

Pressure Balance between a Magnetized
Plasma and a Low Pressure Neutral Gas

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Contents

- 1.) Introduction
- 2.) Assumptions and basic equations
- 3.) Relations between plasma pressure and neutral gas pressure
- 4.) The effects of multiple ionization and anomalous resistivity
- 5.) Some numerical examples
- 6.) Validity of the continuum approach
- 7.) Conclusions

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Abstract

The steady state pressure balance between a plasma and a low pressure neutral gas is calculated by the continuum approach in order to predict the critical density of a neutral gas above which the plasma is drastically affected. The diffusion in the plasma-neutral gas boundary induces a large diamagnetic current which also induces the large pressure difference between a plasma and a neutral gas. This means that a plasma may be affected even by a low density neutral gas.

Contents

- 1.) Introduction
- 2.) Assumptions and basic equations
- 3.) Relations between plasma pressure and neutral gas pressure
- 4.) The effects of multiple ionization and anomalous resistivity
- 5.) Some numerical examples
- 6.) Validity of the continuum approach
- 7.) Conclusions

1) Introduction

The neutral particle penetration into a magnetized plasma is known to have important effects on the spatial distribution of the plasma density and plasma temperature /1/,/2/. When a plasma is sufficiently dense and the plasma radius is sufficiently large, the plasma is impermeable to neutral gas and a "neutral gas blanket" forms around it. From such a layer electrons and ions as well as high energy neutral particles hit the wall, which results in wall recombination and sputtering, followed by an influx of neutral gas and of impurities. These neutral gases are ionized in the boundary and balance the charged particle losses.

Particularly important is the effect of neutral particles on a full scale Tokamak in which the neutral gas accumulates around the plasma. Recently, Lehnert/3/ pointed out that, in recent Tokamak experiments whose plasma density is in the transition region between permeable and impermeable plasma, the collisionless ballooning mode produced by the steepened pressure distribution leads to an instability which may have some relevance for disruption.

In order to have an insight into the effect of the pressure of this neutral gas blanket on the plasma pressure - even when the plasma pressure is compensated by a magnetic field gradient, the steady-state solution of the plasma-neutral gas boundary is solved for a simple case, and then we try to find out at what neutral gas pressure the plasma begins to be affected.

2) Assumptions and basic equations

The following assumptions have been made in the present calculation:

A fully ionized plasma is surrounded by a low pressure neutral gas which contacts a wall located at $x = 0$. At $x \rightarrow \infty$, the uniform fully ionized plasma is sustained. A magnetic field is applied in the z direction. All quantities are independent of the y and z directions. The temperatures of electrons, ions and neutral particles are assumed to be constant but unequal. These constant temperatures are only an approximation of the present problem and represent the temperatures which occur somewhere in the boundary. The magnetic pressure is sufficiently strong with the neutral gas pressure. The electric current along the magnetic field in the boundary is neglected. This assumption may be permitted even in the confined plasma if $|j_z B_y|$ in the boundary layer can be neglected relative to $|j_y B_z|$. The macroscopic fluid equations are applied to each species. This assumption will be discussed in the section 6 of the paper. First, ions are assumed to be singly ionized and there is no anomalous resistivity. But these simple assumptions will be extended in the following section.

The system of multi-fluid equations for electrons, ions and neutral particles is as follows:

for mass conservation:

$$\frac{d}{dx}(N\bar{T}) = -\frac{d}{dx}(N_a \bar{T}_a) = \alpha N_a N' \quad , \quad (1)$$

for momentum conservation in the x direction:

$$-kT_e \frac{dN}{dx} + m_e \gamma_{ea} N N_a (\bar{T}_a - \bar{T}) - eN(E_x + \bar{V}_e B) = 0 \quad , \quad (2)$$

$$-kT_i \frac{dN}{dx} + m_i \gamma_{ia} N N_a (\bar{T}_a - \bar{T}) + eN(E_x + \bar{V}_i B) = 0 \quad , \quad (3)$$

$$-kT_a \frac{dN_a}{dx} + m_e \gamma_{ea} N N_a (\bar{T} - \bar{T}_a) + m_i \gamma_{ia} N N_a (\bar{T} - \bar{T}_a) = 0 \quad , \quad (4)$$

and for momentum conservation in the y direction:

$$m_e \gamma_{ea} N N_a (\bar{T}_a - \bar{T}_e) + m_e \gamma_{ei} N^2 (\bar{T}_i - \bar{T}_e) + eN\bar{T}B = 0 \quad , \quad (5)$$

