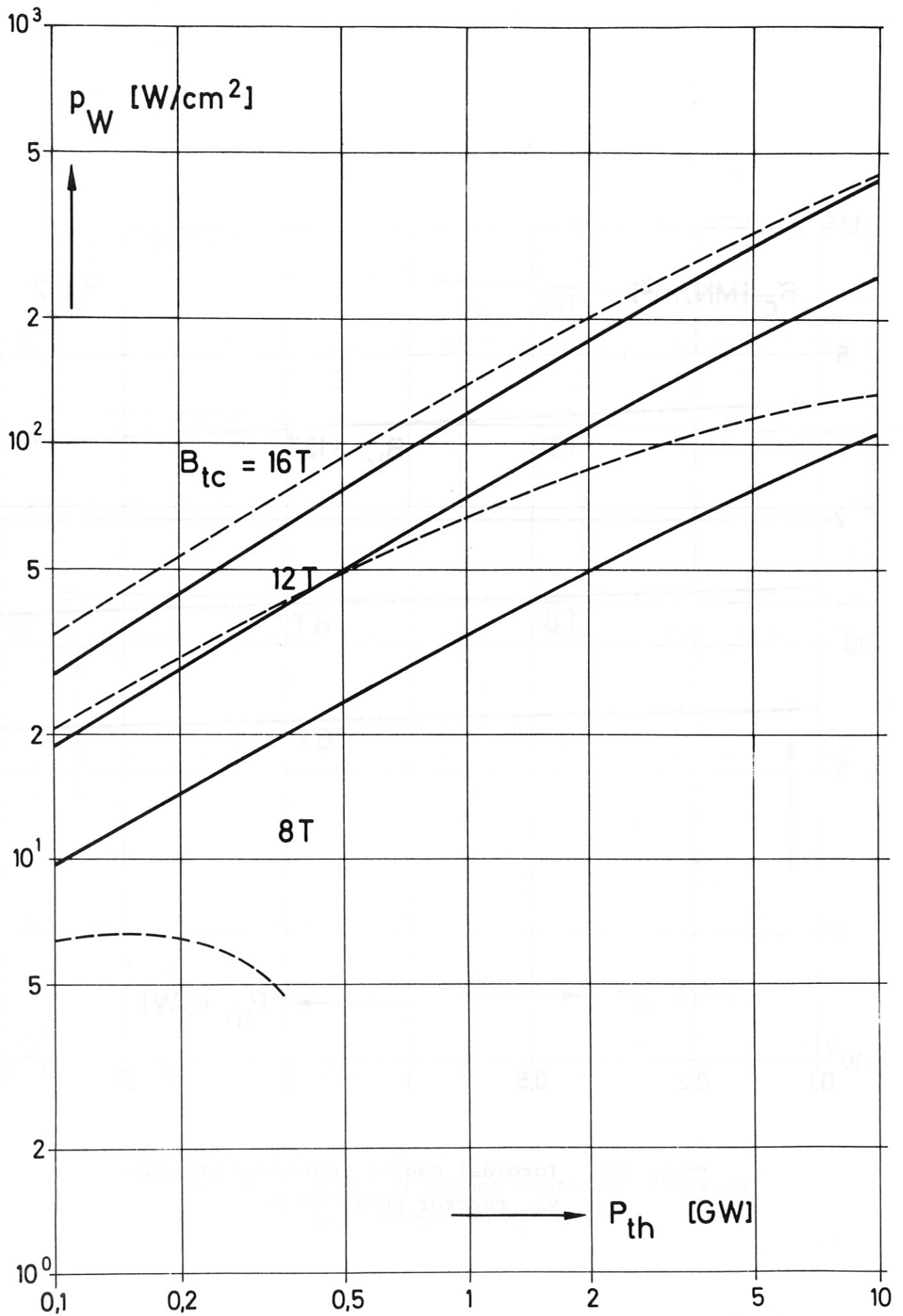


Fig. 11a Thermal power wall loading
vs. reactor power (CCS)

