Turbulent Heating and Quenching of Ion Sound Instability

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Abstract

Turbulent heating and stabilization of the ion sound instability is investigated by 2 D computer simulation. Quasilinear rather than nonlinear effects determine the evolution of the instability. The growth rate is reduced by flattening of the electron distribution and finally the instability is quenched by the formation of a high energy ion tail.

Numerous stabilization mechanisms have been proposed for the current driven ion sound instability, involving quasilinear,[1 2] mode coupling,[3,4,5] trapping and resonance broadening effects.[6,7,8] We have done extensive simulation studies in order to provide a test for these basic predictions.[9] The two dimensional code has been described previously.[10] Specifically for the purpose of testing nonlinear theories of stabilization we have made runs in which the ratio between drift velocity and electron thermal velocity was kept constant, in addition to runs with constant current. The latter case corresponds more closely to the experimental situation. In the same vein we discuss the case of a current perpendicular to a weak magnetic field ($\frac{\Omega_e}{\omega_e} = 0.04$), corresponding to the shock wave case. The magnetic field (perpendicular to the plane of computation) has a very small effect on wave dispersion, but keeps the electron distribution isotropic. (In the case of a current along a magnetic field further complicated dynamical effects are added by the formation of an electron runaway tail). We find that for a wide range of initial parameters the growth phase of the instability is followed by the decay of the wave energy W, the return of the fluctuation level W/nT_e to the thermal level [11] and termination of heating in typically 100-200 ω_i^{-1}, Fig. 1. Clearly, in the case of constant current, the growth phase of the instability must terminate at the latest when the phase velocity reaches the drift velocity, $u \simeq c_s = (T_e/M)^{\frac{1}{2}}$ and taking account of the ion heating $\dot{T}_i \simeq (c_s/u)\,\dot{T}_e$ selfquenching of the instability should occur at drift velocities several times the sound speed.[12] The runs with constant u/c_s however show quenching in much the same way, Fig. 1b. It is seen that in this case the plasma enters a regime in which the macroscopic parameters remain constant. Nonlinear theories of stabilization generally determine a quasisteady fluctuation level $W/nT_e\left[\frac{m}{M}, {}^{u}/v_e, T_e/T_i\right]$ from the condition that the nonlinear damping just balances the linear growth rate, $\gamma = \gamma^L + \gamma^{NL} = 0$. Actually for $\partial/\partial t \ln (W/nT_e) = \dot{W}/W - \dot{T}_e/T_e = 0$ γ must be balanced by the electron heating rate. More generally, for constant $\gamma^L > 0$, an electron heating rate and any nonlinear damping increasing with W/nT_e, saturation at constant W/nT_e would occur, with W and T_e still increasing. A glance at Fig. 1b shows that such a macroscopic description is not adequate, i.e. in the course of time changes in the particle and perhaps spectral distribution occur which lead to a reduction of γ^L and, finally, quenching of the instability. From the observed macroscopic behavior we cannot exclude nonlinear effects but they do not qualitatively change the dynamical character of the instability.

The effective collision frequency, Fig. 2 gives information about the electron wave interaction. From conservation of wave momentum we find[12]

$$\underline{\underline{\nu}} = \frac{1}{V}\sum_{\underline{k}} 2\,\gamma^{e}_{\underline{k}}\, N_{\underline{k}}\, \frac{\omega_{\underline{k}}}{nmu^2}\, \frac{\underline{k}\cdot\underline{u}}{\omega_{\underline{k}}}\,, \tag{1}$$

where $\gamma^e_{\underline{k}}$ is the total (linear and nonlinear) rate of wave dissipation on the electrons and $N_{\underline{k}}$ is the number of waves $\underline{k}$ in Volume V. It follows that we can exclude proposed theories of stabilization by electron trapping or nonlinear Landau damping on the electrons, since for $\gamma^e_{\underline{k}} = 0, \nu \neq 0$ they violate conservation of wave momentum. To obtain friction between electrons and ions at saturation, the electron growth rate has to be balanced by damping on the ions. We find that the collision frequency agrees very well with the quasilinear prediction

$$\nu = \underline{u}\cdot\underline{\underline{\nu}}\cdot\underline{u}/u^2 = eE_o/mu$$

$$\nu/\omega_e = \alpha_2\,(2\pi)^{\frac{1}{2}} \left\langle \frac{\underline{k}\cdot\underline{u}-\omega_{\underline{k}}}{ku}\; \frac{\underline{k}\cdot\underline{u}}{ku}\; \frac{\omega_e}{kv_e} \right\rangle \frac{W}{nT_e} \equiv \rho\,\frac{W}{nT_e}\,, \tag{2}$$

where the average is over the spectrum and α_2 takes account of changes in the electron distribution from a Maxwellian,

$$\frac{\partial F}{\partial w} \equiv -\,\alpha_2 \left(\frac{1}{2\pi v_e^{\,2}}\right)^{\frac{1}{2}} \frac{w}{v_e^{\,2}} \tag{3}$$

where $F(w) \equiv \int dv\, \delta\!\left(\frac{\underline{k}\cdot\underline{v}}{k} - w\right) f_e(\underline{v})$.