$$m_i \gamma_{ia} N N_a (\bar{T}_a - \bar{T}_i) + m_e \gamma_{ei} N^2 (\bar{T}_e - \bar{T}_i) - eN\bar{T}B = 0 \quad , \quad (6)$$

$$m_e \gamma_{ea} N N_a (\bar{T}_e - \bar{T}_a) + m_i \gamma_{ia} N N_a (\bar{T}_i - \bar{T}_a) = 0 \quad , \quad (7)$$

with Ampere's law

$$\frac{dB}{dx} = 4\pi e N (\bar{v}_e - \bar{v}_i) \quad (8)$$

where $N, T, \bar{U}, \bar{V}, \bar{E}, \bar{B}, \alpha, e$ and m are the number density of electrons or ions, the temperatures, the velocity in the x direction, the velocity in the y direction, the electric field, the magnetic field, the ionization coefficient, the electric charge of a proton and the mass, respectively.

γ_{ij} represents the coefficients of mutual momentum exchange between i and j species. The subscripts e, i and a represent electrons, ions and neutral particles, respectively.

We introduce the following normalized quantities:

$$\eta = x / \{ a (\nu_m \nu_I)^{-\frac{1}{2}} \Omega_{ei}^{\frac{1}{2}} \}, \quad n = N / \{ N_{a0} \tau^{-1} \}, \quad n_a = N_a / N_{a0},$$

$$j = J / \{ N_{a0} \tau^{-1} a (\nu_I / \nu_m)^{\frac{1}{2}} \Omega_{ei}^{-\frac{1}{2}} \} \quad \text{and} \quad b = B / B_0, \quad (9)$$

where J is the flux of neutral particles, $\tau = (T_e + T_i) / T_a$,

$a = \{ k(T_e + T_i) / (m_e + m_i) \}^{\frac{1}{2}}, \nu_I = \alpha N_{a0}, \nu_m = (\delta_{ea} m_e + \delta_{ia} m_i) N_{a0} / (m_e + m_i),$

Ω_{ei} is the product of the Hall coefficients of the electrons and ions at $x = 0$ ($= \omega_e \tau_e \omega_i \tau_i$) and the subscript o represents the quantities at $x = 0$. Then the normalized basic equations for the present problem are derived from equations (1) to (8) as

$$\frac{dn}{d\eta} = \frac{J}{\Omega_{ei}} \left\{ 1 + (\tau^{-1} - 1)n + \beta_a^{-1}(1 - b^2) + \frac{\Omega_{ei} b^2}{1 + (\delta \tau^{-1} - 1)n + \beta_a^{-1}(1 - b^2)} \right\}, \quad (10)$$

$$\frac{dj}{d\eta} = - \{ 1 - n + \beta_a^{-1}(1 - b^2) \} n, \quad (11)$$

$$\frac{db^2}{d\eta} = - \frac{\beta_a b^2 j}{1 + (\delta \tau^{-1} - 1)n + \beta_a^{-1}(1 - b^2)}, \quad (12)$$

where β_a is defined by $N_{a0} k T_a / (B_0^2 / 8\pi)$ and $\delta = \Omega_{ei} / \Omega_{ea}$ (Ω_{ei} and Ω_{ea} are the elastic collision cross sections of electrons with ions and neutral particles, respectively). Summation of equations (2) - (4) using equation (8) and the relation $n = 0$ at $\eta = 0$ yields the pressure balance relation,

$$n_a + n + \beta_a^{-1} b^2 = 1 + \beta_a^{-1}. \quad (13)$$

In the present analysis, we are interested in the case where $\beta_a \ll 1$ (the neutral gas pressure is much lower than the magnetic pressure). When $\beta_a \ll 1$, $\tau \gg 1$, $\delta\tau \ll 1$, equations (10) - (12) are approximated by the following equations:

$$\frac{dn_a}{d\eta} = - \frac{j}{2e_i} (\tau^{-1}w + n_a), \quad (14)$$

$$\frac{dw}{d\eta} = \frac{j}{\delta\tau^{-1}w + n_a}, \quad (15)$$

$$\frac{dj}{d\eta} = - n_a (w - n_a), \quad (16)$$

where $w = n + n_a$. In the present problem, the following boundary conditions are imposed on the above equations:

$$n_a = 1 \text{ and } w (= n + n_a) = 1 \text{ at } \eta = 0$$

and

$$n_a = 0 \text{ at } \eta \rightarrow \infty. \quad (17)$$

When we divide equation (15) by equation (14), we have the following equation for Z and N_a :

$$\frac{dw}{dn_a} = - \frac{\Omega e_i}{(\delta\tau^{-1}w + n_a)(\tau^{-1}w + n_a)}. \quad (18)$$

