WALL-STABILIZED HIGH-BETA STELLARATOR

(Proposal)

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Wall-Stabilized High-Beta Stellarator

I. Summary

It is proposed that a wall-stabilized high-beta stellarator (HBS II)be built as a necessary continuation of the work performed by Division 1 in the Institut für Plasmaphysik at Garching and as a further advance towards a fusion reactor. As will be explained in detail later, a wall-stabilized high-beta stellarator is a slender torus with large aspect ratio on which a helical $\ell = 1$ disturbance is periodically superposed as basic disturbance/1-3/. This preserves the basic advantages of the theta pinch, viz. being able to bring plasmas close to the reactor relevant parameter range in a technically simple way with shock waves.

After the essential properties and objectives of the project are described, its reactor relevance and its relation to other projects are dealt with. The physical and technical data that are necessary to achieve the objectives set are presented at the end of this section. A detailed discussion of the points summarized here is given in the following sections.

The HBS may be regarded as an M-&-S-like equilibrium /4/ in which the magnetic surfaces are subjected to essentially helical deformation ($\ell = 1$) for the sake of stability. Stability of the long-wave modes ($m = 1$) thereby requires the proximity of a conducting wall, and so the shock produced plasma may not be further adiabatically compressed. Now that equilibrium has been investigated in experiments without sufficient influence of the walls (ISAR T 1, Scyllac), the objective of the planned experiment will be to investigate experimentally the stabilizing wall effect. The experiment should thereby be sufficiently flexible to allow investigation of the unstable regime, too, in order to ensure extrapolation to future stages.

Besides investigation of the long-wave modes, it will also be possible to observe short-wave modes under conditions much more strongly oriented towards a fusion reactor than hitherto. It will have to be checked whether short-wave modes still do not occur as before. What is required here is to understand the stabilizing influence of finite gyroradii by varying the plasma parameters. The first construction stage of the experiment is primarily directed at testing stability. If this first stage yields positive results, it is intended to increase the confinement time of the plasma in a second stage by lengthening the magnetic field pulse. It will then be necessary to study problems of diffusion and plasma-wall interaction. It is also intended in this stage to test methods of heating the plasma further. Heating with, for example, damped Alfvén waves will also be tested as a specifically high-beta method /5/.

A comparison of the proposed HBS with other projects for investigating toroidal confinement should, on the one hand, take into account the reactor concepts involved in the projects. On the other hand, the present state of development of the various approaches should be compared to establish what funds will be needed to tackle the most important questions in the very near future.

The HBS reactor hitherto drafted as a rough concept would be relatively slender (A ≈ 100) and capable of stationary operation /6/ at high beta (β ≈ 0.7). Compared with reactor concepts involving a relatively compact torus (Tokamak, low-beta stellarator (LBS), belt pinch), this slenderness does admittedly lead to an increase of the blanket and magnetic field volumes, but problems of boiler structure and accessibility such as are connected with, for example, heat transport, are more easily solved in a slender machine. Much more important, however, is the fact that with otherwise comparable reactor parameters a higher beta value appreciably reduces the magnetic field strength and hence superconductor costs despite the larger volume. This advantage over the LBS and Tokamak is further supported by problems of synchrotron radiation.

The Tokamak and belt pinch will probably be pulsed reactors in view of the toroidal plasma current. The HBS, too, has hitherto been intended for pulsed operation as proposed by Ribe et al./7/. As has been shown, however, by comparing superconductor costs, the HBS can easily work in stationary operation. This would obviate a number of problems, which need not be discussed in detail here.

When the individual lines of development to date are compared, it has to be admitted that the Tokamak has a distinct lead with regard to a stable confinement. But these are results at comparable low beta values. This reservation is even more true of the confinement results obtained with the LBS. The reason for working here with beta values well below those conceivable for reactors is the inefficient heating methods used. It seems that the next generation of large experiments (e.g. PLT, WVII) will still not afford any radical improvement in this respect either. Consequently, the important problems of equilibrium and stability at reactor relevant beta values in these lines of development cannot be investigated till a very late date and will thus entail a great deal of expense. The proposed HBS project, on the other hand, would already be capable of operating experimentally at a beta value corresponding to the target set for a reactor.

In the proposed experiment it should not be overlooked that a certain element of risk is entailed owing to the fact that stability has so far not been shown by a sufficient criterion. This, however, is true also for all toroidal configurations with beta values sufficiently high for reactors.

In addition to these basic considerations favouring further investigations aiming at a reactor, due allowance should also be given to the concrete competitive situation in this field.

At IPP experimental work was initiated after the decisive paper presented by Blank, Grad and Weitzner in 1968. A linear helical experiment was already successfully completed barely a year later, the first of its kind in this field. After some modifications of the linear experiment the first complete high-β-stellarator torus was put in operation at ISAR T 1. Different investigations showing the validity of the long wavelength MHD-theory have been carried out on this experiment.

During nearly the same period the Los Alamos group was conducting linear experiments at SCYLLA IV and 120^{o}-sector experiments at the new SCYLLAC-bank. Part of their work was aimed on feedback stabilization in the case of high compression ratio and no wall stabilization. Now experiments with low compression ratio are being planned also, but the project has not assumed a concrete form up to now.

There are some important differences in the choice of helical fields, aspect ratio and the reactor concept followed at Garching and Los Alamos. Thus it is felt that both experiments could supplement each other providing a reliable basis for later stages.

Besides the relation to the Scyllac project, the proposed project could be regarded as in competition with the belt pinch programme at IPP. The belt pinch experiments are not yet so far advanced as to allow a verdict on whether the belt pinch with a highly elongated cross section will permit high betas comparable with those of the HBS, or whether the belt pinch experiments will finally acquire less strongly asymmetric, small cross sections, and hence medium betas, and merge with Tokamak experiments. In the latter case the alternative between the belt pinch and HBS would basically become an alternative between the Tokamak and HBS.

The prospect of a reactor with high beta and the still moderate costs of experimentally investigating the stability question are arguments in favour of further promoting the belt pinch and HBS for the stages now planned. At the same time this should not be allowed to interfere with activities in the low-beta regime in view of the limited costs incurred.

The energy content and other data of the planned device are appropriate to the size and technological experience of Division 1 and the Engineering Division of the IPP, and so the estimates of costs ought to be relatively reliable.

As will be shown in more detail later, the following typical plasma and configuration parameters can be derived from the proposed objective, i.e. investigation of the MHD stability in a collisionless plasma:

Ion temperature	$T_i = 0.5 - 1$ keV
Electron temperature	$T_e = 0.3 - 0.5$ keV
Density	$n_e = 8 \times 10^{20} m^{-3}$
Beta	$\beta = 0.5 - 0.9$
Major torus radius	$R_T = 8$ m
Minor coil radius	$R_S = 10$ cm
Minor plasma radius	$R_p = 4$ cm
Radius of the helix	$R_h = 6$ cm
Helical periodicity number	$m \approx 25$
Magnetic field	$B \approx 0.6$ T
Energy content of the bank	$E \approx 1$ MJ

In spite of its geometrical size the device will be relatively cheap owing to the low magnetic field and the low energy.

Parallel with the planning and construction of the project, it is intended to conduct experiments of limited duration and extent. Although in the ISAR T 1 HBS long-wave modes are not influenced by the wall, present investigations of the MHD properties will be completed because they allow certain properties of the equilibrium and of instabilities to be studied. In ISAR II, a linear experiment, it is also intended to investigate a plasma with low compression ratio.

Besides the experimental work, theoretical studies will have to be intensified. The experiments should be backed up by, for example, calculating equilibria with a diffuse profile to analyse the influence of the stabilizing wall and the stability of short-wave modes.

Because of its brevity this rather general presentation of the HBS project proposal and its justification will be given in detail and elaborated in the following individual contributions. First the present experimental and theoretical situations are reported in Section II "Theoretical and experimental situations of the HBS". Arguments showing that slender high-beta tori are suitable for stationary or pulsed reactors are put forward in III "Reactor concept". In Sections IV "Requirements and parameters of the planned project" and V "Technical aspects and time schedule" the project is precisely defined and the ideas on planning and construction are given. Finally, investigations necessary for the technical and physical preparation of the project are described in VI "Supporting investigations".

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II. Theoretical and experimental status of the high-beta stellarator (HBS)

A) Introduction

The term "HBS" refers to toroidal configurations with the following essential properties:

$\beta = 2 \mu_o p/B_o^2 \approx 1$, no longitudinal current, non-planar magnetic axis, finite rotational transform and M-&-S-like equilibrium structure. The latter means that toroidal equilibrium is achieved by a deformation of the plasma surface similar to the classical M-&-S equilibrium /1/, and therefore compensating currents parallel to magnetic field lines are unimportant in contrast to present low beta stellarators (LBS).

It seems appropriate to include linear, helical equilibria in the HBS complex as simple limiting cases. The classical M-&-S equilibrium may also be regarded as a special case of the HBS with a planar magnetic axis and $\ell = 0$.

It is useful to give a few particulars of nomenclature in advance: For linear configurations we use cylindrical co-ordinates (r, θ, z) and retain them appropriately (z-axis in toroidal direction) in the toroidal case as well, which seems to be reasonable in view of the generally large aspect ratio ($A = R/r_p \gg 1$). The type of equilibrium is accordingly characterized by azimuthal and longitudinal wave numbers ℓ, h and by the respective plasma deformations normalized to the plasma radius $\delta_\ell = (r_{\ell,max} - r_{\ell,min})/(r_{\ell,max} + r_{\ell,min})$ In the case of small $\delta_\ell \ll 1$ the plasma surface is then given by a Fourier expansion (without toroidal correction):

$$r = r_p \left[1 + \sum_\ell \delta_\ell \cos(\ell\theta - hz)\right] \qquad (1)$$

The field period (i.e. "h") is the same for all components, i.e. the combination is no longer helically symmetric.

By analogy plasma disturbances are given by the wave numbers m, k (i.e. $\xi_r \sim \exp i[m\theta - kz]$). The quantity $\epsilon = hr_p$ essentially indicates the degree of slenderness of the plasma.

The theories used in the following were developed mainly in the last few years as extensions of the work of Blank et al. /2/ and Nührenberg /3/.

We shall dispense with a complete representation of all the toroidal HBS versions theoretically possible and concentrate from the outset on the theoretically most favourable case, viz. combination of an $\ell = 1$ stellarator field with additional, but weaker $\ell = 0$ and $\ell = 2$ components.

