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Electron Ring Loaded With
Two Species of Ions

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Abstract

The electrostatic interaction of a relativistic electron ring with the ions contained in it leads to instability under certain conditions, as is well known. Since there are always different ion species present in the ring, the influence of the ion mixture on the instability is investigated in this paper. A dispersion equation is derived in which, besides the interaction of the electrons with one ion species, both the collective interaction of the electrons with a second ion species and the collective interaction between the two ion species are taken into account. As solutions of these equations show, no excessive enhancement or suppression of the instability nor an appreciable displacement of the region of instability due to the additional interaction takes place. For sufficiently small fractional ion loading the region of instability splits into two regions corresponding to the separate interactions of each ion species with the electrons.

The instability of an electron ring arising from its interaction with ions contained in it was treated in several papers theoretically ¹⁾²⁾³⁾ and experimentally ⁴⁾. In Ref. 5 the additional focussing effects which result from cylindrical structures like a "squirrel cage" inside or outside the ring were included in the theory. The present treatment extends this theory to the interaction of the electron ring with two kinds of ions. This same question was dealt with by Nikolaeva and Koshkarev ⁶⁾. In their paper, however, the interaction among the ions was neglected. Their treatment is therefore valid only for small fractional ion loading. Furthermore the magnetic field effect on the ion motion is neglected there which becomes important for the radial instability when the resonance frequencies become very small.

Derivation of the dispersion equation

The equations of radial motions for the electrons and the two kinds of ions are

$$\left. \begin{aligned} \frac{1}{\omega_0^2} \ddot{r}_e &= -q_e^2 r_e + q_{e1}^2 r_1 + q_{e2}^2 r_2 \\ \frac{1}{\omega_0^2} \ddot{r}_1 &= q_{1e}^2 r_e - q_{11}^2 r_1 - q_{12}^2 r_2 \\ \frac{1}{\omega_0^2} \ddot{r}_2 &= q_{2e}^2 r_e - q_{21}^2 r_1 - q_{22}^2 r_2 \end{aligned} \right\} (1)$$

where r_e , r_1 and r_2 are collective radial displacements and ω_0 is the electron revolution frequency. The q 's can be written in analogy to Ref. 5:

$$\left. \begin{aligned}
 q_e^2 &= 1 - n + \rho \left(f_1 + f_2 - \frac{1}{2\alpha_E^2} + \frac{\beta^2}{2\alpha_M^2} \right) \\
 q_{e1}^2 &= \rho f_1 \left(1 - \frac{1}{4\alpha_E^2} \right) \\
 q_{e2}^2 &= \rho f_2 \left(1 - \frac{1}{4\alpha_E^2} \right) \\
 q_{11}^2 &= \rho Q_{g1} \left(1 - f_2 - \frac{f_1}{2\alpha_E^2} \right) + Q_{g1}^2 \\
 q_{12}^2 &= \rho f_2 Q_{g1} \left(1 - \frac{1}{4\alpha_E^2} \right) \\
 q_{1e}^2 &= \rho Q_{g1} \left(1 - \frac{1}{4\alpha_E^2} \right) \\
 q_{21}^2 &= \rho Q_{g2} \left(1 - f_1 - \frac{f_2}{2\alpha_E^2} \right) + Q_{g2}^2 \\
 q_{22}^2 &= \rho f_1 Q_{g2} \left(1 - \frac{1}{4\alpha_E^2} \right) \\
 q_{2e}^2 &= \rho Q_{g2} \left(1 - \frac{1}{4\alpha_E^2} \right)
 \end{aligned} \right\} (2)$$

with

$$\alpha_{E,M} = (R_{E,M} - R) / \sqrt{a \bar{b}}$$

β = electron speed/light speed

$$\gamma = (1 - \beta^2)^{-1/2}$$

$R_{E,M}$ = radius of electric and magnetic images

n = field index = $-(R/B) \cdot (\partial B_z / \partial R)$

$$Q_{g1,2} = Z_{1,2} \gamma m_e / m_{i1,2}$$

$$\rho = r_0 R N_e / (\pi a \bar{b} \gamma)$$

$r_0 = 2.82 \cdot 10^{-15} \text{ m}$ = classical electron radius

R = major radius of the electron ring

a = radial half width of minor ring dimension

b = corresponding axial half width

$$\bar{b} = (a + b) / 2$$

$$f_{1,2} = Z_{1,2} \cdot N_{1,2} / N_e = \text{fractional ion loadings}$$

The coefficients (2) are valid only in the limit of small curvature of the ring as well as the walls. A better approximation done by I. Hofmann ⁷⁾ contains also terms proportional to $\frac{1}{\alpha_E}$. We thought, however, that for our purpose the simpler α_E coefficients (2) are good enough.

