Ion Sound Spectrum and Electron-Wave Interaction in Perpendicular Shocks.

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Abstract:

A theory is presented for the rotation of the ion sound spectrum with respect to the drift velocity as observed in perpendicular shocks. The results of 2D simulation confirm the main points of the theory and represent a direct verification of perturbed orbit theory of strong turbulence.

Recently detailed measurements of the spectrum of ion sound turbulence in perpendicular shocks have been reported [1,2,3]. The main experimental results are: 1) The spectral electric energy density $W(k) = |E(k)|^2/8\pi$ does not seem to agree with the so called Kadomtsev spectrum; 2) there is a pronounced asymmetry in the angular distribution of the spectrum with respect to the direction of the current [1,2]. Previously [4,5] the energy spectrum had been measured only in the small range between $k\lambda_D \simeq 0.7$ and $k\lambda_D \simeq 1$, and the experimental points had been interpolated by the Kadomtsev function $(\alpha/k)\, \ln(1/k\,D)$, choosing for the cut-off $D = \lambda_D$, the electron Debye length. However the Kadomtsev theory is derived in the range $k\lambda_D \ll 1$ and becomes rather ill defined in the vicinity of the arbitrary cut-off length D. In Ref. 3 the measurements have been extended to the long wave length range $k\lambda_D \sim 0.1$, using CO_2 laser scattering. In this range the fluctuation energy is found to be several orders of magnitude smaller than the Kadomtsev prediction. Also the observed cut-off at large $k\lambda_D$ is not as sharp as predicted by the Kadomtsev spectrum[1]. These results had been anticipated by numerical simulations of the cross-field current driven instability[6,7,8]. In all numerical runs it has been found that $W(k)$ is peaked at $k\lambda_D \sim 1$ and that during the nonlinear development of the instability the spectrum is further shifted toward higher values of $k\lambda_D$. Evaluating many simulation runs a theoretical picture has been drawn in Refs. 7 and 8. Growth of long wave length modes is inhibited by linear Landau damping due to a high energy ion tail which extends beyond c_s and thus preferentially damps modes with large phase velocities, $v_{ph} \simeq c_s$ for $k\lambda_D \ll 1$. The shift of the spectrum toward larger values of $k\lambda_D$ is caused by the increase of λ_D due to electron heating, the upper limit of $k\lambda_D$ being determined by Landau damping at the bulk of the ion distribution.

Also the second effect, the asymmetry of the spectrum had been anticipated by numerical simulations[6]. It had been found that the spectrum $W(\underline{k})$ is rotated in the plane perpendicular to B opposite the sense of electron gyration. In the present letter we give a theoretical explanation of this effect by investigating the electron interaction with ion-sound turbulence in a perpendicular magnetic field. Dum and Dupree[9] have shown that turbulence leads to a frequency shift and broadening of the wave particle resonance and that these effects are related to friction and diffusion, respectively, experienced by a test particle in the turbulent plasma. Diffusion across the magnetic field smears out the gyroresonances when the turbulence level exceeds[10]

$$\langle E^2 \rangle / 4\pi n T_e \simeq (\Omega_e/\omega_{pe})^2 \Omega_e / k v_e \qquad (1)$$

where Ω_e and ω_{pe} are the electron cyclotron and plasma frequency, respectively. It has been shown[11] that at this level, which is very low if $\Omega_e \ll \omega_{pe}$, a transition from the beam cyclotron to the unmagnetized ion sound instability occurs. The friction force on a test particle is only compensated in the mean by the applied electric field E_o which maintains the current $j = -n\,|e|\,u$, cf. Fig. 1. Electrons of low speed, which predominantly drive the instability, experience more friction and thus drift in the -x direction. (High speed electrons drift in the opposite direction.) The effective drift velocity for the instability u^* is thus rotated opposite the sense of electron gyration and the angle of rotation should scale as $\mathrm{tg}\,\theta \propto \nu^*/\Omega_e$ where ν^* is the effective collision frequency. This dependence on ν^* and Ω_e is verified by appropriate 2D simulations of the ion sound instability. Computer simulation of a system of test electrons in a given ion sound spectrum is used to verify that electron diffusion (stochastic heating) can be described by the unmagnetized quasilinear diffusion term for fluctuation levels exceeding the level given by Eq. (1).

