Collisionless Plasma Heating at the Ion Cyclotron Frequency in a Toroidal Device

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Abstract

The R.F. power absorbed by the plasma in a toroidal device at the ion cyclotron resonance is evaluated with allowance for the effect of the rotational transform. It is found that the heating rate is a considerable factor larger than that evaluated for purely toroidal magnetic field.

The possibility of using a fast magnetoacoustic wave at the ion cyclotron frequency Ω_i to heat the plasma in toroidal devices beyond the limit of ohmic heating has recently been receiving increasing attention.[1,2,3,4].

In the previous theories the damping decrement of the magnetoacoustic wave was first evaluated for a uniform magnetic field; then the absorption in a toroidal magnetic field was estimated as an average of the local absorption over the plasma volume. The effect of the rotational transform was not taken into account. The resulting heating efficiency was reduced by a considerable factor relative to that evaluated for a uniform magnetic field.

We want to show here that this negative aspect only depends on the incorrect assumptions of the theoretical model, and that the absorption coefficient in the magnetic field of a toroidal device can, in fact, be larger than in a uniform magnetic field.

In the following, we shall consider a straight plasma cylinder confined by a magnetic field of the form:

$$\begin{cases} B_z = B_0 \left(1 - r R_0^{-1} \cos\vartheta\right) \\ B_\vartheta = B_0 \, r / q R_0 \end{cases} \qquad |B_\vartheta| \ll |B_z|$$

And we postalute that all the quantities be periodic in Z, with period $2\pi R$.

To the lowest order in the small quantity r/R_0, particles which are trapped or quasi-trapped can be neglected and the coordinates of the gyrocentres are given by:

$$\begin{cases} r = r_0 + \dfrac{v_\perp^2 + 2 v_\parallel^2}{2 \Omega v_\parallel} \, q \left[\cos(\vartheta_0 + v_\parallel t / R_0 q) - \cos\vartheta_0 \right] \\ \vartheta = \vartheta_0 + v_\parallel t / q R_0 \\ z = z_0 + v_\parallel t \, . \end{cases}$$

In the limit $\varrho_i \ll r$ (ϱ_i ions Larmor radius), the coordinates of the gyrocentres ($\bar{r}$, $\bar{\vartheta}$) can be substituted for r and ϑ in $\underset{\sim}{B}(r, \vartheta)$. Then the equations of motion of a particle can easily be integrated.

By writing the electric field in the form:

$$\underset{\sim}{E} = \Sigma_\ell \, \underset{\sim}{\mathcal{E}}_\ell(r) \, e^{i\ell\vartheta} \, e^{i(kz-\omega t)}$$

the linearized Vlasov equation can now be integrated along the characteristics.

We do not report the long, but straightforward, calculations and write explicitly only the expression for the plasma currents:

$$j_r = -i \, \Sigma_j \frac{e^2}{m_j v_j} \frac{n_j}{\sqrt{\pi}} \Sigma_{\ell n} \quad e^{i\ell\vartheta}$$

$$\times \left[\mathcal{E}_{rn} \left(A^+_{j\ell n} + A^-_{j\ell n}\right) - i \, \mathcal{E}_{\vartheta n} \left(A^+_{j\ell n} - A^-_{j\ell n}\right)\right]$$

$$j_\vartheta = -i \, \Sigma_j \frac{e^2}{m_j v_j} \frac{n_j}{\sqrt{\pi}} \Sigma_{\ell n} \quad e^{i\ell\vartheta}$$

$$\times \left[i \, \mathcal{E}_{rn} \left(A^+_{j\ell n} - A^-_{j\ell n}\right) + \mathcal{E}_{\vartheta n} \left(A^+_{j\ell n} + A^-_{j\ell n}\right)\right]$$

where

$$A^{\pm}_{j\ell n} = \frac{1}{v_j^4} \Sigma_p \int_{-\infty}^{+\infty} e^{-v_{\parallel}^2/v_j^2} (\pm 1)^{\ell - n} J_{\ell - p}\left(\frac{qr\Omega_j}{v_{\parallel}}\right)$$