The electron distribution flattens as a result of diffusion which leads to a reduction of the growth rate and an increase in the effective temperature for wave dispersion, $\lambda_D = \alpha_1\, v_e/\omega_e$ as compared to a Maxwellian of the same energy, $T_e = mv_e^{\,2}$,

$$-P\int du\, \frac{1}{u}\, \frac{\partial F}{\partial u} \equiv \left\langle \frac{1}{v^2} \right\rangle = \frac{1}{\alpha_1^{\,2} v_e^{\,2}}\,, \tag{4}$$

$$\gamma_{\underline{k}}^{L} = \alpha_1^{2} k\lambda_D \left(\frac{1}{1+k^2\lambda_D^2}\right)^{3/2} \left(\frac{\pi}{8}\right)^{\frac{1}{2}} \alpha_2 \left(\frac{\underline{k}.\underline{u}}{k\ v_e} - \frac{u^*}{v_e}\right) \omega_i \ , \qquad (5)$$

where u^* is the critical drift velocity. Quasilinear theory (B=O) [3,13] admits a selfsimilar solution $f_e(v) = C \exp\{-(v/v_o)^5\}$. We observe [13] that $f_e(v_{\underline{1}})$ reaches a selfsimiar form $f_e(v_{\underline{1}}) = C\exp\{-(v/v_o)^x\}$but with x consistently below 5 , x = 3,6-4, $\alpha_2 \simeq 0.33$, $\alpha_1^2 \approx 1.6$, Fig 3.In the 3 D case the effects of changes in $f_e(v)$ are weaker but still appreciable. For x = 5 we have $\alpha_2 = 0.445$, $\alpha_1^2 = 1.45$. For $u/c_s >> 1$ most of the energy delivered to the plasma goes into electron heating, $\dot{T}_e \simeq \nu m u^2$. The energy extracted from the electrons [12]

$$-\left.\frac{\delta E_e}{\delta t}\right| = \frac{1}{V} \sum_{\underline{k}} 2\gamma_k^{e} N_{\underline{k}} \omega_{\underline{k}} \approx \left\langle \frac{\omega_{\underline{k}}}{\underline{k}.\underline{u}-\omega_{\underline{k}}} \right\rangle n\dot{T}_e \qquad (6)$$

goes into wave growth and ion heating. The observed electron and ion heating rates agree very well with these predictions. In particular if we start with a large temperature ratio $T_e/T_i = 50$ than T_e/T_i decreases in the course of time and if we start with a small temperature ratio $T_e/T_i = 2$ than T_e/T_i increases in a predictable way, Table 1.

For quenching of the instability we must turn to the interaction of the waves with the ions. We observe in all cases that the ions develop a two temperature distribution Fig. 3. The bulk of the distribution is heated relatively little and accelerated by the applied electric field at a rate which is close to free acceleration. At small ion temperatures a very substantial fraction of the space averaged bulk temperature is connected with the (reversible) sloshing motion of the ions, as can be determined from the observed difference between space averaged and local ion temperature, $n(T_i - T_i^L) = \langle(\omega_i/\omega)^2\rangle W \approx 2W$. The remaining irreversible bulk heating rate is very small but still about two orders of magnitude larger than expected from nonlinear Landau damping. It must be noted in this connection that the wave spectrum contains a substantial fraction of waves with large kv_e/ω_e and small phase velocity $v_{ph} < c_s$ which can interact directly with the bulk of the distribution. If the initial temperature ratio T_i/T_e is very small this requires a certain minimum fluctuation level, which by sloshing raises the effective ion temperature. Indeed, we observe that tail formation sets in only after the temperature ratio increases to a value such that a nonnegligible but small fraction of particles in the space averaged ion distribution fall into the linear phase velocity range. Then the tail starts extending to larger velocities, up to about $2c_s$ and the number of particles in the tail increases. The process of tail formation is coupled with the evolution of the spectrum, Fig.4. In the course of time it moves to larger and larger values of kv_e/ω_e. This appears to be connected simply with the increase in v_e, whereas the cutoff at large and small kv_e/ω_e is due to damping on ion bulk and tail respectively. The time scale for tail formation $\tau_D \approx 10\text{-}20\ \omega_i^{-1}$ agrees very well with the quasilinear estimate