It should be pointed out that in this notation the classical M-&-S equilibrium represents a combination of an $\ell = 0$ field with symmetric $\ell = +1$ and $\ell = -1$ components. Here the symmetry about $\ell = 0$ produces a planar magnetic axis and closed field lines. A review of the work on the classical M-&-S equilibrium up to the Novosibirsk Conference is presented in /4/. As regards equilibrium, the difference between the two systems is slight. The superiority of the $\ell = 1/2/0$ combination first becomes apparent from the stability study and is due to the special role of the $\ell = 1$ component. The fact that the $\ell = 1$ stellarator has a non-planar magnetic axis dependent on the plasma pressure thereby seems to be of special importance /5/. We shall return to this point later.

In the following, the $\ell = 1/2/0$ equilibrium and its stability, particularly to long-wave $m = 1$ modes, will be investigated in detail. As a basis we take the more or less complete calculations in the surface current model (ideal MHD theory), the essential results of which are available in analytical form.

We shall then deal with a number of problems arising from real plasma properties (diffuse profile, finite gyroradius, finite conductivity, etc.) and in the case of resistive walls. Where experimental results are available, these will be enlisted to check theoretical predictions. Finally, the discussion is intended to reveal possibilities and patterns for future experiments and also provide stimulation for theoretical work.

B) <u>The toroidal $\ell = 1/2/0$ equilibrium and its stability to m = 1 modes</u>

The analytical expressions used in the following are valid for the ideal MHD theory and the surface current model. They are the result of an expansion in the vicinity of the theta pinch. The range of validity is therefore limited to large aspect ratio ($A = R/r_p \gg 1$) and many periods on the circumference. This, of course, by no means implies that there could not be equally favourable equilibria for smaller aspect ratios. Where results with diffuse pressure profile are available, these are briefly presented.

1) <u>Equilibrium</u> /6-9/

As is well known, M-&-S equilibria are characterized by the fact that the inner side of the torus is more strongly corrugated than the outer side, thus ensuring that the pressure balance is locally satisfied. Compensating currents as in the LBS are then inessential.

Such structures are generally obtained by superposing two helical fields with equal field period (equal longitudinal wave number h) and with multiplicity differing by 1 (azimuthal wave numbers ℓ and $\ell \pm 1$).

The condition for toroidal equilibrium with an $\ell = 1$ field in leading order is then:

$$\delta_1 \left[\delta_2 (2-\beta) + \delta_0 (3-2\beta) \right] = \frac{2}{h^2 r_p R} \equiv \frac{2}{\varepsilon^2 A} \equiv \frac{2A}{N^2} \equiv \frac{2}{\varepsilon N} \qquad (2)$$

where N = hR = number of field periods on the circumference,

$$A \gg 1, \quad \varepsilon(\delta_1 + \delta_2 + \delta_0) \ll 1, \quad \varepsilon = h r_p, \quad A = R/r_p, \quad \beta = 2\mu_0 p/B_0^2$$

(e.g. $A = 100$, $N = 20$, $\beta = 1$: $\delta_1 = 1$, $\delta_2 = \delta_0 = 0.25$).

The centre of mass of the plasma column thereby shows no toroidal displacement, i.e. it is concentric with the vessel and windings.

Equilibrium calculations with diffuse pressure profile have been made in linear geometry for $\ell = 0$ and $\ell = 1$ configurations /10, 11, 12/.

Toroidal equilibria with diffuse profile /13/ have also been obtained for the $\ell = 1/2/0$ combination. In this case solutions for arbitrary β and arbitrary profile shape have not yet been found, presumably owing to the special choice of functions, but the toroidal equilibrium condition for the special profiles for which solutions to second order exist is similar in form to that in the surface current model.

Experimentally, the existence of such toroidal equilibria has meanwhile been demonstrated in Garching /14, 15/ and Los Alamos /16/ (ISAR T 1, complete torus: $A \approx 135$, $N = 24$, $\beta \approx 6$ % ... 70 %, $\delta_1 = 0.8 \ldots 2$, $\delta_2 = 0.1 \ldots 0.3$, $\delta_0 = 0.03 \ldots 0.1$; SCYLLAC, 120° sector: $A \approx 250$, $N = 46$(?), only $\ell = 1$ and $\ell = 0$). In SCYLLAC good agreement with the theory is obtained for large aspect ratio; in ISAR T 1 with aspect ratio about half as large and correspondingly greater plasma deformation there are already slight deviations to higher values in the experiment.

It should also be mentioned that in previous experiments on classical M-&-S the agreement with theory was good.

It thus follows that the M-&-S idea in the generalized form can be successfully applied in experiments, and that it provides useful equilibria for arbitrary plasma beta.

2) Stability to long-wave m = 1 modes

A general result of theta-pinch-like experiments is that the most dangerous instabilities are the long-wave $m = 1$ modes. Other modes are only found under special conditions (e.g. $m = 2$ with very fast plasma rotation due to end effects).

a) Predictions of the theory /6,7,8,9,17,18/

The dispersion formula for the m = 1 mode of the above equilibrium is

$$\omega_{m=1}^2 = \Big\{ h^2 v_A^2 \Big[\Big(\frac{\beta^2}{\varkappa^4} - \frac{(2-\beta)(4-3\beta)}{8(1-\beta)} \beta \varepsilon^2 \Big) \delta_1^2 - \beta \frac{(1-\beta)(3-2\beta)}{(2-\beta)} \delta_0^2 - \beta \frac{(2-\beta)}{2} \delta_2^2 \Big] + (2-\beta) k^2 v_A^2 \Big\} \qquad (3)$$

$v_A^2 = B_o^2/\mu_o\rho$ = Alfvén velocity, $\varkappa = r_{wall}/r_p$, valid for:

$$1 \gtrsim \delta_1 \gg \delta_2, \delta_0 \; ; \; \varepsilon\delta_1 \ll 1 \; ; \; A \gg 1 \; ; \; \varkappa^2 \gg 1 \; ; \; \beta \lesssim 0.9$$

First it should be noted that this expression is merely the sum of the terms that are obtained in the linear case for the particular stellarator with a single component. Owing to the large aspect ratio, any influence of the toroidal curvature is not noticeable in this order. For discussion of the individual terms the relation is written for clarity in simplified form:

$$\omega_{m=1}^2 = h^2 v_A^2 \beta \big[\beta \delta_1^2/\varkappa^4 - f_1 \varepsilon^2 \delta_1^2 - f_2 \delta_2^2 - f_0 \delta_0^2 \big] + (2-\beta) k^2 v_A^2$$

$$\text{with } f_{1,0,2}(\beta) = \mathcal{O}(1)$$

The last term is the usual kink mode term. It yields stability for short-wave m = 1 modes.

The $\ell = 0$ and $\ell = 2$ components only have unstable terms ($\sim\delta_0^2$, $\sim\delta_2^2$) and are therefore made small.

On the other hand, in the $\ell = 1$ stellarator in leading order a stabilizing dipole wall term proportional to the plasma pressure occurs ($+\beta\,\delta_1^2/\varkappa^4$), and it thus becomes effective at high beta. In calculations with vanishing plasma pressure, of course, this term is not present. Only in next order ($-\epsilon^2 \delta_1^2$) an unstable term also appears, but owing to $\epsilon\, \delta_1 \ll 1$ this term only provides the same contribution as the small additional components.

Altogether the dispersion relation yields the stability condition:

$$\frac{\beta \delta_1^2}{\varkappa^4} > f_1(\beta)\,\epsilon^2 \delta_1^2 + f_2(\beta)\,\delta_2^2 + f_0(\beta)\,\delta_0^2 \qquad (4)$$

At the same time the equilibrium condition has to be satisfied (eq. 2):

$$\delta_1\left[\delta_2(2-\beta) + \delta_0(3-2\beta)\right] = 2/\epsilon^2 A$$

These two conditions define the m = 1 stable region of this equilibrium.

Optimizing with respect to ϵ, β and δ_0/δ_2 yields for the marginal limit a relation between A, $\varkappa$ and δ_1, which is represented here in Fig. 1 /19/.

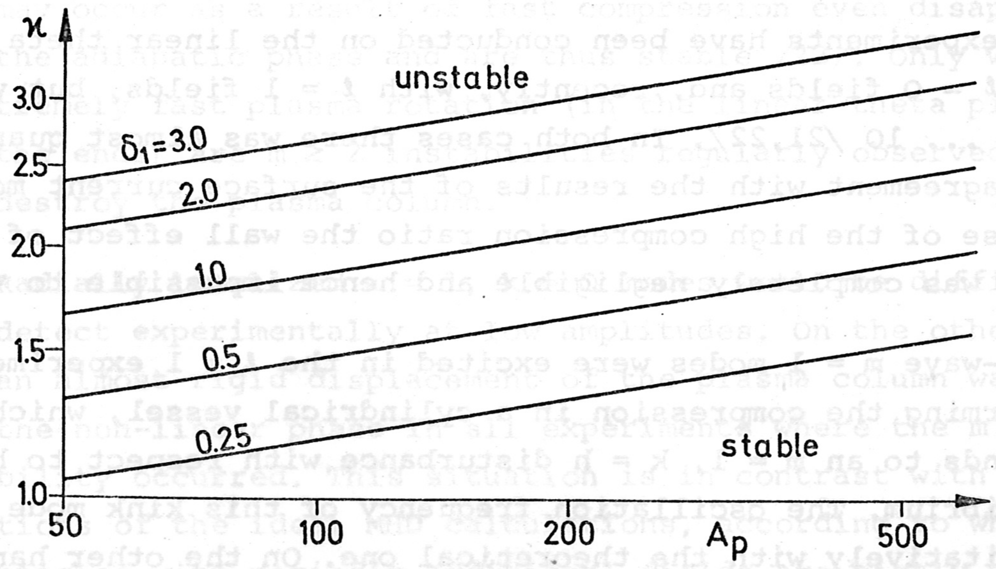

Fig. 1

For a given δ_1 all pairs of values (A, $\varkappa$) below the relevant curve are stable, while above it they are unstable. The optimum values thereby are $\epsilon \approx 0.14$, $\beta \approx 0.8$, $\delta_o/\delta_2 \approx 1$. It should be mentioned here that for $\delta_1 > 1$ the theory is no longer sure to be valid, and that the wall effect for very small $\varkappa$ may be stronger than is indicated by the $\varkappa^4$-dependence in $\omega^2_m = 1$.

Finally, some comment on the diffuse plasma profile is necessary: Where equilibrium and stability calculations with diffuse profile and comparable scaling are available, there is no serious disagreement with the above results /3,12,13,10,11/. As a new result it is found that with diffuse profile (and ideal MHD theory) $m = 1$, $k \approx 0$ instabilities can occur where the radial eigenfunction has one or more nodes /20/. Preliminary calculations /20/ show, however, that the wall effect is retained for these modes.

b) Experimental check of m = 1 stability

As mentioned, the δW of the toroidal system is given by the sum of the δW of the individual components in linear geometry. Linear experiments are therefore justified, provided that end effects are taken into account.

