MEDIUM RESPONSE LIMITATIONS IN SHORT PULSE AMPLIFIERS AND ABSORBERS

G. T. Schappert*)

IPP IV/89 July 1976

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Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

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Abstract

This report reviews some of the basic considerations which determine temporal pulse characteristics in amplifiers and absorbers. To concentrate on analytical results, the amplification or absorption of a step pulse is worked out in detail. It is shown that in the small signal regime the medium response time of T_2 is modified because of propagation effects to $T_2gL/2$ for an amplifier of gain g and length L and $T_2/\alpha L$ for an absorber of loss coefficient α and length L.

Once saturation sets in, general analytical results are no longer possible except in the $T_2 \rightarrow 0$, or rate equation limit. However, combining the finite T_2 small signal results with the rate equation results one can estimate risetime effects.

Coherent effects, such as π pulses and self induced transparency have not yet found applications in high power, high pressure gas lasers.

I N D E X

I. INTRODUCTION

The generation and amplification of short laser pulses play an important role in the whole laser fusion effort. Present generation high intensity short pulse gas laser systems such as CO_2 or iodine lasers produce pulses containing a few hundred Joules in about 1 ns duration. The operating pressures of these devices is typically 1 - 2 atmospheres. This is high enough that response or bandwidth limitations for 1 ns pulses may be neglected. However, with the emphasis to extend these systems into the .1 ns or below regime, bandwidth limitations require some scaling in the pressure, and high pressure operation may complicate the design and impose other limitation on the system. With this in mind, it seems appropriate to look at the physics which limits the response of the medium in short pulse amplifiers and absorbers.

The general pulse propagation analysis is a fairly complex problem and requires involved computer calculations. There are many excellent papers and reviews on this topic /1 - 3/. Our purpose here is to gain some insight into the physics which determines the pulse shape near the leading edge of a pulse and develop some simple criteria which can be used to estimate such rise-time effects in practice. To achieve this, we will study a simple model for which these considerations can be carried out analytically. Unfortunately, only a few special cases for pulse propagation in resonant media are analytically solvable. These include the small signal, or small area pulses /4/, saturation phenomena with pulses of duration long compared to the dephasing time T_2 but short compared to a relaxation or pumping time T_1, where rate equations apply /5,6/, and certain ultrashort pulses ($\tau_p \prec T_2$), as encountered in Self Induced Transparency (S.I.T.) /7/ and other π pulse phenomena /8/.

Since pulses in a high power amplifier chain must first undergo small signal amplification to reach sufficient energy to saturate, the leading edge of a pulse is usually determined by the response of the medium in the small signal regime. At some time later into the pulse, the pulse energy up to that time may be sufficient to saturate the medium and the amplification diminishes. S.I.T. and π pulse effects occur when the electric field of the pulse has enough "Area" ($\mu\int \mathcal{E} \cdot dt$) in a time $t \prec T_2$ to

affect the medium. This condition implies that the pulse energy up to time $t < T_2$ be greater than the saturation energy.

With this in mind, we will concentrate primarily on the small signal response of a simple two level amplifier or absorber to a near resonant pulse. This topic is not new. Most of the results can be found throughout the literature already cited. However, as is often the case, it is difficult to get the physical understanding out of the formalism. It is hoped that this review is of some help in gaining this understanding.

II. BASIC EQUATIONS

As our starting point we use the basic equations for plane wave pulse propagation in a homogeneously broadened two level system. With the usual definition of a slowly varying amplitude and phase

$$\text{Electric field} = \mathcal{E}(z,t)\cos\left[\omega t - kz + \phi(z,t)\right] \tag{2.1a}$$

$$\text{Polarization} = C(z,t)\cos\left[\omega t - kz + \phi(z,t)\right] + S(z,t)\sin\left[\omega t - kz + \phi(z,t)\right] \tag{2.1b}$$

the coupled medium-field equations are

$$\frac{\partial \mathcal{E}}{\partial z} = -\frac{1}{2} k S \tag{2.2a}$$

$$\mathcal{E}\frac{\partial \phi}{\partial z} = -\frac{1}{2} k C \tag{2.2b}$$

$$\frac{\partial S}{\partial \tau} = -\frac{S}{T_2} - \left[(\omega_o - \omega) - \frac{\partial \phi}{\partial \tau}\right] C - \frac{\mu^2}{\hbar}\mathcal{E} N \tag{2.2c}$$

$$\frac{\partial C}{\partial \tau} = -\frac{C}{T_2} + \left[(\omega_o - \omega) - \frac{\partial \phi}{\partial \tau}\right] S \tag{2.2d}$$

and

$$\frac{\partial N}{\partial \tau} = -\frac{N}{T_1} + \frac{\mathcal{E} S}{\hbar} \tag{2.2e}$$

where ω_o, μ, T_2, and T_1 are respectively the resonant frequency, dipole moment, phase and inversion relaxation time of the medium. N is the inversion and τ = t-z/c is the retarded time.

Another form of these equations more convenient for what follows is to introduce the complex quantities

$$E = \mathcal{E} e^{i\phi} \tag{2.3a}$$

and

$$P = -(S + iC) e^{i\phi} \tag{2.3b}$$

in terms of which the set of equations (2.2a) to (2.2e) becomes /1/

$$\frac{\partial E}{\partial z} = \frac{1}{2} k P \tag{2.4a}$$

$$\frac{\partial P}{\partial T} = -\frac{P}{T_2} + i(\omega_o - \omega) P + \frac{\mu^2}{\hbar} E N \tag{2.4b}$$

$$\frac{\partial N}{\partial T} = -\frac{N}{T_1} - \frac{1}{2\hbar}(E P^* + E^* P). \tag{2.4c}$$

The equivalent sets of equations 2.2 or 2.3 are quite general, and describe a variety of effects which can occur in a homogeneously broadened two level system. For real systems, such as CO_2 or Iodine, certain modifications have to be made because we don't have simple two level systems. Essentially, the inversion N has to be replaced with separate equations for the upper and lower level density for each level, resulting in equations for N_j^u and N_j^l. Each transition and level may have its own T_1, ω_{oj} and μ_j. This complicates the analysis, but does not add a significant change in the physics. We will forego these complications to be able to proceed analytically.