We consider the situation given in most experiments on resistive shocks: $\omega_{pe} \gg \Omega_e > \nu^*$, where ν^* is the effective electron collision frequency produced by ion-sound turbulence with $k\,\rho_e \gg 1$, $\rho_e = v_e/\Omega_e$. The plasma is homogeneous, i.e. effects of density gradients are neglected, and the ions are taken unmagnetized, since relevant times are $t < \Omega_i^{-1}$.

We use the diffusion equation for the electrons to determine the average electron distribution f_e and the statistical characteristics of the test particle orbits. In the frame where the mean electron velocity vanishes (the ions drift in the y direction with velocity u, cf. Fig. 1) the diffusion equation for f_e takes the form

$$\frac{\partial f_e}{\partial t} + (\underline{\Omega}_e \times \underline{v}) \cdot \frac{\partial f_e}{\partial \underline{v}} - \frac{\langle \underline{R}_e \rangle}{n\, m_e} \cdot \frac{\partial f_e}{\partial \underline{v}} = \frac{\partial}{\partial \underline{v}} \cdot \underline{\underline{D}}^{(e)} \cdot \frac{\partial f_e}{\partial \underline{v}} \qquad (2)$$

where the components of the frictional force $\langle \underline{R}_e \rangle$ are

$$\langle R_e^{\alpha} \rangle \equiv -n\, m_e\, u_\alpha^* \nu^* = n\, m_e \int dv\, f_e(\underline{v}) \frac{\partial}{\partial v_\beta} \cdot D^{\alpha\beta} , \qquad (3)$$

$\underline{u}^*$ is the effective drift velocity and $\underline{\Omega}_e = (0, 0, |e|\, B/m_e)$.

To simplify the algebra we only consider modes in the plane perpendicular to B, which corresponds directly to the previous and present numerical simulations. Thus f_e may be integrated over v_z; in the following we use the notation $\underline{v} = (v_x, v_y)$. The average distribution $f_e(\underline{v})$ can be expanded in the small parameter ν^*/Ω_e,

$$f_e(\underline{v}) = \overline{f}_e(v) + \tilde{f}_e(v, \emptyset) \qquad v \equiv |\underline{v}| \qquad (4)$$

where to zeroth order $\overline{f}_e$ is the solution of the angle averaged diffusion equation. To first order in ν^*/Ω_e the contour lines of $f_e(\underline{v})$ are shifted circles,

$$f_e(\underline{v}) = \bar{f}_e(w) \simeq \bar{f}_e(v) - \underline{v}_D \cdot \underline{v} \frac{1}{v} \frac{\partial \bar{f}_e}{\partial v} \qquad (5)$$

where $\underline{v} = \underline{w} + \underline{v}_D(w)$.

The shift $\underline{v}_D$, which to lowest order depends only on $w = |\underline{w}|$, can be expressed in terms of the friction force by considering a ring distribution $\bar{f}_e(w') \propto \delta(w' - w)$ and forming the first moment of Eq. (2). We thus obtain

$$\underline{v}_D(w) = - \frac{\underline{R}_e - \langle \underline{R}_e \rangle}{n\, m_e} \times \frac{\underline{\Omega}_e}{\Omega_e^2} \qquad (6)$$

where $\underline{R}_e = \underline{R}_e(w)$ is the average of $\frac{\partial}{\partial \underline{v}} \cdot \underline{\underline{D}}$ over the ring distribution, cf. Eq. (3).

We now derive the dispersion relation $\varepsilon = 1 + \varepsilon_e + \varepsilon_i = 0$ and compute the growth rate for a mode in a plasma with a magnetic field and a given spectrum of ion-sound turbulence. In the perturbed orbit theory of Dum and Dupree[9] ε_j is given by

$$\varepsilon_j(\underline{k},\omega) = \frac{-i\,\omega_{pj}^2}{k^2} \int d\underline{v}\; \underline{k} \cdot \frac{\partial f_j}{\partial \underline{v}} N_j(\underline{v}|\underline{k},\omega), \qquad (7)$$

where

$$N(\underline{v}|\underline{k},\omega) = \int_0^\infty d\tau\; e^{i\omega\tau} \langle \underline{v} | e^{-i\underline{k}\cdot\underline{x}(\tau)} \rangle \qquad (8)$$