$$\times J_{n-p}\left(\frac{qr\Omega_j}{v_{\parallel}}\right) dv_{\parallel} \int_0^{\infty} e^{-v_{\perp}^2/v_j^2} \frac{v_{\perp}^3}{\Delta^{\pm}_{jp}} dv_{\perp} \qquad (1)$$

and

$$\Delta^{\pm}_{jp} = R_o^{-1}\left[(N + pq^{-1} \pm \tfrac{1}{2} q) v_{\parallel} \right.$$

$$\left. \pm v_{\perp}^2 q (4 v_{\parallel})^{-1} \mp (\Omega_{oj} \pm \omega) R_o \right]$$

$$N = k R_o$$

It is easy to see that $A^{\pm}_{e\ell n} \approx \pm \delta_{\ell n} \sqrt{\pi}\, \rho_e /2$, and that

$$A^{+}_{i\ell n} \approx - \delta_{\ell n} \sqrt{\pi}\, \rho_i /4 .$$

In the following, we limit our calculations to the interesting case where the resonance lies on the midplane of the plasma cylinder, i.e. $\omega = \Omega_{oi}$. The integrals which appear in Eq. (1) can be evaluated when $r \gg \rho_i / q$ by using the asymptotic expansion for the Bessel functions (when $l^2 > r/\rho_i q$ this approximation is no longer valid and the corresponding $A_{\ell n}$ decrease rapidly). The real part of $A^{-}_{i\ell n}$ is then zero and the l component of the plasma currents can be written:

$$j_{r\ell} = i\,\frac{\omega_{pi}^2}{16\pi\Omega_0}\Big[\mathcal{E}_{r\ell} + 3i\,\mathcal{E}_{\vartheta\ell} - i\,\frac{3qR_0}{4r}$$

$$\times \sum_{p=0}^{\infty} \frac{p+\delta}{(p+\delta+q^2/4)^{5/2}} \sum_n \left(\mathcal{E}_{r,\ell+2n} + i\,\mathcal{E}_{\vartheta,\ell+2n}\right)(-1^n)\Big]$$

$$j_{\vartheta\ell} = \frac{\omega_{pi}^2}{16\pi\Omega_0}\Big[3\,\mathcal{E}_{r\ell} + i\,\mathcal{E}_{\vartheta\ell} - i\,\frac{3qR_0}{4r}$$

$$\times \sum_{p=0}^{\infty} \frac{p+\delta}{(p+\delta+q^2/4)^{5/2}} \sum_n \left(\mathcal{E}_{r,\ell+2n} + i\,\mathcal{E}_{\vartheta,\ell+2n}\right)(-1)^n\Big]$$

(2)

where $\delta = n_0 + Nq - \frac{1}{2}q^2$ and n_0 is an integral such that $0 \leq \delta < 1$.
Note that if eqs. (2) are combined with Maxwell's equations it is found that the components $\underset{\sim}{\mathcal{E}}_\ell$ of the electric field with l even (odd) are decoupled from those with l odd (even).

We now suppose that the R.F. power is coupled to the plasma by induction coils, and that the current is independent of ϑ. Hence we can assume $\underset{\sim}{\mathcal{E}}_{2s+1} = 0$ (s=0, ±1,...). The current in the coils is approximated by a sheet current distribution:

$$\underset{\sim}{J} = \hat{\vartheta}\, I^\circ\, \delta(r-s)\, e^{i(kz-\omega t)}$$

where s is the radius of the coil and $k = N/R_0$ is a fixed wave number.