$$\omega_i \tau_D = \frac{nM(\Delta v)^2}{4W} , \qquad (7)$$

as do the bulk and tail heating rates. Within the accuracy of our diagnostics the early tail formation process is not inconsistent with a trapping picture either. It must be noted in this connection that the popular trapping estimates of the fluctuation level can always be made to agree since W is proportional to the fourth power of the trapping width. But from a trapping picture we would expect too that the tail would be formed in roughly the same time, after sloshing brings particles up to the phase velocity, and then extend to about $2\ c_s$. Clearly, neither the assumptions for trapping nor for linear Landau damping, which describes the onset of trapping, are fully satisfied, as we do have a wide two dimensional spectrum but at least at the time of tail formation, the energy in the wave motion is a substantial fraction of the unperturbed ion energy. Once the tail is formed, however, or if we start with less extreme temperature ratios, the assumptions for quasilinear interaction with the ions should be satisfied. We are confirmed in this by measuring the (irreversible) ion heating rate $\dot{T}_i^L$ which is found to be proportional to W and in agreement with linear Landau damping. Most of the energy goes into the ion tail. The quasilinear rates of dissipation have been estimated from the slopes of the electron and ion distributions. It is found that the linear growth rate reflects the behavior of the wave energy. Landau damping on the ion tail leads to quenching of the instability. By the same token we can attempt to explain the maximum fluctuation level as a dynamical maximum at which wave growth is overtaken by electron heating. Quasilinear theory gives reasonable estimates and no nonlinear limitation appears necessary. Using (2) and (5)

$$\left.\frac{W}{nT_e}\right|^{max} = \frac{2\gamma^L}{\rho\,(u/v_e)^2} \propto \left(\frac{m}{M}\right)^{\frac{1}{2}} \left(\frac{v_e}{u}\right)\left(1-\frac{u^*}{u}\right) , \tag{8}$$

where the critical drift velocity u^* is essentially determined by the ion tail. In particular, we can explain the observed scaling laws to which we now must turn. Assuming a model ion distribution (two temperature Maxwellian) and that the wave interaction occurs essentially with the ion tail, we can estimate the tail parameters from conservation of wave energy and momentum. It follows that[2] $T_2 \simeq T_e$, $n_2/n \propto (m/M)^{\frac{1}{4}}$ and that the resulting critical drift velocity u^*/v_e scales as $(m/M)^{\frac{1}{4}}$, Table 2. The ion tail becomes increasingly weaker as u/v_e and M/m increase. Accordingly u^* decreases and as seen from (8) this weakens the dependence of $(W/nT_e)^{max}$ on the initial conditions, Table 1-3. One striking feature of turbulent heating experiments has always been the relative constancy of the fluctuation level, $W/nT=10^{-3}-10^{-2}$, under widely different experimental conditions.

In the case of constant current, stabilization by tail formation is always possible and according to (8) the maximum of W and W/nT_e should roughly scale as $(M/m)^{\frac{1}{4}}$ and $(m/M)^{\frac{1}{4}}$ respectively. The mass ratio scaling is born out by our simulation experiments, Table 2. In the case of constant u/v_e there exists a limit $u/v_e|^{max}$ beyond which stabilization by tail formation is not possible, $u/v_e|^{max} \approx 7(m/M)^{\frac{1}{4}}$, which for not too large mass ratios is outside the ion sound regime. In the two stream regime stabilization is due to nonlinear effects on the electrons.

In conclusion, we have shown that the ion sound instability evolves into a damped state and that quasilinear effects are sufficient to explain our simulation experiments. At the observed fluctuation levels other non-linear effects such as nonlinear Landau damping or resonance broadening appear to be rather weak. Our conclusions seem to hold also for a wide range of experiments which develop similar fluctuation levels and effective collision frequencies and for which losses can be neglected during the rapid heating phase[12]. Naturally, the numerical values will be somewhat different in the three dimensional case, but there is no qualitative difference as there is between the 1D (very strong longitudinal magnetic field) and the 2D case. In agreement with our simulation in experiments too termination of heating, even before the current reaches its maximum[12], ion tail formation[12] and flattening of the electron distribution[14] is observed.