The set of equations (1) is solved as usual by the assumption

$$r_{e,1,2} \sim \exp [i (m \varphi - \omega t)]$$

where m is the azimuthal mode number, φ the azimuthal position of the particles taken as being constant for the ions and t for the electrons so that

$$\ddot{r}_e = - (m \omega_0 - \omega)^2 r_e$$

$$\ddot{r}_1 = - \omega^2 r_1$$

$$\ddot{r}_2 = - \omega^2 r_2$$

The equations (1) are reduced to a system of three linear homogenous equations for $r_{e,1,2}$ (with $\nu = \omega / \omega_0$):

$$\left. \begin{aligned} [q_e^2 - (m-\nu)^2] r_e - q_{e1}^2 r_1 - q_{e2}^2 r_2 &= 0 \\ - q_{1e}^2 r_e + (q_1^2 - \nu^2) r_1 + q_{12}^2 r_2 &= 0 \\ - q_{2e}^2 r_e + q_{21}^2 r_1 + (q_2^2 - \nu^2) r_2 &= 0 \end{aligned} \right\} (3)$$

Equations (3) have a nonzero solution only if the determinant of the coefficients of $r_{e,1,2}$ is zero.

This leads to the following sixth order equation in v :

$$\begin{aligned}
 & v^6 - 2m v^5 - (q_e^2 + q_1^2 + q_2^2 - m^2) v^4 + 2m (q_1^2 + q_2^2) v^3 \\
 & + [q_e^2 q_1^2 + q_1^2 q_2^2 + q_2^2 q_e^2 - q_{e1}^2 q_{1e}^2 - q_{12}^2 q_{21}^2 - q_{e2}^2 q_{2e}^2 - m^2 (q_1^2 + q_2^2)] v^2 \\
 & + 2m (q_{12}^2 q_{21}^2 - q_1^2 q_2^2) v \\
 & - q_e^2 q_1^2 q_2^2 - q_{e1}^2 q_{12}^2 q_{2e}^2 - q_{e2}^2 q_{1e}^2 q_{21}^2 + q_e^2 q_{12}^2 q_{21}^2 + q_2^2 q_{e1}^2 q_{1e}^2 + q_1^2 q_{e2}^2 q_{2e}^2 \\
 & - m^2 (q_{12}^2 q_{21}^2 - q_1^2 q_2^2) = 0
 \end{aligned} \tag{4}$$

A program was written which determines whether all six roots of this equation are real or not and which calculates the highest growth rate in case of complex roots.

Influence of ion mixtures on the instability

In the following we want to get an answer to our main question: Does the mixture of different ion species damp the instability or not? Therefore the growth rate of the instability is calculated for various conditions. The presentation of the data is similar to LASLETT's⁸⁾: The quantities for the electron ring are kept constant (and taken as they are typically measured in the Gar-ching SCHUKO electron ring experiment):

$$\begin{aligned}
 N_e &= 5 \cdot 10^{12} & \gamma &= 27 & R &= 2,3 \text{ cm} \\
 a &= 0,3 \text{ cm} & b &= 0,3 \text{ cm} & R_{\text{Squi}} &= 1,55 \text{ cm}
 \end{aligned}$$

The growth rate is determined in a plane of the two parameters: field index n and sum of the two ion loadings $f = f_1 + f_2$. Like in the more usual $Q_1 - Q_i$ - diagrams there is a cusp of the instability area in the $f - n$ -plane for $f \rightarrow 0$. The n -value of this cusp was given by LASLETT⁸⁾:

$$n_{c,1,2} = 2 \sqrt{(\rho + Q_{g,1,2}) Q_{g,1,2}} - (\rho + Q_{g,1,2}) \cdot Q_{g,1,2} - \frac{1}{2} \rho \left(\frac{1}{\alpha_E^2} - \frac{\beta^2}{\alpha_m^2} \right) \quad (5)$$

Tab. 1 a,b,c present this critical n-value for three different cases of electron ring quality as a function of the ion mass and the radius of the "squirrel cage". According to Tab. 1 a,b,c it would be possible to avoid the instability by shifting this cusp to negative n-values which is done by using heavy ions and a strong squirrel cage action.