is the ensemble average propagator for a test particle with initial velocity $\underline{v}$. The perturbed particle orbit consists of a gyration about $\underline{v}_D(w)$ with speed w and diffusive motion which results in resonance broadening $\Delta\omega$. Evaluating Eq. (8) as done in Ref. 9 one obtains

$$N(\underline{v}|\underline{k},\omega) = \int_0^\infty d\tau \, e^{i(\omega + i\Delta\omega - \underline{k}\cdot\underline{v}_D)\tau} \tag{9}$$

$$\times \sum_{n,m} J_n\left(\frac{k\,w}{\Omega_e}\right) J_m\left(\frac{k\,w}{\Omega_e}\right) e^{i(n-m)(\hat{\phi}-\psi)} \, e^{-i\,n\Omega_e\tau}$$

where $\hat{\phi}$ and ψ are the angles of $\underline{w}$ and $\underline{k}$ respectively.

Transforming in Eq. (7) from $\underline{v}$ to $\underline{w}$ coordinates, the $\hat{\phi}$ integration can be performed and the dielectric constant may then be evaluated as in Ref. 11. Since condition (1) is always satisfied for the cases of interest here, the gyroresonances are wiped out by resonance broadening and we obtain

$$\varepsilon_e(\underline{k},\omega) = \frac{\omega_{pe}^2}{k^2} \int d\underline{w} \, \underline{k} \cdot \frac{\partial \bar{f}_e}{\partial \underline{w}} \frac{1}{\omega - \underline{k}\cdot\underline{v}_D(w) - \underline{k}\cdot\underline{w}} \,. \tag{10}$$

Wave growth is determined by

$$\mathrm{Im}\; \varepsilon_e(k,\omega) = -\pi \frac{\omega_{pe}^2}{k^2} \int d\underline{w} \, \underline{k} \cdot \frac{\partial \bar{f}_e}{\partial \underline{w}} \, \delta(\omega - \underline{k}\cdot\underline{v}_D - \underline{k}\cdot\underline{w})$$

$$\simeq -2\pi \frac{\omega_{pe}^2}{k^2} \int_0^\infty dw \frac{\partial \bar{f}_e}{\partial w} \frac{\omega - \underline{k}\cdot\underline{v}_D(w)}{k\,w} \tag{11}$$

$$= -2\pi \frac{\omega_{pe}^2}{k^2} \int_0^\infty dw \frac{\partial \bar{f}_e}{\partial w} \frac{\omega - \underline{k}\cdot\underline{v}_D^*}{k\,w} \,,$$

introducing the effective drift velocity $\underline{v}_D^*$,

$$\underline{v}_D^* \equiv \int_0^\infty dw \, \frac{1}{w} \frac{\partial \bar{f}_e}{\partial w} \underline{v}_D(w) \Big/ \int_0^\infty dw \, \frac{1}{w} \frac{\partial \bar{f}_e}{\partial w} \, . \tag{12}$$

To further evaluate Eq. (11) we need an expression for the differential friction $\underline{R}_e(w)$, since it determines $\underline{v}_D(w)$ by Eq. (6).

The gyroresonances which appear in the diffusion coefficient are smeared out by the same argument as indicated in the derivation of Eq. (10), and we obtain

$$\underline{R}_e(w) \simeq -\underline{c}\left(\frac{u}{w}\right)^3 \quad \text{for} \quad w \gtrsim u \, . \tag{13}$$

The constant $\underline{c}$ is determined by the mean friction coefficient $\langle \underline{R}_e \rangle$ through the condition that in the electron frame the total drift vanishes:

$$\int_0^\infty dw \, w^2 \frac{\partial \bar{f}_e}{\partial w} \left[-\underline{c}\left(\frac{u}{w}\right)^3 - \langle \underline{R}_e \rangle \right] \times \underline{\Omega}_e = 0 \, . \tag{14}$$