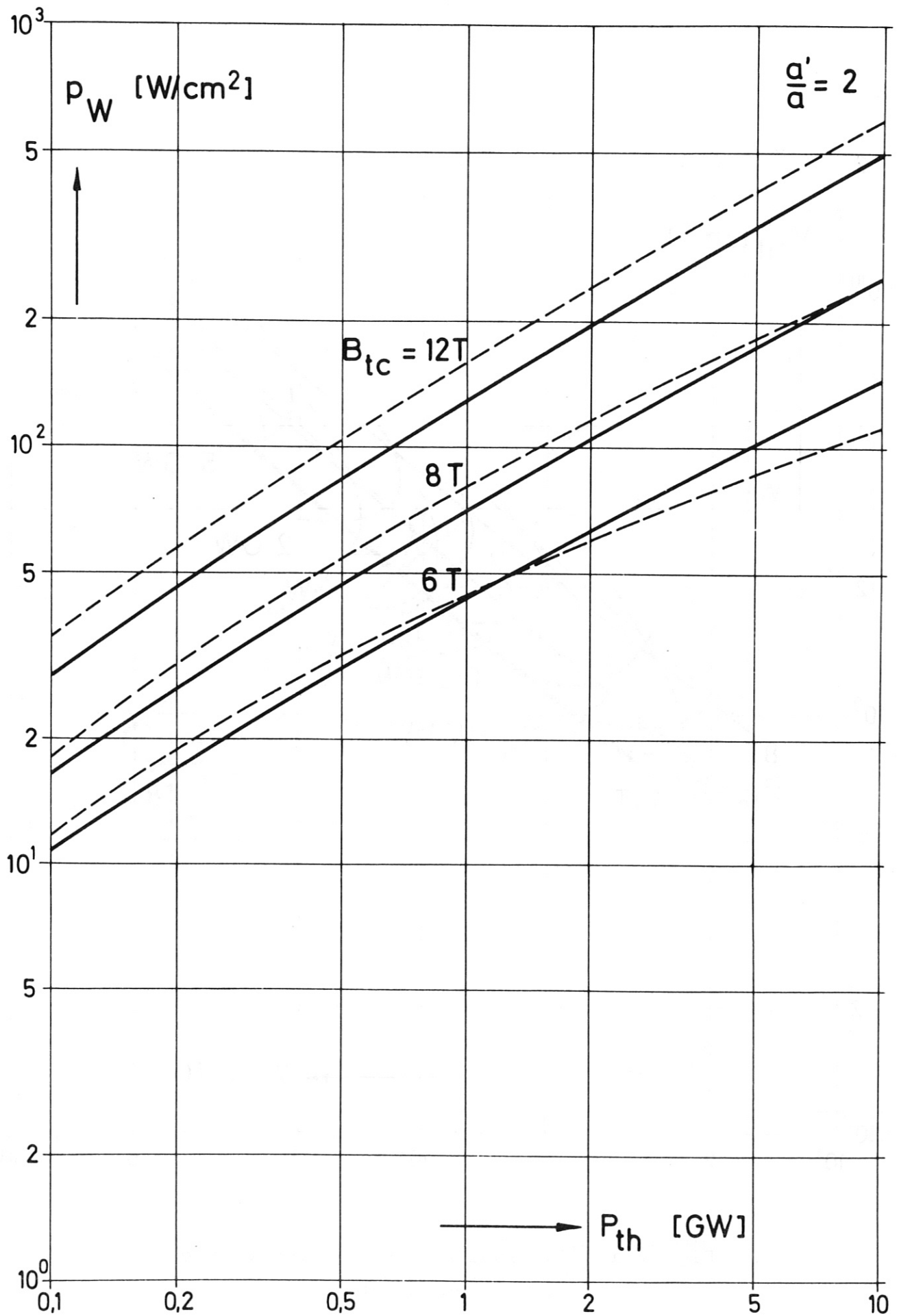


Fig. 11b Thermal power wall loading vs. reactor power (ECS)

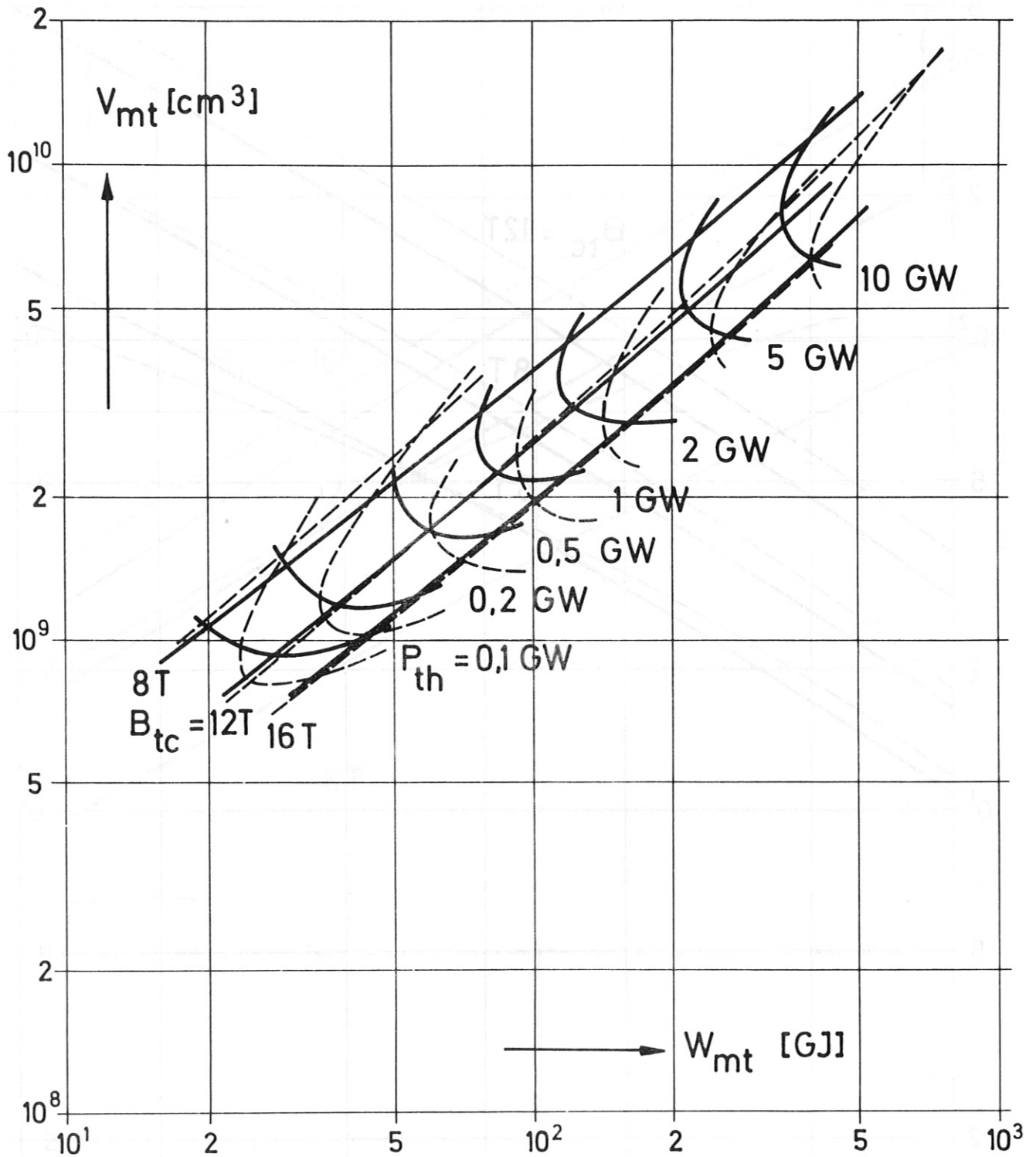


Fig. 12a Toroidal magnet winding volume vs. stored energy (CCS)