3) Relation between pressure and neutral gas pressure

w at $\eta \rightarrow \infty$ (and hence n at $\eta \rightarrow \infty$) can be calculated by integrating equation (18) with the boundary condition $w = 1$ at $n_a = 1$. The following matching procedure is useful to obtain the approximate solution of equation (18); n_a decreases from unity as η increases and finally vanishes at $\eta \rightarrow \infty$. When n_a is close to unity (then $n_a > \tau^{-1}w$ and $n_a > \delta\tau^{-1}w$ are satisfied) equation (18) is approximated by

$$\frac{dw}{dn_a} = - \frac{\Omega_{ei}}{n_a^2} \quad (19)$$

On the other hand, when n_a becomes much smaller than unity, equation (18) is approximated by

$$\frac{dw}{dn_a} = - \frac{\Omega_{ei} \tau^2 \gamma^{-1}}{w^2} \quad (20)$$

These equations, (19) and (20), with the aid of boundary condition (19) are integrated to give

$$n_a = \frac{\Omega_{ei}}{w + \Omega_{ei} - 1} \quad (21)$$

and

$$n_a = \text{const} - \frac{\gamma}{3\Omega_{ei} \tau^2} w^3, \quad (22)$$

respectively. If we match these equations at $w = w^+$ (the values of w^+ and const are adjusted so that n_a and $\frac{dn_a}{dw}$ of both equations (21) and (22) become equal at $w = w^+$), we find that

when $w^+ = \frac{\tau}{\sqrt{\gamma}}$ and $\text{const} = 1 - \frac{\tau}{3\Omega_{ei} \gamma^{1/2}}$ (23)

(this relation is fulfilled for the usual low β_a condition). The relation (23) yields w at $\eta \rightarrow \infty$ as

$$w_\infty (= w^+ \text{ at } \eta \rightarrow \infty) = n_\infty = \left(\frac{3\Omega_{ei} \tau^2}{\gamma} \right)^{1/3} \quad (24)$$

This solution which was obtained by the matching procedure can be shown to be a good approximation to the exact solution, at least when γ is close to unity, by the following procedure; when $\gamma = 1$, equation (18) is easily integrated to give

$$n_a = -\tau^{-1} w - \sqrt{\frac{\Omega_{ei}}{\tau}} + 2\sqrt{\frac{\Omega_{ei}}{\tau}} \left[1 - \exp\left\{ -\frac{2}{\sqrt{\tau\Omega_{ei}}} (w-1) \right\} \frac{1 - \sqrt{\frac{\Omega_{ei}}{\tau}}}{1 + \sqrt{\frac{\Omega_{ei}}{\tau}}} \right]^{-1} \quad (25)$$

which yields for $\eta \rightarrow \infty$

$$\left(\frac{1 + \frac{w_\infty}{\sqrt{\tau\Omega_{ei}}}}{1 - \frac{w_\infty}{\sqrt{\tau\Omega_{ei}}}} \right)^{-1} \exp\left\{ \frac{2}{\sqrt{\tau\Omega_{ei}}} (w_\infty - 1) \right\} = \frac{1 - \sqrt{\frac{\tau}{\Omega_{ei}}}}{1 + \sqrt{\frac{\tau}{\Omega_{ei}}}} \quad (26)$$

Expanding the left hand side of equation (26) in $w_\infty / \sqrt{\tau\Omega_{ei}}$ to the third order, we find that

$$w_\infty = (3\Omega_{ei} \tau^2)^{1/3}, \quad (27)$$

which coincides exactly with the matched solution (24) for $\gamma = 1$.