III. SMALL SIGNAL ANALYSIS

a) Reduction of Equations

By definition, in the small signal regime a pulse cannot affect the population inversion. Hence, for short pulses ($< T_1$), N is assumed to be constant during the pulse duration. Taking the Fourier transform of the polarization and electric field amplitudes

$$\left.\begin{matrix} E(z,\tau) \\ P(z,\tau) \end{matrix}\right\} = \int_{-\infty}^{+\infty} \left.\begin{matrix} E(z,f) \\ P(z,f) \end{matrix}\right\} e^{-if\tau}\, df \qquad (3.1)$$

and applying this to (2.4a) and (2.4b) we get

$$\frac{\partial E(z,f)}{\partial z} = \frac{1}{2} k\, P(z,f) \qquad (3.2a)$$

and

$$-if\, P(z,f) = -\left[\frac{1}{T_2} - i(\omega_0-\omega)\right] P(z,f) + \frac{\mu^2}{\hbar} N\, E(z,f) \qquad (3.2b)$$

The small signal intensity gain or loss coefficient are defined through

$$g \text{ or } \alpha = \frac{\mu^2}{\hbar} k N T_2 \qquad (3.3)$$

with gain for positive N, loss for negative N. Solving the polarization equation for P(z,t), substituting into the field equation and using the definition of the gain coefficient, one obtains

$$\frac{\partial E(z,f)}{\partial z} = \frac{1}{2} \frac{g\, E(z,f)}{1 - i(\omega_0 - \omega + f)T_2} \qquad (3.4)$$

with its solution

$$E(z,\tau) = \int E(z=0,f)\, e^{-if\tau}\, e^{\frac{1}{2} g z \frac{1}{1-i(\omega-\omega_0+f)T_2}}\, df \qquad (3.5)$$

Equation (3.5) is no surprise. Given the spectrum of the electric field envelope E(z = 0,f) at z = 0 and the complex index of refraction of the medium

$$n(f) = 1 + \frac{1}{2} g \frac{c}{\omega} \frac{1}{1 - i(\omega_0 - \omega + f)T_2} \qquad (3.6)$$

the envelope after propagating through a distance z is given by equation (3.5).

The slowly varying envelope approximation implies that the range of f over which the integrand in Eq. (3.5) contributes is small compared to ω and ω_0, but not necessarily small compared to $\omega - \omega_0$, the amount of resonance. The integral (3.5) is to be performed keeping causality in mind.

At this point we make a general observation. From Eq. (3.5) we note that the intensity $|E(z,\tau)|^2$ depends on the complex index, but from Parseval's theorem /9/, the total energy

$$E_N(z) = \int_{-\infty}^{+\infty} I(z,\tau)\, d\tau \sim \int_{-\infty}^{+\infty} |E(z=0,f)|^2 e^{g z \frac{1}{1+(\omega-\omega_0+f)^2 T_2^2}}\, df \qquad (3.7)$$

depends only on the real part, that is, the gain or loss. This is a general result. The energy does not depend on the phase with which the Fourier components are added, but the intensity does!

b) Small Signal Propagation of a Step Pulse

We will now apply Eq. (3.5) to the propagation of an exponentially decaying step pulse with a field envelope as shown below.

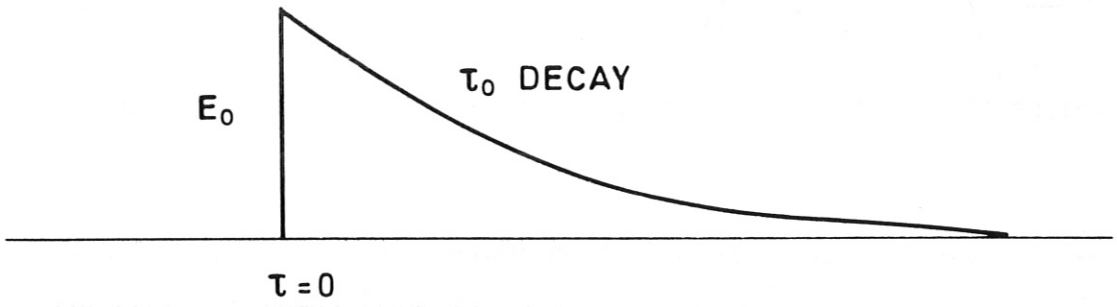


Fig. 3.1 Exponentially decaying step pulse

The analytic expression for the envelope is

$$E(z=0,\tau) = \begin{cases} 0 & \tau < 0 \\ E_0 e^{-\tau/\tau_0} & \tau \geq 0 \end{cases} \qquad (3.8)$$

The choice of this pulse is motivated as follows:

i) Analytic results can be worked out.

ii) There are no causality problems, the field is zero for $\tau < 0$. This is important for risetime questions where a well defined leading pulse edge is desirable.

iii) The zero risetime of this pulse should not be thought to contradict the slowly varying amplitude approximation. We may think that the actual risetime is several optical cycles, which is still quite short compared to T_2 times even in the 10 ps range. Hence, the mathematical convenience of a zero risetime need not contradict the initial assumptions of this model. We will return to this point later on in this section.

iv) In the small signal regime, where because of the linearity of the equations the superposition principle applies, the exponentially decaying step pulse is very suitable for constructing other interesting pulses via superposition.