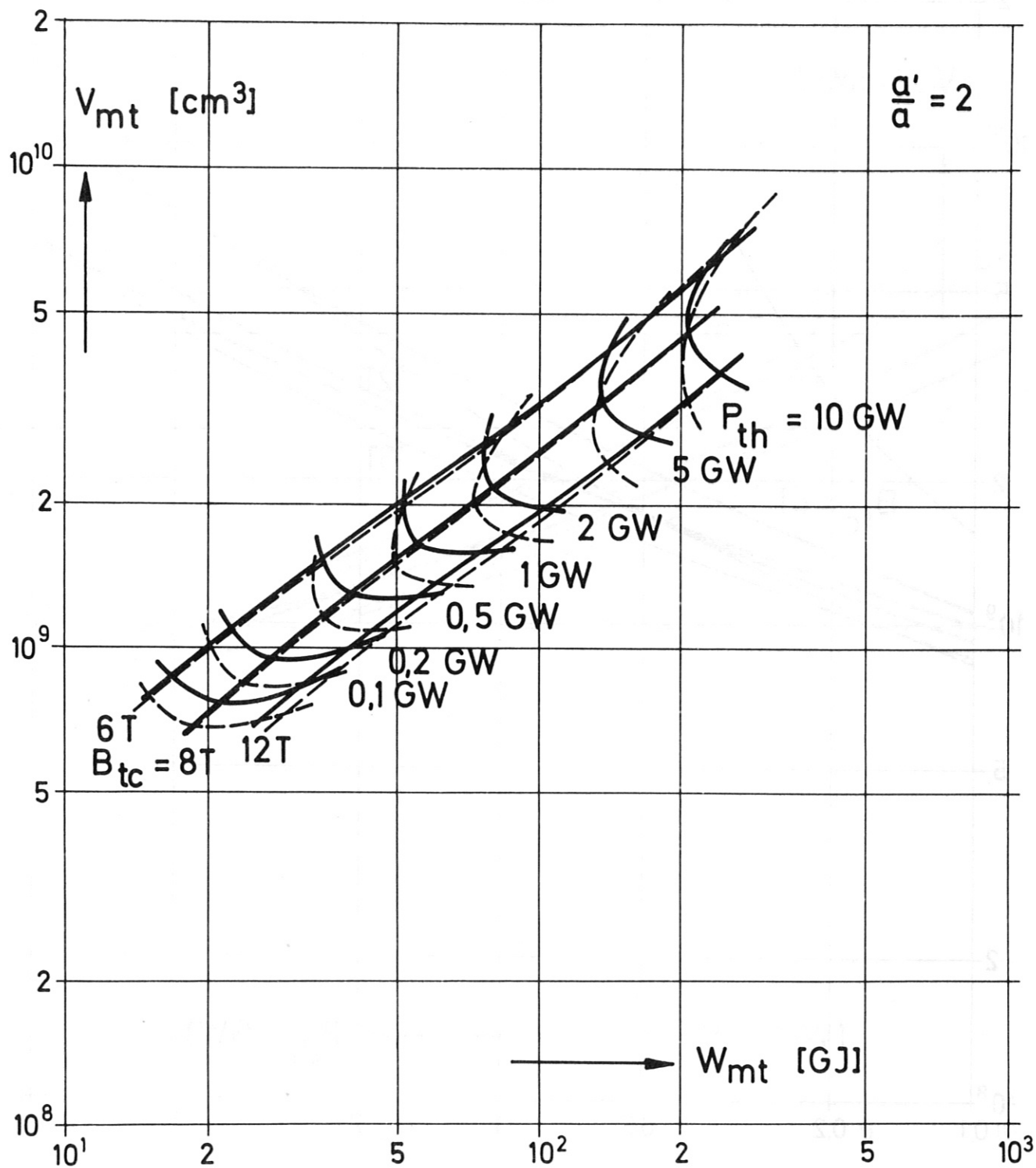


Fig. 12b Toroidal magnet winding volume vs. stored energy (ECS)

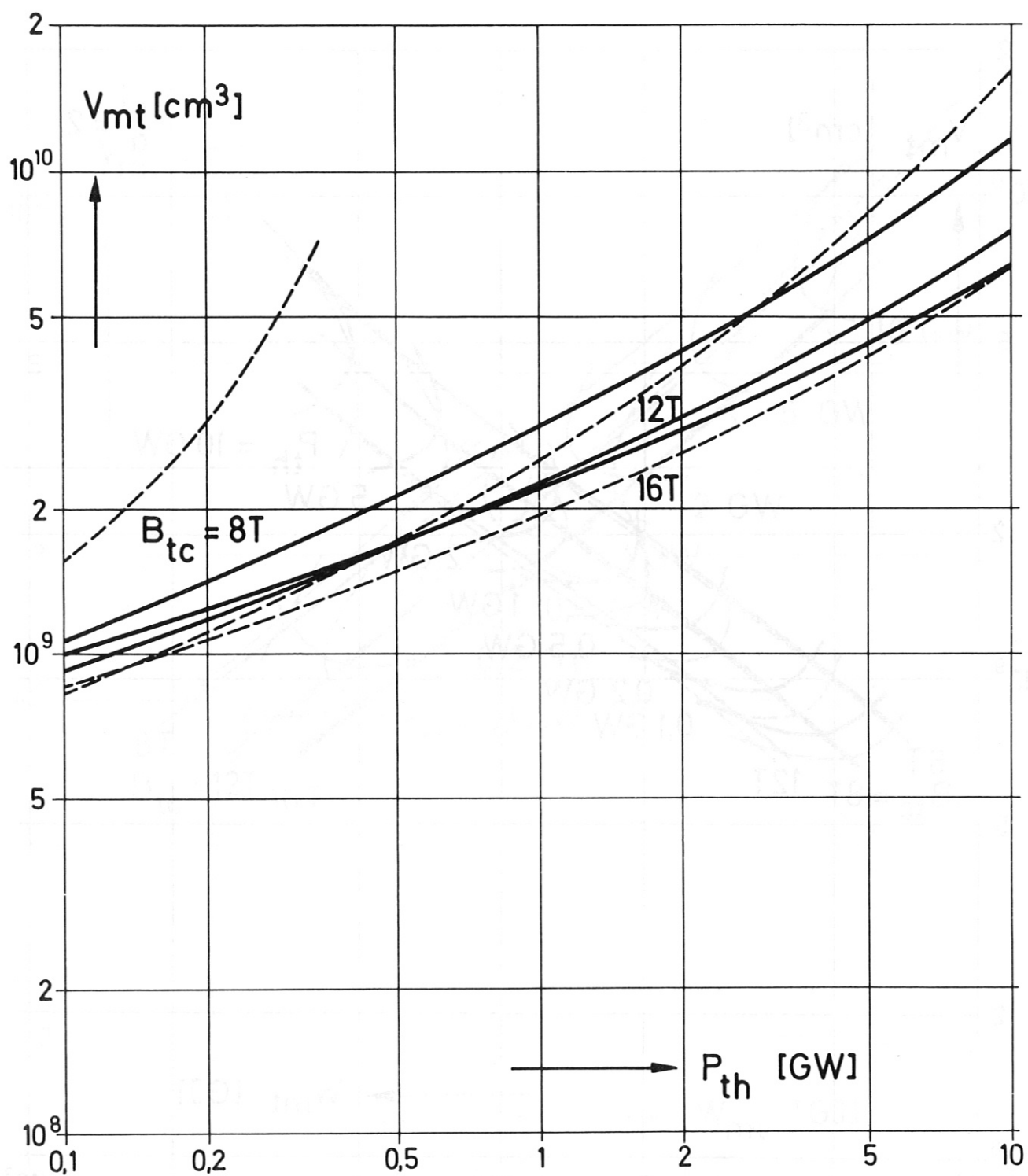


Fig. 13a Toroidal magnet winding volume
vs. reactor power (CCS)

