Linewidth Considerations in Small Signal Gain or Loss Measurements with Pulses

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Abstract

This report discusses the difference between c.w. and pulse small signal gain or loss measurements. The spectrum of the gain or loss medium must be compared to the spectrum of the probe signal. First we discuss the formulation of the problem and apply the results to a Gaussian pulse propagating through a single line Lorentzian gain or loss medium. Second, we calculate the pulse energy gain or loss for the six overlapping lines of the iodine spectrum.

In practice, the small signal gain or absorption coefficient is often measured using a pulse as a probe signal. If the spectral width of the pulse is not sufficiently narrow compared to the linewidth of the transition, the measured gain or absorption coefficient represents a certain overlap between the two spectra.

In this note, we analyze this situation for a Gaussian pulse in a gain or loss medium characterized by a single Lorentzian line and by the complete 6 overlapping lines iodine spectrum.

A) Theory

The mathematical description of these considerations is quite simple, although there are some subleties which are easily overlooked. Consider a medium described by a complex gain coefficient for the electric field

$$g(\nu) = g_R(\nu) + i\, g_i(\nu) \tag{1}$$

where g_R and g_i are the real and imaginary part respectively. $g_R(\nu)$ is the real gain, and $g_i(\nu)$ is related to the index of refraction. To satisfy causality, g_R and g_i are related through the Kramers-Krönig dispersion relations. Next, consider an electric field pulse given by

$$\mathcal{E}_o(\tau) = \int_{-\infty}^{+\infty} \mathcal{E}_o(\nu)\, e^{i 2\pi\nu\tau}\, d\nu \tag{2}$$

where τ = (t - z/c) is the retarded time. If such a pulse propagated through a distance L in a medium characterized by the above complex gain, the exit pulse is

$$\mathcal{E}(\tau, L) = \int_{-\infty}^{+\infty} \mathcal{E}_o(\nu)\, e^{(g_R(\nu) + i g_i(\nu))L}\, e^{i 2\pi\nu\tau}\, d\nu \tag{3}$$

and the observed intensity is

$$I(\tau, L) = \frac{c}{4\pi} \left| \mathcal{E}(\tau, L) \right|^2 . \tag{4}$$

The total exit energy / cm^2 becomes

$$E(L) = \int_{-\infty}^{+\infty} d\tau \; I(\tau, L) \tag{5a}$$

$$= \frac{c}{4\pi} \int_{-\infty}^{+\infty} \left| \mathcal{E}(\tau, L) \right|^2 d\tau \tag{5b}$$

$$= \frac{c}{4\pi} \int_{-\infty}^{+\infty} \left| \mathcal{E}_0(\nu) \right|^2 e^{2 g_R(\nu) L} \, d\nu . \tag{5c}$$

The step from 5b to 5c follows from Parseval's theorem /1/. We note that the intensity I (τ, L) depends on both the gain and the index of refraction, but the energy only on the gain. In general, a gain measurement based on comparing the peaks of the incident and exit intensity will not be the same as comparing the corresponding energies. The exit intensity temporal pulse shape may be quite different from the entrance pulse shape because of the dispersion in $g(\nu)$. Only if $\mathcal{E}_0(\nu)$ has a narrow spectrum compared to that of $g(\nu)$ will the exit pulse be a scaled entrance pulse, and the intensity gain be the energy gain. If this is not the case, the dispersion in the gain usually broadens a pulse, the dispersion in the loss may shorten it.

For gain or absorption coefficient measurements with short pulses, the resolution of the detection system is often insufficient to follow the temporal behavior of the intensity, and an integrated energy is measured. In this case Eq. 5c describes the energy gain.

B) Energy Gain or Loss in a Two Level System

1. Gain

A simple example for which we can demonstrate these effects is a two level system, characterized by a Lorentzian line. Hence, we take

$$2g_R(\gamma) = g_0 \frac{(\Delta\gamma_A/2)^2}{\gamma^2 + (\Delta\gamma_A/2)^2} \tag{6}$$

where g_0 is the line centre intensity or energy small signal gain coefficient, $\Delta\gamma_A$ the full width at 1/2 max. and γ the frequency relative to line centre.

For the incident pulse we take a normalized Gaussian, on line centre, such that

$$\frac{c}{4\pi} |\mathcal{E}_0(\gamma)|^2 = \frac{1}{\frac{\Delta\gamma_p}{2}\sqrt{\pi}} e^{-\left(\frac{\gamma}{\Delta\gamma_p/2}\right)^2} \tag{7}$$

where $\Delta\gamma_p$ is the full width 1/e of the spectrum of the pulse, related through

$$\Delta\gamma_p(\text{fw } 1/e) = \frac{4}{\pi} \frac{1}{\tau_p(\text{fw } 1/e)}$$

to the temporal width of the Gaussian pulse. The small signal energy gain from Eq. 5c now becomes

$$G = \frac{E(L)}{E(0)} = \int_{-\infty}^{+\infty} \frac{d\gamma}{\frac{\Delta\gamma_p}{2}\sqrt{\pi}} e^{-\left(\frac{2\gamma}{\Delta\gamma_p}\right)^2 + g_0 L \frac{\Delta\gamma_A^2}{4\gamma^2 + \Delta\gamma_A^2}} \tag{8}$$

Fig. 1 shows the spectra of the components which enter into this calculation. All spectra are normalized to unity. The dashed curve shows a Gaussian with $\Delta\nu_D$ = 4 GHz (fwl/e), the other two a Lorentzian of 4 GHz (fwl/2) and the exponential of the Lorentzian with a gL = 6, showing the effect of gain narrowing.

We note that a change of variables lets us write Eq. 8 in the simple form

$$G = \int_{-\infty}^{+\infty} \frac{dx}{\sqrt{\pi}} e^{-x^2} e^{gL \frac{R^2}{x^2 + R^2}} \leq e^{gL} \tag{9}$$

where R = $\Delta\nu_H / \Delta\nu_D$. Fig. 2 shows a plot of the small signal gain, normalized to exp(gL), as a function of the bandwidth ratio R for various values of gL. This is obtained by numerically integrating Eq. 9. As expected for large R we obtain the full small signal gain. For $R \lesssim 1$, we observe considerable reduction in the gain, particularly for large gL values, where gain narrowing is greater. Besides the obvious inequality in Eq. 9, another useful estimate can be derived. One can easily show that

$$G > \int_{-\infty}^{+\infty} \frac{dx}{\sqrt{\pi}} e^{-x^2} e^{gL\left(1 - \frac{x^2}{R^2}\right)} = \frac{e^{gL}}{\sqrt{1 + gL/R^2}} . \tag{10}$$

A comparison of the numerical integration and the above inequality for the case gL = 6 is shown on Fig. 3. We note that the estimate is quite good in the chosen range.