To calculate the propagation of the pulse (3.8), we first need the Fourier transform of the incident pulse

$$E(z=0, f) = \frac{-E_0}{if - 1/\tau_0} \tag{3.9}$$

and then from Eq. (3.5) get $E(z,\tau)$. The integral to be evaluated has a first order singularity at $f = -i/\tau_0$ from the incident field and an essential singularity at $f = -(\omega_0-\omega) - i/\pi_2'$ from the exponential which gives the propagation. Both singularities are in the lower half of the complex f plane as demanded by causality. From the calculus of residues one obtains after some tedious algebra /4/

$$E(z,\tau) = \begin{cases} 0 & \tau < 0 \\ E_0 e^{i\Delta\tau - \tau/\pi_2'} \sum_{n=0}^{\infty} x^n \begin{cases} I_n(y) & \text{amplifier} \\ J_n(y) & \text{absorber} \end{cases} & \tau \geq 0 \end{cases} \tag{3.10}$$

where $J_n(y)$ and $I_n(y)$ are Bessel functions

and $$x = \sqrt{\frac{2\pi_2'\tau}{(g \text{ or } \alpha)z}} \left(\frac{1}{\pi_2'} - \frac{1}{\tau_0} - i\Delta \right) \tag{3.11}$$

$$y = \sqrt{\frac{2\tau z\,(g \text{ or } \alpha)}{\pi_2'}} \tag{3.12}$$

for an amplifier with small signal intensity gain g or an absorber with loss α . The amount off resonance is given by $\Delta = \omega_0 - \omega$.

As it stands, the analytic solution (3.10) is not very transparent. However, several points can be made.

a) Amplifier:

i) The medium cannot respond to the sharp leading edge. When $\tau = 0$, $E(z, \tau = 0) = E_0$.

ii) Using the sum given in Appendix I.1 one can show that for

$$\tau_0 \gg T_2 , \quad \tau \to \infty$$

$$E(z, \tau \to \infty) = E_0 e^{-\tau/\tau_0} e^{\frac{gz}{2} \frac{1}{1 - i\Delta T_2}} . \tag{3.13}$$

This is what one would expect for the off-resonance electric field amplification when the medium responds with the full small signal gain. For the intensity one has

$$I(z, \tau \to \infty) = I(z=0, \tau \to \infty) e^{\frac{gz}{1+\Delta^2 T_2^2}} . \tag{3.14}$$

iii) An interesting result follows for long step pulses on resonance, $\tau_0 \to \infty$, $\Delta = 0$. For $2\tau/gLT_2 = 1$, x = 1 and the sum for the amplifier in Eq. (3.10) can be carried out (Appendix I.2) with the result

$$E\left(\tau = T_2 \frac{gL}{2}, z = L\right) = E_0 e^{gL/2} \left[\frac{1 + e^{-gL} I_0(gL)}{2}\right] . \tag{3.15}$$

For practical gL values, say gL = 5, one has

$$E\left(\tau = T_2 \frac{gL}{2}, L\right) \approx (.5 - .6) E_0 e^{gL/2} , \tag{3.16}$$

that is, one reaches about .5 of the full electric field gain after $t = T_2\ gL/2$, or only about 1/4 - 1/3 of the full intensity small signal gain.

It is tempting to formulate this result as a rule of thumb for estimating the medium response times in the small signal regime

for the electric field:

$$\tau \text{ (Rise to 1/2 Max)} = T_2 gL/2 \qquad (3.17)$$

The meaning of this rule is that even though the local medium-response-time is T_2, propagation effects lengthen the effective response.

iv) Another simple result obtains for on resonance pulses with $\tau_0 = T_2$. In this case

$$E(\tau, z) = E_0 e^{-\tau/\tau_2} I_0\left(\sqrt{\frac{2gz\tau}{\tau_2}}\right). \qquad (3.18)$$

Such a pulse never reaches its full small signal amplification, it is simply too short!

v) Figure 3.2 shows a plot of the amplification of an on-resonance E field step pulse ($\tau_0 \to \infty$, $\Delta = 0$) as a function of time. The curves are normalized to t/T_2 and $E(z,\tau)/E_0 e^{gL/2}$. Hence, full amplification is unity. Curves for different values of the parameter $G = gL/2$ are plotted. Notice the progressively greater time required to reach .5 - .6 of the full gain as G increases. The crosses represent the field values determined from Eq. (3.15).

b) Absorber:

i) Again the medium cannot respond to the sharp leading edge of the pulse and $E(z, \tau = 0) = E_0$.

ii) Using the sum listed in Appendix I.3 one can show that

$$E(\tau \to \infty, z) = E_0 e^{-\tau/\tau_0} e^{-\frac{\alpha z}{2}} \frac{1}{1 - i\Delta T_2} \qquad (3.19)$$

as expected for off-resonance electric field attenuation when the medium responds with the full small signal loss.

iii) For the long on resonance step pulse one can show that near $\tau = 0$ the dominant term in Eq. (3.10) at z = L is