The dimensional form of N_{∞} is obtained from the relation (9):

$$N_{\infty} = \left(3 \frac{\Omega_{ei}}{\delta \tau}\right)^{1/3} N_{a0}$$

or

$$k(T_e + T_i) N_{\infty} = \left(\frac{3 \Omega_{ei} \tau^2}{\delta}\right)^{1/3} k T_a N_{a0}, \quad (29)$$

which means that the plasma pressure is $\left(\frac{3 \Omega_{ei} \tau^2}{\delta}\right)^{1/3}$ times larger than the neutral gas pressure. Because Ω_{ei} is proportional to N_{a0}^{-2} , the relation between N_{∞} and N_{a0} is written as

$$\frac{N_{a0}}{N_{\infty}} = \left(\frac{N_{\infty}}{N^*}\right)^2, \quad (30)$$

where $N^* = \left(3 \frac{\Omega_{ei}}{\delta \tau} N_{a0}^2\right)^{1/2} = \left\{3 \delta^{-1} \tau^{-1} \frac{\pi c}{8} \frac{e^2 B^2}{(m_e m_i)^{1/2}} \frac{1}{k T} \frac{1}{\alpha_{ea} \alpha_{ia}}\right\}^{1/2}. \quad (31)$

The assumption $\Omega_{ei}/\delta \tau \gg 1$ means that $N^* \gg N_{a0}$ because

$$N^*/N_{a0} = \left(3 \frac{\Omega_{ei}}{\delta \tau}\right)^{1/2}.$$

4) The effects of multiple ionization an anomalous resistivity

When ions have a charge of eZ , the basic equations (1) - (8) should be changed to

$$\frac{d}{dx}(N_e \bar{v}) = - \frac{d}{dx}(Z N_a \bar{v}_a) = dZ N_a N_e, \quad (1)'$$

$$-k T_e \frac{dN_e}{dx} + \frac{m_e \delta_{ea}}{Z} N_e Z N_a (\bar{v}_a - \bar{v}) - e N_e (E_x + \bar{v}_e B) = 0, \quad (2)'$$

$$- \frac{k T_i}{Z} \frac{dN_e}{dx} + \frac{m_i \delta_{ia}}{Z^2} N_e Z N_a (\bar{v}_a - \bar{v}) + e N_e (E_x + \bar{v}_i B) = 0, \quad (3)'$$

$$- \frac{k T_a}{Z} \frac{d(Z N_a)}{dx} + \frac{m_e \delta_{ea}}{Z} N_e Z N_a (\bar{v} - \bar{v}_a) + \frac{m_i \delta_{ia}}{Z^2} N_e Z N_a (\bar{v} - \bar{v}_a) = 0, \quad (4)'$$

$$\frac{m_e \delta_{ea}}{Z} N_e Z N_a (\bar{v}_a - \bar{v}_e) + \frac{m_e \delta_{ei}}{Z} N_e^2 (\bar{v}_e - \bar{v}_i) + e N_e \bar{v} B = 0, \quad (5)'$$

$$\frac{m_i \delta_{ia}}{Z^2} N_e Z N_a (\bar{v}_a - \bar{v}_i) + \frac{m_e \delta_{ei}}{Z} N_e^2 (\bar{v}_e - \bar{v}_i) - e N_e \bar{v} B = 0, \quad (6)'$$

$$\frac{m_e \delta_{ea}}{Z} N_e Z N_a (\bar{v}_e - \bar{v}_a) + \frac{m_i \delta_{ia}}{Z^2} N_e Z N_a (\bar{v}_i - \bar{v}_a) = 0, \quad (7)'$$

and

$$\frac{dB}{dz} = 4\pi e N_e (V_e - V_i) \quad (8)'$$

We know that equations (1)' - (8)' are reduced to the equations (1) - (8) when the following substitutions are carried out:

$$N \rightarrow N_e, \quad N_a \rightarrow Z N_a, \quad T_i \rightarrow \frac{T_i}{Z}, \quad T_a \rightarrow \frac{T_a}{Z},$$

$$m_i \delta_{ia} \rightarrow \frac{m_i \delta_{ia}}{Z^2} \quad \text{and} \quad m_e \delta_{ea} \rightarrow \frac{m_e \delta_{ea}}{Z} \quad (32)$$

These replacements change the characteristic parameters in the following way

$$\Omega_{ei}' = Z^3 \Omega_{ei}, \quad \alpha' = \left\{ \frac{k(T_e + \frac{T_a}{Z})}{m_e + m_i} \right\}^{1/2},$$

$$\nu_m' = (m_e \delta_{ea} + \frac{m_i \delta_{ia}}{Z}) N_{ao} / (m_e + m_i),$$

$$\tau' = \frac{Z T_e + T_a}{T_a} \quad (33)$$

where the superscript' denotes the new parameters. Then we know from (28) that N_{ao} is modified to the following form:

$$N_{eo} = Z N_{ioo} = \left(\frac{3 \Omega_{ei}'}{\gamma \tau'} \right)^{1/3} Z^2 \left(\frac{T_e + T_a}{Z T_e + T_a} \right)^{1/3} N_{ao}, \quad (28)'$$

where Ω_{ei} is the same as that defined in §2.