We now insert the selfsimilar solution of the angle averaged quasi-linear equation [12]

$$\bar{f}_e(w) = A \, e^{-\left(\frac{w}{w_o}\right)^5} \, , \tag{15}$$

which yields

$$\underline{v}_D^* = \frac{\nu^*}{\Omega_e^2} \left[\underline{u}^* \times \underline{\Omega}_e \right] \left(\frac{\Gamma(1/5)\,\Gamma(7/5)}{\Gamma(4/5)^2} - 1 \right) . \tag{16}$$

Thus the total effective drift as seen from the ions is $\underline{u} + \underline{v}_D^* = \underline{u}^*$ and the maximum growth rate is rotated out of the $\underline{u}$ direction by an angle Θ:

$$\tan \Theta = \frac{v_d^*}{u^*} = \frac{\nu^*}{\Omega_e} \left(\frac{\Gamma(1/5)\,\Gamma(7/5)}{\Gamma(4/5)^2} - 1 \right)$$

$$\simeq \frac{2\nu^*}{\Omega_e} . \qquad (17)$$

Inserting numbers from a typical computer run $\nu^*/\omega_{pe} \simeq 0.5$ W/n T_e, W/n $T_e \simeq 1.4 \times 10^{-2}$, $\Omega_e/\omega_{pe} = 0.04$, we obtain $\Theta \simeq 20^\circ$ in excellent agreement with the numerically obtained value, Fig. 2. In addition to the simulations reported in Ref. 6 further computer runs of the self consistent electron -ion systems have been performed to investigate the Ω_e dependence of the rotation angle Eq. (17). Increasing Ω_e/ω_e from 0.04 to 0.1 does not noticeably change ν^* (as stated earlier, ion sound turbulence is rather independent of the magnetic field as long as $\Omega_e \ll \omega_{pe}$), but Θ is reduced to 10°, again in agreement with Eq. (17). The contour lines of the electron distribution are shifted circles as predicted by Eq. (5). Furthermore, as predicted, the experiments[1] show the rotation angle to be independent of the magnitude of k . In order to verify that electron diffusion can be described by the unmagnetized quasilinear diffusion tensor, a 2D stochastic acceleration model has been run, where a set of test electrons were moving in an prescribed isotropic spectrum W(k), $\omega = k\, v_{ph}$, v_{ph} = constant. Quasilinear theory predicts that the distribution function relaxes to the selfsimilar form (15) and that the electron heating rate is given by the relation

$$\frac{\partial T_e^{5/2}}{\partial t} = \frac{5}{2} (2\pi)^{1/2} \alpha_2\, \omega_{pe}^2\, v_{ph}^2 \left\langle \frac{1}{k} \right\rangle W \frac{m_e^{3/2}}{n} , \qquad (18)$$

where $\langle 1/k \rangle = \left[\int d\underline{k}\; W(\underline{k}) / k \right] / W$, $W = \int d\underline{k}\; W(\underline{k})$, and α_2 is a form factor depending on the shape of the distribution function[8], $\alpha_2 = 1$ for a Maxwellian and $\alpha_2 \simeq 0.285$ for Eq. (15).

The numerical results agree well with these predictions; they are virtually independent of the shape of the spectrum W(k), the magnetic field B $(\Omega_e/\omega_{pe} = 0 - 0.1)$ and diffusion times are inversely proportional to the field energy W $(W/n\,T_{eo} = 2\cdot 10^{-2} - 6\cdot 10^{-2})$. We find that after a short time a selfsimilar distribution f_e is established, $f_e \propto \exp\left\{-(v/v_e)^x\right\}$ with $4.7 \leq x \leq 4.9$, only insignificantly smaller than the theoretical value 5 (in previous self-consistent electron-ion simulations a somewhat smaller value of x has been found[8]; this effect is probably due to the nonnegligible electron collision rate, which is much smaller in the stochastic acceleration model). Simulations and theory agree even better with regard to the heating rate. Fig. 3 shows the electron temperature as a function of time for a typical simulation run. The numerical curve starts with the inclination corresponding to the initial Maxwellian, but soon becomes practically identical with the theoretical asymptotic solution.

To summarize, the numerical simulations verify the main points of the theory: the prediction for the rotation of the spectrum with respect to the drift velocity and that electron diffusion can be described by the unmagnetized quasi-linear diffusion tensor, even for rather high levels of ion sound turbulence. The results represent a direct verification of orbit perturbation theory first introduced by Duprèe. The theory is easily extended to the 3-D case and the basic conclusions still hold. Details of the theory will be published in a forthcoming paper[13].