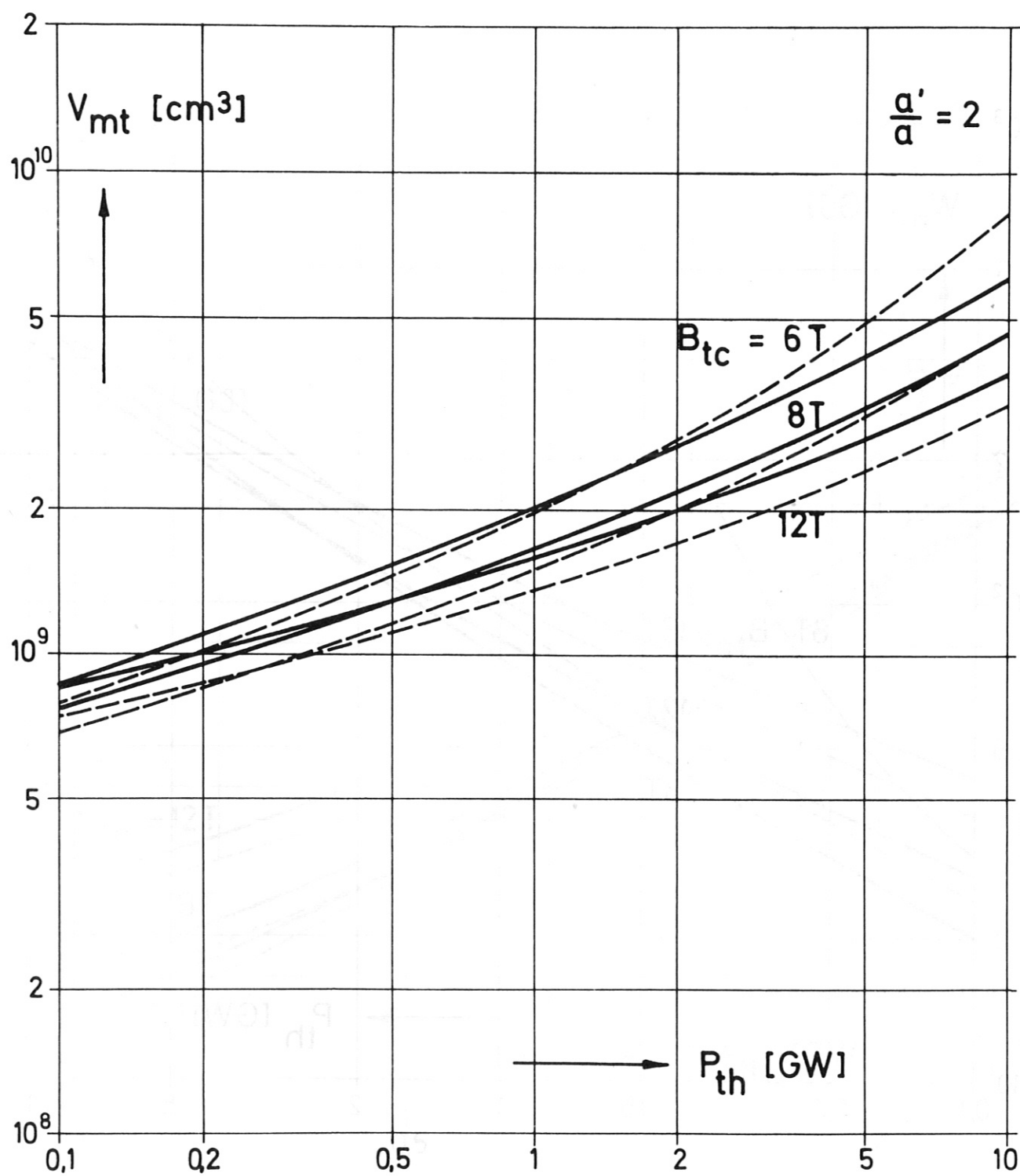


Fig. 13b Toroidal magnet winding volume
vs. reactor power (ECS)