2. Absorption

The only change we have to make to study the two level Lorentzian absorber is to replace the g with $-a$, where a is the absorption coefficient, and we get the pulse energy transmission

$$T = \int_{-\infty}^{+\infty} \frac{dx}{\sqrt{\pi}} e^{-x^2} e^{-aL \frac{R^2}{x^2+R^2}} . \tag{11}$$

As a function of the linewidth ratio R, T(R) satisfies

$$T(R\to\infty) = e^{-aL} \le T(R) \le 1 = T(R\to 0). \tag{12}$$

Fig. 4 shows a plot of 1/ T(R) normalized to 1/ exp(-aL). On such a plot, the full small signal absorption ($R \to \infty$) approaches unity. From Fig. 4 we note that the bandwidth limitations for the absorber are even more severe than for the amplifier.

One can also derive some inequalities for the transmission T(R), but they do not seem as useful as the ones for the gain medium. Similarly to Eq. 10, we now have

$$T(R) < \int_{-\infty}^{+\infty} \frac{dx}{\sqrt{\pi}} e^{-x^2} e^{-aL\left(1-\frac{x^2}{R^2}\right)} \tag{13a}$$

provided $aL/R^2 < 1$, otherwise the integral diverges. Similarly one has

$$T(R) > \int_{-\infty}^{+\infty} \frac{dx}{\sqrt{\pi}} e^{-x^2} e^{-aL\frac{R^2}{x^2}} = e^{-2\sqrt{aL}\,R} . \tag{13b}$$

The two inequalities, compared to the numerical integration, are indicated on Fig. 5 with aL = 6.

C) Application to Iodine Amplifier and Absorber

To apply the previous results to the iodine $^2P_{1/2}$ - $^2P_{3/2}$ transition, one must modify the calculation to take into account the multiline nature, due to the hyperfine structure of that transition. This does not change anything in principle, but as we shall see, because of the line spacings, the results are no longer functions of $R = \Delta\nu_A / \Delta\nu_P$ alone.

We characterize the iodine gain or absorption spectral line shape by a superposition of 6 Lorentzian lines, each with the same pressure broadened line width. The lineshape is normalized to unity at the centre of the 3-4 transition, which is approximately at the maximum of the spectra. The analytic expression for this normalized spectral shape is

$$F(\nu) = \frac{\sum_i \dfrac{g_i A_i \left(\frac{\Delta\nu_I}{2}\right)^2}{(\nu-\nu_i)^2+\left(\frac{\Delta\nu_I}{2}\right)^2}}{\sum_i \dfrac{g_i A_i \left(\frac{\Delta\nu_I}{2}\right)^2}{(\nu_{3-4}-\nu_i)^2+\left(\frac{\Delta\nu_I}{2}\right)^2}} \qquad (14)$$

where the sum goes over the 6 transitions at frequencies ν_i, with statistical weight g_i and transition rate A_i. These parameters are listed below /2/.

Line	Spacing (GHz)	A(sec^{-1})	g_i
2 - 3	- 14.86	2.4	5/12
2 - 2	- 12.89	3.0	5/12
2 - 1	- 12.15	2.3	5/12
3 - 4	0	5.0	7/12
3 - 3	+ 4.22	2.1	7/12
3 - 2	+ 6.19	.6	7/12

Figs. 6 - 8 show the normalized spectral shapes $F(\nu)$ (solid curves) for three values of the single line linewidth $\Delta\nu_I$ = .5,4 and 20 GHz (fwl/2). Also shown (dashed) for each case is the normalized gain narrowed spectrum $e^{gLF(\nu)}/e^{gL}$ for a gain-length product gL = 6. For $\Delta\nu_I$ = .5 GHz, the 6 lines are still resolvable, for 4 GHz two groups are resolved, and at 20 GHz one observes complete overlap.

The small signal energy gain or loss is now computed by numerically integrating

$$G \text{ or } T = \int_{-\infty}^{+\infty} \frac{d\nu}{\frac{\Delta\nu_P}{2}\sqrt{\pi}} \, e^{-\left(\frac{2\nu}{\Delta\nu_P}\right)^2} e^{(gL \text{ or } -aL)F(\nu)} \quad . \qquad (15)$$

From the form of $F(\nu)$ we note that the result is no longer a function of $R = \Delta\nu_I/\Delta\nu_P$ alone. Fig. 9 shows the pulse gain in iodine for a single transition linewidth of 4 GHz (fwl/2) as a function of R for various values of gL. Again, the curves are normalized to exp(gL). Fig. 10 shows the same calculation for the pulse transmission in an absorber for a single transition linewidth of $\Delta\nu_I$ = 1 GHz (fwl/2).

The choice of 4 GHz for the amplifier and 1 GHz for the absorber is based on the approximate operating conditions of the Garching iodine amplifiers and saturable absorbers.

The numerical integrations and plots were performed on a HP 9820A calculator and plotter, and are easily extended to other operating conditions.

In conclusion, this note should serve as a useful guide in comparing small signal measurements made with pulses and the limiting c.w. values.

Acknowledgement

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References:

/1/ Morse and Feshbach,
Methods of Theoretical Physics, Vol. I

/2/ Zuev et al.
Soviet Physics JETP 35, 5 pg. 870, 1972

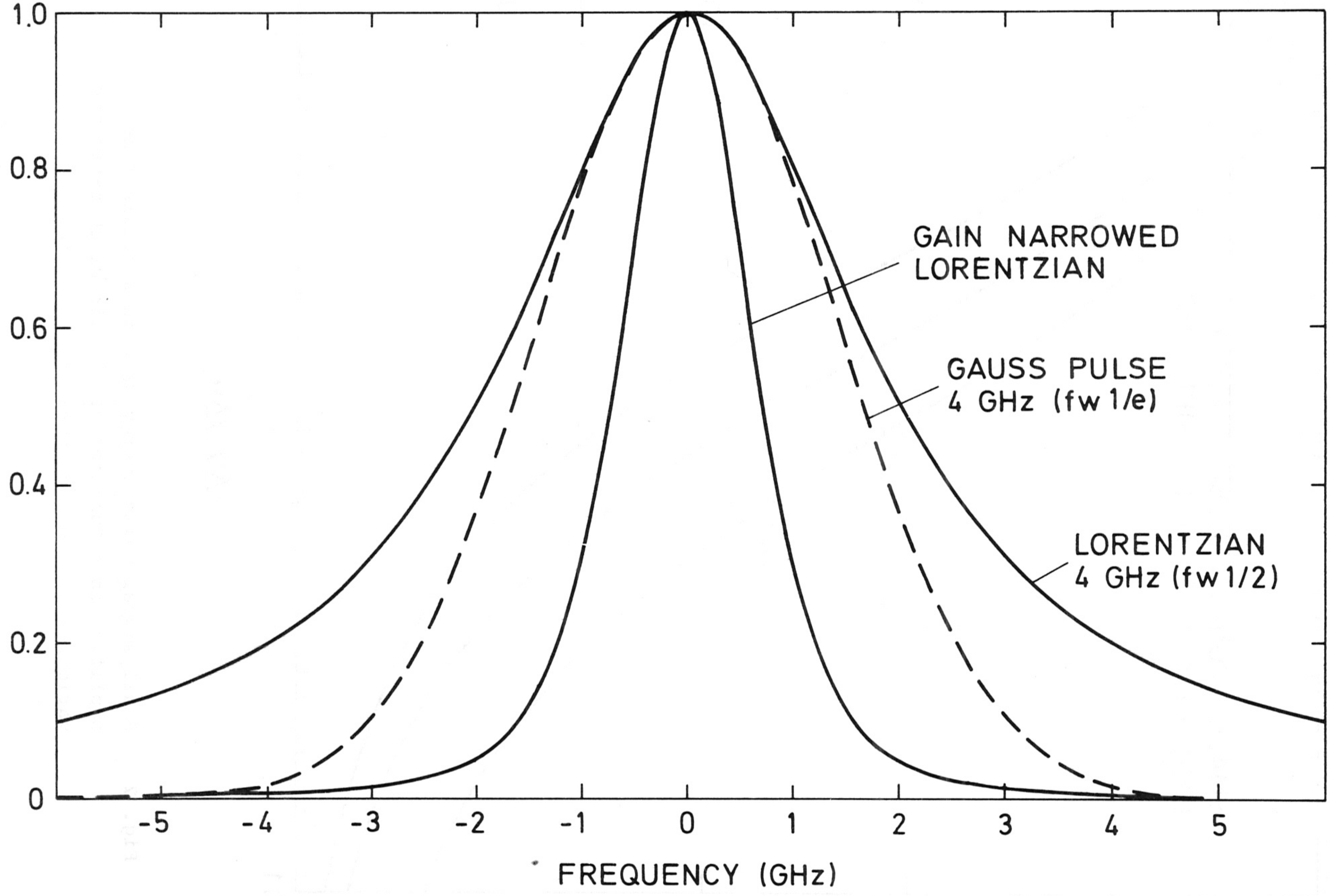

Fig. 1 Normalized Spectra of a Gaussian, Lorentzian, and gain narrowed Lorentzian with $gL = 6$.

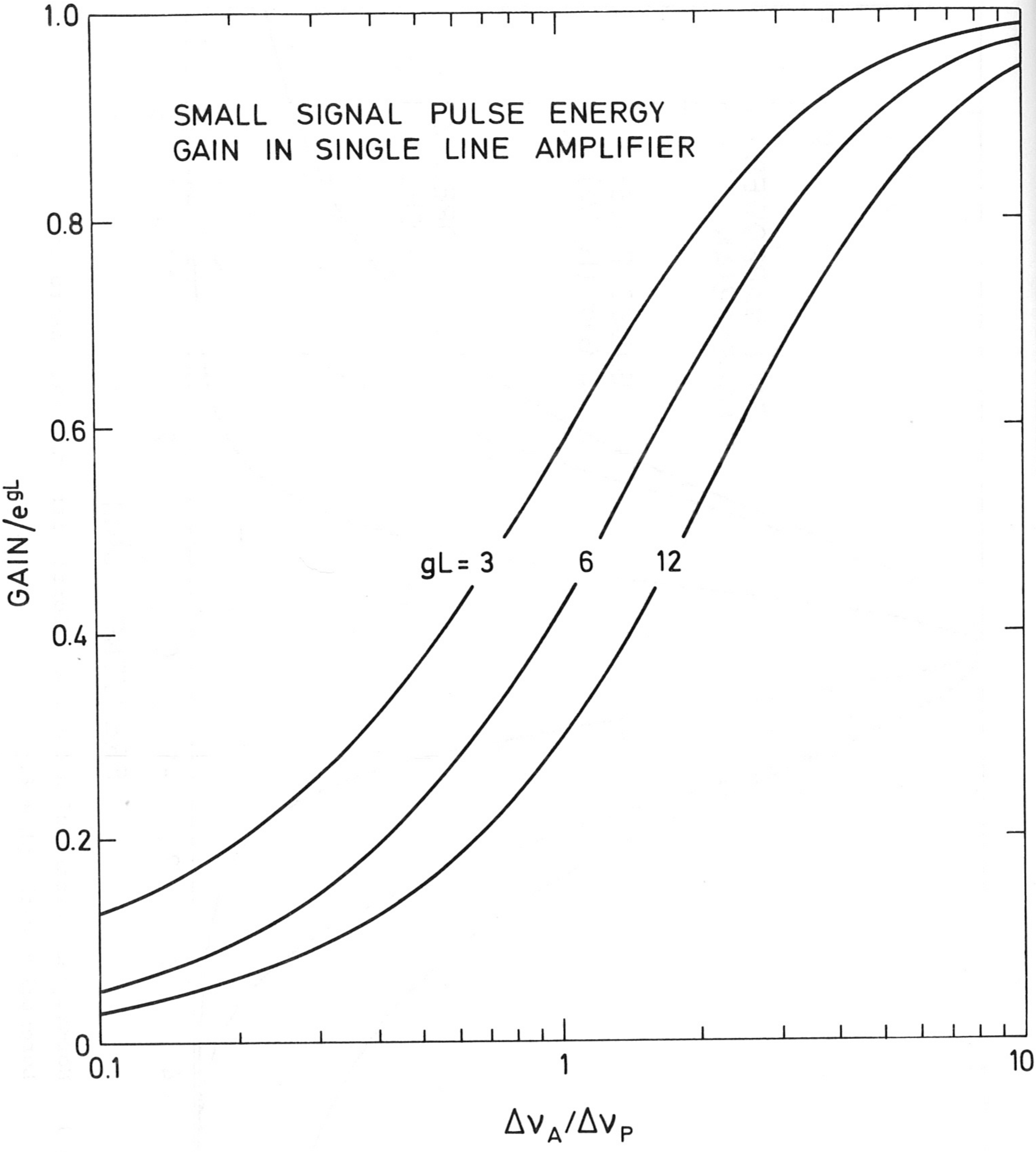

Fig. 2 Small signal pulse energy gain in a single line amplifier as a function of the linewidth ratio for various values of the small signal gain.