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11 Particularly for small mass ratio runs the thermal fluctuation level is not negligible. We have roughly accounted for this by assuming that W^{th} is given by W at an early stage (W_o) as $W^{th}/nT_e \propto 1/n\lambda_D^2$ in 2D.

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Scaling Laws

Table 1 Dependence on the initial temperature ratio $T_i/T_e|^o$ (M/m=100)

$T_i/T_e\|^o$	$W/nT_e\|^{max}(10^{-2});\omega_i t$	$T_i/T_e\|^f$	$T_e/T_e^o\|^f$	$\omega_i t^f$
0.02 ($u=v_e^o$)	1.40; 34	0.134	4.5	140
0.08 ($u=v_e^o$)	0.78; 27	0.114↑ *	2.2↑	53
0.5 ($u=\sqrt{3}\, v_e^o$)	0.87; 13.3	0.208↓	5.0↑	69

Table 2 Dependence on the mass ratio ($T_i/T_e|^o$=0.02)

M/m	$W^{max}(10^{-2}nT_e^o);\omega_i t$	$W/nT_e\|^{max}(10^{-2});\omega_i t$	$n_2/n\|^f$	$T_i/T_e\|^f$	$T_e/T_e^o\|^f$	$\omega_i t^f$
a $u=v_e^o$						
100	3.7; 51	1.4; 34	0.2	0.134	4.5	140
400	6.6; 68	1.25;41.6	0.13	0.093	9.0	137
1600	9.0; 91	1.04;50	0.08	0.046↑	14↑*	114
b $u=0.75v_e$						
100	3.9; 65	1.35;40	0.20	0.106	6.18	117
1600	2.7↑;64	1.0; 40	0.05	0.022↑	38.5↑	64.5

Table 3 Dependence on the drift velocity (M/m=100; $T_i/T_e|^o$=0.02)

	$W/nT_e\|^{max}(10^{-2});\omega_i t$	$T_i/T_e\|^f$	$T_e/T_e^o\|^f$	$\omega_i t^f$
$u=0.75v_e^o$	1.29;40	0.142	3	140
$u=v_e^o$	1.4 ;34	0.134	4.5	140
$u=0.6v_e$	1.19;52.3	0.126	4.13	131.5
$u=0.75v_e$	1.35;40	0.106	6.18	117

* ↑↓ : still increasing (decreasing) at end of computation, t^f.