The following tables show the unstable parameter areas in the f-n-plane. The upper halves of the tables give the real part of the complex frequency (or a zero if all solutions of equations 3 are real), the lower halves give the imaginary part. In case of more than one pair of complex conjugate solutions the most unstable case is selected. The critical field index n_c as calculated from equation 5 is indicated at the bottom of each table. In both halves of the tables not the values of the frequencies themselves but their logarithms are printed out according to the relation

$$\text{Printed value} = \text{INTEGER} (100 + 10 \lg (\omega_{r,i} / \omega_0)).$$

To see whether the ion mixture reduces the growth of the instability it is thought of as a gedanken experiment: Without changing any other parameters the field index n is changed at a constant rate. As a relative measure for the total amplitude growth when the cusp is crossed we use the approximate expression $\Delta n \sum \omega_i(n) \propto \int \omega_i(t) dt$ although this applies only under rather stringent conditions (sufficiently small values of $\dot{\omega}_r$, $\dot{\omega}_i$ and sufficiently constant ratios of r_1/r_e and r_2/r_e). To get the above expression the imaginary parts are summed up in horizontal rows of these tables, multiplied with the step width Δn and printed in the last columns, also in a logarithmic form as above.

We choose three examples:

1. A mixture of N^+ and N_2^+ to see if something extraordinary might happen when different kinds of ions of the same gas are present (Tab. 2a,b,c, and 3a,b,c). This corresponds to the situation in a "waiting room" with different ionization states of the same ions.
2. A mixture $^{14}N^+$ and $^{16}O^+$ to see a possible stabilizing effect of similar charge to mass ratios (Tab. 4a,b,c, and 5a,b,c).
3. A mixture of $^1H^+$ and $^{16}O^+$ to see whether harmonic resonances occur between the ions (Tab. 6a,b,c, and 7a,b,c).

Discussion

The comparison of the tables for single species and their mixtures shows no stabilizing effect of the mixing of masses. In order to clarify this point further some selected data of Tab. 2 to 7 are collected to Tab. 8 where some numbers for the growth of the instability are rewritten. Tab. 8 clearly shows that the mixture of different ions does not stabilize the electron-ion ring against the dipole instability¹⁰⁾. The table shows that the instability growth is the bigger the lighter the ions (compared for same fractional loading). For a loading of 2% for example the growth for H^+ ions is about 5 times the growth for N_2^+ ions, and about twice the growth for He^+ ions. So a method to make the instability less dangerous would be the use of heavier ions. This is the same conclusion as drawn above concerning the position of the critical field index. The use of heavy ions (or better: ions with a small charge to mass ratio) is impractical, however, because of

the limited holding power of the ring during acceleration. The increase of the ring quality seems therefore necessary not only for a higher holding power but also for suppression of the electron ion resonances.

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References

1. F.E. MILLS, Symposium on Electron Ring Accelerators, UCRL-18103, Feb. 1968, p. 448
2. D.G. KOSHKAREV and P.R. ZENKEVICH, Particle Accelerators 3, 1 (1972)
3. W. DOMMASCHK, MPI für Plasmaphysik, Garching, Report IPP O/19 (Dec. 1973)
4. C. ANDELFINGER et al., 9th Int.Conf.on High Energy Accelerators, May 1974, Stanford, Cal., USA
5. W. OTT and L.J. LASLETT, ERAN 234 (March 1974)
6. L.P. NIKOLAEVA and D.G. KOSHKAREV, On three-component electron-ion rings, Report No. ITEP-79, 1973
7. I. HOFMANN, private communication
8. Appendix of Ref. 5
9. Formula (A-8;b) in Ref. 5
10. L.J. LASLETT came to a similar result:
that the resonances arising, in the region of small ion loading, may be rather precisely accounted for by the superposition of resonant regions obtained by considering just one ion species to be present at one time.