If it is assumed that $\mathcal{E}_r^\circ + i\,\mathcal{E}_\vartheta^\circ \approx \gamma\,\mathcal{E}_\vartheta^\circ$, with $|\gamma| \ll 1$, as one can expect for the magnetoacoustic wave, one obtains from the Maxwell equations:

$$\frac{d^2}{dr^2}\mathcal{E}_\vartheta^\circ + \frac{1}{r}\frac{d}{dr}\mathcal{E}_\vartheta^\circ - \left(\frac{1}{r^2} - \frac{\omega_{pi}^2}{c^2}\nu\right)\mathcal{E}_\vartheta^\circ = -\frac{i}{2}\frac{\omega_{pi}^2}{c^2}\nu\gamma\mathcal{E}_\vartheta^\circ \tag{3}$$

and

$$\sum_m \left(\mathcal{E}_r^{2m} + i\,\mathcal{E}_\vartheta^{2m}\right) \approx 8\nu r\,\mathcal{E}_\vartheta^\circ / 3qR_\circ\Sigma \tag{4}$$

where Σ stands for $\sum_{p=0}^{\infty}\frac{p+\delta}{(p+\delta+q^2/4)^{5/2}}$ and $\nu = 1 - 2k^2c^2/\omega_{pi}^2$.

One can now determine the quantity $\gamma(r)$. In fact, as only the azimuthal mode m = 0 is coupled to the plasma from the external current and the plasma column does not radiate, only the mode m = 0 contributes to the energy flux on the plasma surface; the Poynting theorem then takes the form:

$$-\frac{\nu}{16\Omega_\circ}\int_0^p \gamma(r)\left|\mathcal{E}_\vartheta^\circ\right|^2 r\,dr = \frac{1}{2}\,\mathcal{R}e\int_0^p r\,dr\int_0^{2\pi}\underset{\sim}{\mathcal{E}}^*\cdot\underset{\sim}{j}\,d\vartheta$$

where p is the plasma radius.

From Eq. (2) and Eq. (4) one obtains:

$$\mathcal{R}e\int_0^{2\pi}\underset{\sim}{\mathcal{E}}^*\cdot\underset{\sim}{j}\,d\vartheta = \omega_{pi}^2 r\nu^2\left|\mathcal{E}_\vartheta^\circ\right|^2/3\Omega_\circ qR_\circ\Sigma.$$

So that, since $\gamma(r)$ is independent of p, one finally obtains:

$$\gamma \approx -8\nu r/3qR_\circ\Sigma\,; \tag{5}$$

as we have throughout considered $r/R \ll 1$, one has $|\gamma| \ll 1$, in agreement with our assumption.

A consequence of Eq. (3) and Eq. (5) is that when $\omega = \Omega_0$ the damping decrement of the magnetoacoustic wave in a toroidal device is independent of the ion temperature. This rather surprising result can, however, be supported by a simple physical argument. In fact, the damping decrement is determined by the fraction of the total plasma current which is left-handed polarized, i.e. in the direction of the ion gyromotion. In a toroidal device, owing to the rotational transform all particles go through the resonance if $\omega = \Omega_0$; the left-handed plasma current is then proportional to the fraction of time which each particle spends in a zone where $\Omega = \Omega_0 (1 + r R_0^{-1} \cos\vartheta)$ is sufficiently close to ω. This time is proportional to R_0 / r and does not depend on the ion parallel velocity. A further important consequence of Eq. (5) is that the absorption coefficient for K = 0 is comparable to or larger than that for K ≠ 0, whereas according to the previous theories it should vanish for K going to zero.
A comparison with the plane case of Ref. 1 shows the importance of the rotational transform. According to Ref. 1 the damping of the wave is approximately given by $\omega_{pi} \nu^{3/2} k \rho_i / 4 c \sqrt{\pi}$. The ratio of this quantity and the corresponding damping obtained from Eqs. (3) and (5) is approximately $k \rho_i q R_0 / 5 r$. It follows that the damping in a toroidal geometry is larger than that in a plane geometry as soon as $r R_0^{-1} > 0.2 \, k \rho_i q$.

The solutions of Eq. (3) that are regular at $r = 0$ are given by a solution regular at $r = 0$, multiplied by an arbitrary constant. We first find a solution numerically and then determine the free multiplying factor by the boundary conditions at $r = p$ (p plasma radius), $r = s$ and $r = w$. (w wall radius). We do not report the long, but straigthforward, calculations.