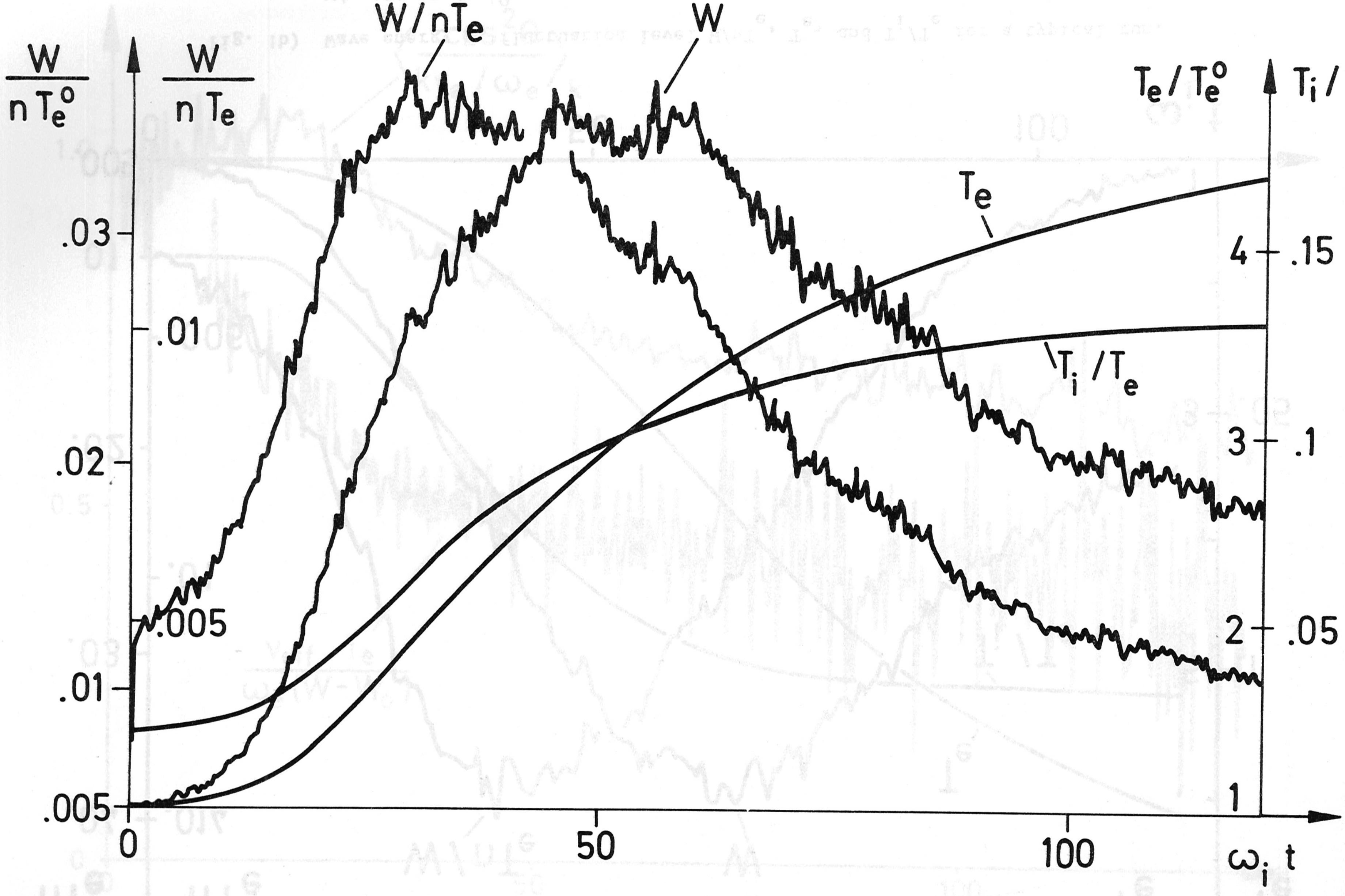

Fig. 1a) Wave energy W, fluctuation level W/nT_e, T_e, and T_i/T_e for a typical run. $M/m=100$, $T_i/T_e|^o=0.02$, const $u=v_e^o$