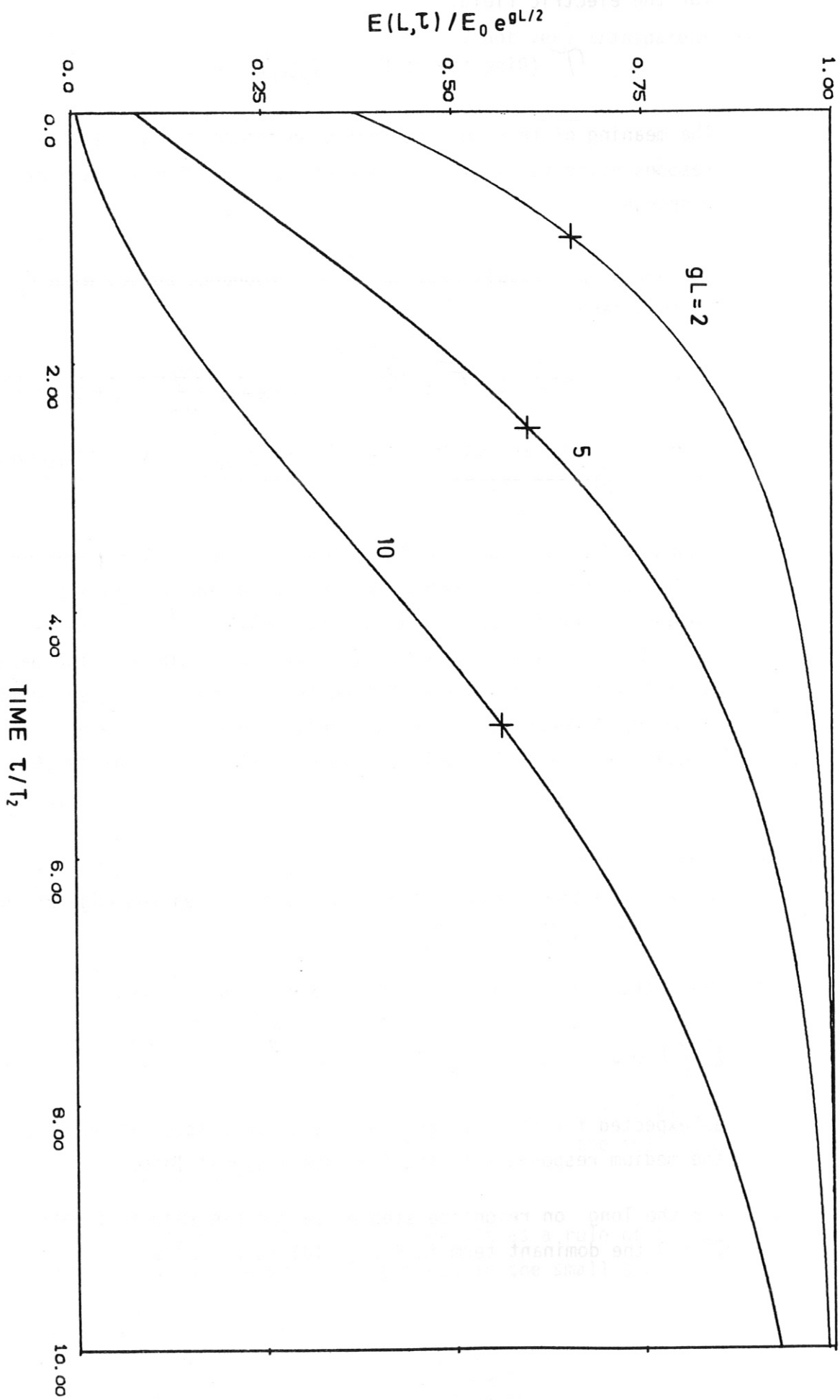


Fig. 3.2 Small signal amplified step pulse

$$E(\tau, L) \approx E_0 \, J_0\left(\sqrt{\frac{2\tau\alpha L}{\tau_2}}\right) \qquad (3.20)$$

which determines the transient decay of the leading edge. The time constant ($J_0(y) \approx .5$) is $\tau \approx T_2/\alpha L$. Again one can give a rule of thumb for the response time for the electric field:

$$\tau \text{ (absorber response)} \approx T_2/\alpha L \qquad (3.21)$$

iv) Figure 3.3 shows a plot of the absorption of an on-resonance E-field step pulse as a function of time. The curves are normalized to τ/T_2 and $E(z,\tau)/E_0$. Hence, all the curves start from unity at the leading edge of the pulse. Curves for different values of the parameter $A = \alpha L/2$ are plotted. Note the transient pulse width obtained is roughly consistent with the $T_2/\alpha L$ rule. However, the electric field amplitudes can now go negative, and the approach to the steady state absorption is not monotonic, but contains ringing.