Such experiments have been conducted on the linear theta pinch with $\ell = 0$ fields and, recently, with $\ell = 1$ fields, but with $\varkappa \approx 5 \ldots 10$ /21,22/. In both cases there was almost quantitative agreement with the results of the surface current model. Because of the high compression ratio the wall effect of the $\ell = 1$ was completely negligible and hence impossible to verify.

Short-wave $m = 1$ modes were excited in the $\ell = 1$ experiments by performing the compression in a cylindrical vessel, which corresponds to an $m = 1$, $k = h$ disturbance with respect to helical equilibrium. The oscillation frequency of this kink mode agreed quantitatively with the theoretical one. On the other hand, the strong damping observed could only be ascribed to phase mixing in the diffuse profile (possible plasma heating for such equilibria?).

In the ISAR T 1 toroidal HBS the question of stability has not yet been finally clarified since definite identification of the instability is difficult because of the complex geometry and the time dependence of the equilibrium parameters. The lifetime of some torus drift times would, however, be consistent with the theoretically expected growth rate of an $m = 1$, $k = 0$ mode. Owing to $\varkappa \approx 10$ the wall effect is again completely negligible and in ISAR T 1 is impossible to check.

The SCYLLAC sector experiments yield very similar results and so do the older M-&-S tori, but the latter, unlike the $\ell = 1/2/0$, do not promise sufficient wall effect even with small wall distance.

A future experiment will therefore definitely have to determine the existence and size of the positive wall effect in the $\ell = 1$ stellarator.

C) <u>$m \geq 2$ modes and radially localized modes; finite gyroradius</u>

First it should be recalled that $m \geq 2$ modes and radially localized modes are not observed in the theta-pinch-like experiments. $m \geq 2$ deformations of the plasma column that may occur as a result of fast compression even disappear in the adiabatic phase and are thus stable /22/. Only with extremely fast plasma rotation (in the linear theta pinch due to the ends) are $m \geq 2$ instabilities regularly observed, which destroy the plasma column.

Radially localized $m = 1$, $k \approx 0$ modes would be difficult to detect experimentally at low amplitudes. On the other hand, an almost rigid displacement of the plasma column was found in the non-linear phase in all experiments where the $m = 1$ instability occurred. This situation is in contrast with the predictions of the ideal MHD calculations, according to which theta pinches with periodic additional fields are highly unstable for $m \geq 2$.

Furthermore, a mode analysis /10,11,20/ for the periodically deformed, linear theta pinch has shown that radially localized $m = 1$ modes may occur for diffuse profiles. With the bumpy theta pinch ($\ell = 0$) these do not exhibit the wall effect, which, of course, is very small in any case. For $\ell = 1$ symmetry, on the other hand, first results /20/ show that the positive properties, particularly the wall effect, are practically preserved, this probably being due to the finite rotational transform and shear of the magnetic field in the region of the pressure drop (in the pure $\ell = 1$ vacuum field the rotational transform is small, the shear almost zero).

The discrepancy between experiment and theory is probably due to the influence of the finite ion gyroradius ($r_p/r_g \approx 5 \ldots 10$). In fact, both general estimates and calculations for special geometries /24/ yield stabilization or at least a decrease in the $m \gtrsim 2$ growth rates by several orders of magnitude. Gyrostabilization of radially localized $m = 1$ modes has also been theoretically demonstrated for the case of the rotating theta pinch /29/.

Another explanation might be deduced from a recent calculation of Nührenberg and Herrnegger /30/. They investigate straight, diffuse $\ell = 1$ high beta equilibria using various local criteria. One result is, for instance, that at high beta the Mercier unstable region is reduced to the vicinity of the magnetic axis, that is, Mercier's criterion is fulfilled practically all over the plasma volume.

D) Special problems

1) Diffusion in the toroidal HBS

According to Nührenberg the HBS with $\ell = 1$ as main component belongs to a class of equilibria that have essentially no enhanced diffusion (Pfirsch-Schlüter effect). No problems thus arise with respect to the attainment of the equilibrium state in the case of finite conductivity /25/.

Under fusion conditions corrections in the sense of neoclassical diffusion are expected. It seems doubtful, however, whether present neoclassical theories are valid for high beta, M-&-S-like equilibria.

In the present state of the theory additional transport processes (pseudo-classical diffusion?) cannot be ruled out. However, the occurrence of, for example, drift instabilities seems less probable under theta-pinch-like conditions than in the Tokamak /26/.

2) Wall stabilization of the $m = 1$ mode with a resistive wall

This question is treated in /27/ and the result suggests the following model:

If an ideally conducting wall has to be located at $r_C = r_W$ according to stability condition (4), a resistive wall would have to be a distance of Δ closer to the plasma ($r_R = r_W - \Delta$) if all modes are to be stabilized with a growth time $\tau < \Delta / v_D$, where v_D is the diffusion velocity of the magnetic field in the resistive wall and the wall thickness is greater than Δ. The $m = 1$ mode is therefore not totally stabilized, but the growth rate is reduced to the value $\frac{1}{\tau} \approx v_D/\Delta$. Under realistic conditions it is quite possible to achieve values in the millisecond range.

3) Feedback and dynamic stabilization

From the foregoing it is expected that the long-wave $m = 1$ mode (and possibly the $m = 2$ mode as well) will have a growth rate in the millisecond range. According to estimates made in connection with current experiments both dynamic and feedback stabilization are easy to perform in the millisecond range, whereas stabilization in the microsecond range seems to be at least extremely uneconomic. If the growth rate of the instability is decreased by, for example, three orders of magnitude, the necessary ~~rise time~~ dB/dt of a feedback correction field is reduced by six orders of magnitude.

There already exist ideas for feedback systems (e.g. $\ell = 0$ or $\ell = 2$ fields with free phase relation to $\ell = 1$ /7/) and for dynamic stabilization /28/.

E) Summary and conclusions

The foregoing results impose a number of requirements on theory and experiment:

The crucial gap in previous experiments has been the failure to detect the $\ell = 1$ wall effect. This is due to the fact that all previous theta pinch systems work with strong, adiabatic compression and hence with $\varkappa \gg 1$. To achieve $\varkappa \leq 2$, pure shock heating has to be applied instead (magnetic field rise time $\approx$ compression time). A simple estimate reveals that the wall effect in linear geometry can only be verified with a very long coil, and so toroidal experiments should be given preference from the outset. Moreover, many special questions of equilibrium and stability ($m \geq 2$ modes, relaxation processes, long-term behaviour, marginal limits, etc.) have still to be clarified in detail.

On the theoretical side, there seems to be a need for extending and elaborating the existing toroidal equilibria, particularly with diffuse profile. For these equilibria a mode analysis of the dangerous low-m modes should be made.

In addition, it is essential to develop the theoretical models further towards large deviations from the theta pinch. In terms of present developments around the theta pinch this means that terms of higher order previously neglected as small will become important.

There is reason to believe that in this respect the $\ell = 1$ symmetry shows positive properties owing to the non-planar magnetic axis. According to Shafranov /5/, in this case there is a potential well which increases with plasma pressure, which improves stability with increasing β, and which does not exist at $\beta = 0$ (see also /30/).

The above discussed models already point in this direction as well (large δ_1, stability at high β). Also belonging to this regime are the helical equilibria which are stable according to sufficient criteria /28/.

It is hoped that extending the models to large deviations from the theta pinch will lead to an increase of the stable parameter range. This could mean that, for example, at the same beta the requirements for the aspect and compression ratios would be reduced.

At present, known results have to be enlisted as a guideline for designing an experiment. Then, as additional results become available, such an experiment should and can be modified in the direction required.

To sum up, the following should be mentioned:

There are toroidal, net-current-free high-β equilibria with M-&-S-like structure which should be m = 1 stable in an interesting parameter range. Except for the wall effect, these theoretical predictions have been experimentally checked and a large measure of agreement has been found. As for other instabilities, particularly $m \geq 2$ modes, it has so far been found that they do not occur under the usual experimental conditions. This disagrees with the simple ideal MHD models, but can be accounted for, in part at least, by the finite ion gyroradius and by diffuse high beta profiles.

Experimentally, the main objective in the next phase will be to prove the existence of the wall effect and hence of the m = 1 stable range. At the same time an attempt will have to be made to improve the theoretical models and, hopefully, to increase the stable range of operation.

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III. Reactor aspects of the high-beta stellarator

1) Introduction

Reactor studies, particularly those concerning economic aspects, and the restriction to output powers of less than about 20 GW that is imposed by the power grid and by the environment have led to the view that slender toroidal reactors with the aspect ratio $A \geqslant 100$ would not have much chance.

To ensure the most economic operation possible, it seems appropriate to optimize the mean output of a reactor boiler over the boiler volume with the boundary condition of given material load. Carruthers / 1 / has applied these arguments to toroidal fusion reactors under certain conditions and shown that optimum conditions will be obtained when the (minor) plasma radius is equal to the blanket thickness. The maximum possible load on the wall closest to the plasma thus already fixes the reactor power per unit length of the circumference. This is in the region of $100 \frac{MW}{m}2$, and so reactors with a large circumference (aspect ratio) fail because of too high power.

These arguments for reactors with a typical beta of $\beta \leqslant 10\%$ cannot, however, simply be applied to high-beta reactor concepts and used for comparisons of high and low beta. The costs for the reactor magnet, which constitute the major share of the total boiler costs, directly depend on the magnetic induction, and hence also on beta and, in a more complicated way, on the reactor dimensions.

To allow comparisons between toroidal reactors, all reactor parameters in the following will be expressed by the quantities which have to be given for economic, technical and physical reasons. Magnetic field costs, blanket costs and dimensions for various reactor types can thus be compared. It will be shown that slender reactors of the HBS type can compete with Tokamak and LBS reactors in a certain respect.

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2) Definition of suitable parameters for describing a toroidal fusion reactor

Economic factors and power grid conditions govern the reactor power L (thermal power in megawatts). Technological investigations will determine the maximum permissible load on the first reactor wall P_w, expressed in MW/m^2. The blanket thickness D, measured in metres, is prescribed by the moderation length of 14 MeV neutrons and by the shielding thicknesses. The conditions of the plasma physical equilibrium configuration can be roughly characterized by the aspect ratio

$$A = \frac{R_t}{R_p} \text{ and the beta } \beta = \frac{2nkT}{B_o^2/2\mu_o}$$

where R_t and R_p are the major torus radius and the minor plasma radius. Also needed is the plasma-to-wall distance, which is represented by the compression ratio $\varkappa = \frac{R_W}{R_p}$, where R_w is the minor radius of the first wall. Finally, it is useful to introduce a derived length $D_o = \frac{1}{2\pi}\sqrt{\frac{L}{P_w}}$. All other reactor parameters can be expressed by these six quantities: L, P_w, D, A, β and $\varkappa$, provided that the plasma temperatur is fixed. The latter is assumed in the following to be $T_e = T_i = 15$ keV on the basis of the maximum of the reaction parameter for the D-T reaction.