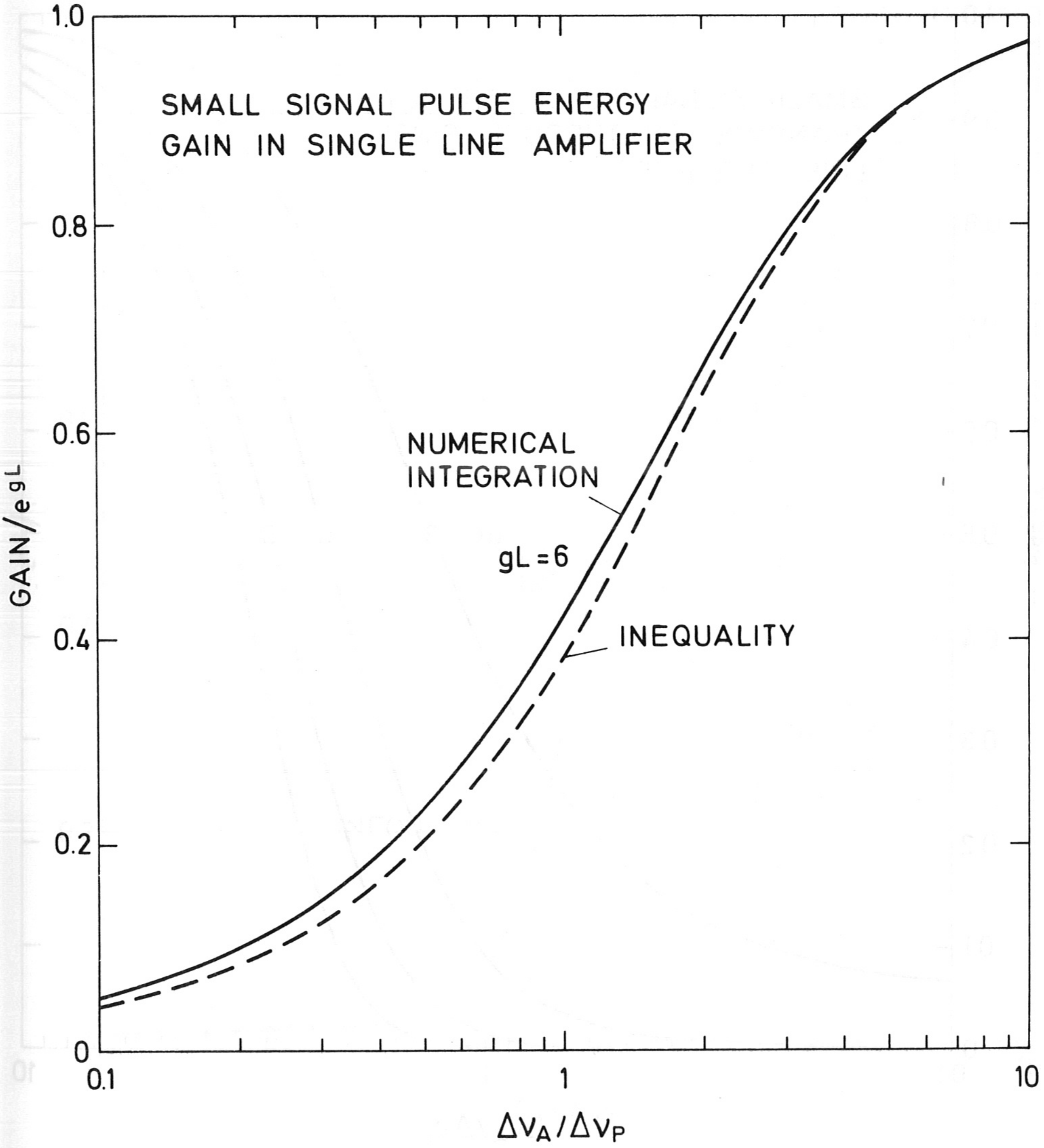

Fig. 3 Comparison of numerical integration and the inequality given in Eq. 10

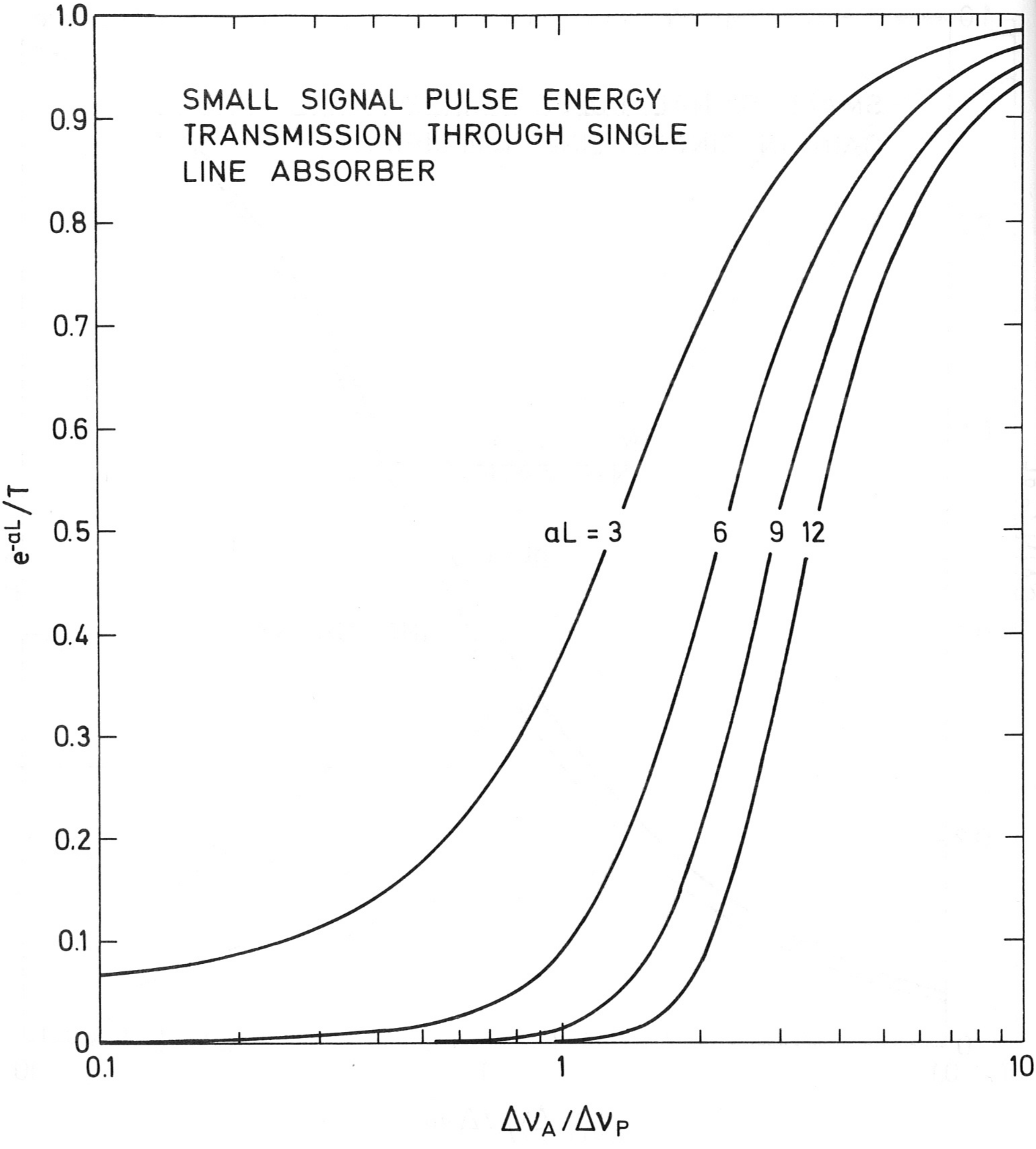

Fig. 4 Small signal pulse energy transmission through a single