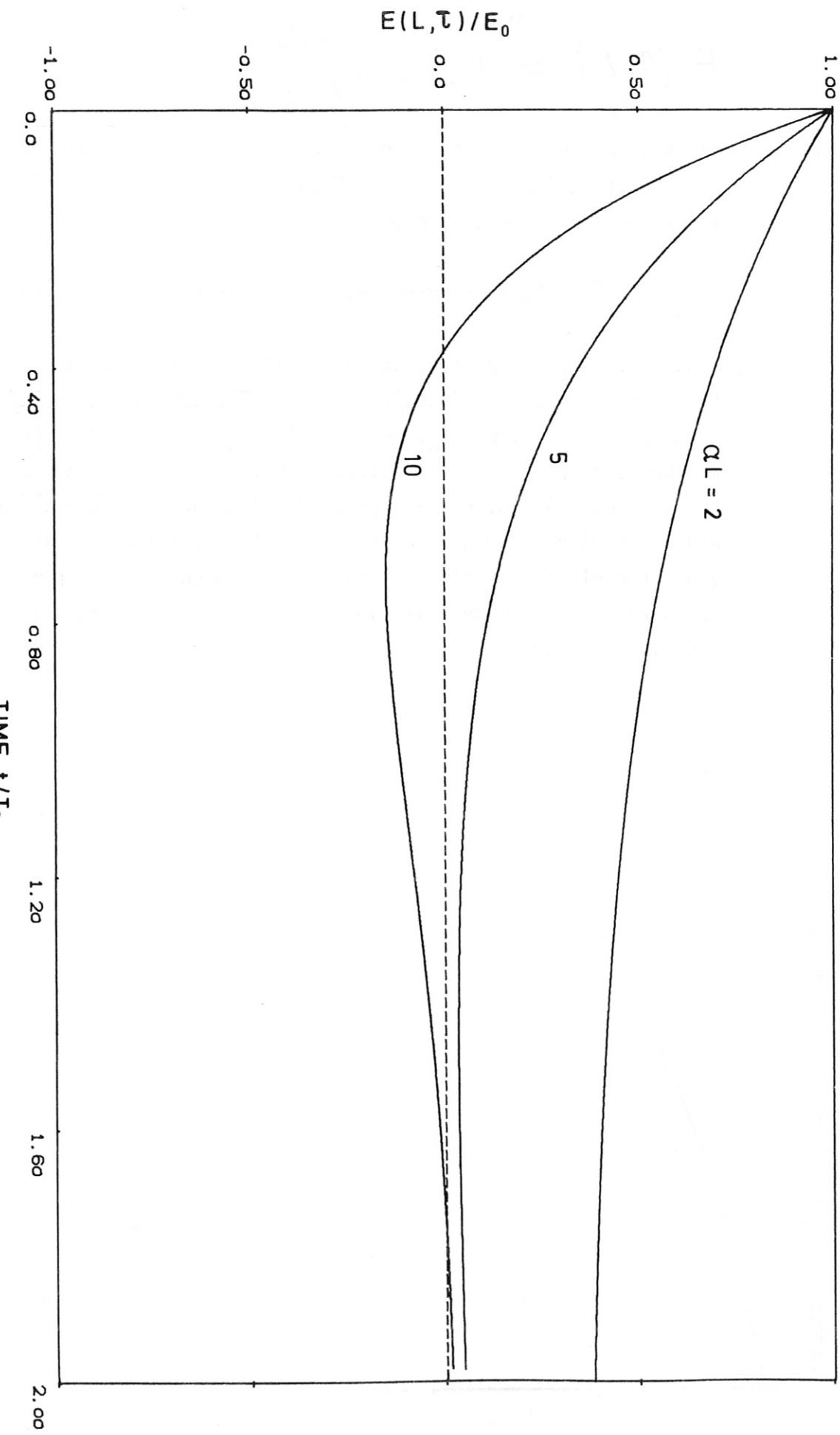


Fig. 3.3 Step pulse after small signal absorber

c) Discussion

The analytical results obtained above can be understood as follows: in the amplifier, energy is put into the back of the pulse, in the absorber it is taken out of the back of the pulse. To understand physically why the effective response times are T_2 times gL or divided by αL, we consider the following, first for the amplifier.

The medium response time is T_2, or its bandwidth is $\Delta\nu \sim 1/T_2$. However, this Lorentzian frequency response appears in the exponential because of the pulse propagation. This exponential may be written (Eq.(3.5)) on resonance in the form

$$e^{\frac{1}{2}gz\left\{\frac{1}{1+T_2^2 f^2} + \frac{iT_2 f}{1+T_2^2 f^2}\right\}} \qquad (3.22)$$

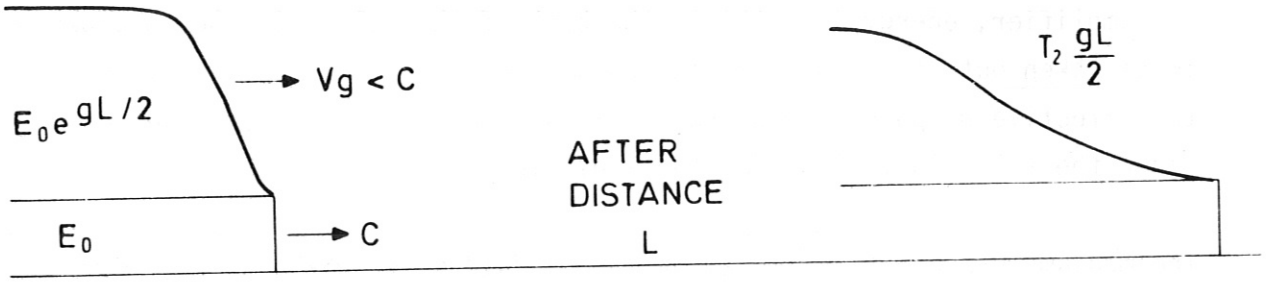
GAIN — ANOMALOUS INDEX OF REFRACTION

which explicitly shows the real gain and anomalous index of refraction. The exponential gain has a narrower bandwidth than the Lorentzian alone (gain narrowing), which leads to a longer effective response time. The anomalous index leads to a group velocity $V_g = C/(1 + T_2 cg/2)$ /10/. Since the leading edge of the pulse propagates with the speed of light (high Fourier components) but the main body of the pulse with the group velocity, a further stretching of the rise time results.

Precisely the opposite is true for the absorber. The effective bandwidth is broadened and the group velocity $V_g > C$. This has the effect of producing a short pulse behind the leading edge. The two cases are depicted on Figure 3.4, a and b.

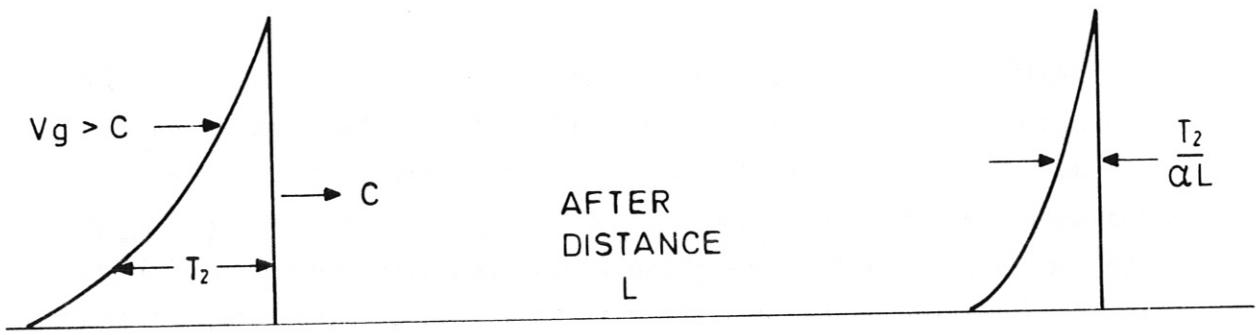
In connection with the preceeding physical argument one might ask whether the effective narrowing of the gain (broadening of the loss) or the group velocity has the major effect in shaping the pulse. To decide this, let us write the complex index in the form

GAIN NARROWING



a) SMALL SIGNAL AMPLIFIER

LOSS BROADENED



b) SMALL SIGNAL ABSORBER

Fig. 3.4 Gain narrowing or loss broadening and group velocity effects on pulse shapes in amplifiers and absrobers

$$\left\{ \frac{1/2}{1 - i f T_2} + \frac{1/2}{1 + i f T_2} \right\} \frac{aL}{2} +$$

$$\left\{ \frac{1/2}{1 - i f T_2} - \frac{1/2}{1 + i f T_2} \right\} \frac{bL}{2}$$

If a = b, we have the correct complex index, but a and b separately determine the real part (gain or loss) and the imaginary part (anomalous index). The condition $a \neq b$ is not very meaningful, since it does not obey causality. Nevertheless, the above breakup of the index shows that in the limit a = b, our calculation would give results involving the combination (a + b)/2. This implies that the real and the imaginary part of the index contribute equally in distorting the pulse.