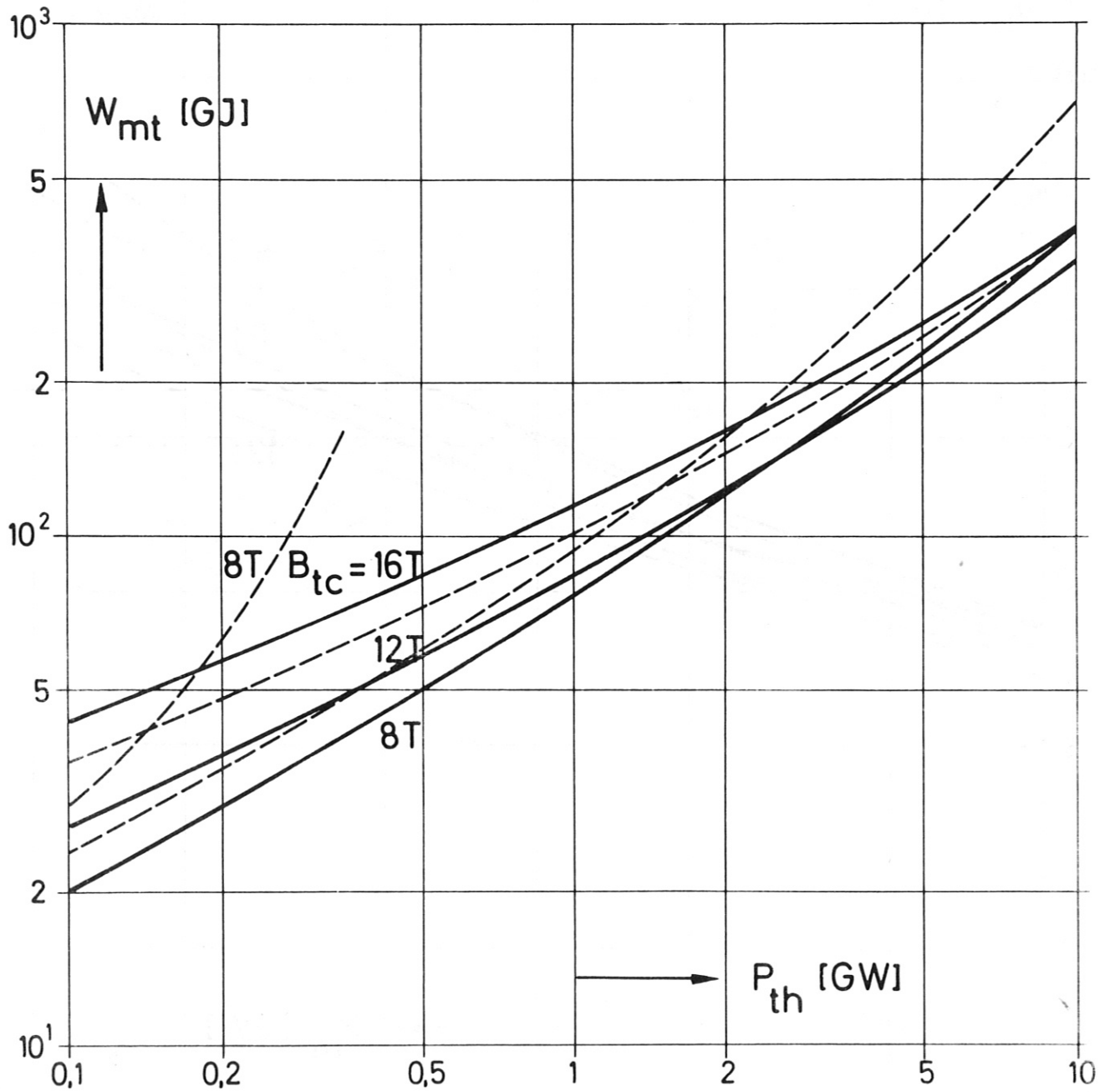


Fig. 14a Toroidal magnet stored energy
vs. reactor power (CCS)

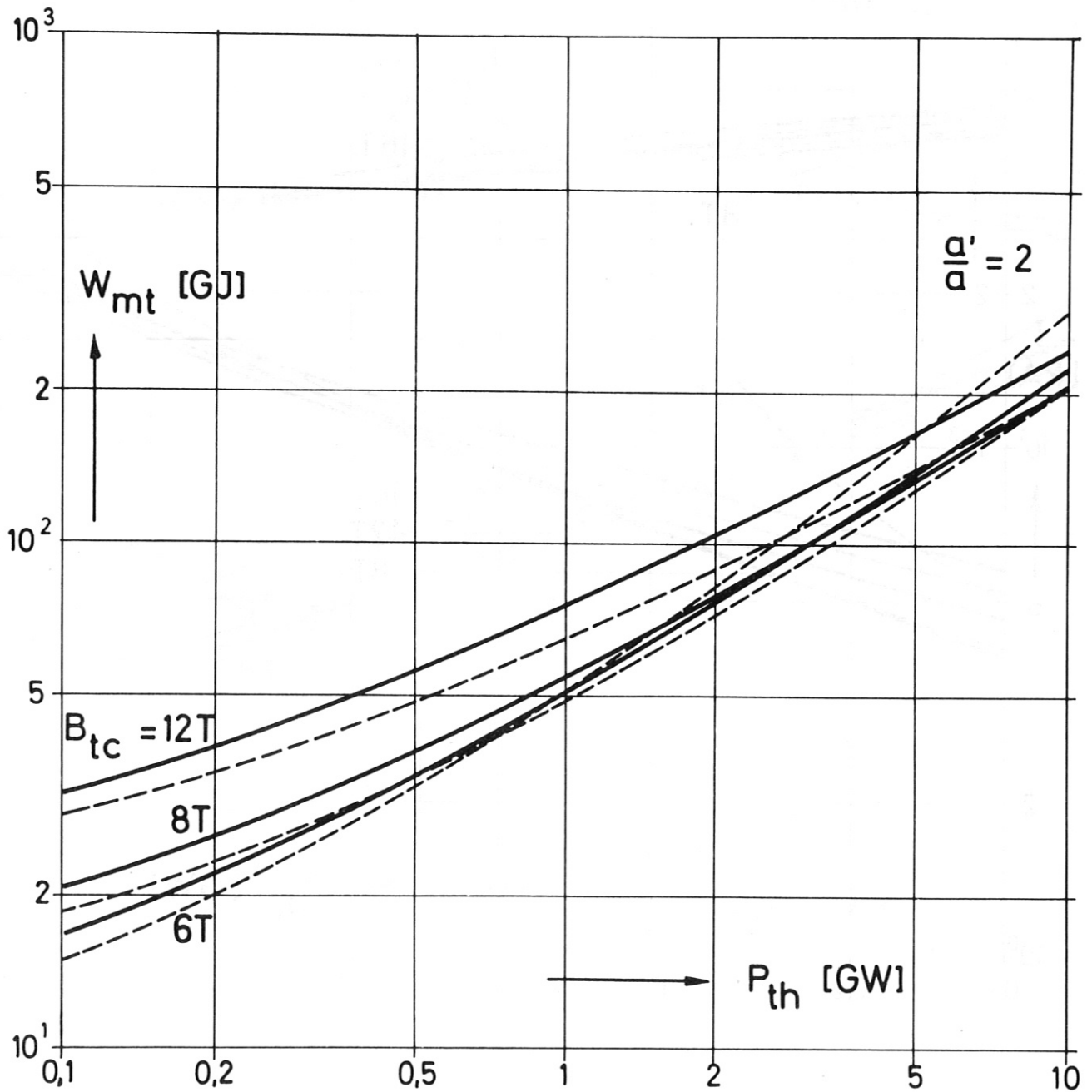


Fig. 14b Toroidal magnet stored energy
vs. reactor power (ECS)