3) Magnetic field and blanket costs

A detailed analysis of the costs of superconducting magnets for fusion reactors has been given by Lubell and co-workers / 2 /. These include the costs for material, production, design of supports, thermal and electrical insulation, and finally for refrigeration. It was found that the numerous cost formulae can readily be combined into one approximation, according to which the magnetic field costs are

$$C_B(\$) = 3.48 \cdot 10^6 \, E_B^{4/5} \quad (1)$$

where E_B, the magnetic energy in gigajoules, ist

$$E_B(GJ) = 1.57 \cdot 10^{-2} \, B_o^2(T) \, R_t^3(m) \cdot \left(1 - \sqrt{1 - \left(\frac{r}{R_t}\right)^2}\right) \quad (2)$$

where R_t is the torus radius and r the minor coil radius. In the quantities introduced above r is $r = D_o\sqrt{\frac{\kappa}{A}} + D$. As the approximation formula according to / 2 / will be applied here to larger values, particularly with regard to the aspect ratio, than can probably be expected of it according to / 2 /, the approximation (1) was compared with the results of the more exact formulae in the ranges $3 \leq A \leq 200$ and $2 \leq B \leq 8$ tesla. This did not yield any deviations larger than those in Fig. 3 of / 2 /.

The specific magnetic field costs C_B, i.e. with reference to the reactor power L, are now calculated to be

$$(3) \quad C_B^*\left(\frac{\$}{MW}\right) = \frac{C_B}{L} = 1.93 \cdot 10^3 \frac{\kappa}{\beta^{4/5} P_w^{3/5} A^{1/5}} \left(1 + \frac{D}{D_o}\sqrt{\frac{A}{\kappa}}\right)^{8/5}$$

The dependence on β is obvious, while the dependence on the aspect ratio is proportional to $A^{-1/5}$ for small aspect ratios and proportional to $A^{3/5}$ for large A, depending on whether $\frac{D}{D_o}\sqrt{\frac{A}{\kappa}}$ is small or large relative to unity. Reactors are quite possible in the intermediate range, and so the complete expression has to be used.

The dependences of the relations (3) are represented in Fig. 1 in the case of a reactor with a thermal power of $L = 10^4$ MW and a wall load of $P_w = 10$ MW/m^2.

The solid curves represent isomagnetic field costs as a function of β and A for $\kappa = 2$ while the dashed curves show the costs for $\kappa = 1.25$. For the hatched regions theoretical predictions on the stability and properties of toroidal equilibria are already available from MHD approximations.

For stellarator reactors larger costs are expected than relation (3) yields due to additional investments for the helical field components. In case of an HBS the main $\ell = 1$ distortion can be produced by an helical displacement of the toroidal magnet coils from its circular axis. The cost increase should then enter into relation (3) mainly via the by a factor $\sqrt{1+(\delta_1 \varepsilon)^2}$ enlarged magnetic field volume. Since $\delta_1 \approx 1$ and $\varepsilon \approx 0.15$ (see chapter II) the helical field components add only a few per cent to the total magnet costs.

/ 2 / Lubell, M.S., H.M. Long, J.N. Luton jr., W.C.T. Stockdart; Oak Rige Nat. Lab. Rep. ORNL-TM-3927.

Schmitter / 3 / has also reported a formula for the costs of superconducting magnets which was obtained under certain optimizing conditions. According to this formula the costs of magnets scale in proportion to the surface area of the magnet, in proportion to B^2 and a factor $(1 + \frac{A_m}{A_m - 1})$, where A_m is the aspect ratio of the magnet. This relation somewath reinforces the influence of β relative to $B^{8/5}$ in Ref. / 2 / because B^2 appears here.

Taking a simplified view, it is possible to represent the blanket costs in proportion to the blanket volume V_B / 4 /:

$$(4) \quad V^*_{Bl}\left(\frac{m^2}{MW}\right) = \frac{D}{P_w}\left(1 + \frac{D}{2D_o}\sqrt{\frac{A}{\kappa}}\right)$$

To arrive at the costs $C_{Bl} = \bar{\lambda} V^*_{Bl}$, a factor $\bar{\lambda}$ of about 10^4 S/m^3 would have to be chosen according to very rough estimates. This corresponds roughly to the case A 1 of the blanket calculated by Steiner / 4 /. The blanket costs for the examples in Fig. 1 are thus almost always below the magnetic field costs, and so the high-beta stellarator represents an advantageous solution as regards the sum of the magnet and blanket costs.

4) Reactor and plasma parameters

It is also convenient to express the other reactor and palsma parameters by means of L, P_w, D, β, A and κ. As already stated, the numerical factors are abtained for $T_e = T_i$ = 15 keV and for a half deuterium and half tritium plasma.
($\langle \sigma v \rangle_{DT}$ = 2.84 · 10^{-16} cm^3 sec^{-1}).

This yields:

external magnetic field

$$(5) \quad B_o(T) = 1.625\,\beta^{-1/2}\left(\frac{A P_w \kappa^3}{L}\right)^{1/8}$$

electron density

$$(6) \quad n_e(m^{-3}) = 2.15 \cdot 10^{20}\left(\frac{A P_w \kappa^3}{L}\right)^{1/4}$$

/ 3 / Schmitter, K.H.; "Optimization of Superconducting Toroidal magnets for Tokemak Fusion Reactors", Garching, Report IPP 4/108, Jan 1973

/ 4 / Steiner, D.; Oak Ridge Nat. Lab. Rep. ORNL-TM-2360 plasma radius

(7) $$R_p(m) = 0.163 \left(\frac{L}{A P_w \kappa}\right)^{1/2}$$

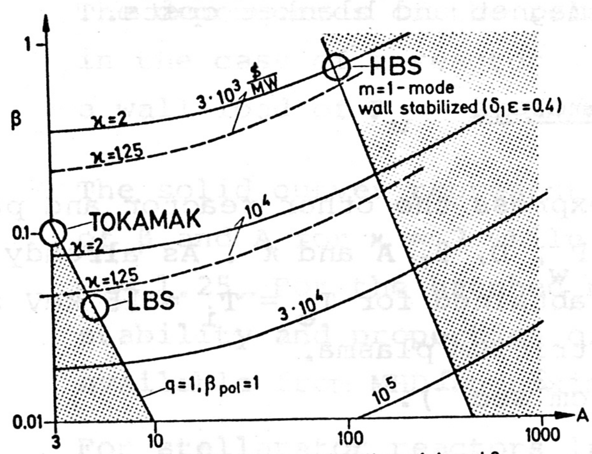
plasma volume

(8) $$V_p(m^3) = 8.55\cdot 10^{-2}\, A^{-1/2} \left(\frac{L}{P_w \kappa}\right)^{3/2}$$

plasma surface area

(9) $$O_p(m^2) = \frac{L}{P_w \kappa}$$

Two reactors types, HBS and Tokamak (or LBS), for a power of 10 GW and a wall of 10 MW/m^2 may serve to illustrate the difference in dimensions, parameters and costs. In the case of low beta two versions, one with $\kappa = 2$ and one with $\kappa = 1.25$, are treated. The field strength E is discussed in the following section.



Costs ot superconductors as a function of A and β
(L = 10 GW P_w = 10 $\frac{MW}{m^2}$ D = 1.5 m κ = 2 und κ = 1,25)

Fig. 2

Tabel I.

fusion reactor thermal power L = 10 GW
wall power loading $P_\omega = 10\ \frac{MW}{m^2}$
plasma temperature $T_e = T_i = 15$ keV

	A = 100, β = 0.7, κ = 2	A = 10, β = 0.1, κ = 2	A = 10, β = 0.1, κ = 1.25
$n\ (m^{-3})$	$6.5\cdot 10^{20}$	$3.6\cdot 10^{20}$	$2.5\cdot 10^{20}$
$B_0\ (T)$	3.35	6.7	5.6
$R_p\ (m)$	0.36	1.15	1.45
$C_B^*\left(\frac{\$}{MW}\right)$	$2.57\cdot 10^3$	$8.8\cdot 10^3$	$6.4\cdot 10^3$
$C_{Bl}^*\left(\frac{\$}{MW}\right)$ $(\bar\lambda \approx 10^4 \frac{S}{m^3})$	$3\cdot 10^3$	$2\cdot 10^3$	$2.1\cdot 10^3$
$E\left(\frac{kV}{cm}\right)$ $(T_i = 6\,keV)$	4.5	—	—

5) Comments on the question of pulsed or stationary HBS reactor

Los Alamos has developed the concept of a periodically pulsed reactor on the HBS principle. This concept, which has been modified several times, was last described in / 5 /.

The essential characteristics are: shock heating followed by adiabatic compression and refuelling in every cycle. The advantageous features of this prosposal are: shock heating, no divertor problem and no pumping losses in lithium. Uncertain elements are: normally conducting coil for the shock field in the blanket, necessity of economic storage of magnetic energy, momentary overloading of the wall and periodic temperature fluctuations in the blanket.

The points raised in Sub-section 3) have shown that as regards cost structure a stationary HBS reactor need not be at a disadvantage compared with low-beta concepts. For the start it is possible to apply shock heating up the kiloelectron volt region, in so far as a technically sound solution under reactor conditions can be found. Heating of a reactor with shock waves alone is technically hard to imagine, as the following relation, derived from the snowplough model, for the electric field strength required on the circumference of the first wall shows:

$$E\left(\frac{kV}{cm}\right) = 0.87 \cdot T_i(keV)\left(\frac{AP_w^3}{Lk^5}\right)^{1/8} \tag{10}$$

Attaining $T_i \approx 6$ keV for the reactor data used in Sub-section 3) would require field strengths of at least 5 kV/cm, which are technically almost impossible to control. For producing high beta in toroidal reactors, however, the initial heating to about 1 keV shock waves may be very important because the temperature range in which diffusion and hence the mixture of magnetic field and plasma are appreciable has to be quickly overcome. Further heating up to the final temperature can then be applied for longer times, e.g. by the heating methods discussed in low-beta concepts. These methods have to raise the temperature to at least

/ 5 / Burnett, S.C., W.R. Ellis, T.A. Oliphant, F.L. Ribe; "A Reference Theta-Pinch-Reactor", Los Alamos S.L. Report, LA-5121 MS (1972).

the ignition point. Various heating cycles are being calculated at present with the aid of MHD computer models.

An HBS reactor in stationary operation should not involve any more serious problems than a compact LBS reactor. For the same output power and wall load the same quantities of fuel and ash have to be injected and removed respectively through the same wall surface area.