For both the amplifier and the absorber we have found that the infinitely fast risetime of the step pulse is preserved. However, we know that within the envelope approximation this risetime must be at least several optical cycles, and the question arises as to what happens to a still fast, but not a step rise. In this case one can show quite generally that if the leading edge of the incident pulse rises faster than $T_2/\alpha L$ or T_2/gL for a small signal absorber or amplifier respectively, it propagates unchanged. The proof is given in Appendix II.

IV. APPLICATIONS

The basic equations (2.2) or (2.4) are linear in the small signal regime where $N = N_0$ = constant. This permits the use of the superposition principle to get new solutions for pulses which are superpositions of our basic decaying step. We look at two examples:

i) To obtain a pulse with an exponential rise and decay we take

$$E(t) = \begin{cases} 0 & t < 0 \\ E_0 \left(1 - e^{-t/t_R}\right) e^{-t/t_F} & t \geq 0 \end{cases} \qquad (4.1)$$

For $t_R < t_F$, t_R is the rise time, E_0 approximately the peak amplitude, and t_F the decay time. In terms of our step with exponential tail, the above pulse is the difference between two such steps, one with decay constant $1/t_F$, the other with $1/t_F + 1/t_R$. The resulting output is easily expressed in terms of solutions of type (3.5).

ii) Another interesting example is to send a long pulse, which after some time is truncated to zero, through an absorber. This can be considered as the superposition of a step pulse, having started some time ago, and a negative step, starting at time t = 0. Near t = 0, the output of the absorber then consists of the first pulse, having reached its equilibrium absorption $\exp(-\alpha L/2)$, and a negative step, near its leading edge. The expression for the output electric field can be written as

$$E_{TOT}(T, L=Z) = E_0 e^{-\alpha L/2} - E(T, Z) \qquad (4.2)$$

where $E(z,T)$ is the absorber solution from Eq. (3.10). For large αL, this is a spike as shown below on Fig. 4.1.

This particular scheme was first used by Yablonovitch /11/ to create ultrashort CO_2 laser pulses. The fast truncation of a long pulse is achieved through a gas breakdown shutter.

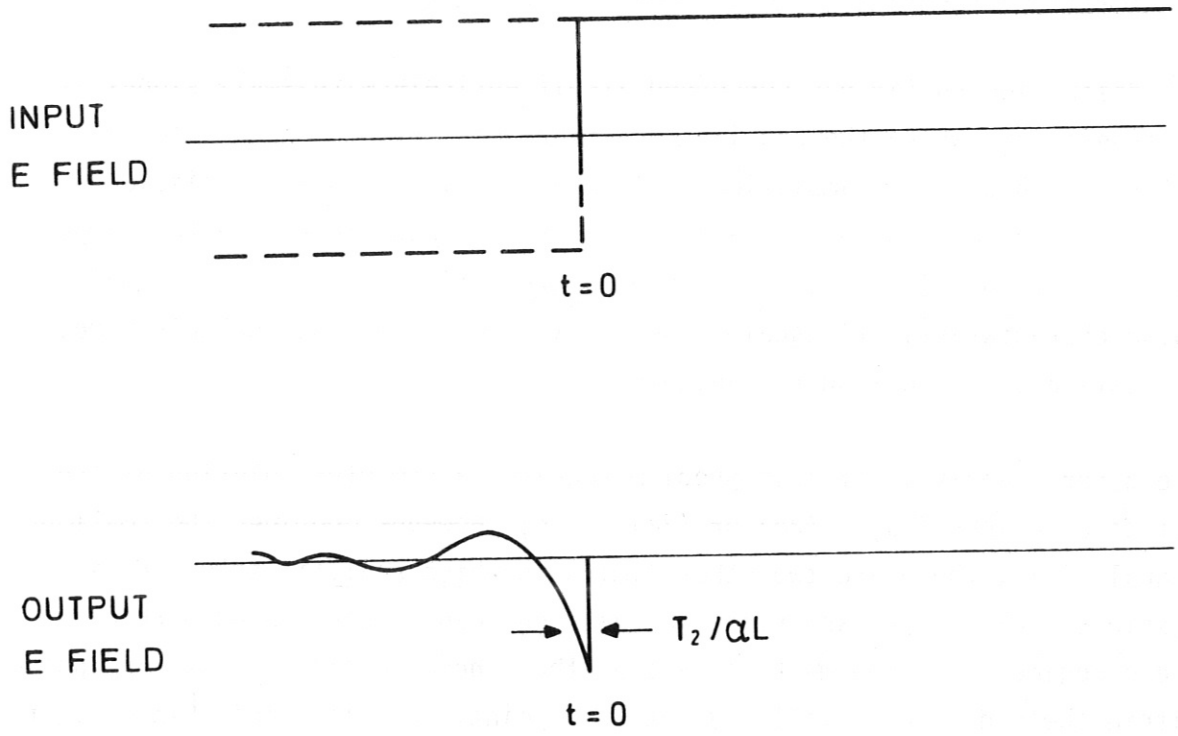


Fig. 4.1 Free induction decay pulse as a superposition of two step pulses

The above effect with a truncated pulse is often explained as a free induction decay. From this point of view, the absorber is said to radiate a field out of phase with the input to destructively interfere with the input. When the input is abruptly terminated, the absorber polarization cannot decay instantaneously, but takes a time T_2. Propagation effects then shorten this free induction decay to a spike of width $T_2/\alpha L$.

V. LIMITATIONS: SATURATION AND LARGE AREA EFFECTS

The discussion so far has concerned itself entirely with small signal considerations where the population difference stays constant. Sooner or later in a high power amplifier this condition is no longer satisfied and even the leading edge of a pulse may have enough intensity to change the populations. Clearly, if T_2 gL or $T_2/\alpha L$ estimate some small signal pulse characteristic timescale, then if α or g changes during this time, the estimate is no longer meaningful.