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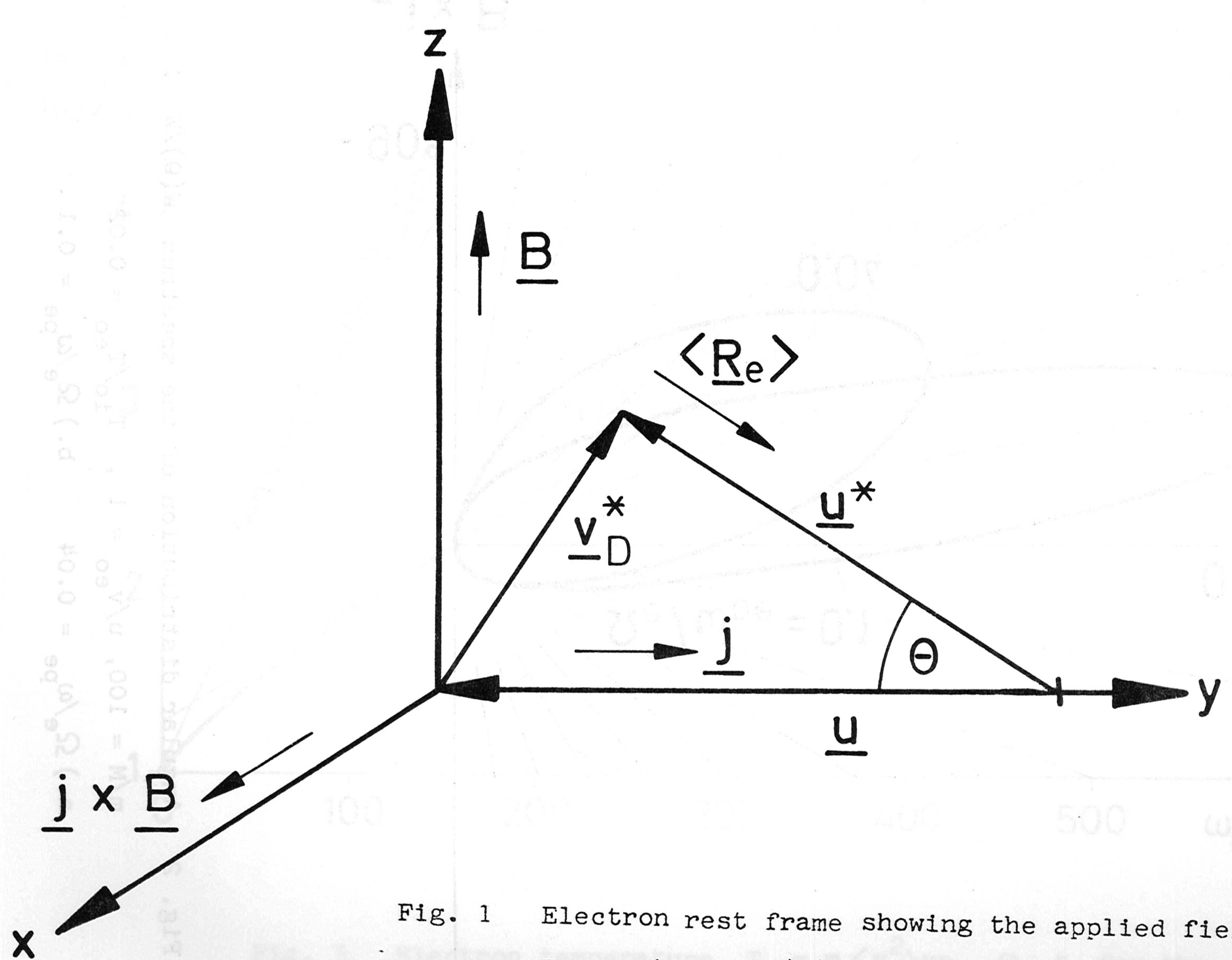

Fig. 1 Electron rest frame showing the applied field $\underline{E}_o = \langle R_e \rangle / n|e|$, the electron-ion drift velocity $u = u_e - u_i$ and the effective drift velocity u^*.