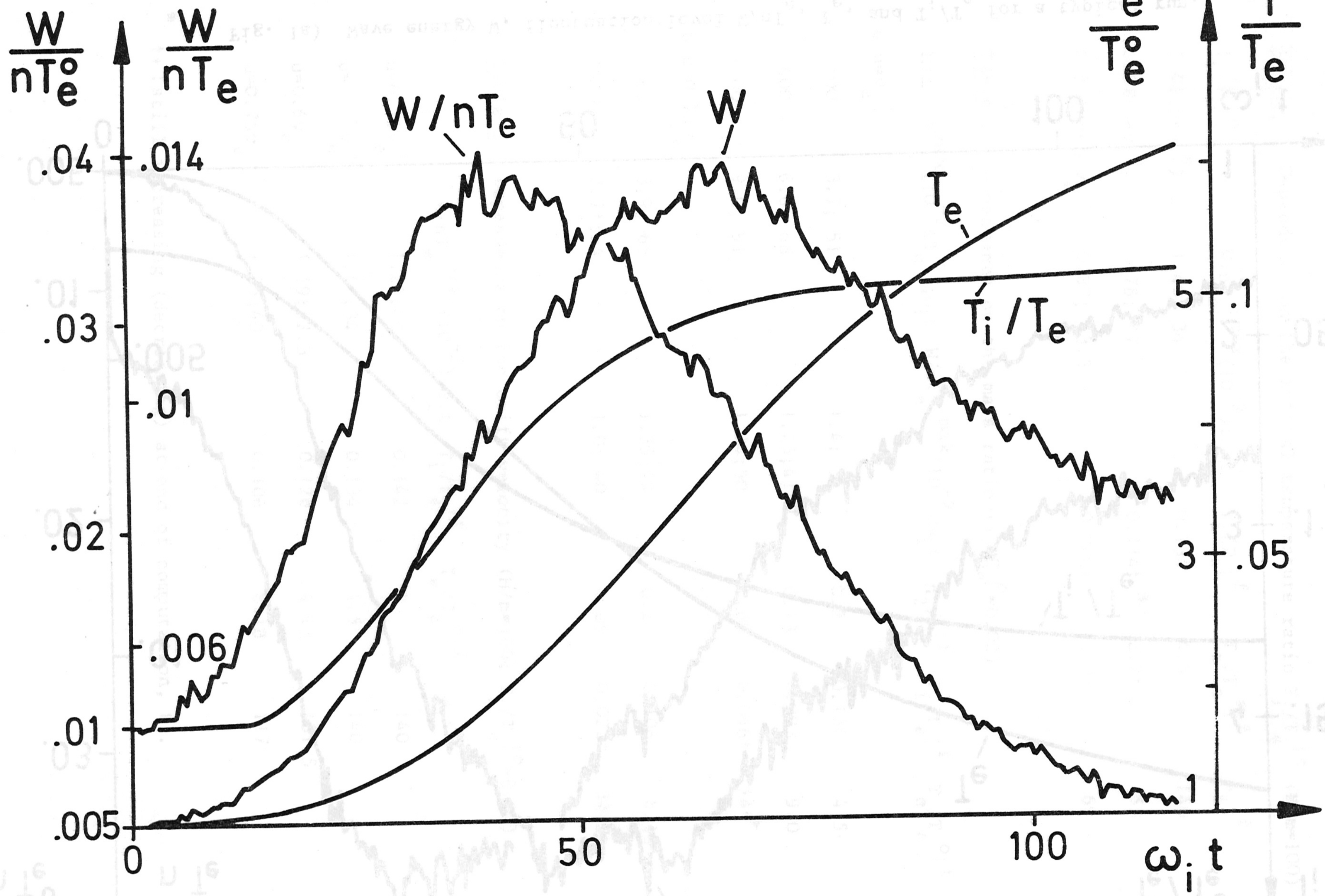

Fig. 1b) Wave energy W, fluctuation level W/nT_e, T_e, and T_i/T_e for a typical run.