line absorber as a function of the linewidth ratio for various values of the small signal loss

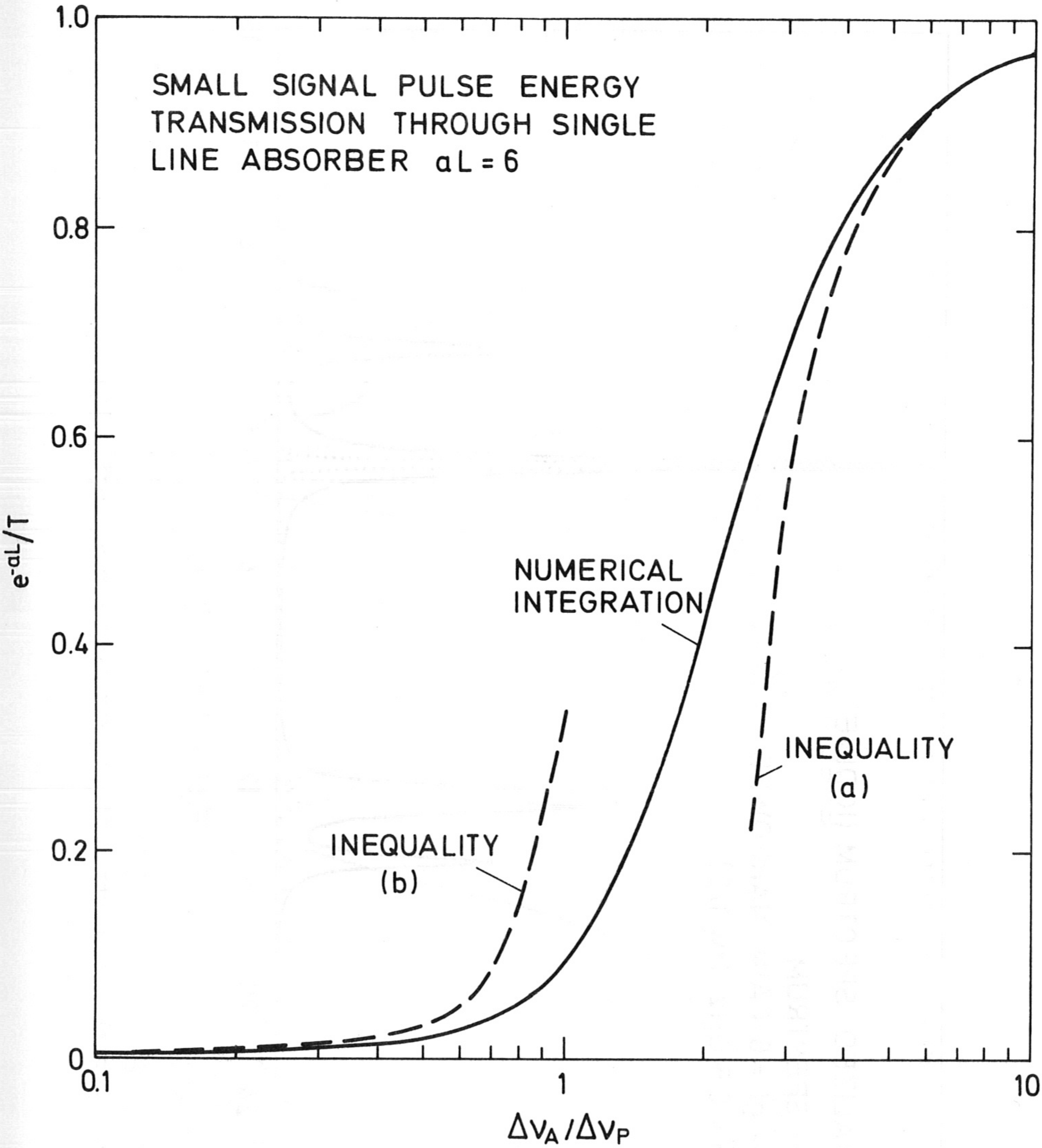

Fig. 5 Comparison of the integration and the inequalities given in Eqs. 13a and 13b