This relations means that

$$\frac{N_{ao}}{N_{eo}} = \left(\frac{N_{eo}}{N^{*'}} \right)^2 \quad (30)'$$

where $N^{*'} = Z^3 \left(\frac{T_e + T_a}{Z T_e + T_a} \right)^{1/2} N^*$.

It is shown that N_{ao} is reduced by factor Z^2 when N_{ioo} is the same as that for singly ionized ions and $Z \gg 1$.

If the plasma has an anomalous resistivity, equation (12) changes to

$$\frac{db^2}{d\eta} = - \frac{\delta_1^{-1} [\beta_a b^2]}{1 + (\gamma \tau'^{-1}) n + \beta_a^{-1} (1 - b^2)}, \quad (12)'$$

where δ_1 represents in an empirical way anomalous effects perpendicular to the magnetic field that are due to non-

classical diffusion and other phenomena. Equation (12)' changes the equation (18) to

$$\frac{dn_e}{dn_a} = - \delta_1^{-1} \Omega_{ei} \frac{1}{(\delta \tau^{-1} n_e + n_a)(\tau^{-1} n_e + n_a)}, \quad (18)'$$

which gives $N_{e\infty}$ as

$$N_{e\infty} (= \sum N_{i\infty}) = \delta_1^{-\frac{1}{3}} \sum^2 \left(\frac{T_e + T_i}{\sum T_e + T_i} \right)^{\frac{1}{3}} \left(\frac{3 \Omega_{ei}'}{\delta \tau} \right)^{\frac{1}{3}} N_{a0}, \quad (28)''$$

$$\frac{N_{a0}}{N_{e\infty}} = \left(\frac{N_{e\infty}}{N^{*''}} \right)^2 \quad (30)''$$

where $N^{*''} = \delta_1^{-\frac{1}{2}} \sum^3 \left(\frac{T_e + T_i}{\sum T_e + T_i} \right)^{\frac{1}{2}} N^*$. N_{a0} increases by a

factor δ_1 owing to the reduction of the diamagnetic current by the anomalous resistivity.

5.) Some numerical examples

From equations (1) - (8), the thickness of the boundary layer L_b can be estimated:

$$L_b \sim a \tau^{-\frac{1}{2}} \left(\frac{v_m v_I}{N_a \tau^2} \right)^{-\frac{1}{2}} \frac{1}{N_{e0}} = \tau^{-\frac{1}{2}} \frac{1}{N_{e0} \Omega_{ia}} \left(\frac{v_m}{v_I} \right)^{\frac{1}{2}}. \quad (34)$$

If the plasmaradius R exceeds L_b (this means that the plasma density exceeds $\tau^{-\frac{1}{2}} \Omega_{ia} R^{-1} \left(\frac{v_m}{v_I} \right)^{\frac{1}{2}}$), the neutral particles accumulate outside the plasma core. Then we can define the critical plasma density $N_{cr} \sim a \omega \left(\tau^{\frac{1}{2}} \Omega_{ia} R \left(\frac{v_m}{v_I} \right)^{\frac{1}{2}} \right)^{-1}$. For the typical condition that $\tau \sim 10^2$, $\Omega_{ia} = 4 \times 10^{-15} \text{ cm}^{-2}$, $R = 10 \text{ cm}$, and $\left(\frac{v_m}{v_I} \right)^{\frac{1}{2}} = 10^{-1}$, N_{cr} is found to be $2.5 \times 10^{13} \text{ cm}^{-3}$. This critical plasma density N_{cr} is the criterion which determines only whether neutral particles accumulate around the plasma or not.

If the neutral particle density around the plasma exceeds N_{a0} given by (30) or (30)'', a steepening of the plasma pressure profile should take place in the outer plasma. This may provide a driving force for instabilities. For $\gamma = 10^2$, $\tau = 10^2$, $T_e = T_i = 10^6 \text{ K}$, $\Omega_{ea} = 2 \times 10^{-16} \text{ cm}^{-2}$, $\Omega_{ia} = 2 \times 10^{-15} \text{ cm}^{-2}$,

$m_i = 5 \times 10^{-24}$ gr and $B = 5$ tesla, for example, N^* is found to be $5,3 \times 10^{15} \text{ cm}^{-3}$. When $\delta_i = 10^3$ and $Z = 5$, this value of N^* yields $N_{a0} = 4.5 \times 10^{11} \text{ cm}^{-3}$ and $3.5 \times 10^{12} \text{ cm}^{-3}$ when $N_{i\infty} = 5 \times 10^{13} \text{ cm}^{-3}$ and 10^{14} cm^{-3} , respectively.