ELEKTROJNENZAHLE, GAMMA: 5.000D 12 2.700D 01
 RINGGEOMETRIE: R, a, b (CM): 2.300D 00 3.000D-01 3.000D-01
 RHO = 4.245D-01

squirrel cage radius (cm)	1	2	4	8	16	32	64	128	256
4.00	0.1477	0.1029	0.0712	0.0486	0.0326	0.0212	0.0131	0.0073	0.0033
3.95	0.1473	0.1025	0.0708	0.0482	0.0322	0.0208	0.0127	0.0069	0.0029
3.90	0.1468	0.1021	0.0703	0.0478	0.0317	0.0203	0.0122	0.0065	0.0024
3.85	0.1463	0.1016	0.0698	0.0473	0.0312	0.0198	0.0117	0.0060	0.0019
3.80	0.1458	0.1011	0.0693	0.0467	0.0307	0.0193	0.0112	0.0055	0.0014
3.75	0.1452	0.1005	0.0687	0.0461	0.0301	0.0187	0.0106	0.0049	0.0008
3.70	0.1445	0.0998	0.0681	0.0455	0.0294	0.0180	0.0099	0.0042	0.0001
3.65	0.1438	0.0991	0.0673	0.0447	0.0287	0.0173	0.0092	0.0035	-0.0006
3.60	0.1430	0.0982	0.0665	0.0439	0.0279	0.0165	0.0084	0.0026	-0.0014
3.55	0.1421	0.0973	0.0656	0.0430	0.0270	0.0155	0.0075	0.0017	-0.0024
3.50	0.1410	0.0963	0.0645	0.0420	0.0259	0.0145	0.0064	0.0007	-0.0034
3.45	0.1398	0.0951	0.0634	0.0408	0.0247	0.0133	0.0052	-0.0005	-0.0046
3.40	0.1385	0.0938	0.0620	0.0394	0.0234	0.0120	0.0039	-0.0018	-0.0059
3.35	0.1370	0.0922	0.0605	0.0379	0.0219	0.0104	0.0024	-0.0034	-0.0075
3.30	0.1352	0.0904	0.0587	0.0361	0.0201	0.0087	0.0006	-0.0052	-0.0092
3.25	0.1331	0.0884	0.0566	0.0341	0.0180	0.0066	-0.0015	-0.0072	-0.0113
3.20	0.1307	0.0860	0.0542	0.0316	0.0156	0.0042	-0.0039	-0.0096	-0.0137
3.15	0.1279	0.0831	0.0514	0.0288	0.0127	0.0013	-0.0068	-0.0125	-0.0166
3.10	0.1244	0.0797	0.0480	0.0254	0.0093	-0.0021	-0.0102	-0.0159	-0.0200
3.05	0.1203	0.0756	0.0438	0.0213	0.0052	-0.0062	-0.0143	-0.0200	-0.0241
3.00	0.1153	0.0706	0.0388	0.0162	0.0002	-0.0112	-0.0193	-0.0250	-0.0291
2.95	0.1091	0.0643	0.0326	0.0100	-0.0060	-0.0174	-0.0255	-0.0313	-0.0353
2.90	0.1012	0.0565	0.0247	0.0022	-0.0139	-0.0253	-0.0334	-0.0391	-0.0432
2.85	0.0911	0.0464	0.0147	-0.0079	-0.0240	-0.0354	-0.0435	-0.0492	-0.0533
2.80	0.0779	0.0331	0.0014	-0.0212	-0.0372	-0.0486	-0.0567	-0.0625	-0.0665
2.75	0.0600	0.0152	-0.0165	-0.0391	-0.0551	-0.0666	-0.0746	-0.0804	-0.0845
2.70	0.0349	-0.0098	-0.0416	-0.0642	-0.0802	-0.0916	-0.0997	-0.1054	-0.1095
2.65	-0.0016	-0.0464	-0.0781	-0.1007	-0.1168	-0.1282	-0.1362	-0.1420	-0.1461
2.60	-0.0579	-0.1027	-0.1344	-0.1570	-0.1731	-0.1845	-0.1926	-0.1983	-0.2024

Tab. 1a

critical n-value

ion mass

ELEKTRONENZAHL, GAMMA: 5.0000 12 2.7000 01
 RINGGEOMETRIE: R, a, b (CM): 2.3000 00 2.0000D-01 2.0000D-01
 RHO = 9.551D-01