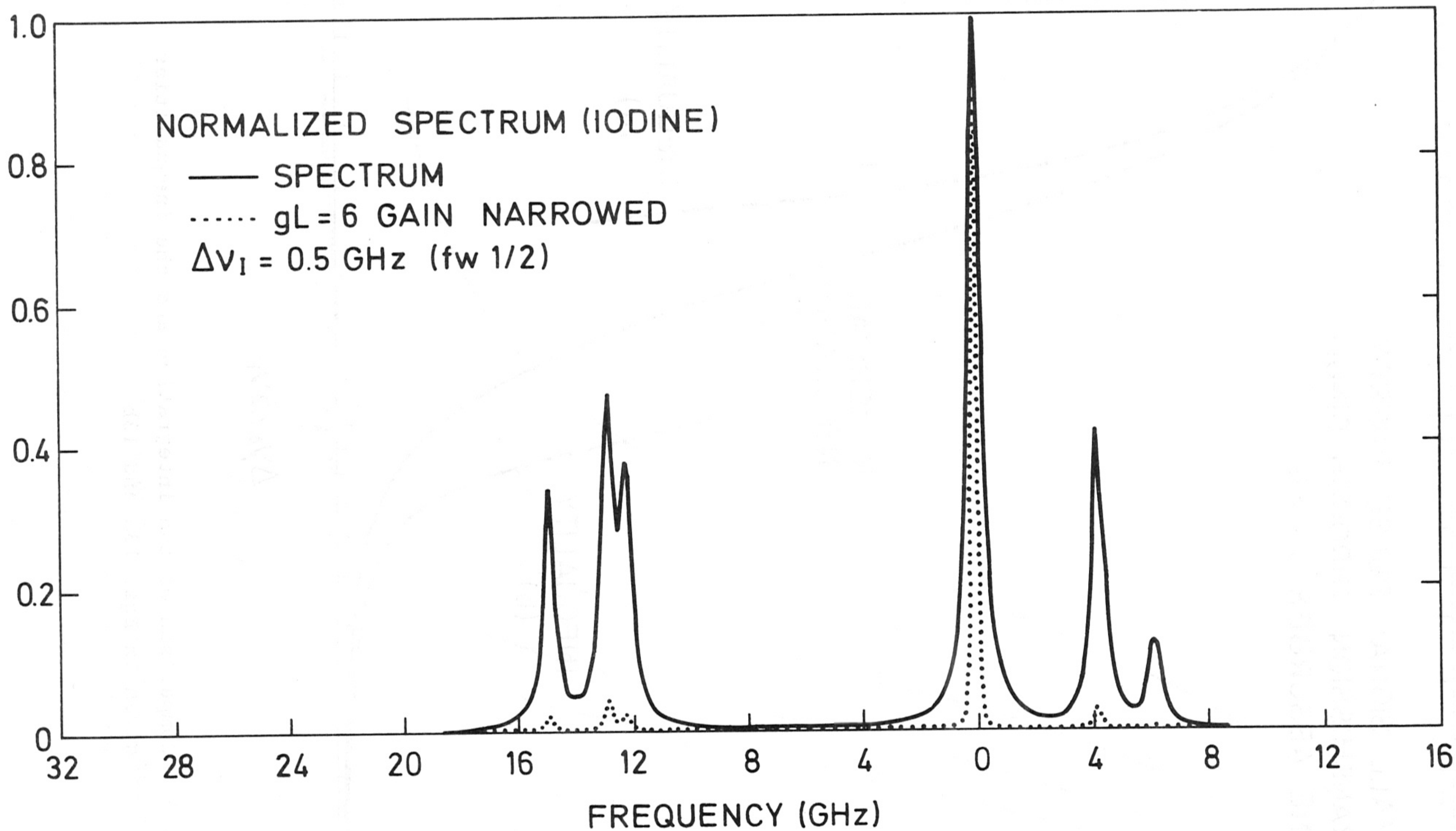

Fig. 6 Normalized spectrum of the six overlapping iodine lines

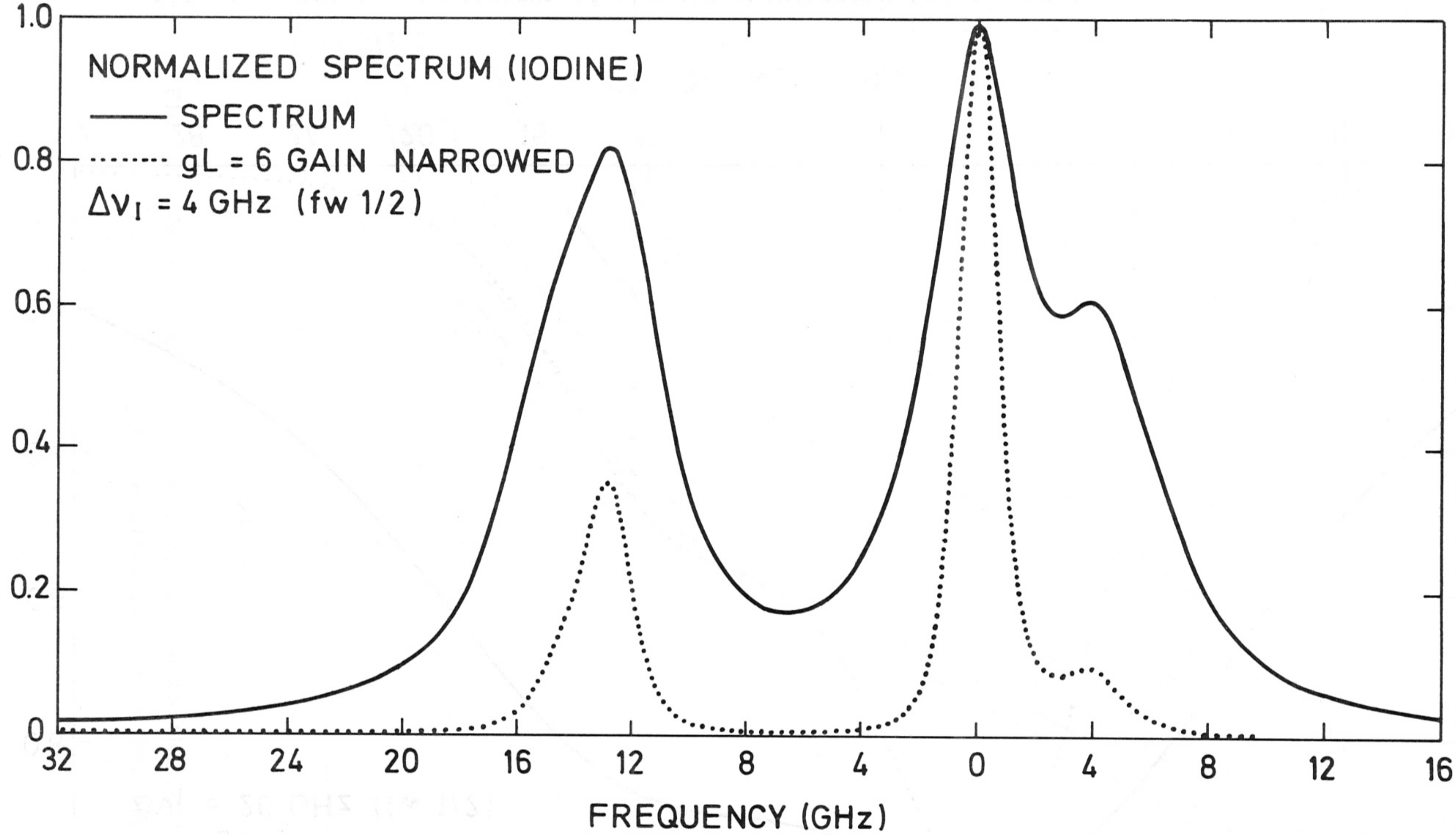

Fig. 7 Normalized spectrum of the six overlapping iodine lines

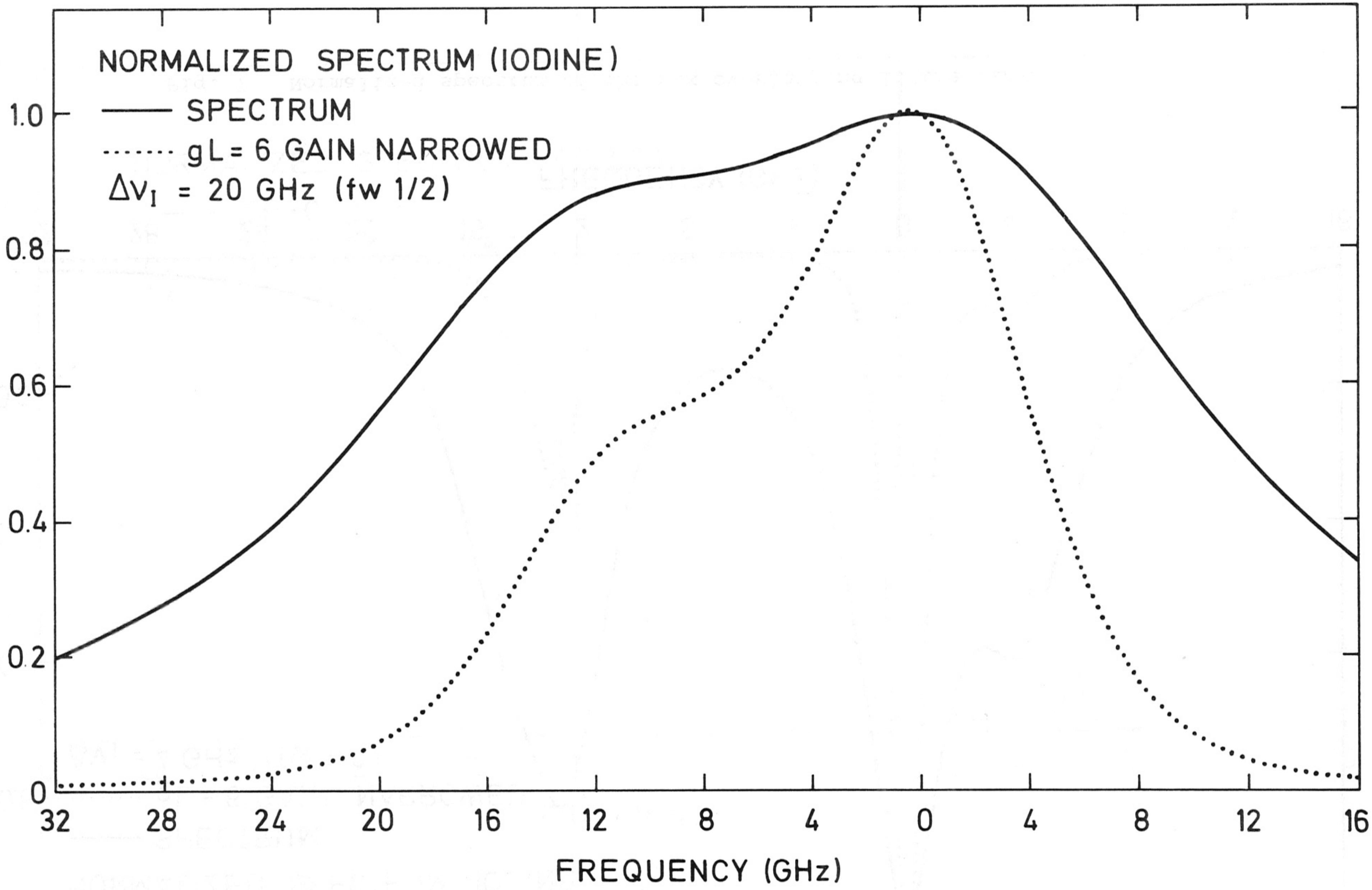