Estimates show that in theory neo-classical diffusion dominates, and that, despite the smaller plasma radius, it is not greater for the HBS reactor than for Tokamak reactors. However, the extent to which the plasma losses in the individual concepts differ, i.e. the amount of power that has to be transferred via divertors, can only be shown by future experiments.

In any case the wall surface with less curvature in the major torus radius would entail only slight design problems and be amenable to the idea of modular design.

6) Remarks on synchrotron radiation

With respect to synchrotron radiation high-beta configurations should always be given preference first. According to Trubnikov and Kudryavtsev / 6 / the α particle energy dominates synchrotron radiation losses in a plasma optically thin to all frequencies at $T_e = T_i = 15$ keV beyond $\beta \geq 0.25$. According to simple calculations taking the radiation transport into account / 6,7 / the question below what frequency ω^* a plasma is to be regarded as optically thick is of prime importance in estimating the total power emitted. If ω^* is known, the synchrotron radiation is calculated from

$$(11) \qquad Q_B = \int_0^{\omega^*} S_B(\omega)\, d\omega + \int_{\omega^*}^{\infty} \sum_n I_n(\omega)\, d\omega$$

where S_B is the Rayleigh-Jeans approximation of black-body radiation and I_n is the intensity of the n-th harmonic which is emitted per m^2 from a disc of thickness ℓ. Calculations of I_n/S_B, normalized to a quantity $\mathcal{L} = \frac{\omega_p^2 \ell}{\omega_b c}$, are contained in. for example, / 7 /. Here ω_p denotes the plasma frequency and $\omega_b = \frac{eB}{m_e}$. For the length ℓ it is

/ 6 / Trubnikow, B.A., V.S. Kudryavtsev; Proc. Sec. United Nat. Int. Conf. on the Peaceful Uses of Atomic Energy, Geneva, Vol.31, p. 93 (1958).

convenient to put the plasma diameter 2 R_p. For given values of the frequency ω^* (for $\alpha_n \ell = 1$) can be found from the calculated values $\frac{I_n}{S_B} = \frac{\ell \alpha}{\mathcal{L}}$

In the reactor parameters introduced $\mathcal{L}$ becomes

(12) $$\mathcal{L} = 2.6 \cdot 10^3 \left(\frac{\beta}{1-\beta}\right)^{1/2} \left(\frac{L^3}{A^3 P_W K}\right)^{1/8}$$

The synchrotron radiation and the quantities $\mathcal{L}$, $\frac{\omega^*}{\omega_b}$ and ω_b necessary for calculating it are contained in Table II for the examples discussed here. The figurs show that in the HBS case the plasma remains longer optically thin, measured in ω_b units than for the other cases. Since, however, ω_b is much smaller than in the low-beta cases and since $\int S_B(\omega) d\omega \sim \omega^{*3}$, the radiation power obtained for the HBS is more than an order of magnitude smaller. This direct comparison is possible because reactors with the same output, wall load and compression ratio have the same plasma surface area, if the plasma temperatures are also equal.

Trubnikov and Kudryavtsev / 6 / also introduced a so-called transparency factor $K_{\mathcal{L}}$, which gives the ratio of the synchrotron radiation emitted from an opaque palsma to the synchrotron radiation that would be emitted if the plasma were optically thin. When expressed by the quantities used here the transparency factor is

(13) $$K_{\mathcal{L}} = 20.6 \frac{\beta}{1-\beta} \frac{Q_B}{P_W K}$$

Here use was made of the approximation given in / 6 / for the integral synchrotron radiation in the optically thin case. Again expressed in reactor parameters, this can be written.

(14) $$Q_{B(\text{ vacuum })}\left(\frac{MW}{m^2}\right) = 0.57 \frac{1-\beta}{\beta} \left(\frac{A P_W K^3}{L}\right)^{1/2}$$

Values of K for the reactor examples are contained in Table II.

/ 7 / Hirschfield, J.L., D.E. Baldwin, Sanborn C. Brown; Phys. Fluids, Vol 4 N.2. p.198 (1961)

Table II

	A = 100 $\beta = 0.7,\ \chi = 2$	A = 10 $\beta = 0.1,\ \chi = 2$	A = 10 $\beta = 0.1,\ \chi = 1.25$
Q_B ($\frac{MW}{m^2}$)	$5{,}25 \cdot 10^{-3}$	$1.72 \cdot 10^{-1}$	$1.06 \cdot 10^{-1}$
ω_b (sec^{-1})	$3 \cdot 10^{11}$	10^{12}	$9 \cdot 10^{11}$
$\frac{\omega^*}{\omega_b}$	6	4.7	4.7
$\mathcal{L}$	$1.4 \cdot 10^{4}$	$2.5 \cdot 10^{3}$	$2.6 \cdot 10^{3}$
$K_{\mathcal{L}}$	$1.28 \cdot 10^{-2}$	$1.9 \cdot 10^{-2}$	$1.9 \cdot 10^{-2}$

IV. Determination of the project parameters

In determining the parameters of the proposed project it should be borne in mind that the question of MHD stability is the main object of the experiment. It shall be proved in this experiment, that the wall can stabilize the plasma as predicted by theory. There is a chance here of clarifying the important question of stability at a reactor relevant beta value with comparatively little effort and at a relatively early date. If a positive result could be obtained, a very important requirement for a fusion reactor (see Section III) would be satisfied.

Quantitatively, a check of MHD stability requires that sufficient characteristic times for the growth of instabilities are provided by the length of the magnetic field pulse. It requires further sufficient flexibility to investigate both the stable and unstable regimes. The need to work in the vicinity of the marginal limit of the (m = 1) mode results in a relation between the relative quantities: aspect ratio A, compression ratio $\varkappa$ and the normalized helical deformation δ_1. This relation is represented in Fig.1 of section II. To obtain a low value of A suitable for a reactor, $\varkappa$ and δ_1 habe to be chosen small and large respectively. There are, however, limits to reducing $\varkappa$, mainly owing to the increased technical outlay involved. With the same requirements on the plasma parameters extremely steep current pulses will be needed if $\varkappa$ is to become smaller. An estimate of the capacitor bank costs as a function of $\varkappa$ shows that below $\varkappa = 2.4$ there is a steep rise in costs*) (see Fig.3). Hence $\varkappa = 2.4$ was taken for standard conditions. A variation of $\varkappa$ in the experiment is still possible by an adiabatic expansion or compression after the first shock implosion (see Sect. V).

*) The compression ratio was determined by a free particle code. With fixed values for T and n the bank voltage U has to be increased to get lower $\varkappa$ values. The costs were set proportional to U^2.

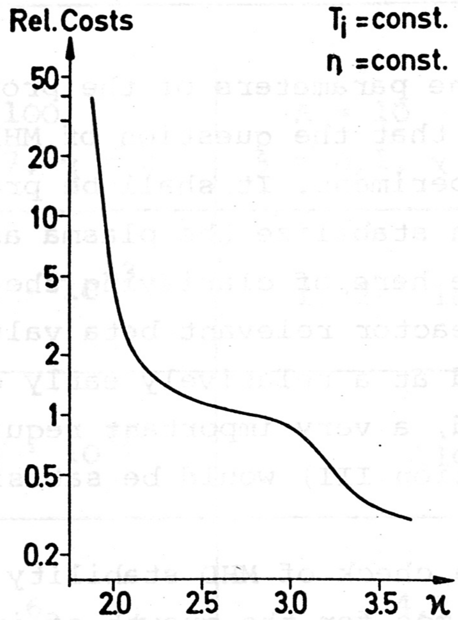


Fig. 3

In determining the helical deformation δ_1 the situation is different. According to our present knowledge, increasing δ_1 should take us into a generally favourable regime (e.g. smaller aspect ratios) and experiments in this direction should certainly be tried. On the other hand, if δ_1 is chosen noticeably larger than 1, this takes us out of the regime in which the present theories are valid. Therefore, for the first stage of the proposed project we should choose $\delta_1 = 1.5$. This value would then yield a plasma aspect ratio of 200 and an aspect ratio of approx. 8o with reference to the inner coil surface.

Once the relative parameters habe been settled the absolute parameters: temperature, density and a linear dimension, have to be fixed. The initially imposed requirement, i.e. the need to check the MHD stability, has to be made more stringent by postulating collisionless conditions.

Since collisionless conditions are not expected to have any influence on very long wavelengths, the requirement "mean free path of the ions approximately equal to the periodicity length of the helical disturbance" represents a reasonable compromise. As the linear dimensions can only be varied within limits for other reasons, this requirement results in siffuciently high temperature and suffiviently low density.

There are various arguments against appreciably reducing the density and hence the requirement imposed on the temperature and consequently the expenditure:

1) Preionization at low densities proves to be difficult. If a high degree of ionization of the neutral gas is needed, the long ionization time at low densities leads to widespread wall contact by the plasma (the possibility of magnetic field confinement during preionization is a matter of further development). Consequently, there would be a high degree of impurity.

2) Because it is hardly completely preionized, the plasma should be post-ionized during the first compression. A strong neutral gas background such as has occurred in pinch experiments with very low densities would cause appreciable disturbance in a confinement experiment. This requirement is much more stringent than requirement 1.

3) If the temperature is too low, this would lead to rapid cooling of the plasma as a result of radiation losses. With an impurity concentration of, for example, 1% oxygen the temperature must be above about 100 eV/1/.

4) The density and temperature should not be too far removed from the target values of a reactor ($n \approx 2 \cdot 10^{21} m^{-3}$, $T_i \approx 15$ keV; see Section III) since a number of problems which have not been discussed in detail here (diffusion, recycling, etc) will have to be investigated in a reactor relevant regime as early as possible.

The requirement of sufficiently long mean free path with allowance for the previously determined relative parameters is expressed quantitatively as follows:

(1) $T_i\,[\,keV\,] \sim (\,n[10^{21} m^{-3}]\; R_s\,[m])^{1/2}$

T_i: ion temperature, n: density, R_s: minor coil radius,
The additional need for complete ionization of the plasma during the first compression*) (point 2) leads to the following relation:

(2) $T_i\,[\,keV\,] \leqslant 170\;(n\,[10^{21} m^{-3}]\; R_s\,[m])^2$

Combining eqs. (1) and (2) yields for the ion temperature

(3) $T_i \geqslant 200$ eV

and for the lower limit of the product of the density and radius

(4) $n\,R_s \geqslant 4\,.\,10^{19} m^{-2}$.

As the temperature is still very close to the limiting value at which radiation losses quickly lead to cooling of the plasma (point 3), and further more in order to have a sufficient working range, the attainable temperature should be fixed at 500 eV at least. This reserve is specially necessary, if $\varkappa$ is lowered by adiabatic expansion (see. page 31). For a final temperature $T =$ 500 eV a two-dimensional ion temperature of $T_{i\perp} \approx 1$ keV has to be attained by the fast compression. This value was taken as a basis for the technical concept.