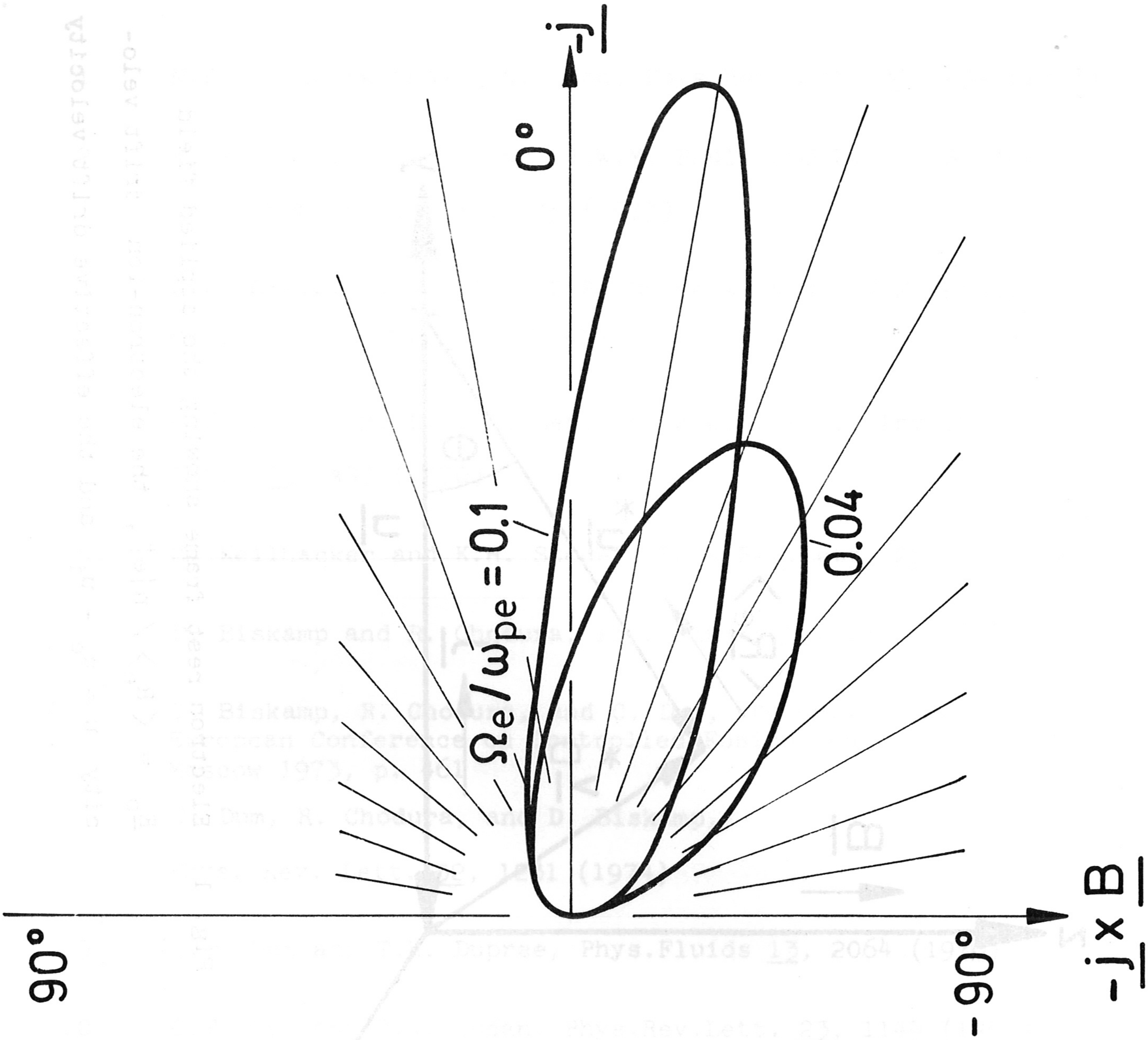

Fig. 2 Angular distribution of the spectrum $W(\Theta)/W$;
$m/M = 100$, $u/v_{eo} = 1$, $T_{io}/T_{eo} = 0.02$
a.) $\Omega_e/\omega_{pe} = 0.04$ b.) $\Omega_e/\omega_{pe} = 0.1$.