$M/m=100$, $T_i/T_e|^o=0.02$, const. $u/v_e=0.75$

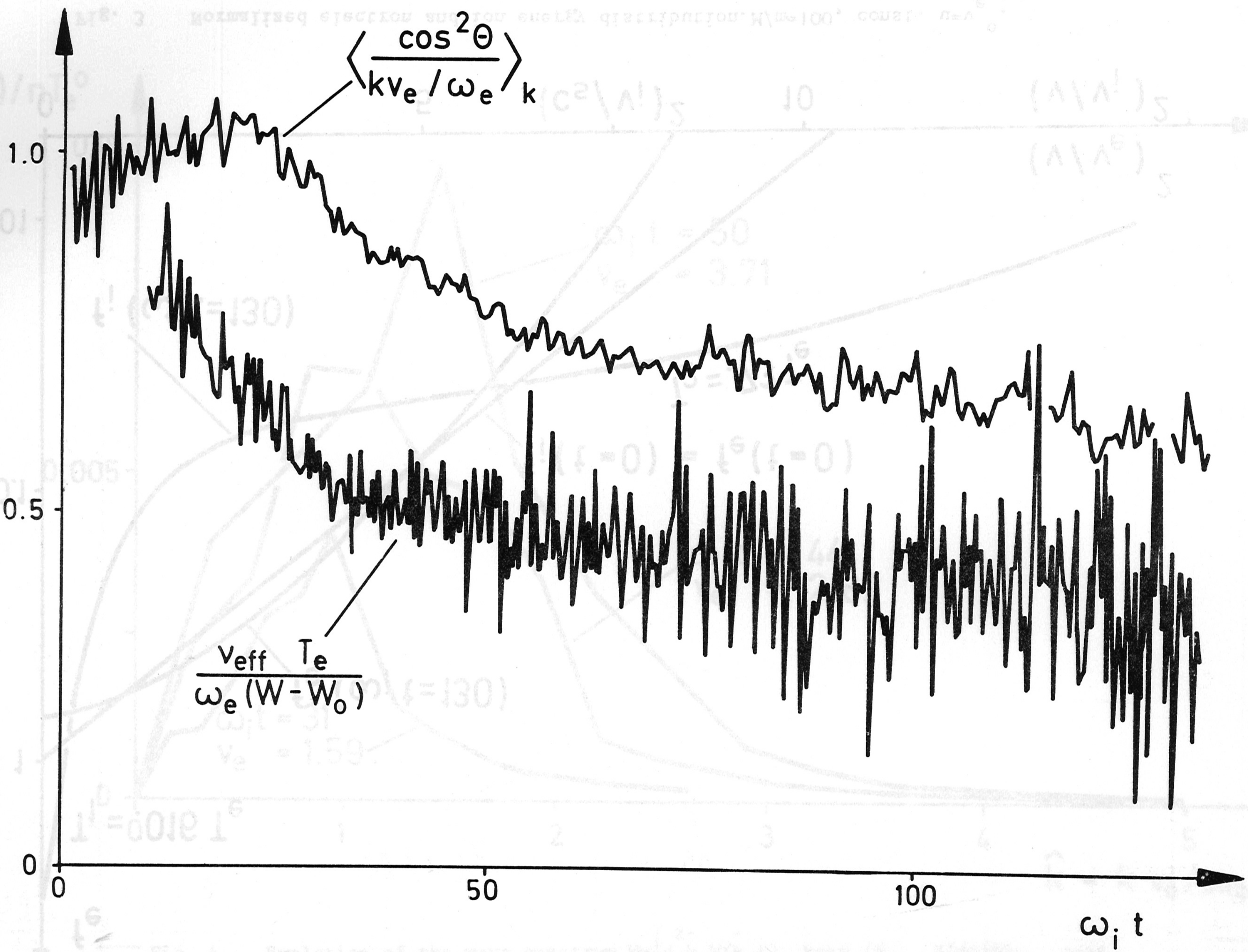

Fig. 2 Ratio between effective collision frequency and non thermal fluctuation level; Spectral form factor. M/m=100, $T_i/T_e|^o = 0.02$ $u=v_e^{\ o}$.

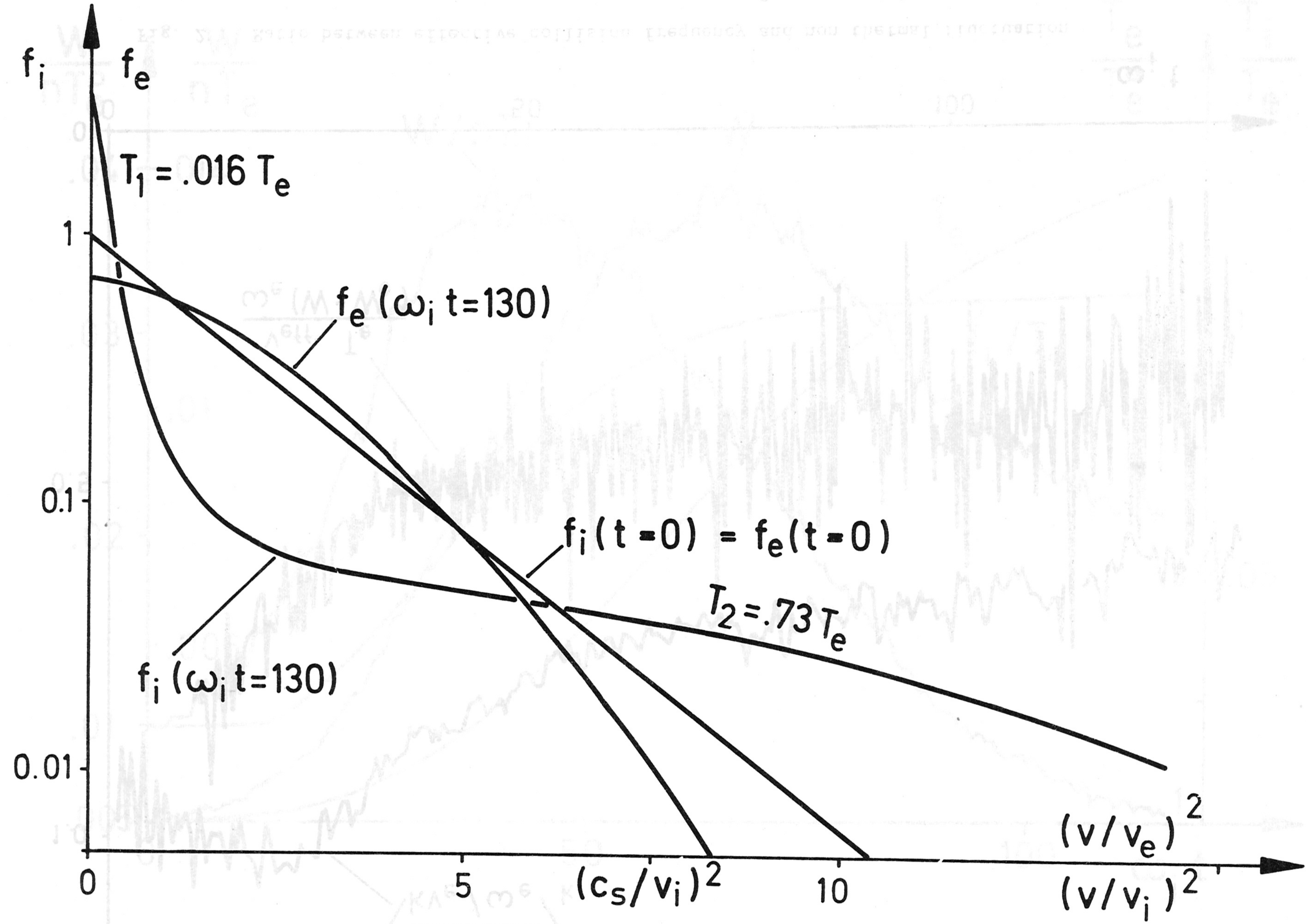

Fig. 3 Normalized electron and ion energy distribution. $M/m=100$, const. $u=v_e^{\,o}$. $T_i/T_e|^o=0.02$.