6) Validity of the continuum approach

In order to check the validity of the present analysis, we should compare L_b with some kinds of mean free paths. First, L_b is compared with the mean free path of the neutral particles $l_a (= N_{a0}^{-1} Q_{ia}^{-1} (1 + Z^2)^{-1})$:

$$\frac{l_a}{L_b} = \frac{\tau^{1/2}}{1 + \tau^{1/2}} \left(\frac{v_I}{v_m} \right)^{1/2} \quad (35)$$

Because v_m contains v_I (the ionization of neutral particles is directly connected with ion production), we can conclude that $l_a < L_b$; The boundary layer is always thicker than the mean free path of the neutral particles.

Next, we compare L_b with the mean free path of the ions l_{ia} (collision of ions with neutral particles):

$$\begin{aligned} \frac{l_{ia}}{L_b} &= \left\{ \frac{N_a}{N_{a0}} a \tau^{-1/2} (v_m v_I / N_{a0}^2)^{-1/2} Q_{ia} \right\}^{-1} \\ &= \left(\frac{N^*}{N_{a0}} \right)^2 \tau^{1/2} \left(\frac{v_I}{v_m} \right)^{1/2} \quad (36) \end{aligned}$$

Because $N^* > N_{a0}$, l_{ia} is larger than L_b ; ions are almost free from collisions with neutral particles in the boundary. But this conclusion does not exclude the continuum approach because, as will be shown below, the ion cyclotron radius r_c is usually smaller than L_b :

$$\begin{aligned} \frac{r_c}{L_b} &= \frac{a \omega_i^{-1}}{N_{a0}^{-1} a \tau^{-1/2} (v_m v_I / N_{a0}^2)^{-1/2}} \\ &= \frac{\tau^{1/2} (v_I / v_m)^{3/2} (m_i / m_e)^{1/2} (Q_{ia} / Q_{ei})}{\omega_i \tau_i} \quad (37) \end{aligned}$$

For the typical condition of $B = 5$ tesla, $T_e = T_i = 10^6$ K, the condition $r_c < L_b$ is satisfied when $N_{a0} < 10^{15} \text{ cm}^{-3}$.

Finally, we compare the mean free path of ion-ion collisions l_{ii} with L_b :

$$\frac{l_{ii}}{L_b} = \frac{(N_{a0} Q_{ii})^{-1}}{N_{a0} a \tau^{-\frac{1}{2}} (v_{im} v_I / N_{a0}^2)^{-\frac{1}{2}}} \quad (38)$$

$$= \tau^{\frac{1}{2}} (v_I / v_m)^{\frac{1}{2}} (Q_{ia} / Q_{ii}).$$

Because $Q_{ii} \gg Q_{ia}$ and $v_m > v_I$, l_{ii} is normally shorter than L_b . This ensures a Maxwellian velocity distribution of the ions with their temperature T_i , and the boundary condition that $N = 0$ at $x = 0$.

To conclude this section, the fact that the mean free path of the neutral particles and the ion cyclotron radius are shorter than the thickness of the boundary layer confirms the validity of the continuum approach, and sufficiently frequent ion-ion collisions in the boundary should make the ion velocity distribution Maxwellian and also validates the boundary condition at the walls used in this analysis.

7) Conclusions

When the plasma density and/or the plasma radius increases above the critical value, the neutral particles tend to accumulate around the plasma and the neutral gas-plasma boundary layer is formed. A steady-state pressure balance between the plasma and the neutral gas was calculated for the case where the neutral gas pressure is much lower than the magnetic pressure. The large difference of pressure between the plasma and the neutral gas is induced by the diamagnetic effect of diffusion. This is promoted when the magnetic field is large and the neutral gas pressure is low. This means that the plasma pressure profile may be influenced by a rather low neutral gas pressure. This critical number density of neutral particles above which the plasma is much affected is given in the relation (30) including the effects of multiple

ionization and the anomalous resistivity. A typical example of this critical neutral particle density is about 1 % of the ion number density for the case of $B = 5$ tesla, $\bar{J}_L = 10^3$, $Z = 5$, $T = 10^6$ K and $N_{i\infty} = 5 \times 10^{13} - 10^{14} \text{ cm}^{-3}$.

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