The absolute linear dimension is essentially determined from experimental arguments. An analysis of the capacitor bank costs shows that they decrease with decreasing radius although the density will increase according to relation (4). The limit here is primarily

---

*) We set time of ionization ti = time of compression tc, with ti = $(\frac{n}{2}<\sigma\, v>)^{-1}$, $<\sigma\, v> \approx 10^{-14}\, m^3\, s^{-1}$, and tc = $R_{vessel}/V_{th,ion}$.

set by largely fixed technical dimensions. Vessel wall thicknesses and insulation thicknesses hardly allow rescaling and, since theses parameters have to be negligible - the metal wall has to be close to the plasma -, a lower limit is set.

The necessarily short rise time of the magnetic field also impose a lower limit on the radius. Below a few hundred nanoseconds new problems are incurred in the start and crowbar spark gaps (e.g. the heating time for the spark becomes reasonable).

In consideration of these arguments the inner vessel radius should be fixed at 8 cm and the mean coil radius at 10 cm. If a density of about $8 \cdot 10^{20} m^{-3}$ in the compressed state is then aimed at, condition (4) is satisfied with a sufficient safety margin.

The last of the important experimental parameters to be determined for the first stage is the observation time allowed by the pulse length of the magnetic field. The required observability of m = 1 instabilities should be expressed quantitatively as follows: If it is assumed that the stabilizing wall influence were absend, then it should be possible to observe at least ten growth rates. The data already determined and the expression for ω^2 (eq. (3) in Section II) yield a minimum time of $\tau_b \geq 40$ µs.

If the pure MHD theory is assumed, shorter times can be derived for high m modes, and so there is no further increase in stringency.

The standard parameters of the project are compiled in the summary (s. page 5).

/1/ Düchs, D., W. Engelhardt, W. Köppendörfer, publication under preparation.

V. Technical Concept

1. General considerations

Compared with older theta-pinch experiments the proposed experiment is different from the technical point of view mainly in two respects:

1.) The rise time of the magnetic field is only about o.5 /us. Afterward the field has to be maintained for at least 4o /us, during this time the field variation should not exceed 1o% of the mean value.

2.) The magnetic field peaks at only o.6 T.

Though the magnetic field is fairly moderate compared to previous experiments the short rise time leads to an 8o kV charging voltage of the start battery. Among the different circuit approaches to achieve the constant current for a 4o-5o /us period an active crowbar system utilizing a delay line network was chosen.

Both the start and crowbar battery can be designed with elements of known reliability, as fas as capacitors are concerned. For the start and crowbar switches, present day technology offers different types for 4o and 8o kV developed inhouse, as well as in other CTR-facilities. The concept given here will be tested in two stages, the first to select under the given variety of elements and the second a scaled down e.g. 1 m Sector module for a check of the reliability of all elements. Stage I is under way and first results will be given at the end of section V.

The power crowbar circuitry offers some flexibility in plasma control. Different charging voltages of the crowbar delay will result in either a flat, a rising or a decaying field after the first rise.

This will allow to vary the plasma radius over a wide range by adiabatic expansion or compression.

No serious problems shall be expected with the mechanical design of the toroidal collector and coil. Due to the moderate field below 1 T, the mechanical forces and the low current densities at contacts will allow the use of standard materials.

The given cost analysis is based on prices from known elements proposed for the system and inhouse experiences with previous experiments.

2. Design principles of a low inductance bank system

The bank design is based on the proposed parameters of the experiment:

a) Plasma system

Toroidal coil		
Torus diameter	D_T	= 16 m
Coil diameter [1)]	d_{coil}	= o.2o m
Vacuumvessel	d_{vessel}	= o.16 m
Coil inductance	L'_{coil}	= 39.5 nH
(1 m section, without plasma)		

b) Magnetfield

Risetime	t_c	= o.5 /us
Amplitude	B	= o.6 T
Pulse Duration	t_D	= 4o - 5o /us
Compression ratio (with respect to the tube wall)	K	≈ 2
Field ripple	$\frac{\Delta B}{B}$	$\approx$ o.1

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1) For the bank layout a circular crosssection was assumed, however as discussed later on the final system will have an elliptical crosssection.

The layout of the main energy storage system was calculated to fullfill the above parameters. A simplified model was adapted for the plasma interaction on the resulting coil inductance. According to the "snow plough model" /1/ the plasma compression takes place with nearly constant velocity and this leads to a simple time relation for the combined system inductance:

$$L_{load} = \frac{4\,\pi^2}{l_{coil}} \left[r^2_{coil} - r^2_{vessel} \left(1 - \frac{t}{2t_c}\right)^2 \right]$$

This assumes sufficiently preonized plasma and the magnetic flux contained only between plasma boundary and coil wall. After determining the technical parameters the electrical circuit together with the plasma was calculated in a selfconsistent model. The plasma ions were treated in this model in a particle code while the electrons were assumed to be a fluid.

After preionization of the plasma, the start battery discharges through the coil, the current and hence the magnetic field rise within the time t_c to its maximum values in a damped sinusoidal shape. In Fig. 1 the principle bank circuitry is shown. At the time t_c the crowbar switches are closed. The stored energy in the "active crowbar" system now holds the current constant during the characteristic delay line time

$$t_D = m \cdot \sqrt{L \cdot C} \approx 4o - 5o\,\mu s$$

The efficient energy transfer from the bank to the load calls for a low impedance start- and crowbar system and consequently a subdivision into n-units is necessary. With known switching capability of already developed sparkgaps n may vary from 2oo - 1ooo. If 8o kV pressurized switches each for 5o - 6o kA current maximum are assumed, a total number of 5oo subunits is necessary. A possible layout of the bank system is showed in fig. 2.

/1/ Köppendörfer, Dimensionierung und Vergleich von Z - Pinch, Antipinch und Thetapinch .... IPP 1/28, 1964

The bank parameters have been calculated on the basis of the wanted plasma parameters. In Table 1 the electrical data of the HBS-16 m torus experiment has been listed together with the data of the running ISAR T I experiment.

The technical design of the main field battery will be based on the results of stage I and II development phase. During this phase an extensive testing of elements (capacitors and switches) will take place. Especially on the switch side the known principle of simultaneous triggering /2/ will be evaluated for three electrode pressurized sparkgaps. The use of multichannel switches field distortion sparkgaps /3/ will allow a low inductance bank design at tolerable costs.

The theoretical network analysis for the power crowbar circuit /4/ has been carried out /5/ for the proposed bank design. Wave forms have been found to be very similar for both, the simplified "snow plough" model and the more detailed "free particle model". In fig. 3a the computed current wave form is shown and compared with the wave form obtained experimentally in a test unit which supplied a 1o cm coil section. The 8o kV test unit was built from standard 4o kV metal case capacitors and 8o kV pressurized sparkgaps. The use of simple design sparkgaps /6/ in a 4o kV elementary unit results in less problems in the trigger circuitry, but the at least two-fold increase in the number of switches will lead to a greater complexity of the bank arrangement. A time forced test procedure during the two development stages I and II shall result in the final bank design according the time schedule given in following sections.

/2/ Barnes, Gruber, James; High Current Switching using low inductance field distortion sparkgaps closely connected in parallel. CLM-R71, Culham Report 1967; Journ. Scient. Inst. 1967, Vol. 44, pg. 544

/3/ James, Gruber; The initiation of multiple arc breakdowns in a 60 kV low inductance sparkgap.
VIII the Int. Conf. Phenom. Ionized Gases, 1967, Vienna

/4/ Durand, Klüber, Wulff; Eine Anlage zur Erzeugung kurzzeitig konstanter Magnetfelder.
Zeitsch. Angew. Phys. Bd. 12, H.9., S. 393

/5/ Preis; Die Analyse transienter Vorgänge in linearen elektr. Netzwerken, IPP 4/87/1971

/6/ Klement, Wedler; Switch Systems for High-ß-Experiments 7th Symp. on Fusion Technology, Grenoble 1972,
EURATOM EUR 49 38 e. Pg. 373

TABLE 1 Bank Parameters Comparison ISAR I - HBS

			High-ß-Experiments		Techn. Development Program	
			I S A R I 2,7 m TORUS	H B S 16 m TORUS	Stage I Basic Unit	Stage II 1 m Coil - Test Rigg
Compression System (Start Battery)	Stored energy	kJ	2600	400	0,8 1 2	8
	Capacitor voltage	kV	40	2x40	2 x 40	2 x 40
	Capacity	µF	3270	125	0,25 0,31 0,62	2,5
	Bankinductance	nH	2,53	0,32	100 80 40	16
	Current	kA	24.10^3	23.10^3	46 58 116	460
	Risetime	µs	8	0,5	0,5	0,5
	Number of Basic Units		252	200 400 500	1 1 1	4 8 1o
	Current per Unit	kA	95	116 58 46	46 58 116	1160 580 460
Sustained field Syst. (Delay Line PFN)	Stored energy	kJ	-	1550	3,1	31
	Impedance, $Z=\sqrt{\frac{L}{C}}$	Ω	-	6.10^{-5}	30.10^{-3}	3.10^{-3}
	PFN-Voltage	kV	-	2,7	2,7	2,7
	Pulse Length	µs	passive Crowbar: 150 us	50	50	50
	Number of Basic PFN's		-	50 x 10	1	10
Load	Geometry		$\emptyset_T$ =2,7m d=22cm	Ø=16m d=22cm	linear coil	linear 1 m coil
	Inductance L_{To}/nH		5.6 (without plasma)	0,66	330 264 132	33
	Magnetic Field	Tmax	4,5	0,6	~0,3 - 0,5	~0,55