squirrel cage radius (cm)	4.00	3.95	3.90	3.85	3.80	3.75	3.70	3.65	3.60	3.55	3.50	3.45	3.40	3.35	3.30	3.25	3.20	3.15	3.10	3.05	3.00	2.95	2.90	2.85	2.80	2.75	2.70	2.65	2.60	2.55	2.50
	0.2180	0.2175	0.2172	0.2167	0.2161	0.2155	0.2149	0.2141	0.2133	0.2124	0.2114	0.2102	0.2088	0.2073	0.2055	0.2035	0.2010	0.1982	0.1948	0.1907	0.1856	0.1794	0.1716	0.1615	0.1482	0.1303	0.1052	0.0687	0.0124	-0.0810	-0.2529
	0.1546	0.1542	0.1538	0.1533	0.1527	0.1521	0.1515	0.1507	0.1499	0.1490	0.1480	0.1468	0.1454	0.1439	0.1421	0.1401	0.1376	0.1348	0.1314	0.1273	0.1222	0.1160	0.1082	0.0981	0.0848	0.0669	0.0418	0.0053	-0.0510	-0.1444	-0.3163
	0.1087	0.1082	0.1078	0.1073	0.1068	0.1062	0.1055	0.1048	0.1040	0.1030	0.1020	0.1008	0.0995	0.0979	0.0962	0.0941	0.0917	0.0888	0.0854	0.0813	0.0763	0.0701	0.0622	0.0521	0.0389	0.0209	-0.0041	-0.0407	-0.0970	-0.1904	-0.3623
	0.0756	0.0752	0.0747	0.0742	0.0737	0.0731	0.0724	0.0717	0.0709	0.0699	0.0689	0.0677	0.0664	0.0648	0.0631	0.0610	0.0586	0.0557	0.0523	0.0482	0.0432	0.0370	0.0291	0.0190	0.0058	-0.0122	-0.0372	-0.0738	-0.1301	-0.2234	-0.3954
	0.0518	0.0514	0.0510	0.0505	0.0500	0.0494	0.0487	0.0480	0.0472	0.0462	0.0452	0.0440	0.0427	0.0411	0.0394	0.0373	0.0349	0.0320	0.0286	0.0245	0.0195	0.0132	0.0054	-0.0047	-0.0180	-0.0359	-0.0609	-0.0975	-0.1538	-0.2472	-0.4191
	0.0349	0.0345	0.0341	0.0336	0.0330	0.0324	0.0318	0.0310	0.0302	0.0293	0.0283	0.0271	0.0257	0.0242	0.0224	0.0204	0.0179	0.0151	0.0117	0.0076	0.0025	-0.0037	-0.0115	-0.0216	-0.0349	-0.0528	-0.0779	-0.1144	-0.1707	-0.2641	-0.4360
	0.0229	0.0224	0.0220	0.0215	0.0210	0.0204	0.0197	0.0190	0.0182	0.0172	0.0162	0.0150	0.0137	0.0121	0.0104	0.0083	0.0059	0.0030	0.0004	-0.0045	-0.0095	-0.0158	-0.0236	-0.0337	-0.0469	-0.0649	-0.0899	-0.1265	-0.1828	-0.2762	-0.4481
	0.0143	0.0139	0.0134	0.0129	0.0124	0.0118	0.0111	0.0104	0.0096	0.0087	0.0076	0.0064	0.0051	0.0036	0.0019	0.0003	0.0027	0.0055	0.0090	0.0131	0.0181	0.0243	0.0322	0.0423	0.0555	0.0734	0.0985	0.1350	0.1913	0.2847	0.4566
	0.0082	0.0078	0.0073	0.0069	0.0063	0.0057	0.0051	0.0043	0.0035	0.0026	0.0015	0.0004	0.0010	0.0025	0.0043	0.0064	0.0088	0.0116	0.0150	0.0192	0.0242	0.0304	0.0383	0.0483	0.0616	0.0795	0.1046	0.1411	0.1974	0.2908	0.4627

1 2 4 8 16 32 64 128 256

critical n - value ion mass

Tab. 1b

ELEKTRONENZAHL, GAMMA: 5.000D 12 2.700D 01
 RINGGEOMETRIE: R, a, b (CM): 2.300D 00 1.000D-01 1.000D-01
 RHO = 3.820D 00

squirrel cage
 radius (cm)