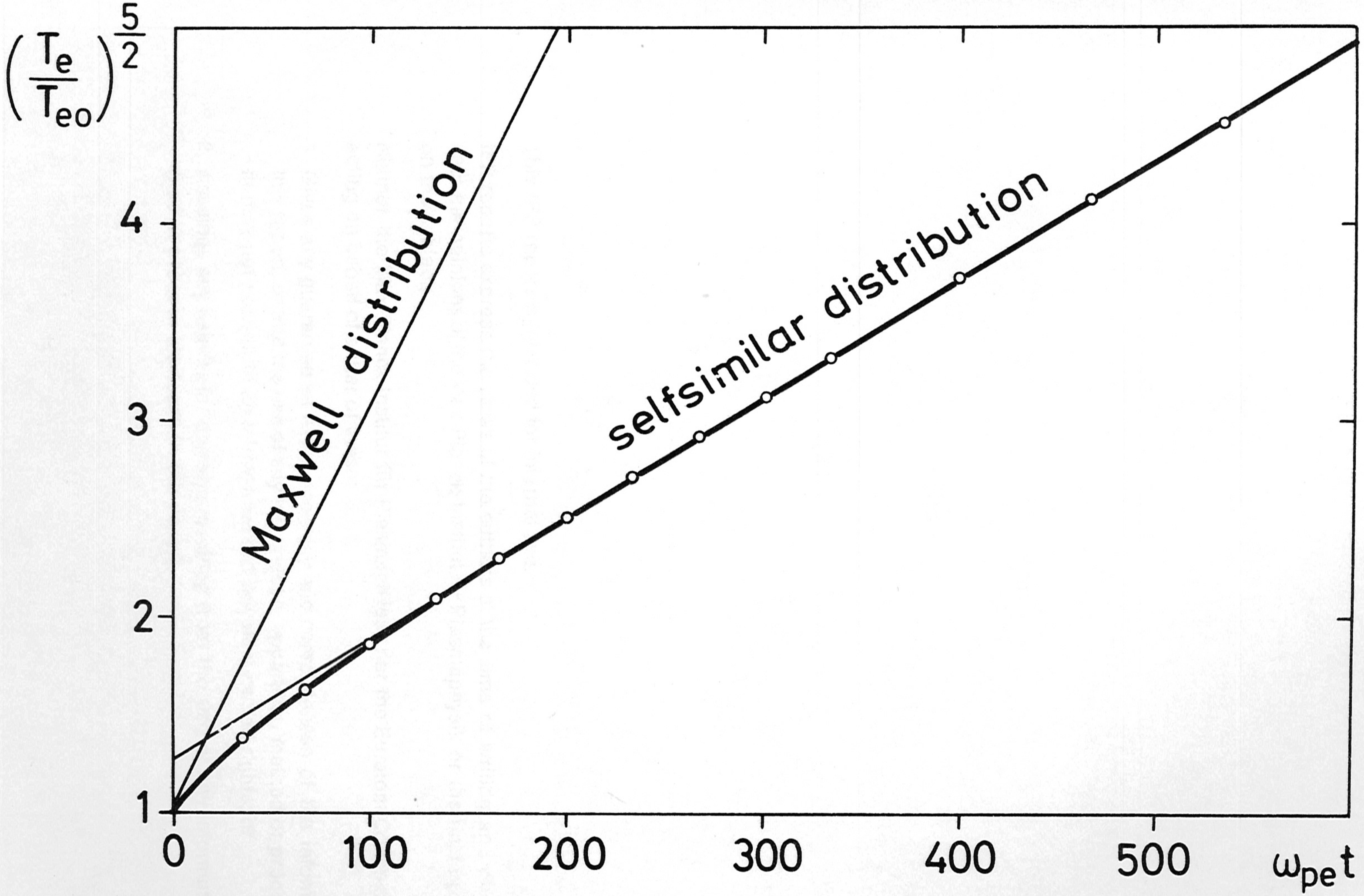

Fig. 3 Electron temperature $T_e = m_e \langle v_e^2 \rangle$ vs. $\omega_{pe} t$ for the stochastic acceleration model, $v_{ph} = 0.5\ v_{eo}$, $W/n\ T_{e0} = 2 \times 10^{-2}$.