Fig. 8 Normalized spectrum of the six overlapping iodine lines

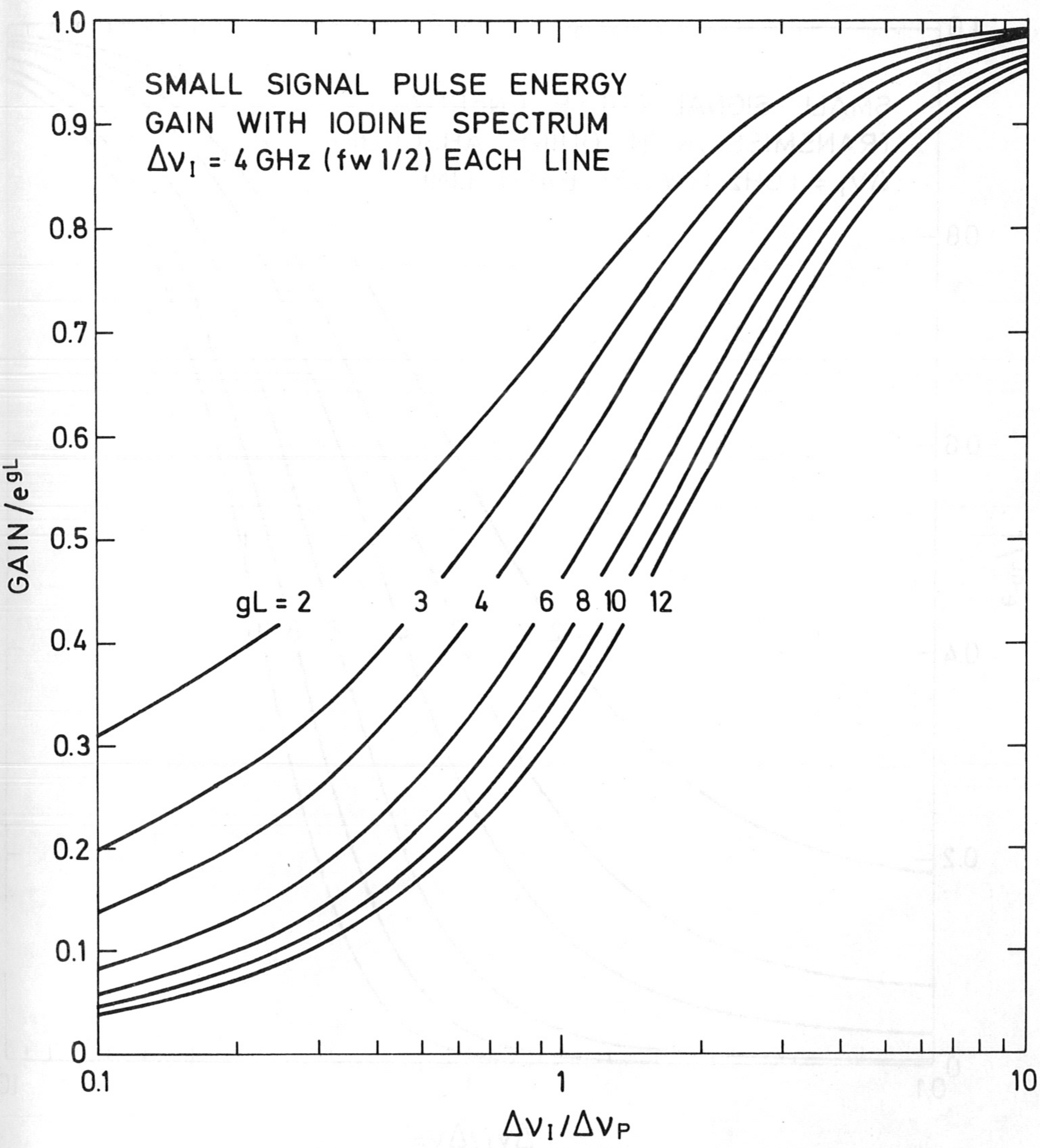

Fig. 9 Small signal pulse energy gain with the iodine spectrum as a function of the linewidth ratio for