In a real experiment the most interesting quantity is the fraction of the total R.F. power which can be coupled to the plasma. Therefore, we evaluate the ratio Ψ of R.F. power absorbed by the plasma to the ohmic losses in the coil and vessel wall. For this purpose we evaluate the sum of the power absorbed by the plasma and the wall as follows:

$$P_s = -\frac{1}{2} \Re e \{ \underset{\sim}{E}(s) \cdot \underset{\sim}{J}^* \} .$$

The ohmic losses in the wall are evaluated from

$$P_w = \frac{1}{2} \sigma_w \int_w^\infty |E_\theta|^2 \, dr$$

(σ_w being the electrical conductivity of the wall) and the ohmic losses on the coil from:

$$P_c = 2\pi^2 s \omega \delta_c I^{o2} c^{-2}$$

where it has been assumed that the current in the coil is uniformly distributed over a skin depth δ_c .

The ratio $\Psi = \dfrac{P_s - P_w}{P_c + P_w}$ is then given by:

$$\Psi = \sqrt{2}\, w\, p^{-1} \delta_w^{-1} \left[I_1(ks) K_1(kw) - I_1(kw) K_1(ks) \right]^2$$

$$\times \left(\mathcal{E}_R \mathcal{E}_I' - \mathcal{E}_I \mathcal{E}_R' \right) \left[F^2(ps) + F^2(pw)\, w\, \delta_c / s\, \delta_w \right]^{-1}$$

where

$$\mathcal{E}_R = \mathcal{R}e\{\mathcal{E}_\vartheta(r)\}_{r=p} \quad ; \quad \mathcal{E}_I = \mathcal{I}m\{\mathcal{E}_\vartheta(r)\}_{r=p}$$

$$F(p,s) = \left[I_1'(kp) K_1(ks) - I_1(kp) K_1'(ks) \right] \mathcal{E}_R$$

$$+ \left[K_1(kp) I_1(ks) - I_1(kp) K_1(ks) \right] \mathcal{E}_R' .$$

Here the derivaties are with respect to r and δ_w is the skin depth of the wall.

Figure 2 shows the quantity $\Psi\, \delta_w/p$ as a function of $\eta^2 = \omega_{pi}^2 p^2 c^{-2}$ for q = 3, $R_o/p = 10$, N=0 and different values of w/p. The coil radius s is chosen to be midway between the plasma and the vessel radius (s = (w+p) /2) and it is assumed that the electrical conductivity of the coil and the vessel are equal: $\delta_c = \delta_w = \delta$. Figure 3 shows the same quantity for $N = R_o/p$

As p/δ_w can be of the order of 10^4 in present-day experiments (and much larger in a fusion reactor), a large fraction of R.F. power can be coupled to the plasma outside the maxima of the absorption as well. This reduces the requirements imposed on the coupling system.

For example, for $p = 15\ cm$, $w/p = 1.2$, $B_0 = 30\ K\Gamma$, $\sigma_w = 5.10^6 \frac{mhos}{m}$ (steel) we have $p/\delta \approx 5 \cdot 10^3$ and the power absorbed by the plasma is at least twice as high as the ohmic losses in the whole plasma density range $n \gtrsim 10^{13} cm^{-3}$. It is concluded that the absorbed R.F. power is larger than that evaluated in previous papers [1,2], where only the R^{-1} dependence of the toroidal field was taken into account. We also find the interesting result that in a toroidal device with rotational transform the R.F. power can be coupled to the plasma when k=0 as well, the absorption efficiency being comparable with that found for $k \neq 0$.

Literature:

1) G. Cattanei, Phys. Rev. Letters 27, 980 (1971)

2) F.W. Perkins, Varenna Symposium on Plasma Heating and Injection (1973), P. 20

3) Perkins et al. (Third International Symposium on Toroidal Plasma Confinement, Garching 1973, paper B-8

4) J.C. Hosea and W.M. Hooke, Phys. Rev. Letters 31, 150 (1973)

Figure Captions

Fig. 1 - Coordinate system.