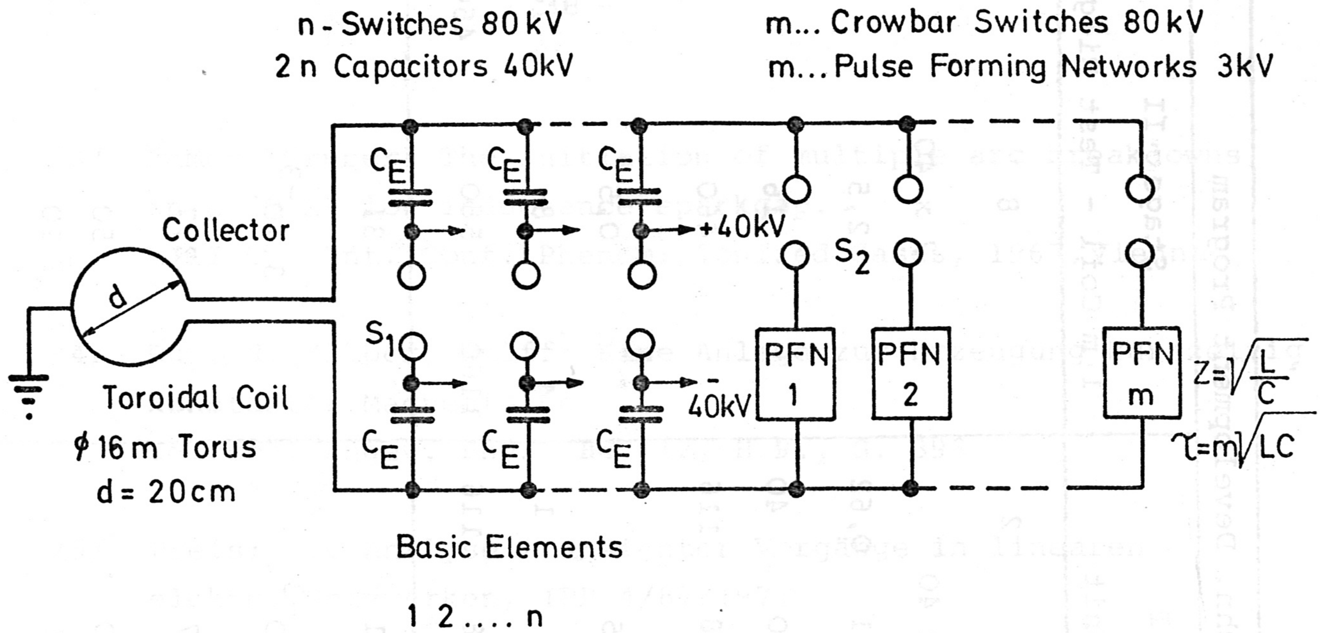


Fig.1 Principle Circuit of 80/3 kV Power Crowbar System

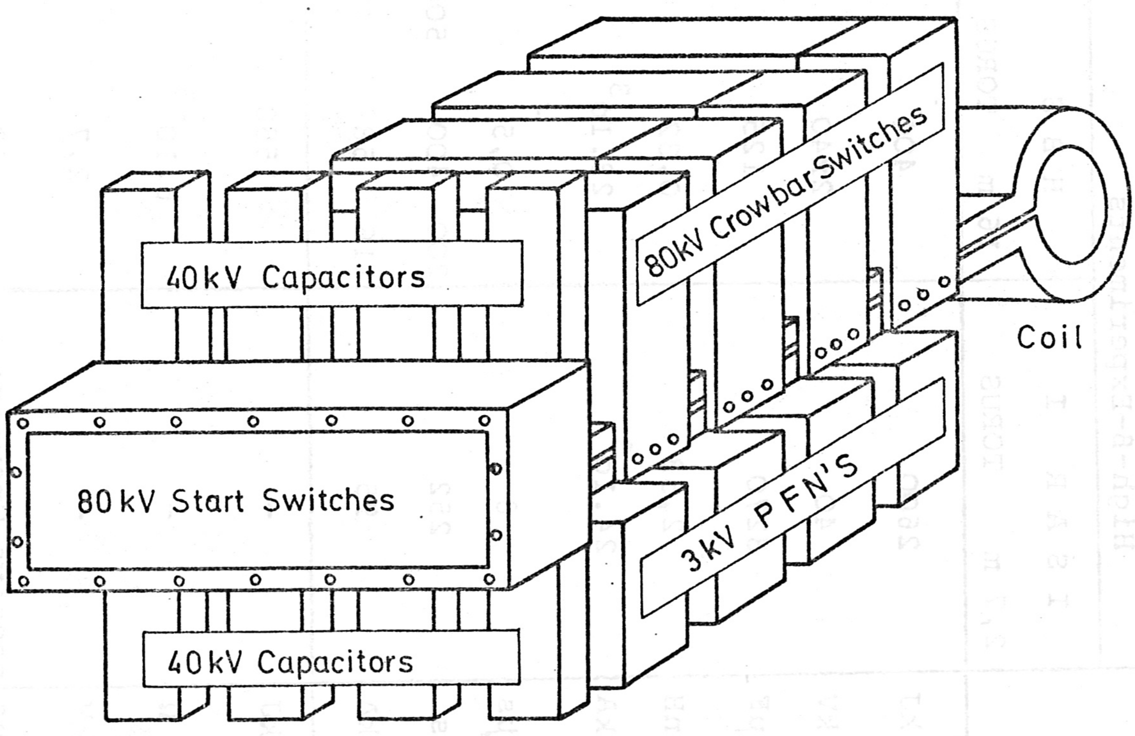
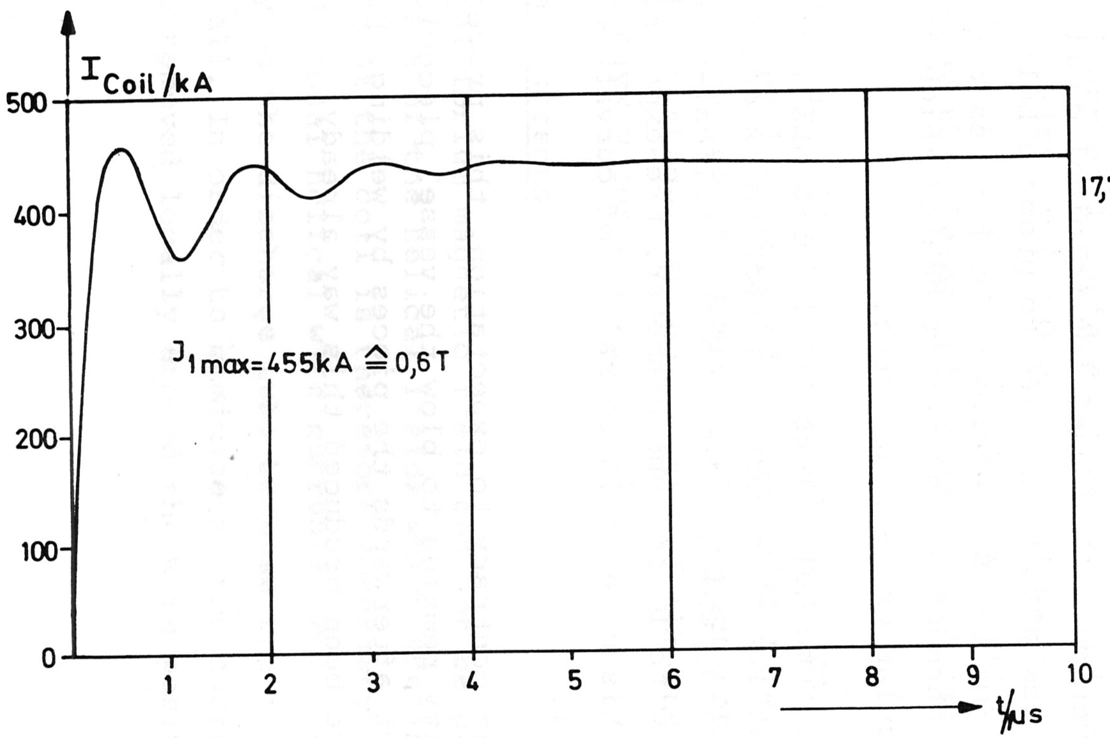
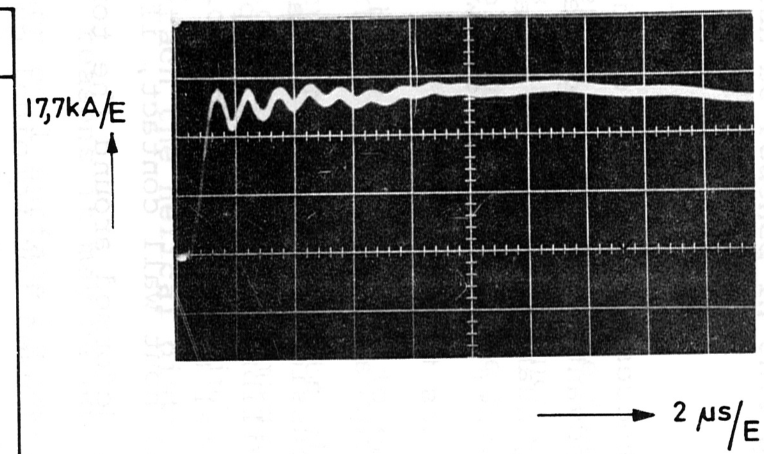


Fig.2 Proposed Design Layout for a Sector Bank Module (e.g. for a 1m Coil)



a) Computed Curve



b) Measured Current of a Testunit for a lo cm Coil. (Stage I Basic Unit o,8 kJ, 2x4o kV; PFN Z = 3 mOhm, 3 kV)

Fig. 3 Current Wave form

3. Special components

3.1 Discharge tube

Quartz tubes were successfully used for high energy pinch experiments. The disadvantage of the high melting temperature of quartz in producing quartz vessels is balanced by its small thermal expansion coefficients which allows to weld larger pieces of quartz tubes together. This has already been done successfully in case of the ISAR T 1 toroidal vessels.

The preceeding discussion in Chapter IV has shown that the proposed plasma equilibrium has small compression ratio and a considerable helical distortion. Such a plasma can only be produced and kept without wall contact, if the discharge tube itself is helically deformed around the toroidal axis.

The production method used for toroidal quartz tubes on ISAR T 1 can also be applied to produce helical-toroidal vessels. For the production of such tubes a quartz cylinder is drawn through a ring burner behind which it is conducted on a circular or another curve.

In near future helical-toroidal tubes shall be used on ISAR T 1. At present for this purpose a machine is constructed at the IPP Garching, which allows to form helical-toroidal tube sections with 10 cm tube diameter. Pre-experiments for bending tube of this size to 25 cm radius of curvature were already successful.

Should, in contrary to expectation, this procedure fail, the possibility remains to blow the vessel piecewise into form and to connect afterwards the pieces by welding. Fat toroidal tubes have been produced this way already.

After a cleaning process very clean discharge conditions (oxygen $\approx$ 0.1 %) can be reached in this quartz tubes.

3.2 Discharge coil

It is intended to produce the $\ell = 1$ and the smaller $\ell = 0/2$ field components by shaping the inner surface of the coil. Only a small and variable fraction will be produced by helical currents (see 3.3).

Although the coil system is shaped complicately by the helical and toroidal curvatures, its costs will be low compared with the total installation costs. The low magnetic field and the therefore little forces allow the use of aluminium casted coils. In this case it is easy to attach the helical distortions.

By several reasons it may be advantageous to cover distinct areas of the inner coil surface by semi-conducting materials, e.g. by graphite. Firstly $\ell = 0$ and $\ell = 2$ modulations can be damped in the early phase of the discharge, and secondly an adiabatic expansion of the plasma column can be obtained if a uniform cover is used.