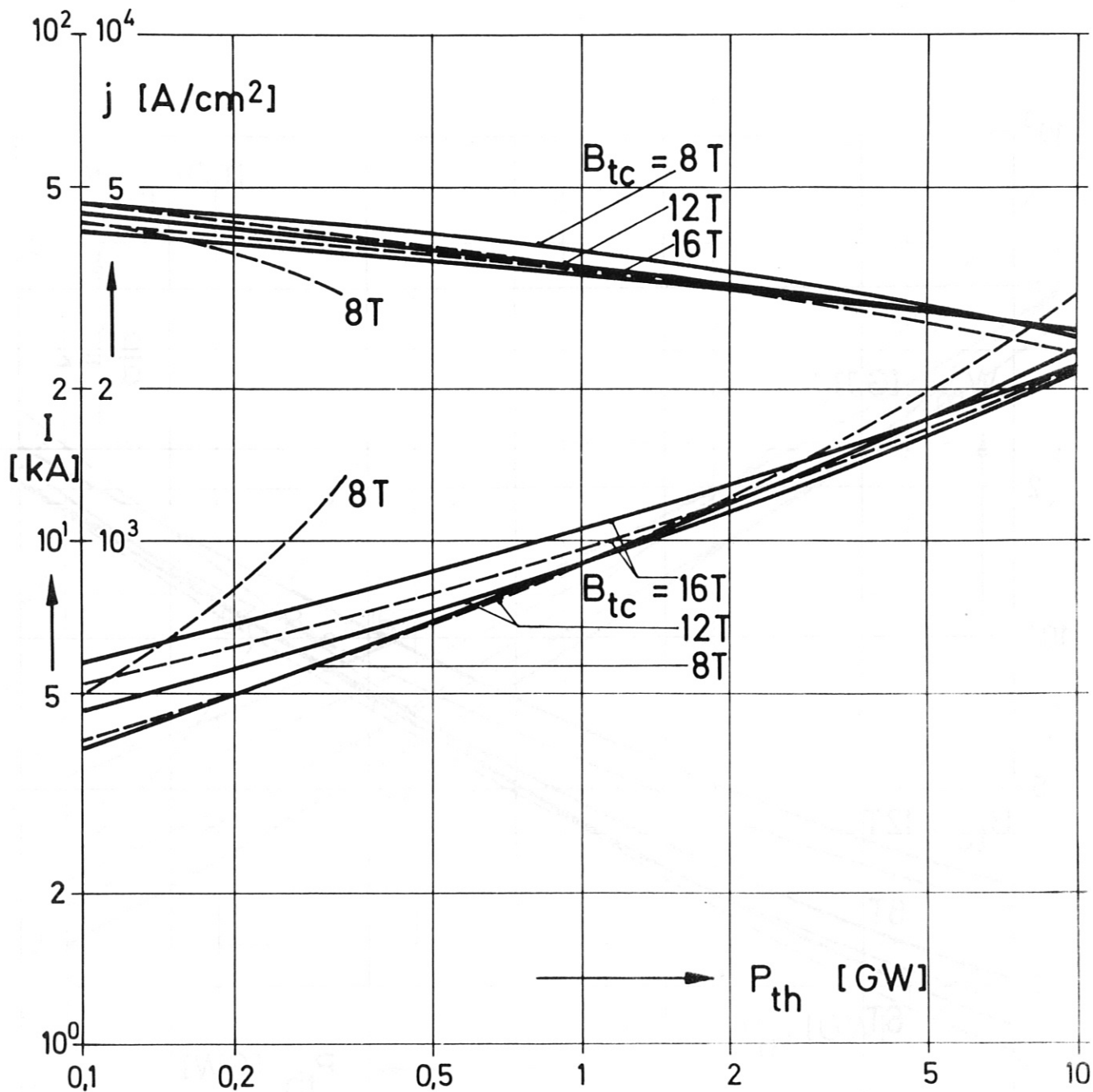


Fig. 15a Toroidal magnet current density and current vs. reactor power (CCS)

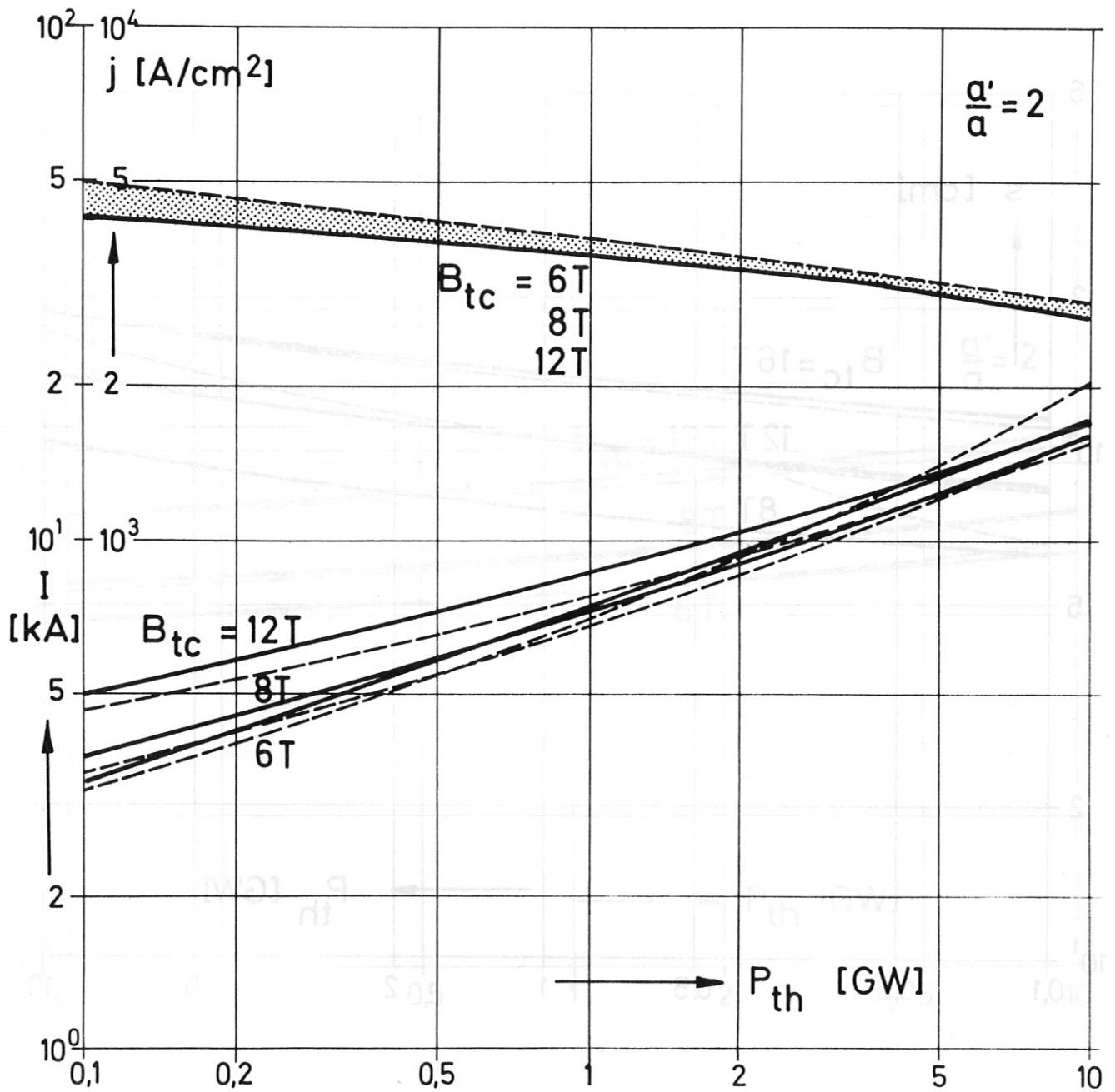


Fig. 15b Toroidal magnet current density and current vs. reactor power (ECS)