4.00	0.4121	0.3010	0.2167	0.1542	0.1086	0.0756	0.0520	0.0350	0.0230
3.95	0.4117	0.3006	0.2163	0.1538	0.1082	0.0752	0.0516	0.0346	0.0226
3.90	0.4113	0.3001	0.2158	0.1533	0.1077	0.0748	0.0511	0.0342	0.0221
3.85	0.4108	0.2996	0.2153	0.1529	0.1073	0.0743	0.0506	0.0337	0.0216
3.80	0.4103	0.2991	0.2148	0.1523	0.1067	0.0738	0.0501	0.0332	0.0211
3.75	0.4097	0.2985	0.2142	0.1517	0.1061	0.0732	0.0495	0.0326	0.0205
3.70	0.4090	0.2978	0.2135	0.1511	0.1055	0.0725	0.0488	0.0319	0.0199
3.65	0.4083	0.2971	0.2128	0.1503	0.1047	0.0718	0.0481	0.0312	0.0191
3.60	0.4074	0.2963	0.2120	0.1495	0.1039	0.0709	0.0473	0.0303	0.0183
3.55	0.4065	0.2953	0.2110	0.1486	0.1030	0.0700	0.0463	0.0294	0.0174
3.50	0.4055	0.2943	0.2100	0.1475	0.1019	0.0690	0.0453	0.0284	0.0163
3.45	0.4043	0.2931	0.2088	0.1464	0.1008	0.0678	0.0441	0.0272	0.0152
3.40	0.4030	0.2918	0.2075	0.1450	0.0994	0.0665	0.0428	0.0259	0.0138
3.35	0.4014	0.2902	0.2059	0.1435	0.0979	0.0649	0.0412	0.0243	0.0123
3.30	0.3997	0.2885	0.2042	0.1417	0.0961	0.0631	0.0395	0.0226	0.0105
3.25	0.3976	0.2864	0.2021	0.1396	0.0940	0.0611	0.0374	0.0205	0.0084
3.20	0.3952	0.2840	0.1997	0.1372	0.0916	0.0587	0.0350	0.0181	0.0060
3.15	0.3923	0.2811	0.1968	0.1344	0.0888	0.0558	0.0321	0.0152	0.0032
3.10	0.3889	0.2777	0.1934	0.1310	0.0854	0.0524	0.0287	0.0118	-0.0002
3.05	0.3848	0.2736	0.1893	0.1268	0.0812	0.0483	0.0246	0.0077	-0.0044
3.00	0.3798	0.2686	0.1843	0.1218	0.0762	0.0433	0.0196	0.0027	-0.0094
2.95	0.3735	0.2624	0.1781	0.1156	0.0700	0.0370	0.0134	-0.0036	-0.0156
2.90	0.3657	0.2545	0.1702	0.1077	0.0621	0.0292	0.0055	-0.0114	-0.0235
2.85	0.3556	0.2444	0.1601	0.0977	0.0521	0.0191	-0.0046	-0.0215	-0.0335
2.80	0.3423	0.2312	0.1469	0.0844	0.0388	0.0058	-0.0178	-0.0348	-0.0468
2.75	0.3244	0.2132	0.1289	0.0665	0.0209	-0.0121	-0.0358	-0.0527	-0.0647
2.70	0.2994	0.1882	0.1039	0.0414	-0.0042	-0.0371	-0.0608	-0.0777	-0.0898
2.65	0.2628	0.1516	0.0673	0.0049	-0.0407	-0.0737	-0.0974	-0.1143	-0.1263
2.60	0.2065	0.0953	0.0110	-0.0514	-0.0970	-0.1300	-0.1537	-0.1706	-0.1826
2.55	0.1131	0.0020	-0.0823	-0.1448	-0.1904	-0.2234	-0.2470	-0.2640	-0.2760
2.50	-0.0588	-0.1700	-0.2543	-0.3167	-0.3623	-0.3953	-0.4190	-0.4359	-0.4479
2.45	-0.4302	-0.5414	-0.6257	-0.6881	-0.7337	-0.7667	-0.7904	-0.8073	-0.8193
2.40	-1.4914	-1.6025	-1.6868	-1.7493	-1.7949	-1.8279	-1.8515	-1.8695	-1.8805

1 2 4 8 16 32 64 128 256

ion mass

critical n-value

Tab.8: GROWTH OF INSTABILITY

Comparison of selected data

sum of fractional loadings	100%	50%	0%	N ⁺ N ₂ ⁺
	0%	50%	100%	
1%	61	60	60	with SQC
2%	64	63	63	
1%	61	61	60	without
2%	65	64	63	

	100%	50%	0%	N ⁺ O ⁺
	0%	50%	100%	
3%	66	66	66	with SQC
15%	73	73	73	
45%	78	78	77	
3%	66	66	66	without
15%	73	73	73	
45%	78	78	78	

	100%	50%	0%	H ⁺ He ⁺
	0%	50%	100%	
1%	67	65	64	with SQC
2%	70	69	67	
4%	73	72	70	
10%	77	76	74	
20%	80	79	77	
1%	67	66	64	without
2%	70	69	67	
4%	73	72	70	
10%	77	76	74	
20%	80	79	77	