The general analysis of this problem requires a computer solution of the set of equations (2a) - (2e) or (3a) - (3c). However, besides the small signal limit, there are two other limits in which analytic solutions are possible. Pulse shape and risetime questions can be considered within these regimes, but one must be careful that these considerations are applied within their domain of validity. At this point, we will briefly investigate the two cases.

a) Rate Equations

If one assumes that any change in E, P, and N is slow compared to T_2, the polarization equation (3b) implies that P is always instantaneously in equilibrium with the product EN, and (3b) is easily solved for P. Substituting this P into (3a) and (3b) results in the rate equation approximation, which can be formulated so that only $|E|^2$ and N are left. If one further assumes that T_1 is large compared to the pulse duration of interest ($T_1 \to \infty$), the two coupled equations for $|E|^2$ and N can be solved analytically /5/. We quote only the result

$$I(\tau, z) = I_0(\tau) \Big/ \left(1 - \left(1 - e^{-az}\right) e^{-\int_0^{\tau} \frac{I_0(t)\,dt}{E_s}}\right) \qquad (5.1)$$

where $I_0(\tau)$ is the intensity of the incident pulse, a is the gain g or loss $-\alpha$ of the amplifier or absorber espectively. E_s is the saturation energy

$$E_s = \frac{\hbar\omega}{2\sigma(\omega)} \qquad \text{Joules/cm}^2 \qquad (5.2)$$

where the stimulated emission cross section is

$$\sigma(\omega) = \frac{k\mu^2 T_2}{\hbar} \frac{1}{1+T_2^2(\omega-\omega_0)^2} \cdot \tag{5.3}$$

Equation (5.1) may be integrated over all time to relate the input and output energy densities E_{in} and E_{out}

$$E_{out} = E_s \ln\left[1 + e^{az}\left(e^{E_{IN}/E_s} - 1\right)\right] \tag{5.4}$$

$$a = g \text{ or } -\alpha .$$

which is the familiar and much used equation in the short pulse laser business.

Equation (5.1) may be used to predict the pulse shape. It has the property that the leading edge of the pulse near $\tau = 0$ propagates with the full small signal gain or loss. Later on in the pulse saturation sets in and the gain or loss is reduced. This has the effect of distorting the pulse shape. In an amplifier saturation tends to push energy towards the front of the pulse, in an absorber towards the back. After propagating through a certain length of saturated medium, significant changes in the pulse shape during times comparable to T_2 result, and the model no longer applies. The full interplay between T_2 response times effects and saturation must then be considered. This situation can be understood from the following example. Consider again a step pulse into an amplifier. The expected output is shown on Fig. 5.1.

The curve marked rate equations is a plot of Eq. (5.1) with gL = 5 and since we are measuring time in units of T_2 the incident intensity was chosen such that $I_o T_2 = .1 E_s$ for arbitrary values of these parameters. The incident step pulse is too small to be seen on this graph. We notice the full small signal gain at the leading edge and the subsequent gain saturation characteristic of the rate equation solution.

The curve marked small signal response is taken from Fig. 3.2 gL = 5, but the square of the ordinate is taken, since we are comparing intensities

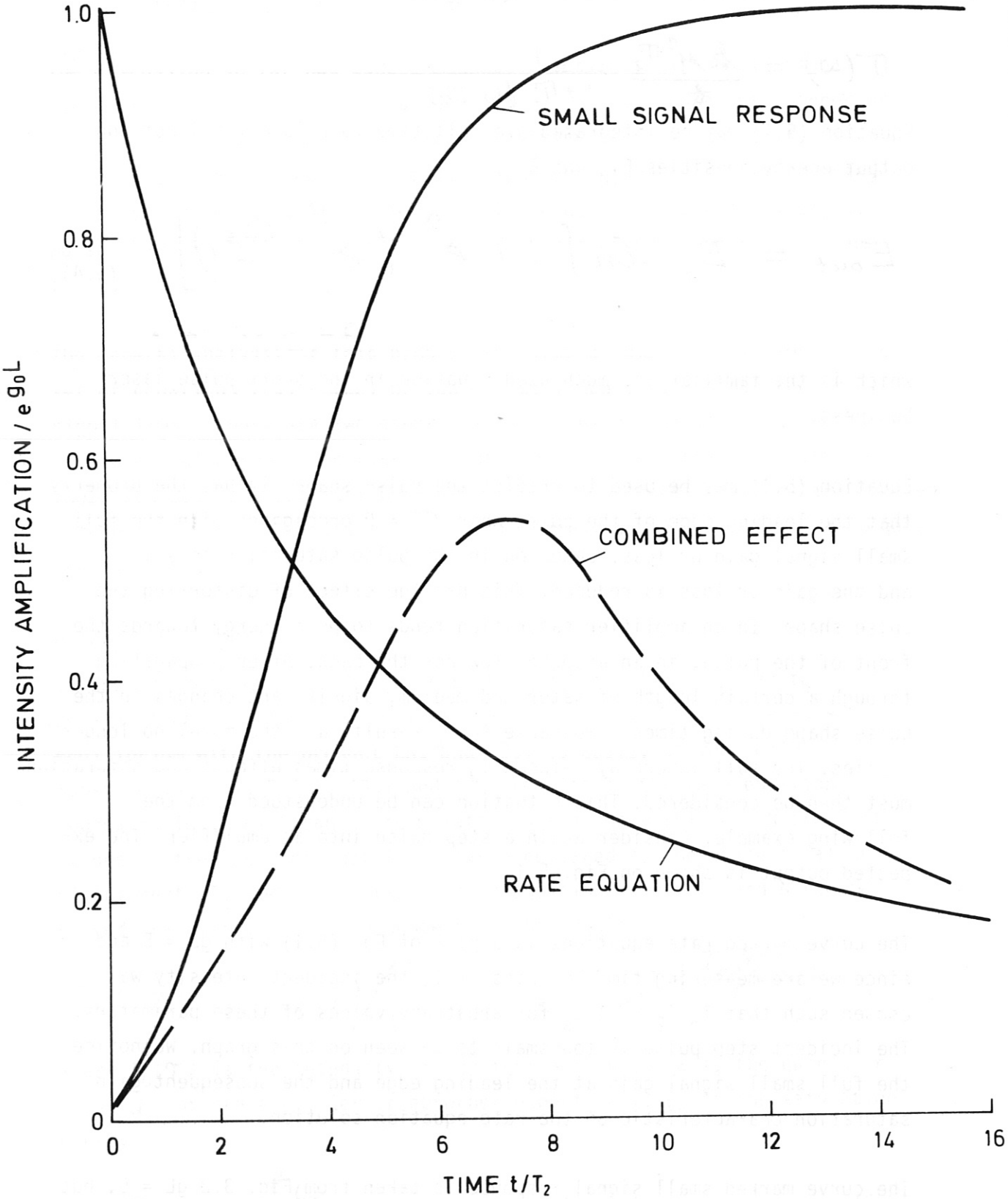


Fig. 5.1 Combined effects of small signal response and rate equation saturation in step pulse amplification

rather than E fields. Note the approximate 1/2 E field point at $t/T_2 = gL/2$ becomes the approximate .25 full intensity point on this figure.