The coil and collector costs have been estimated to be 420.000 UC, again a sum low compared to the total investment. It appears possible therefore, to diminish the aspect ratio of the experiment in a later stage. The components of the energy storage bank are suited anyway for rearrangement.

3.3 Helical currents

The equilibrium magnetic field can not be precisely computed. A variation of the helical fields which are in principle proved by shaping of the coil is therefore necessary and can be achieved by additional helical wire currents. The wires can be wound directly on the discharge tube because the forces acting on them are small. In order to mimimize mirror currents the coil surface can be removed locally around the wire position.

The field energy produced by the additional wire currents is about one per cent of the toroidal magnetic field, so the costs for these auxiliary banks are small.

3.4 Toroidal currents

As a consequence of the helical fields a poloidal flux appears which surrounds the plasma the short way. During the fieldrise a toroidal voltage is induced therefore reaching 90 kV for the proposed project. In order to suppress the toroidal current, it is necessary to induce a voltage in the opposite direction. This can be achieved as in the ISAR T 1 experiment: the coil sections will be insulated and a toroidal voltage induced by a current loop. The relatively small necessary energy can be taken from the main bank.

3.5 Preionization

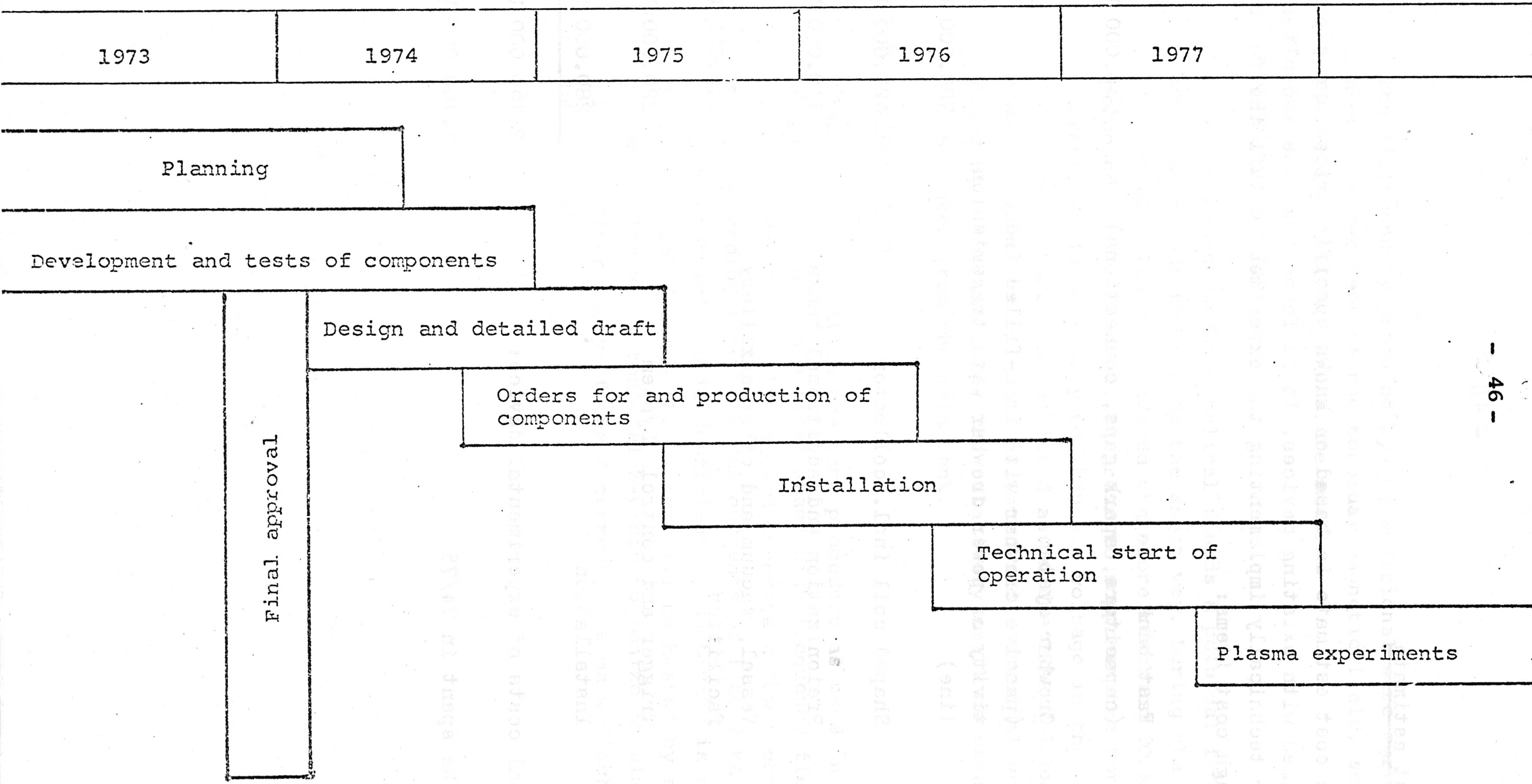
Also for preionization the same procedure as used in ISAR T 1 shall be applied. In a first stage high frequency fields (10 - 20 MHz, 20 - 40 kV) should generate a low electron density. Succeeding slow toroidal currents of 1 to 2 kA will heat the plasma ohmicly until sufficient conductivity is reached. In a final third stage the plasma will be heated by a fast current pulse via a toroidal Z-pinch to ionize it almost completely. After decay of the Z-current, the main discharge will be applied.

4. Cost estimate

The cost estimate is based on known specific prices and experience gained with existing devices. It is found that the two alternatives for technically implementing the experiment entail the following rough cost items:

1.	Fast bank (capacitors, spark gaps, connections)	560.000 UC
2.	Crowbar system (passive crowbar with iron-filled inductivity or power crowbar with transmission line)	700.000 UC
3.	Shaped coil incl. collector	420.000 UC
4.	Preionization and additional banks	140.000 UC
5.	Vessel, vacuum, and other auxiliary facilities	140.000 UC
6.	Trigger and control System	220.000 UC
7.	Installation	280.000 UC
Total costs of experimental device:		2.460.000 UC
To be spent in 74/75		1.100.000 UC

6. Time schedule for the H3S - experiment



5. Man-power

For the proposed project HBS II together with the supporting experiments the following man-power will be needed:

physicists	
experimentalists	7
theoreticians	3
engineers	4
technicians	10

VI. Supporting experiments

1. Experiment on ISAR T 1

1.1 Present status of the experiment

A complete high-beta stellarator torus was put in operation at ISAR T 1 at the end of 1972. In the first version, which is still working, the magnetic field configuration is produced by a simple toroidal theta coil together with helical windings and small inserts. A non-helical quartz torus is used as a vacuum vessel.

The existence of M&S like toroidal high-beta stellarator equilibria has been demonstrated for the $\ell=1/\ell=2$ system and for the theoretically more favourable $\ell=1/\ell=2/\ell=0$ combination. The plasma life time seemed to be limited by a long wavelength ($\lambda > 4$ m) $m = 1$ instability in rough agreement with theory. The exact form of the instability, probably combination of several fundamental modes with similar growth rates, has not yet been completely resolved. *)

No long wave $m \geq 2$ instabilities were observed during the plasma life time of several toroidal drift times. Short wavelength $m = 0$, $m = 1$ and $m = 2$ modes produced by shock compression in the simple quartz torus turned out to be stable and strongly damped.

Some phenomena, not all predicted by the simple sharp boundary models, have been observed (e.g. transverse plasma drift, some non-linearity in the equilibrium condition, dependence of the interference force on plasma dynamics, transverse magnetic fields, and induced toroidal plasma currents etc.). Some of these effects must be further examined in order to get a reliable basis for future experiments.

The main disadvantage of the present arrangement is that the vessel is not adapted to the helical symmetry of the equilibrium. This causes strong dynamic effects as mentioned above

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*) Meanwhile a detailed measurement showed quantitative agreement with the $m = 1$ mode theory.

Although helical fields produced by helical windings provide great flexibility they lead to high induced voltages and strong magnetic forces acting on them. By these reasons the experiments had usually to be limited to 0.5 MJoule bank energy and plasma temperatures were consequently not larger than about 100 eV (at 20 micron filling pressure). The 2.6 MJ available in ISAR T 1 could thus not be used.

1.2 Planned experiment on ISAR T 1

In the next step these disadvantages of the present experiment will be overcome by using a helically shaped theta coil ($\ell=1$, $\ell=2$, $\ell=0$) and a helical, screw-like quartz vessel following the main $\ell=1$ deformation of the plasma column. Some flexibility will be retained by using helical correction windings for one helical component ($\ell=2$) with small helical current only.

A strong reduction of dynamic effects and much higher temperature are expected. In addition the helix radius can be made large (e.g. helix radius $\approx$ 3 x plasma radius). This seems to be favourable from general stability considerations, although it is beyond the range of validity of present sharp boundary models. Therefore experimental results would be very helpful. Of course, all the effects observed in the present experiment will be further examined with the improved system at higher temperatures, especially the dynamic effects and the transverse drift.

Special inserts for a large bore theta coil have been developed and tested, which provide the shaped inner coil surface. They can be easily exchanged. An apparatus for the production of the shaped quartz vessel is now under construction (10 cm small diameter). It should be mentioned that for the proposed wall stabilized high-beta stellarator a shaped vessel is necessary in every case. Therefore the experience how to produce such a vessel will be very useful.

In addition to the main experimental programme, we are preparing an experiment to investigate heating by Alfvén waves.

2. Experiments on ISAR II

In order to prepare for the proposed wall stabilized toroidal stellarator the investigation of linear theta pinch discharges leading to small compression ratios ($\varkappa \leqslant 2.5$) are of great help. Parallel to certain experiments carried out at Los Alsmos and in Jülich the linear theta pinch ISAR II shall be used to produce in a 1 meter long coil with 20 cm diameter a plasma with parameters of the above proposal and a compression ratio of $\varkappa \approx 2.5$. In order to achieve this the ten parallel circuits of ISAR II will be switched in such a way that a suitable current wave form, i.e. sharp rise within 0.5 microseconds and then constancy for about 4 microseconds, results.

Especially the following probelms will be investigated:

1) the efficiency of plasma compression at filling densities of $n_H \approx 3 \cdot 10^{14} cm^{-3}$, after suitable preionization,
2) the damping of compression oscillations,
3) the radial pressure distribution in equilibrium and
4) the plasma density between the dense core and the tube walls.

The experience gained from these investigations together with the results obtained in Jülich and Los Alamos will shorten the first experimental period in the HBS experiment, concerning plasma production and heating, appreciably and will allow to enter the investigation of toroidal equilibrium and stability problems as soon as possible.