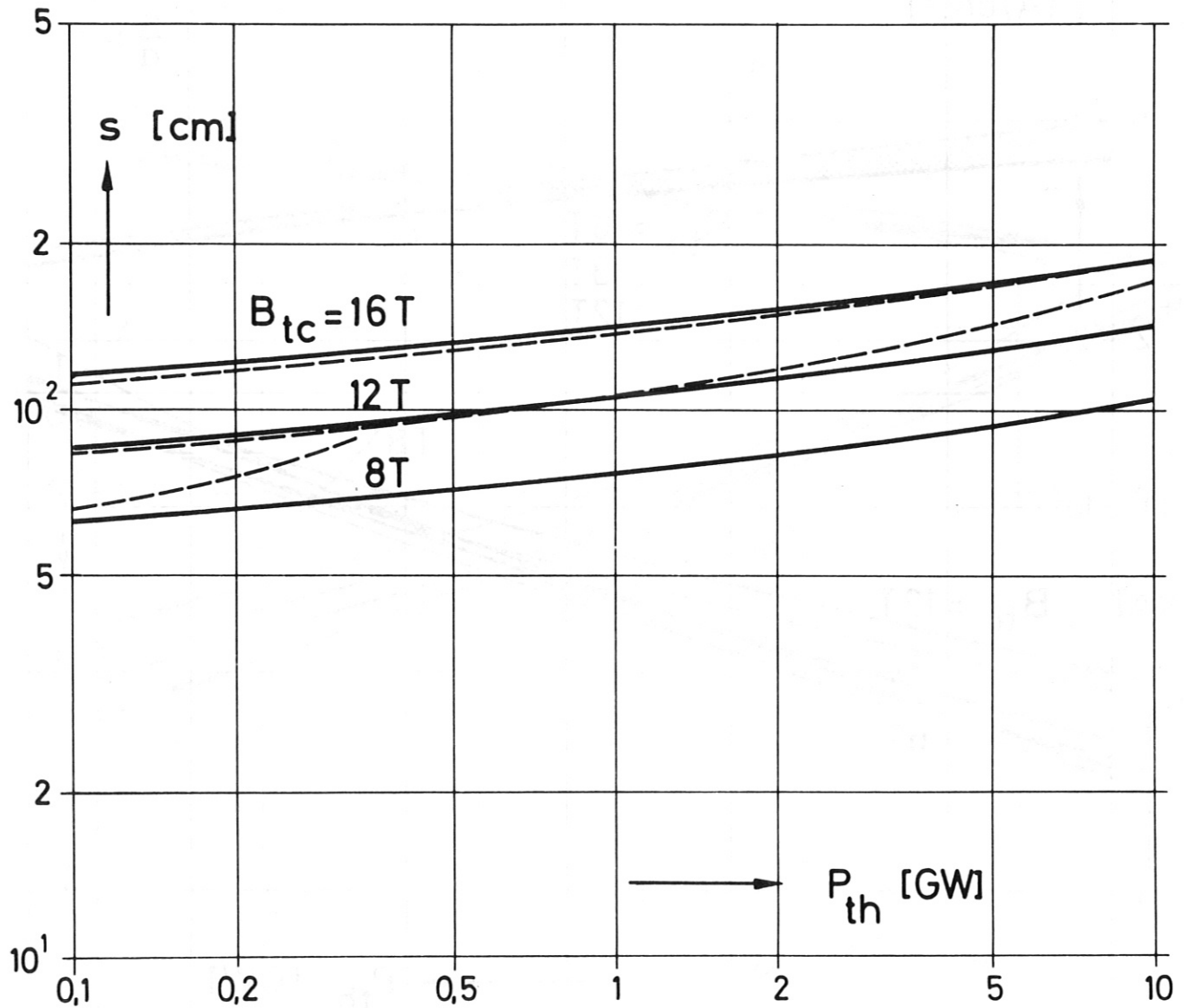


Fig. 16a Torus coils radial overall winding thickness vs. reactor power (CCS)

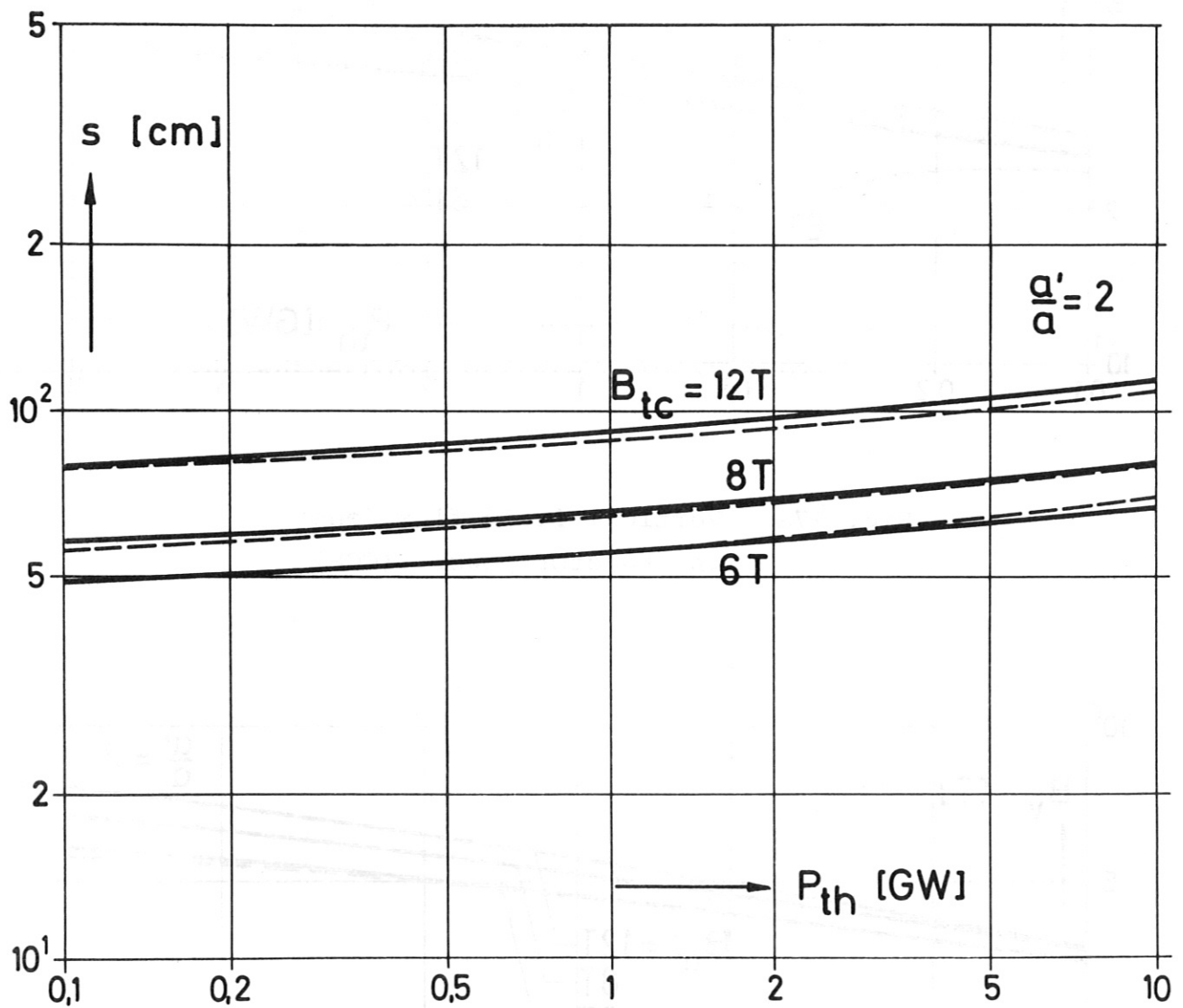


Fig. 16b Torus coils radial overall winding thickness vs. reactor power (ECS)