The dashed curve is what one might expect from the combined response time and saturation effect. The curve is not calculated, but simply a sketch of a pulse which initially tries to follow the small signal curve, but gradually falls behind because of saturation. It reaches a peak, but then must approach the rate equation result for large t/T_2.

b) Coherent Effects

A special situation arises when the time integral of the electric field multiplied by the dipole moment

$$\psi(z,t) = \frac{\mu}{\hbar}\int_0^t \mathcal{E}(z,t')\,dt' \tag{5.5}$$

becomes appreciable for times $t < T_2$. This can arise for ultra short pulses ($\tau_p < T_2$) or in the leading edge of a pulse with a steep rise time. As before, we must of course have pulses or rise times larger than several optical cycles to satisfy the envelope approximation.

To understand this condition, consider Eqs. (4a) - (4c) in the limit when both T_1 and $T_2 \to \infty$. In this case, the pulse spectrum is much broader than the medium response spectrum so that without loss of generality we can set $\omega = \omega_0$ and hence chose P and E real. Equations (4b) and (4c) then read

$$\frac{\partial P}{\partial \tau} = \frac{\mu^2}{\hbar} EN \tag{5.6a}$$

and

$$\frac{\partial N}{\partial \tau} = -\frac{EP}{\hbar} \tag{5.6b}$$

with the solution

$$N = N_0 \cos \psi(\tau, z) \tag{5.7a}$$

$$P = \mathcal{M} N_0 \sin \psi(\tau, z) \tag{5.7b}$$

where $\psi(\tau, z)$ is the above defined quantity.

The total "pulse area" is defined as

$$\Psi(z) = \frac{\mathcal{M}}{\hbar} \int_{\text{pulse duration}} \mathcal{E}(z, t')\, dt' \tag{5.8}$$

and when this quantity is 2π, equation (5.7a) tells us that the population was left unchanged. This is self induced transparency (S.I.T.).

For our purposes here we wish to point out an interesting condition. The small signal limit here implies from Eq. (5.7a) that $\psi \ll 1$. For an incident step pulse we may write $\psi(t) \sim \mathcal{E}_0 t$ where t is the time measured from the leading edge into the pulse. Hence, the small signal condition becomes

$$\left(\frac{\psi}{2\pi}\right)^2 = \frac{E_N(t)}{E_S} \frac{t}{\pi T_2} \ll 1 \tag{5.9}$$

where we have used Eqs. (5.2), (5.3), and the fact that the energy/cm² in a step up to time t is given by

$$E_N(t) = \frac{c\, \mathcal{E}_0^2\, t}{8\pi} . \tag{5.10}$$

Since $t < T_2$ in this regime, pulses with $E_N(t)/E_S > 1$ may still propagate in the small signal region.

Together with the equation for the field

$$\frac{\partial E}{\partial z} = \frac{1}{2} k P \tag{5.11}$$

one then has the starting point for various interesting topics such as self induced transparency /7/ and other π pulse problems.

So far these effects have received little attention in the high power gas laser business. One reason is that pulse lengths below T_2 of a high pressure device are hard to come by. Second, and more important, is that once S.I.T. and π pulse coherent effects set in, the field intensity is high enough to completely dominate the medium with little effect of the medium on the pulse energy. This is not exactly a good power amplifier. Another problem comes from the fact that CO_2 and iodine laser transitions are degenerate (angular momentum 2J + 1) which greatly complicates matters. S.I.T. and π pulse effects tend to be washed out in degenerate media. Nevertheless, this area of research on pulse propagation effects in large power amplifiers may require more attention in the future.

APPENDIX I

1) Hansen /12/; formula 57.6.2 pg. 385

$$\sum_{K=0}^{\infty} c^K J_{K+a}(z) = z^{-a} e^{cz/2} \int_0^z t^a e^{-ct^2/2z} J_{a-1}(t)\,dt$$

2) Same; formula 58.5.2. pg. 414; similar formula for I(z)

3) Same; formula 58.1.2 pg. 411

$$\sum_{K=1}^{\infty} I_n(x) = \frac{1}{2}\left(e^x - I_0(x)\right)$$

4) Same; formula 57.1.2 pg. 377

$$\sum_{K=1}^{\infty} J_K(z) = \frac{1}{2}\left[1 - J_0(z) + \int_0^z J_0(t)\,dt\right]$$

APPENDIX II

The statement made at the end of Section III that pulse risetimes faster than $T_2/\alpha L$ or T_2/gL are preserved during propagation can be shown as follows.

If the Fourier transform of a function q(t) is F(f), then there is a relationship between q(t) $t \rightarrow 0$ and F(f) $f \rightarrow \infty$ /9/. From Eq. (3.5), the propagation exponential may be expanded for large f

$$e^{\frac{1}{2} g L \frac{1}{1 - i(\omega - \omega_0 + f)T_2}} = 1 + \frac{gL}{2T_2}\left(\frac{i}{f}\right) \text{-----}$$

and in the time domain, this factor then contributes

$$1 + \frac{1}{2} gL \frac{t}{T_2} \text{ ----}$$

to the original pulse shape, which has no effect for $t < T_2/gL$.

ACKNOWLEDGEMENT

The author wishes to thank the Max-Planck-Gesellschaft, Projektgruppe für Laserforschung, particularly Drs. Witkowski and Brederlow for the hospitality extended to him during his visit. He also wishes to acknowledge the many fruitful discussions with his colleagues in the Iodine Laser Group and thank Fr. Brändlein for the computer plots.

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