Fig. 2 - $\psi\delta/p$ Versus $\eta^2 = \frac{\omega_{pi}^2}{c^2} p^2 \approx 2 \cdot 10^{-15} p^2 n$

For $K = 0$ and:

a) $w/p = 1.1$

b) $w/p = 1.2$

c) $w/p = 1.4$

d) $w/p = 1.6$

Fig. 3 - $\psi\delta/p$ Versus $\eta^2 = \frac{\omega_{pi}^2}{c^2} p^2 \approx 2 \cdot 10^{-15} p^2 n$

For $K p = 1$ and:

a) $w/p = 1.1$

b) $w/p = 1.2$

c) $w/p = 1.4$

d) $w/p = 1.6$

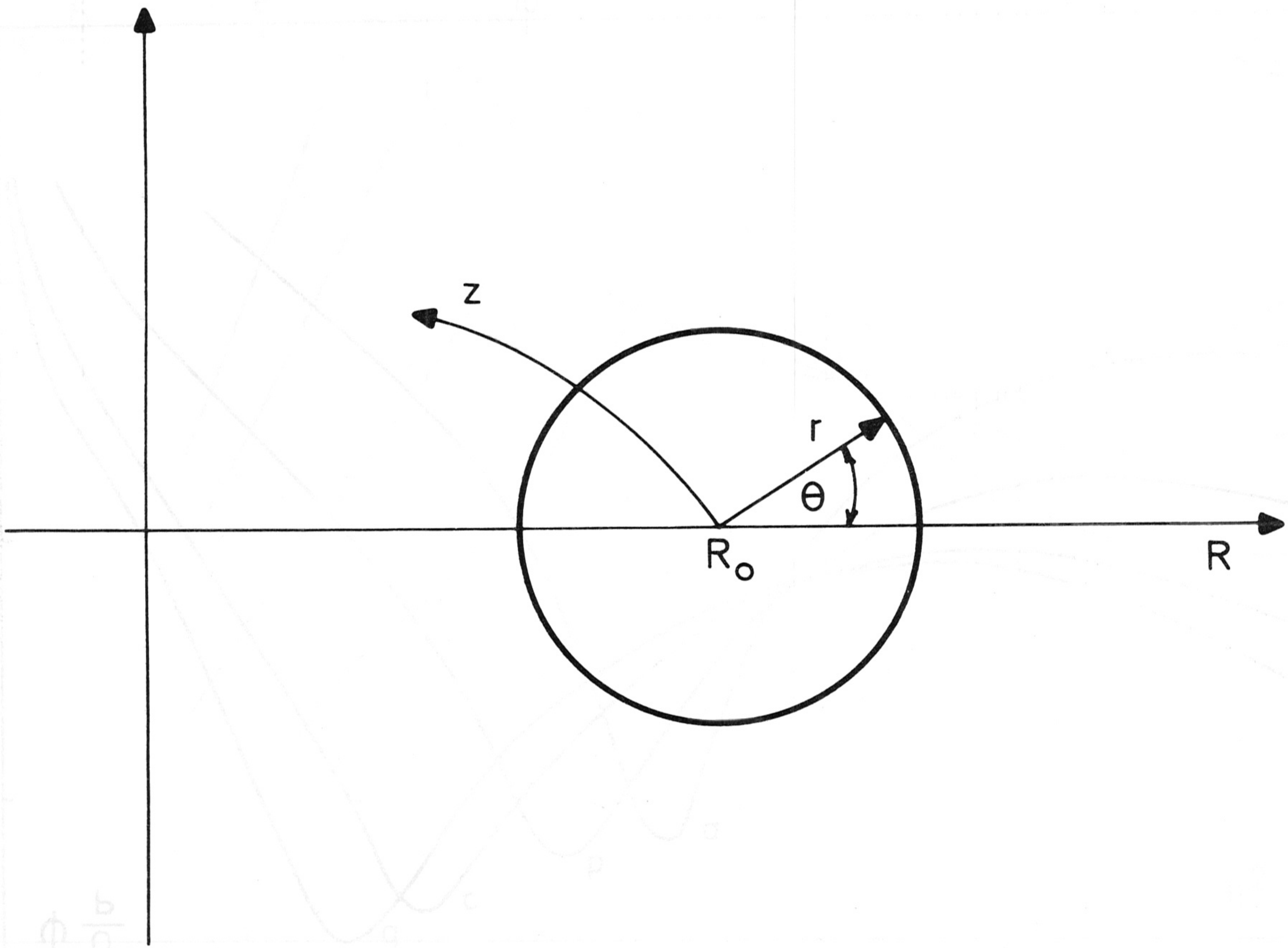

Fig. 1

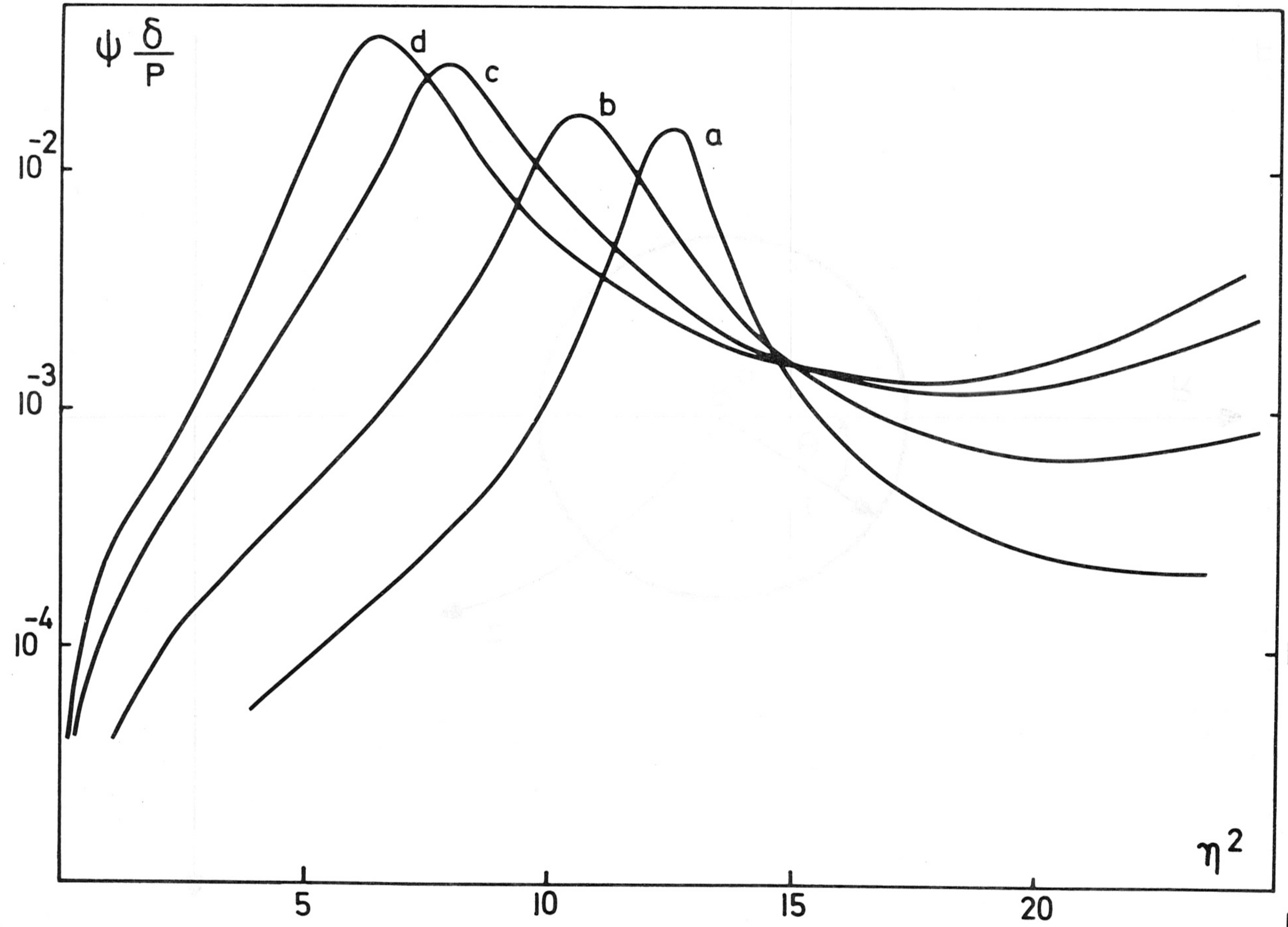

Fig. 2

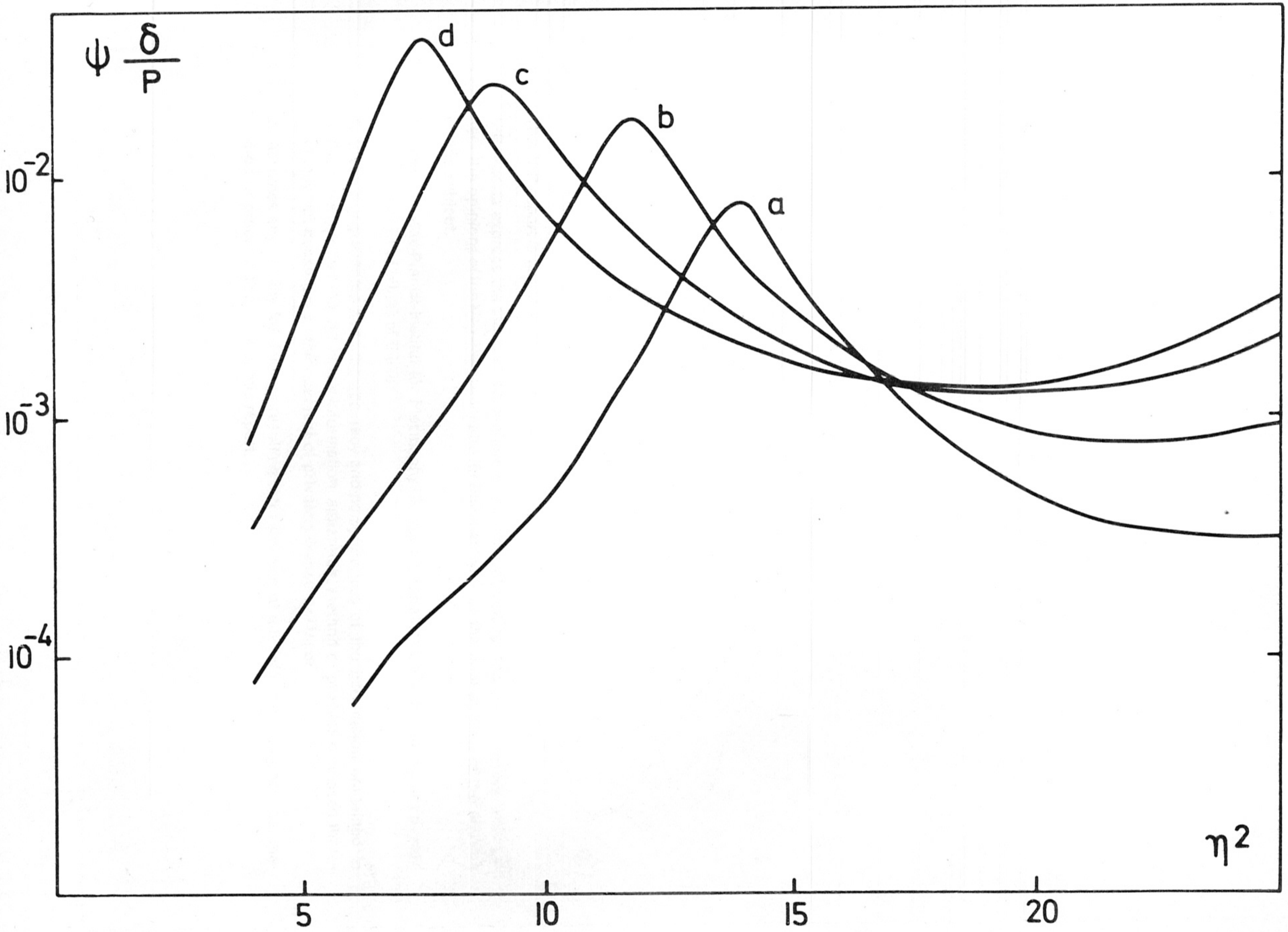

Fig. 3