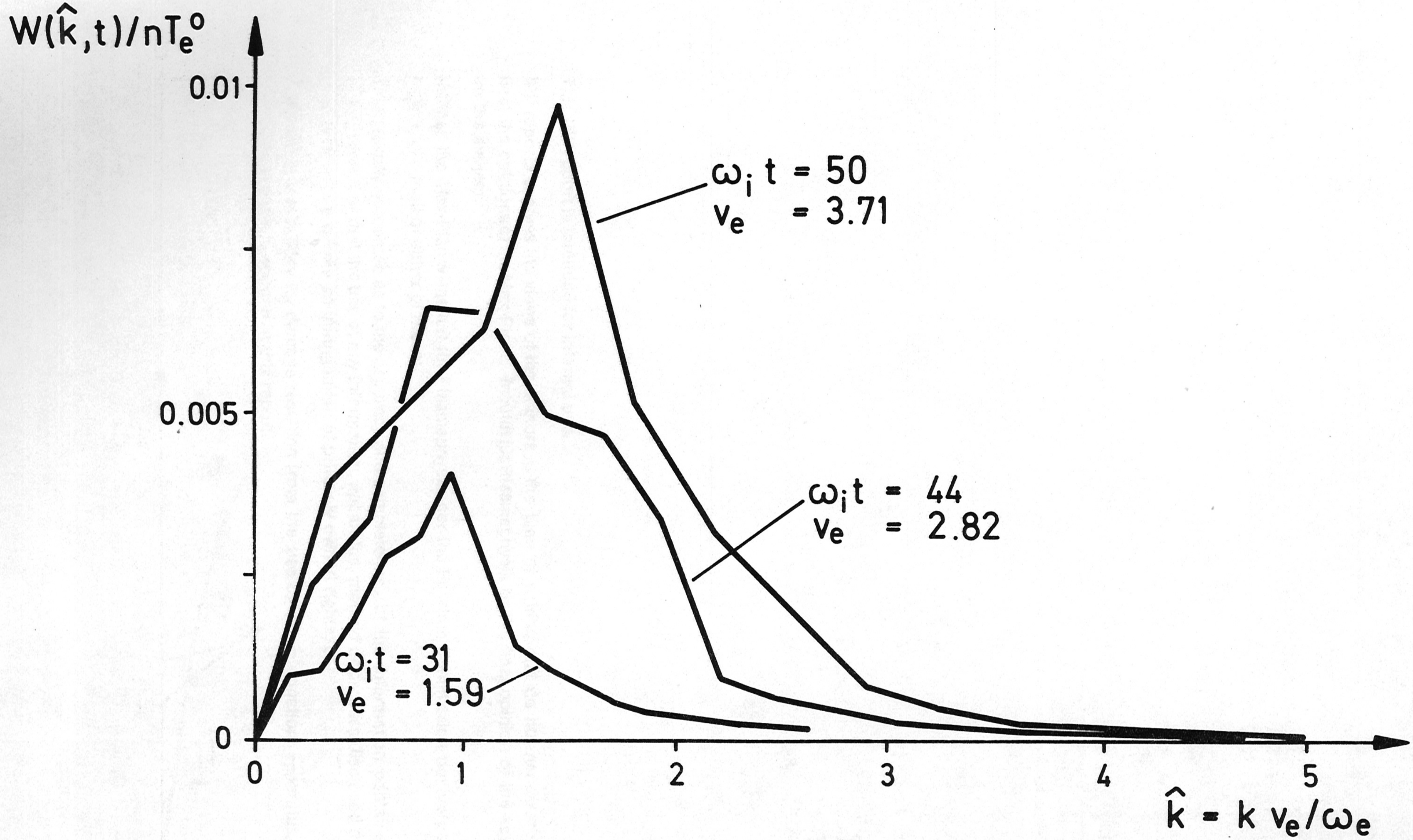

Fig. 4 Evolution of the wave spectrum $W=\int d^2\hat{k}\, W(\hat{k},t)$, $\hat{k}=kv_e/\omega_e$. M/m=1600, const. $u/v_e=0.75$, $T_i/T_e|^o=0.02$.