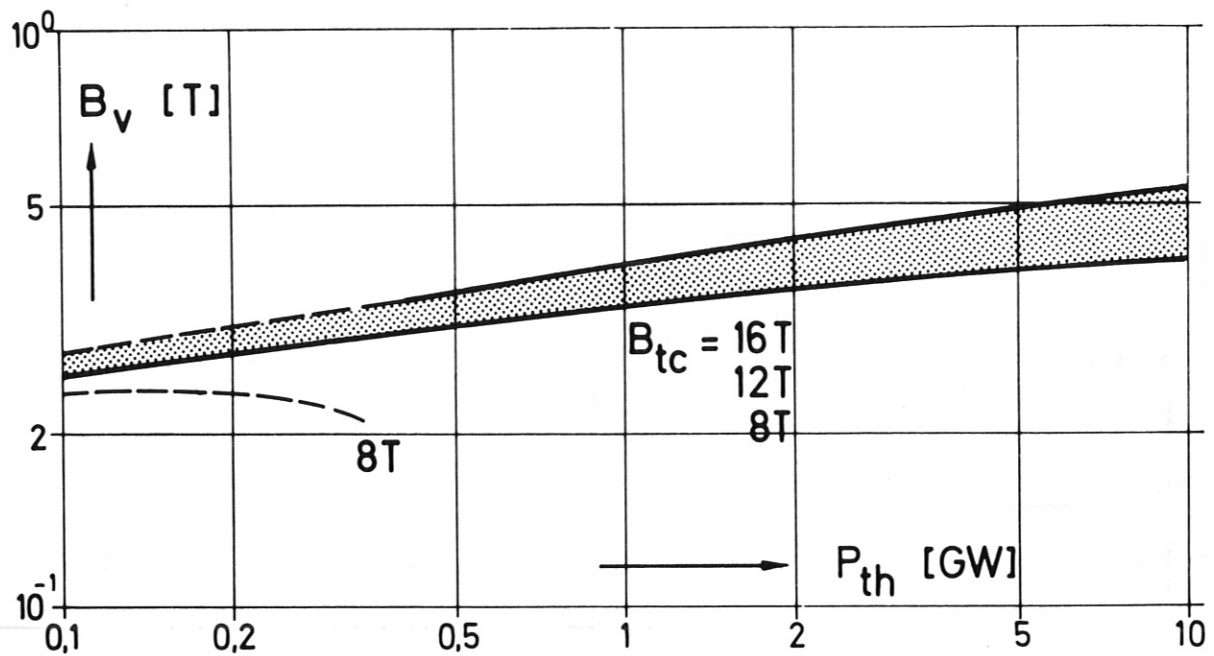


Fig. 17a Vertical field flux density
vs. reactor power (CCS)

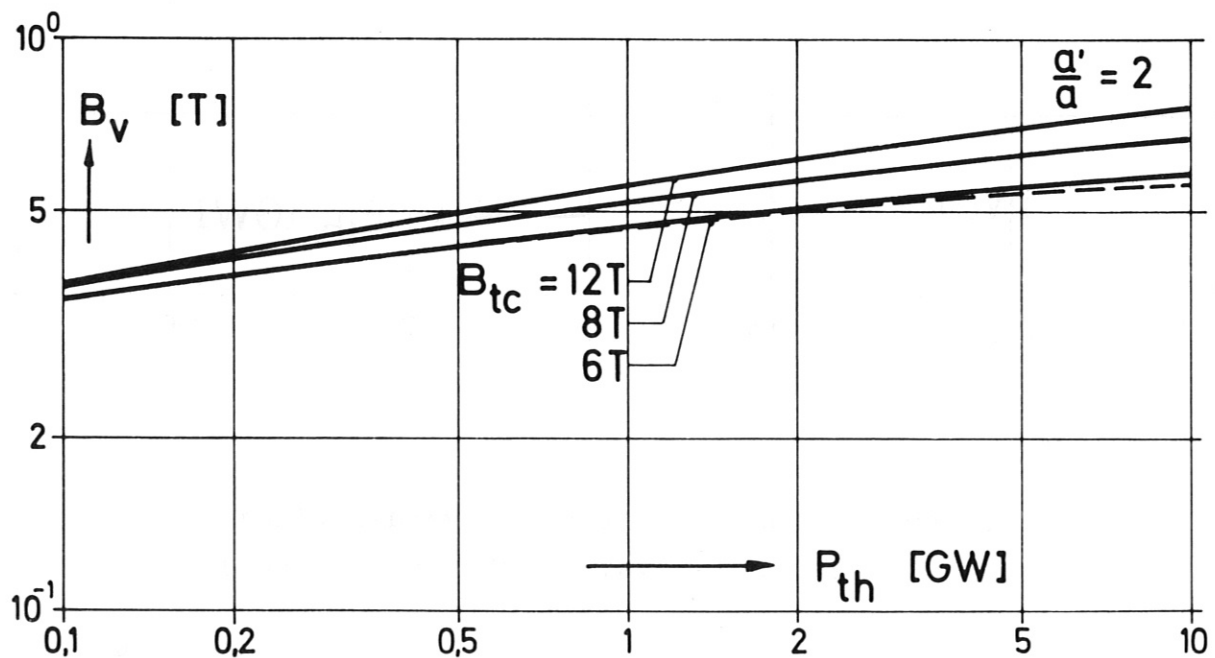


Fig. 17b Vertical field flux density
vs. reactor power (ECS)

References

- [1] KNOBLOCH, A.F., Proc. 6th Symp. on Eng. Probl. of Fus. Res., San Diego 1975.
- [2] SPANO, A.H., Nucl. Fusion 15 (1975), page 915.
- [3] MADDOCK, B.J. et al., Proc. IEEE Vol. 115, April 1968, page 543.
- [4] DEAN, S.O. et al., WASH 1295, UC-20, page 63.
P.H. Rutherford, private communication.