various values of the small signal gain

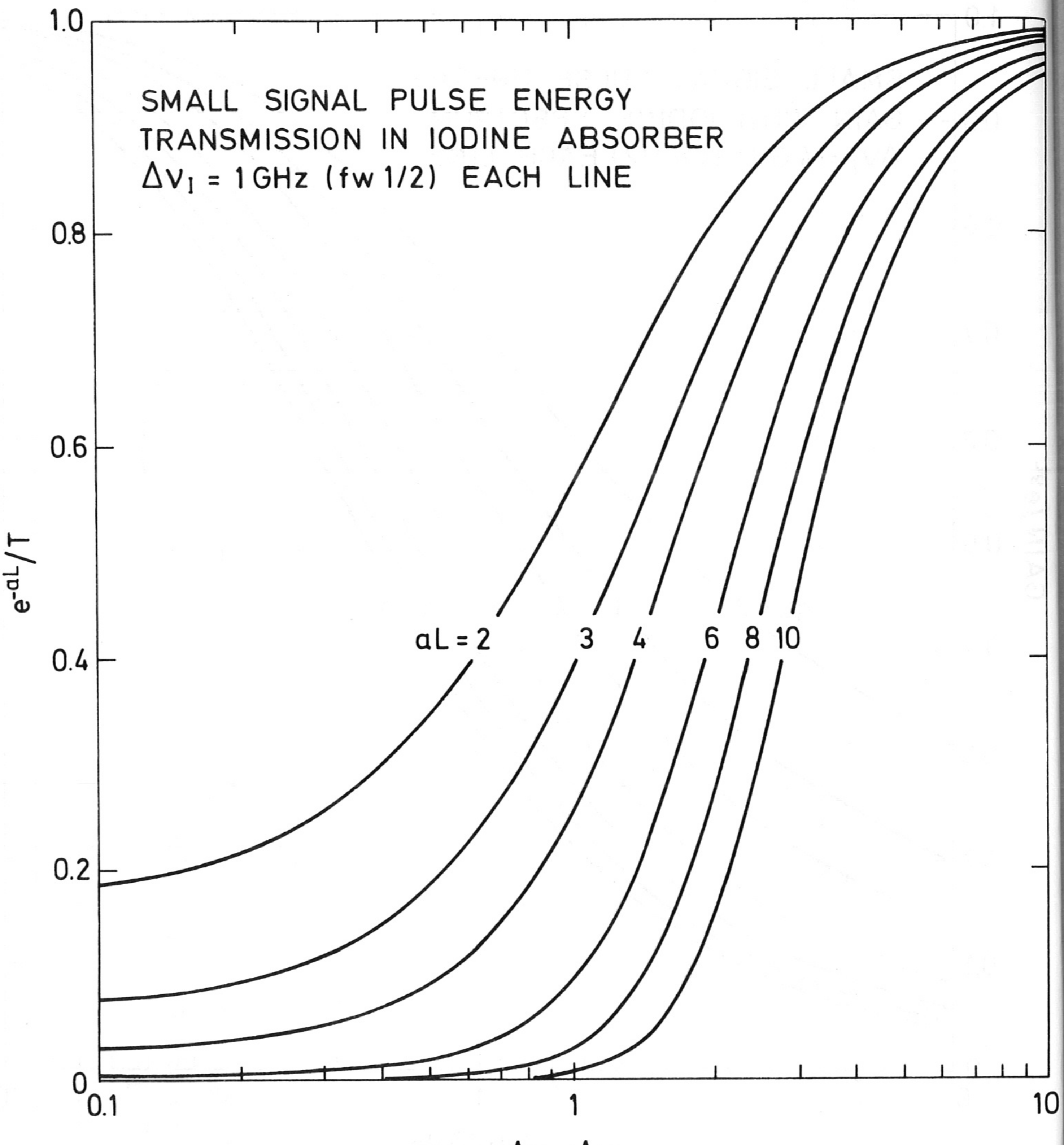

Fig. 10 Small signal pulse energy transmission with the iodine spectrum as a function of the linewidth ratio for various values of the small signal loss.